Monopolistic competition: Critical evaluation the theory of monopolistic competition with specific reference to the seminal 1977 paper by Dixit and Stiglitz

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Abstract

This paper revisits the D-S (Dixit-Stiglitz) model. It's a simple general monopolistic model with n monopolistic goods, and a numeraire good Labour ($w = 1$); aggregation for all goods in the economy. We have considered in our paper constant elasticity of substitution case(CES).On the supply side, the assumption is that the labour is perfectly mobile factor of production across the sectors, so as a result in our model there is single wage rate which we denote as W in the other sectors than monopolistic there is constant returns to scale and we can specify the production function: The Dixit-Stiglitz model of monopolistic competition works only when n is large; from the functions of the productions best when one applies linear production function. Under increasing returns to scale monopolistic competition will lead to a greater degree of product differentiation than it is socially optimal.

Key words: Monopolistic competition, CES, Dixit-Stiglitz model, product differentiation

Introduction

The assignment should revisit the model of monopolistic competition build Dixit and Joseph Stiglitz .The basic model has been used to study optimum product diversity .It's a simple general monopolistic model with n monopolistic goods, and a numeraire good Labour $(w = 1)$; aggregation for all goods in the economy.

Constant elasticity case*

Then model simplifies economy on two sectors. The first sector produces a homogenous good under constant returns to scale and is perfect competition model and the second sector consists of a large group of monopolists who produce under increasing returns to scale. The utility function of household is represented homogenous quasi-concave*

$$\max_{X_0,X_i} u = U(X_0, [\sum_{i=1}^{n} x_i^{\rho}] \xrightarrow{1/\rho}$$

x_i Is the consumption of variety; ρ * is **constant elasticity of substitution** inside the group of monopolistic goods $i = 1,2 \ldots \ldots., n$ or alternatively and utility is separable between $x_0 nummeraire$ and $x_i$other commodities.

(2) $(\int c(\rho)^{\alpha} dF)^{\frac{1}{\alpha}}$, (1)

$C(\rho)$ is a consumption of good produced with productivityρ, and $\alpha \in]0,1[$

The constant elasticity of substitution in this case is $\frac{\alpha}{1-\alpha}$[1]

[1] [1] Koeniger Winfried, Licandro IZA Omar European University Institute and FEDEA January 2004 ;Substitutability and Competition in the Dixit-Stiglitz Model*
see appendix point 1.2:
seeappendix 3.3

Consumer there with demand for characteristics faces budget constraint:

(3)$I = \sum_{i=0}^{n} p_i\, x_i$ Where I the income is $P_i$are the prices of the commodities, or alternatively

(4) $x_0 + \sum_{i=1}^{n} p_i x_i = I^*$.Next given in order to maximize utility , household buys necessary set of goods, and what is left after buying is equally spent in constant proportions on various goods, in the model that two stage budgeting process is accepted so Dixit and Stiglitz model defines quantity and price indexes:

(5) $x_i = \left(\frac{p_i}{q}\right)^{-s} \frac{Y}{q}$ demand for good i s *intrasector elasticity of substitution*

(6) $q = \left(\sum_{i=1}^{n} P_i^{\,1-s}\right)^{1/(1-s)}$ →Price index

Next in the analysis it's supposed to compute optimal level of quantity index y, the consumption of composite goods, and the optimal level of x_0 , as the numeraire good w=1

(7)$y = \left(\sum_{i=1}^{n} x_i^{\,(s-1)/s}\right)^{s/(s-1)} = \frac{Y}{q} = \alpha(q)\frac{I}{q}$

(8)$x_0 = [1-\alpha(q)]I, \quad x_i = I\frac{S(q)}{q}\left(\frac{q}{P_i}\right)^{\sigma}$ (2) demand for good

$\alpha(q)$is the marginal propensity to consume in the monopolistic sector for composite goods so now we move on further to the **supply side** of the model. The assumption is that the labour is perfectly mobile factor of production across the sectors, so as a result in our model there is single wage rate which we denote as W in the other sectors than monopolistic there is constant returns to scale and we can specify the production function:

(9) $x_0 = \ell_0$ (3)

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* ρ is the substitution parameter

