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Stability analysis in a Cournot duopoly with managerial sales delegation and bounded rationality

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Abstract The present study analyses the dynamics of a Cournot duopoly with managerial sales delegation and bounded rational players. We find that when firms' owners hire a manager and delegate the output decisions to him, the unique Cournot-Nash equilibrium is more likely to be destabilised (through a flip bifurcation) than when firms maximise profits. Moreover, highly periodicity and deterministic chaos can also occur as the managers' bonus increases.

Keywords Bifurcation; Chaos; Cournot; Duopoly; Managerial incentive contracts

JEL Classification C62; D43; L13

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1. Introduction

While traditional economic theories assume that the single aim of competing firms is profit maximisation, a more recent literature argues that, at least in large companies, ownership and management are separated (see, e.g., Fama and Jensen, 1983). Indeed, starting from the seminal paper by Baumol (1958), managers may be driven by motives different than pure profit-maximisation, so that a sales-maximisation model may be a more realistic alternative to be dealt with to describe oligopoly markets. On empirical grounds, there exists evidence (see, e.g., Jensen and Murphy, 1990) suggesting that the owners of firms try to motivate their managers through incentive contracts with the aim to gain a competitive advantage with respect to the rivals.

The strategic use of managerial incentive contracts in models with imperfect competition in the product market has been introduced in economics by the pioneering works by Vickers (1985), Fershtman (1985), Fershtman and Judd (1987) and Sklivas (1987) (VFJS henceforth). According to them, each owner has the opportunity to compensate a manager through a bonus based on a weighted sum of profits and sales ("sales delegation"),[1] as an incentive device to conduct a more aggressive behaviour in the market.[2] More recently, static oligopoly models have been developed to study problems with unions and managerial incentives in a Cournot duopoly (see Bughin, 1995), or strategic delegation under quality competition (see Ishibashi, 2001), and managerial schemes and loan commitments (see Coscollá and Granero, 2003).

The present study, in particular, focuses on the case of a dynamic nonlinear Cournot duopoly. As is known, the Nash equilibrium in a duopoly with quantity competition and standard linear demand and cost functions is stable if expectations of each firm are of the "naïve" type (i.e., each firm expects that the output produced today by the rival equals the output produced in the previous period),[3] as shown by Theocharis (1960). The use of static expectations, however, has been criticised because it overestimates the importance of past values. Indeed, the rational expectations[4] revolution, which has initiated in macroeconomics with the works by, amongst many others, Lucas (1972) and Sargent and Wallace (1973), prescribes that (*i*) agents form expectations on the value take on a certain variable in the future using information in the most efficient way, and (*ii*) the expectations of a single economic agent on a certain variable (i.e., the subjective probability distribution of the events) tend to be distributed in accordance with the prediction of the prevailing economic theory (i.e., the objective probability distribution of the events). However, also the hypothesis of rational expectations has been subject to some criticisms (see, e.g., Burmeister, 1980), because it seems to overestimate the ability of agents to predict the behaviour of prices and quantities.

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[1] Jansen et al. (2007, 2009) argue that this combination can be rewritten as a combination of profits and output volume. In this paper we follow such authors and assume such a formula to compute the manager's bonus.
[2] Indeed, an important work by Grossman and Stiglitz (1977) exists that attempts to unify the theories of conventional profit maximisation of competitive firms and the managerial literature.
[3] As is known, in a duopoly game each firm must forecast the behaviour of the competitor in order to make the optimal output choice. It seems more realistic to consider mechanisms through which players form their expectations on the decisions of the competitors rather than assume that firms are able to perfectly infer the choices of the other firms. Cournot (1838) was *de facto* the first to use naïve expectations in a duopoly market.
[4] See Muth (1961) for the notion of rational expectations.

The burgeoning interest in nonlinear dynamic models has therefore renewed the interest in the use of expectations formation mechanisms at all different from the scheme of rational expectations. Indeed, as claimed by Agliari et al. (2006, p. 527), "When one takes into account the fact that nonlinear dynamical systems can produce dynamic paths that are not so regular and predictable, one of the major arguments against adaptive expectations does not seem so strong.", because linear models represent an approximation of nonlinear models (see, amongst others, Chiarella, 1986, 1990; Bischi et al., 2010).

Therefore, if expectations formation mechanisms of one or both firms on the quantity produced by the rival in the future are those of the type suggested in the most part of the recent dynamic oligopoly literature, see, e.g., Dixit (1986), i.e., firms are assumed the have bounded rational expectations (see Simon, 1957, for the notion of bounded rationality), and then increase/decrease outputs tomorrow, according to a certain degree or speed of adjustment, depending on information given by the marginal profit in the current period, then the Nash equilibrium in a Cournot duopoly with standard linear demand and cost functions may be destabilised when the speed of adjustment of each firm's output is fairly high and, in particular, complex dynamics can also be observed, as shown by, e.g., Puu, 1991, 1998; Kopel, 1996; Agiza and Elsadany, 2003, 2004; Zhang et al., 2007; Tramontana, 2010).