ℓ_i is quantity of labour employed to produce good i $(i = 0,1 \ldots\ldots, n)$,and $\beta = 1$ characterizes globally constant returns to scale. Nominal wage rate is $w = 1$.Each Firm tries to maximize the profit and, first order condition is when $MR = MC$ in terms of our model[2]

(10)$p_i\left(1 - \frac{1}{\varepsilon_i}\right) = wC'(x_i)$

ε_i is the price demand elasticity for good point elasticity $\varepsilon_i[(\partial x_i/\partial p_i)(p_i/x_i)]$,and the cross elasticity of demand for a firm is $\varepsilon_{ij}[\equiv (\partial x_i/\partial p_j)(p_j x_j)]$.In the symmetric equilibrium with $p_i = p_j\ \forall_{i,j}$thatis our equilibrium price for zero profit condition. In the Dixit-Stiglitz model there are made two important assumptions: Each monopolist ignores the cross-price elasticity of demand for a given variety of goods $(\varepsilon_{ij} = 0)$.And the second assumption that the influence on the individual price change[3] on the general price index is ignored ; $(\partial_q/\partial p_i = 0)$(see Yang Heijdra (1993) pp.296This two assumptions are related when $\varepsilon_{ij} = 0 \Leftrightarrow \varepsilon_i = -\sigma$; σ is the inter sectoral elasticity of substitution between two differentiated products. All these assumptions to hold there must be large$n$; large n is outcome from the model, it's not assumption of the model; if and only first of the assumptions hold there will be monopolistic competition.$\sigma = 1/(1+\rho)$ [*].Now σ rises with

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[2] [2] (2) d'Aspremont Claude; Rodolphe Dos Santos Ferreira; Louis-André Gérard-Varet(Jun., 1996), **On the Dixit-Stiglitz Model of Monopolistic Competition** pp:624

(3)**See** Xiaokai Yang; Ben J. Heijdra (Mar., 1993), **Monopolistic Competition and Optimum Product Diversity: Comment**
The American Economic Review, Vol. 83, No. 1. pp. 299

(4)Xiaokai Yang; Heijdra J. Ben (Mar., 1993), **Monopolistic Competition and Optimum Product Diversity: Comment**
The American Economic Review, Vol. 83, No. 1. pp. 297.

(5) Lancaster Kelvin (Sep., 1975), **Socially Optimal Product Differentiation** pp571 *The American Economic Review*, Vol. 65, No. 4.

(6Breakman)Steven an and Heiydra J. Ben (2004)***The Monopolistic competition Revolution in Retrospect*** Cambridge:Cambridge university press.Chapter 6

(7) Koeniger Winfried, Licandro IZA Omar European University Institute and FEDEA January 2004 ;Substitutability and Competition in the Dixit-Stiglitz Mod el*

the number of available varieties; $\sigma = \sigma(n)$ and estimates that $\varepsilon_{ij} \to 0 \; as \; n \to \infty$ if $\sigma(n) \; rises \; at \; slower \; rate \; than$ (4) .So as number of firms gets larger cross elasticity of demand will tend to be zero. There can be distinguished several equilibriums when firms in symmetric equilibrium charge $p_i = p_j = p \; andproduce \; x_i = x_j = x$. And $v_i = F(Q_i)$ (5) which mean resource requirements for all goods are the same; implying constant returns to scale $F(Q_i)$ is the input function. Under constant returns to scale the optimal the optimal number of goods in DS model is unbounded because preferences form continuum. $Marginal \; preference \; for \; diversity = \frac{1}{\sigma - 1}$ (6) .Market equilibrium is decentralized Chamberlain monopolistic competition .The representative agent there maximizes $\int_{\Gamma} p(\rho) c(\rho) dF = 1 + \Pi$ (7)·Left side is income which is equal to profit plus labour endowment is1. Since labour is only factor o production $1 + \Pi = \frac{1}{\alpha}$ profit measured in labour units is $1 - \frac{1}{\alpha}$ (7) $\frac{1}{\alpha}$ is the industry markup.

$(11) \; p_i = c\left(\frac{\varepsilon_i}{1+\varepsilon_i}\right)$ Where c is marginal costs and with the symmetry present in the model this implies zero profit all of the firms work at breakeven point. In DS model it is assumed fixed demand curve for the entire monopolist, goods have to be perfect substitutes amongst themselves but not for the goods outside of the group.Recall $x_0 + \sum_{i=1}^{n} p_i x_i = I; \; x_0 = 1;$ $I = 1$ From the original model $\frac{s\left(p_e \; n_{\overrightarrow{e}} - \beta\right)}{p_e n_e} = \frac{a}{\beta c}$ as n increase the whole expression decrease break even goes down; firms have to reduce their capacities. Next constrained social optimum has the same price as market equilibrium; same break even constraint a number of firms is the same this optimum is obtained in absence of subsidies to cover the losses $(p < AC)$.Unconstrained social optimum*4 where lump-sum subsidies are allowed

4 *Technology and labour are constraints here

(8) in the model $u = x_0^{1-\gamma}\{\sum_i v(x_i)\}^{\gamma}$

9)(10) Xiaokai Yang; Heijdra J .Ben (Mar., 1993), **Monopolistic Competition and Optimum Product Diversity: Comment** *The American Economic Review*, Vol. 83, No. 1. pp. 300.

*$\theta(q)[\equiv (\partial s/\partial q)/q/s]$ is elasticity of share function

$I = 1 - an$.Output remains at the social optimal level t but the number of firms increase an are the lump sum subsidies that cover the variable cost .Dixit-Stiglits conclusion in constant elasticity case is $n_u > n_c = n_e (n\ is\ number\ of\ firms)$

Model's extension

Now, assume variable elasticity Utility function is now Cobb-Douglas [8]. $s(q) = \gamma$ $\theta(q) = 0,$* and $w(x) = I * s(q)$Production function is $x_i = \ell^{1/\beta}$ $\beta < 1$; break even condition on left side assumes linear cost function $px = a + cx$ but, we find the equilibrium number of firms using also the DD curve i.e. $DD = w(x)$ .Now $\varepsilon_{ij} = \frac{\rho - \beta}{(1-\beta)(1-\rho)}$[9]. The model will only hold if $\rho = \beta$ from which we have$\rho(x) \lesseqgtr 0$.ρ is the elasticity of utility in DS model the higher x would give us lower n equilibrium includes lower number of big firms with larger fixed cost than the constrained optimum or "vice versa" situation

(12) $n_c \gtreqless n_e\ according\ as\ x_c \lesseqgtr x_e$ * that is because equilibrium is

Symmetrical .In the unconstrained optimum where firms face lower price and resource is most efficient .In DS model $MR = \rho P$; $MR = \left[\frac{\rho(n-1)}{(n-\rho)}\right] P$ [10] First case if $\rho = \beta$ we cannot find the number of firms the second term tell us if P is lower expression in the brackets has to be higher so that n is higher and elasticity of the utility of the two groups is the same .Unconstrained social optimum has more bigger firms but less variety than constrained social optimum. Asymmetric demand and costs open the possibility of production of incorrect commodities market may be biased unlike equilibrium. Leading to loss in social welfare.

Conclusion

The Dixit-Stiglitz model of monopolistic competition works only when n is large; from the functions of the productions best when one applies linear production function. Under increasing returns to scale monopolistic competition will lead to a greater degree of product differentiation than it is socially optimal.

APPENDIX 1

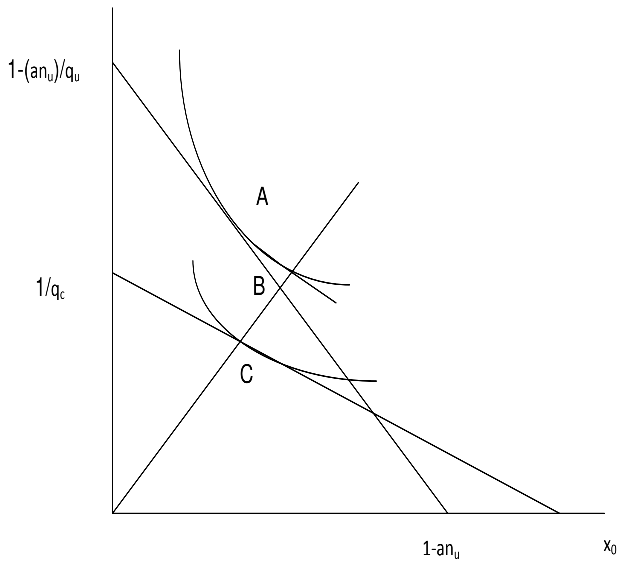

Figure 1 The Dixit- Stiglitz model equilibriums (when constant returns to scale)