As regards the topic of the present paper, in spite of the widespread use of the assumption of managerial incentive contracts à la VFJS in the industrial economics literature, a stability analysis of a duopoly game in which the owners of firms negotiate their manager's incentive contract as a linear combination of profits and sales, has not been so far tackled on, at the best of our knowledge. The aim of the present paper, therefore, is to fill a gap in the existing literature on nonlinear oligopolies by considering a simple Cournot duopoly with managerial sales delegation and bounded rational players. In particular, we focus on the dynamic role played by the weight of sales in the managers' bonus, and we address the following questions: how the departure from the pure profit maximisation objective represented by the weight on sales in the manager's objective affects stability outcomes?

We can summarise the main results of the paper in the following sentence: under managerial sales delegation, the Cournot-Nash equilibrium is more prone to be destabilised than when firms maximise profits: for instance, starting out with a stable Cournot-Nash equilibrium without managerial incentive contracts, a rise in the weight on sales in the managers' bonus may cause a loss of stability (through a flip bifurcation), ceteris paribus as regards the costs of production. Interestingly, these results are qualitatively robust to the homogeneity or heterogeneity between duopolists as regards the type of expectations formation (see Footnote 7 below).

The rest of the paper is organised as follows. Section 2 introduces the model and the nonlinear dynamic system. Sections 3 analyses the local stability properties of the unique positive Cournot-Nash equilibrium showing the local bifurcations and the emergence of regular and complex dynamics with numerical simulations. Section 4 concludes.

2. The model

The model is outlined in accordance with the standard line of research that follows the approach by VFJS. Specifically, as regards both the assumptions on the normalised inverse demand and specification of the manager's bonus, we strictly follow the recent papers by Jansen et al. (2007, 2009) and van Witteloostuijn et al. (2007).

We consider a normalised Cournot duopoly for a single homogenous product with inverse demand given by $p = 1 - Q$,[5] where p denotes the price and Q is the sum of outputs q_1 and q_2 produced by firm 1 and firm 2, respectively. The average and marginal costs for each single firm to provide one additional unit of output in the market are equal and constant at $0 < c < 1$.[6] We assume that the owners of both firms hire a manager and delegate output decision to him. Each manager receives a fixed salary and a bonus offered in a publicly observable contract. In particular, we assume that the bonus of the manager hired by firm $i = \{1,2\}$ is $U_i = \Pi_i + b_i q_i$, where $\Pi_i = (p - c)q_i$ denotes the profits of the ith firm and b_i is a non-negative parameter that weights output volume in the manager's objective. The bonus of the manager of the ith firm can therefore be rewritten as $U_i = (1 - Q - c + b_i)q_i$.

An equilibrium of the market game satisfies the following system of first-order conditions, obtained by the maximisation of the managers' objectives with respect to the quantity produced by the firm where she is delegated, that is:

$$\frac{\partial U_1}{\partial q_1} = 0 \Leftrightarrow 1 - 2q_1 - q_2 - c + b_1 = 0 , \tag{1.1}$$

$$\frac{\partial U_2}{\partial q_2} = 0 \Leftrightarrow 1 - q_1 - 2q_2 - c + b_2 = 0 . \tag{1.2}$$

Therefore, the reaction- or best-reply functions of managers hired by firms 1 and 2 are determined by solving Eqs. (1.1) and (1.2) with respect to q_1 and q_2, respectively, that is:

$$q_1(q_2) = \frac{1}{2}(1 - q_2 - c + b_1), \tag{2.1}$$

$$q_2(q_1) = \frac{1}{2}(1 - q_1 - c + b_2). \tag{2.2}$$

Let $q_{i,t}$ be the firm i's quantity produced at time $t = 0,1,2,\ldots$. Then, $q_{i,t+1}$ is obtained through the following optimisation programmes:

$$q_{1,t+1} = \arg\max_{q_{1,t}} U_{1,t}\left(q_{1,t}, q^{e}{}_{2,t+1}\right), \tag{3.1}$$

$$q_{2,t+1} = \arg\max_{q_{2,t}} U_{2,t}\left(q^{e}{}_{1,t+1}, q_{2,t}\right). \tag{3.2}$$

where $q^{e}{}_{i,t+1}$ represents the quantity that the rival (i.e., the firm where manager j is delegated) today (time t) expects will be produced in the future (time $t+1$) by the firm where manager i is delegated. In this paper we assume that each manager has bounded rational expectations about the quantity to be produced in the future. Therefore, each player uses information on the current manager's objective in such a way to increase or decrease the quantity produced at time $t+1$ depending on whether the marginal manager i's bonus (i.e., $\partial U_{i,t} / \partial q_{i,t}$) is either positive or negative.

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[5] Note that the standard inverse demand model $p' = A - BQ'$ can be normalised using the transformed variables $p = p'/A$ and $Q = (B/A)Q'$.