As it can be seen in figure 1 unconstrained optimum is marked with A unconstrained optimum C-constrained optimum B equilibrium each firm moves from C to B and then to A that will increase the quantity index while X will remain the same .Because of the presence of the lump-sum subsidies at the unconstrained optimum appears lowest level of price $q_u < q_c = q_e$ and for the number of firms $n_u > n_c = n_e$(see Avinash K. Dixit; Joseph E.

Stiglitz (Jun., 1977), Monopolistic Competition[5] and Optimum Product Diversity Vol. 67, No. 3.pp.302)

1.1 **Linearity**

What Dixit-Stiglitz made in their model are the assumptions that economic relations can be expressed in terms of linearity .Demand for one good x_i in terms of prices $p_i \ldots\ldots, p_n$ and income r .The demand for x_i can be found using equation which represents the expansion of a polynomial function of the nth degree(Taylor series)expansion around point $x = x_i this\ is\ the\ case\ x = 1$ which we call linear function is $y = f(x);$ $x_i = x^0{}_i + \left(\frac{\partial x_i}{\partial p_i}\right)^0 (p_i - p_0{}^0) + \cdots . + \left(\frac{\partial x_i}{p_n}\right)^0 (p_n - p_0{}^0) + \left(\frac{\partial x_i}{\partial r}\right)(r - r_0) + u$.This tell us if we have small changes in pricesand income error term is small(see Paul Samuelson 1947 Some implications of linearity) *Econometrica* .And consumer has two choose between two necessary sets of goods ,with given budget constraint ,and the consumer will always choose the indifference curve with highest utility in the given budget constraint as Samuelson makes note. $(x_i, \bar{x}_2, \ldots . \bar{x}_n) = (-\gamma_1, -\gamma_2 \ldots\ldots, -\gamma_n)$ (11)

1.2 **Utility maximization revisited**

Maximize $U = U(x_0, \ldots\ldots\ldots . x_n)$

Subject to $P_1 x_1 \ldots\ldots . + P_n x_n \leq B$

And $X_1, \ldots\ldots\ldots, x_n \geq 0$

(a) The function $f(x)\ should\ be\ differentiable\ and\ concave\ and\ nonegative$

(b) The constraint $g_x{}' = P_1 X_1 \ldots\ldots + P_n X_n\ is\ also\ linear\ differentiable\ and\ convexnonnegative$

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5 (11) **Samuelson A .Paul**, 1947, "Some implications of linearity " *Econometrica*

(c) The points $\bar{x}$ satisfies the Kuhn Tucker conditions(see point 1.1 Linearity)[13]

The budget line where expenditure exactly equals the income is

$P_1X_1 + P_2X_2 = I$ [(*)]

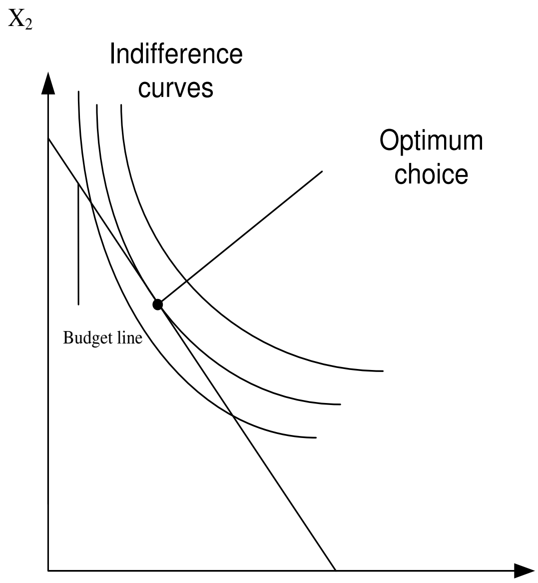

Fig.1.1-The consumers optimum choice

$$U(x_1, x_2) = constant$$

x_1, x_2 Combination is preferred bundle of goods $U(x_1, x_2)$ is the utility level of the consumer [6]. x_1, x_2 are the two goods respectively and we have prices of the two goods P_1, P_2 respectively budget constraint is as we stated $P_1X_1 + P_2X_2 = I$ if our utility function is $U(x_1, x_2)$now if we do semi-log function

$U(x_1, x_2) = \alpha ln(x) + \beta ln(y)$, α *and* β are positive constants ln is natural logarithm. Now the whole expression we can write in Lagrangian terms.

$L(x_1, x_2, \lambda) = \alpha ln(x_1) + \beta ln(x_2) + \lambda(I - P_1X_1 - P_2X_2)$

[*]Chiang C.Alpha (1984),Fundamental Methods of Mathematical Economics

Now if we write that : $\frac{dln(x)}{dx} = \frac{1}{x}$

The first order conditions for maximization are partial derivatives to be equal to zero

$\frac{\partial L}{\partial x_1} = \alpha/x_1 - \lambda p_1 \equiv 0, \frac{\partial L}{\partial X_2} = \beta/x_2 - \lambda p_2 \equiv 0$ and $\frac{\partial L}{\partial \lambda} \equiv I - \quad P_1 X_1 - P_2 X_2 = 0$

From the first two equations we have : $x_1 = \frac{\alpha}{\lambda p_1}$,$x_2 = \frac{\beta}{\lambda p_2}$ and then we substitute in the equation three :

$I - \frac{\alpha}{\lambda p_1} * P_1 + \frac{\beta}{\lambda p_2} * P_2 = 0 = I - \frac{\alpha}{\lambda} + \frac{\beta}{\lambda} = 0 = I - \frac{\alpha+\beta}{\lambda} = 0 = I\lambda - \alpha + \beta = 0 \Rightarrow \lambda = \frac{\alpha+\beta}{I}$

or then if we substitute in the previous two equations $x_1 = \frac{\alpha}{\frac{\alpha+\beta}{I} p_1} = \frac{\alpha I}{(\alpha+\beta)p_1}$ and then identical $x_2 = \frac{\beta I}{(\alpha+\beta)P_2}$. Alternatively we can express the share of income spent $\frac{x_2 p_2}{I} = \frac{\beta}{(\alpha+\beta)}$ or for first equation $\frac{x_1 p_1}{I} = \frac{\alpha}{(\alpha+\beta)}$.This shares of income spent on two goods are constants for the utility function maximization.

$\tilde{U} = x_1{}^{\alpha} x_2{}^{\beta}$ if we use log again we can turn product into a sum and the best is to assume $\alpha + \beta = 1$ The multiplier for this problem is $\tilde{\lambda} = \frac{\alpha^{\alpha}\beta^{\beta}}{(\alpha+\beta)^{\alpha+\beta}} \frac{I^{\alpha+\beta-1}}{P_1{}^{\alpha} P_2{}^{\beta}}$ if $\alpha + \beta = 1$ then $\tilde{\lambda} = \frac{\alpha^{\alpha}\beta^{\beta}}{P_1{}^{\alpha} P_2{}^{\beta}}$ that is the multiplier in this case. [7]

But if but if , $\alpha + \beta > 1$ then $\tilde{\lambda}$ increase with I but in constant elasticity case if $\alpha + \beta = 1$ then $I^{\alpha+\beta-1} = I^0 = 1$

Or let's get back to our :

$L(x_1, x_2, \lambda) = \alpha ln(x_1) + \beta ln(x_2) + \lambda(I - \quad P_1 X_1 - P_2 X_2)$ now since our utility level is $U(x_1, x_2)$ than we can write

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[13] Dixit A.K. (Dec.1989), **Optimization in economic theory** ,Oxford university press,Chapter 2, pp20
* Chiang C.Alpha (1984),Fundamental Methods of Mathematical Economics

$U_1, U_2 > 0$ and in our Lagrangian form we can write like

$L(U_1, U_2, \lambda) = U(x_1, x_2) + \lambda(I - \quad P_1 X_1 \; - P_2 X_2)$After dong partial derivatives of the elements in the equation we can find that if the consumer is indifferent to the two goods than this ratio holds :

$\frac{U_{x_1}}{P_1} = \frac{U_{x_2}}{P_2} = \lambda$ .About the indifference curve $dU = U_{X_1} dx_1 + U_{X_2} dx_2$.And we can represent like in figure 1.1*

Appendix 2

Product differentiation curve

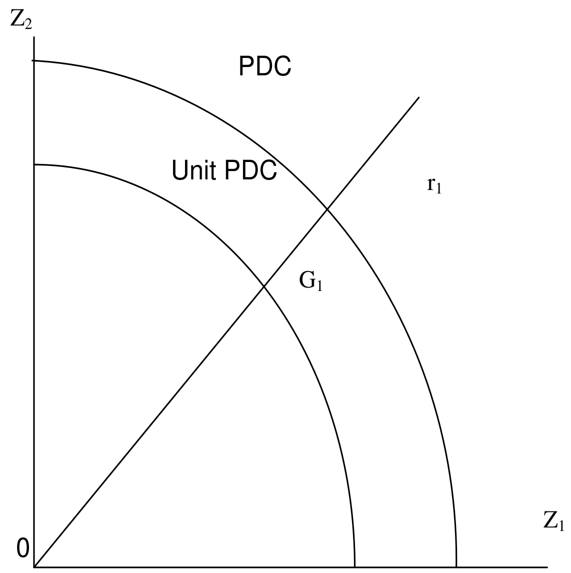

Fig 2 .Product differentiation curve *8

[14] Lancaster Kevin (Sep., 1975), **Socially Optimal Product Differentiation** *The American Economic Review*, Vol. 65, No. 4.pp 569

The set of all the characteristics combinations, producible from a given level of resources previously determined output; actually the maximum output of the good with given ratio of characteristics is predetermined. And it can be plotted as a curve on a characteristics space. That kind of curve we call "Product differentiation curve".

Input function is all for the same goods

$v = f(Q_i)$;v is the resource requirement and also this functional relationship between

$v\ and\ Q$ will be the same for all goods i, j.We bring different goods to the same measure with

defining unit quantity of any good to be that quantity which can be produced with unit resources. That is given unit product differentiation curve. If there is constant returns to scale, linear production function , linear product differentiable curve ,and also assumption for linear utility function the number of goods that are needed to achieve the social optimum is lower than what is optimal to produce when we have decreasing returns to scale.

Now ,to find what are the conditions for optimal differentiation and to find optimal configuration which means producing goods with certain characteristics and distribution of those goods over customers and assuming minimum use of resources. Now, this is the simple model

$R_1, \dots\dots, R_{n-1}$ These are the characteristics ratios and by $r_1 \dots\dots\dots, r_n$ the characteristics ratios of n goods. Optimal choice for r_i is the one which minimizes Q_i, which fulfils the assumption for minimum resources.

Now our optimum condition is

$\partial Q_i(r_i, R_i, R_{i-1})/\partial r_i = 0$ this is first optimum condition .Now we introduce $Q_i{}^*$which is optimized Q_i.

$Q_i{}^* = Q_i{}^*(r_i, R_i, R_{i-1})$.To optimize this we must minimize the total resource use given by:

$v = \sum F_i\left(Q^*{}_i\right) = V(R_1, \dots\dots, R_{n-1})$ .Now, the optimum conditions for R are as follows :

$\frac{\partial V}{\partial R_i} = \frac{\partial Q_{i+1}}{\partial R_i} F_{i+1}{}' + \frac{\partial Q_i}{\partial R_i} F_i{}' = 0$ [15(9)]

2.1 Enlarged model

Now Claude d'Aspremont et al.(jun 1996) presented enlarged model of the Dixit-Stiglitz original model that has been used since it's own publishing with numeraire as non produced good aggregating the rest of the economy outside of the monopolistic sector. Now in the model W is the money endowment of labour. In this model Dixit-Stiglitz model is just a partial equilibrium on the monopolistic side of the market and , labour which also determines the demand for labour as a relationship between aggregate employment and money wage. The equilibrium on market in this model happens by equalizing labour demand and labour supp $I = [M + W\sum_{j=1}^{n} C(x_j)] + \sum_{j=1}^{n}[p_j x_j - wC(x_j)] = M + Y$.Income equals distributed profits and representative consumer endowment M money endowment Y is the given level of expenditure. Demand for good is $x_i = \left(\frac{p_i}{q}\right)^{-s} \frac{\alpha(q)}{[1-\alpha(q)]q} M$.And demand elasticity is a sum of two elasticity's intra and inter- sector elasticities and also depends on ratio of price in monopolistic sector and price index q, $\varepsilon_i = \left[1 - \left(\frac{p_i}{q}\right)^{1-s}\right] s + \left(\frac{p_i}{q}\right)^{1-s} \sigma(q)$, Inter-sectoral elasticity of substitution is constant .Now, authors assume price ,output ,and number of firms at equilibrium however conclusion that point elasticity in Dixit-Stiglitz model point elasticity of the demand for product i is $e_i = s$ is squal to intrasector elasticity and $s \geq \sigma$whatever number of firms when intra is equal to inter sectoral utility. And the equilibrium number of firms is found $n^* \approx \frac{s-\sigma}{s-1/(1-\beta)}$ [9(*)]

Appendix 3

3.1Asymmetry

When there is asymmetry we assume two goods besides the numeraire or let say two groups of goods ,and those constitute the utility function of representative consumer and the utility

[15(9)] Lancaster Kevin (Sep., 1975), **Socially Optimal Product Differentiation** *The American Economic Review*, Vol. 65, No. 4.
[15,10(*)]Claude d'Aspremont; Rodolphe Dos Santos Ferreira; Louis-André Gérard-Varet(Jun., 1996), On the Dixit-Stiglitz Model of Monopolistic Competition pp:628

function is now Cobb-douglas,the two sets of commodities are perfect substitutes between each other and have constant elasticity of substitution ρ.Now we presume that only one good will be produced but Nash equilibrium exist only if for one firm it does not pay to produce the good of the second firm which means $\bar{q}_1 < \frac{s-c_2}{s-a_2}$.Where c respectively is marginal cost and $a$is fixed cost. And break - even point is given $q_i = p_i n_i^{-\beta_i}$ .What this equation tell us is that if we have lower elasticity of commodities of group i, it means that price index for the group of firms will be higher and since P_i is the price that firm sets for its own product it will cover losses because they are inevitably and since $q_i = p_i n_i^{-\beta_i} = c_2(1+\beta_2)^{1+\beta_2}\left(\frac{a_2}{s}\right)^{-\beta_2}$.It will cover some of the variable cost since break even is inverse with fixed costs if we assume only one firm that will satisfy the demand for a product with those ratio of characteristics then we assume higher fixed costs because for one firm it doesn't pay to produce the good that other firm does .Also products with lower elasticity have higher earning possibilities over variable costs, they also have significant consumer surpluses .

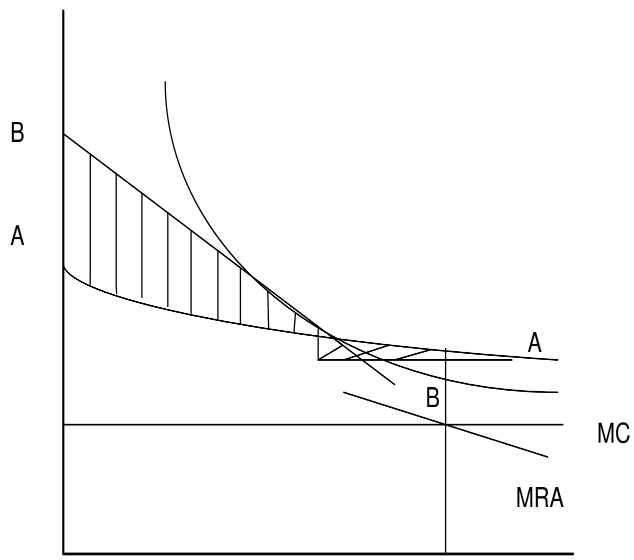

Figure 3(*)

Here is an example of preciously[10] said Product A is more elastic than product B and also has lower consumer surplus .A is produced and B is not in monopolistic competition although it is preferred one socially.

3.2 **Application of the canonical model of monopolistic competition in International trade theory**

Krugman (1980) model is special form of Dixit-Stiglitz model $s(q) = \gamma = 1$; which reminds us of part two our model extension .The results can be obtained by using the solutions in the extended model .In the Krugman's model I is the size of the economy. In the open economy the international trade will lead to increase the size of I and therefore which influence the level of diversification, price level, and the level of output.

$$p_e = \frac{c}{\rho}\left(\frac{\gamma I}{\gamma I - a}\right) \Rightarrow \frac{\partial p}{\partial I} < 0;$$

$$x_e = \frac{\rho(\gamma I - a)}{c\left[\frac{\gamma I}{a}(1-\rho) + \rho\right]} \Rightarrow \frac{\partial x_e}{\partial I} > 0$$

$$n_e = \frac{\gamma I}{\alpha}(1-\rho) + \rho \Rightarrow \frac{\partial n_e}{\partial I} > 0 \quad (17)$$

The left side of this expressions shows that price and output do depend on the size of the economy ,while the right sides tell opposite which can be case in the right side also if and only if $s(q) = \gamma = 1$

The share function is equal to 1.Although this Dixit-Stiglitz model has wide range of applications Growth theory, macroeconomics etc.

[16, 11] [(*)] Avinash K. Dixit; Joseph E. Stiglitz (Jun., 1977), **Monopolistic Competition and Optimum Product Diversity** *The American Economic Review*, Vol. 67, No. 3.pp307

3.3 **CES Production function**

The equation of CES production function is as it is generally accepted

$Q = A[\delta K^{-\rho} + (1-\delta)L^{-\rho}]^{-1/\rho}$ and about the parameters in the equation

$(A > 0; 0 < \delta < 1; -1 < \rho \neq 0)$

K and L represent the two factors of production capital and labour and $A; \delta Aand\ \rho$ are the parameters in the equation. If we multiply the function with J each variable in the function we will show however that the function is homogenous with the degree one. Now we will multiply K and L with j

$$Q = A[\delta(JK)^{-\rho} + (1-\delta)(JL)^{-\rho}]^{-1/\rho} = A\{J^{-\rho}[\delta K^{-\rho} + (1-\delta)L^{-\rho}]\}^{-\rho} = (J^{-\rho})^{-1/\rho} Q$$

$$= \sqrt[\rho]{\left(\frac{1}{J^{\rho}}\right)^{-1}} * Q = \sqrt[\rho]{J^{\rho}} * Q = JQ$$

So that proves that function is homogenous on first degree which implies constant returns to scale/Now about the inpust the optimal input ratio implies

$\left(\frac{K}{L}\right) = \left(\frac{\delta}{1-\delta}\right)^{1/(1+\rho)} \left(\frac{P_L}{P_K}\right)^{1/(1+\rho)}$ Now, if we replace $\left(\frac{\delta}{1-\delta}\right)^{1/(1+\rho)} = c$; then ,

(17)See Xiaokai Yang; Heijdra J. Ben ,(Mar., 1993), **Monopolistic Competition and Optimum Product Diversity: Comment***The American Economic Review*, Vol. 83, No. 1. pp. 299.

$$\left(\frac{\bar{K}}{\bar{L}}\right) = c\left(\frac{P_L}{P_K}\right)^{1/(1+\rho)}$$

Elasticity is ratio of marginal and average function, this input function ratio is a function of the two inputs prices Marginal function we find by definition like a ratio of the marginal changes of the two sides of the equation

$$marginal\ function = \frac{d(\bar{K}/\bar{L})}{d(P_L/P_K)} = \frac{c}{1+\rho}\left(\frac{P_L}{P_K}\right)^{1/(1+\rho)-1}$$

$$\text{average function} = \frac{\bar{K}/\bar{L}}{P_L/P_K} = c\left(\frac{P_L}{P_K}\right)^{1/(1+\rho)-1}$$

Elasticity of substitution is $\sigma = \frac{marginal\ function}{average\ function} = \frac{1}{1+\rho}$ *11

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