[6] This hypothesis implies that firm i produces through a production function with constant (marginal) returns to labour, that is $q_i = L_i$, where L_i represents the labour force employed by the ith firm (see Correa-López and Naylor, 2004).

Following Dixit (1986), the adjustment mechanism of quantities over time of the ith bounded rational player is described by:

$$q_{i,t+1} = q_{i,t} + \alpha_i q_{i,t} \frac{\partial U_{i,t}}{\partial q_{i,t}}, \tag{4}$$

where $\alpha_i > 0$ is a coefficient that captures the speed of adjustment of firm i's quantity with respect to a marginal change in the manager's bonus when q_i varies at time t. Therefore, through the use of Eq. (4), the system that characterises the dynamics of this simple duopoly game is:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha_1 q_{1,t} \dfrac{\partial U_{1,t}}{\partial q_{1,t}} \\ q_{2,t+1} = q_{2,t} + \alpha_2 q_{2,t} \dfrac{\partial U_{2,t}}{\partial q_{2,t}} \end{cases}. \tag{5}$$

Then, substituting out the marginal bonus Eqs. (1.1) and (1.2) into (5), and assuming, for simplicity, $\alpha_1 = \alpha_2 = \alpha$, we get:

$$\begin{cases} q_{1,t+1} = q_{1,t} + \alpha q_{1,t}\left(1 - 2q_{1,t} - q_{2,t} - c + b_1\right) \\ q_{2,t+1} = q_{2,t} + \alpha q_{2,t}\left(1 - q_{1,t} - 2q_{2,t} - c + b_2\right) \end{cases}. \tag{6}$$

Equilibrium implies $q_{1,t+1} = q_{1,t} = q_1$ and $q_{2,t+1} = q_{2,t} = q_2$. Then, the dynamic system defined by (6) can be reduced to:

$$\begin{cases} \alpha q_1\left(1 - 2q_1 - q_2 - c + b_1\right) = 0 \\ \alpha q_2\left(1 - q_1 - 2q_2 - c + b_2\right) = 0 \end{cases}. \tag{7}$$

Fixed points $E\left(q^*_1, q^*_2\right)$ of the two-dimensional system (6) are characterised by the following non-negative solutions of (7):

$$E_0 = (0,0), \quad E_1 = \left(0, \frac{1}{2}\left(1 - c + b_2\right)\right), \quad E_2 = \left(\frac{1}{2}\left(1 - c + b_1\right), 0\right), \tag{8.1}$$

and

$$E_3 = \left(\frac{1}{3}\left(1 - c + 2b_1 - b_2\right), \frac{1}{3}\left(1 - c + 2b_2 - b_1\right)\right). \tag{8.2}$$

Eq. (8.2) defines the unique interior Cournot-Nash equilibrium of a duopoly game with managerial sales delegation. Then, by assuming for simplicity a uniform bonus $b_i = b$,[7] the equilibrium outcomes (8.2) can be reduced to:

$$E_3 = \left(\frac{1}{3}\left(1 - c + b\right), \frac{1}{3}\left(1 - c + b\right)\right). \tag{8.3}$$

corresponding to which both firms produce the same outcome $q^*_1 = q^*_2 = q^*$. Using Eq. (8.3), the price and both the profit and manager's bonus corresponding to the equilibrium outcomes q^* are respectively given by:

$$p^* = \frac{1 + 2c - 2b}{3}, \tag{9}$$

[7] We abstract from an “optimal” determination of the equilibrium bonus by the firms' owners as in the static game à la VJFS, because the present dynamic duopoly inherently represents a disequilibrium adjustment process. Since the existence of managerial incentive contracts is a stylised fact in actual economies, we investigate the stability effects of the existence of manager's bonuses irrespective of the reasons why firms' owners stipulate such contracts.

$$\Pi^*_1 = \Pi^*_2 = \Pi^* = q^*\left(q^* - b\right), \tag{10.1}$$

$$U^*_1 = U^*_2 = U^* = \frac{1}{9}(1 - c + b)^2 . \tag{10.2}$$

From Eqs. (8.3)-(10.2), it can easily be seen that an increase in b causes the equilibrium levels of outputs and manager's bonuses to raise, while reducing the market price and profit of both firms. Moreover, from Eqs. (9) and (10.1) the conditions $c < \bar{c} := 1 - 2b$ and $0 < b < 1/2$ should hold to guarantee that $p^* > 0$ and $\Pi^* > 0$, and an increase in b monotonically reduces $\bar{c}$, that is $\partial\bar{c}/\partial b < 0$ for any $0 < b < 1/2$. If $b = 0$, i.e. firms are profit-maximising, then $\bar{c} = 1$ and thus $p^* > 0$ and $\Pi^* > 0$ always hold in such a case.

3. Local stability analysis

In order to investigate the local stability properties of the Cournot-Nash equilibrium E_3 under the hypothesis of bounded rational players,[8] we build on the Jacobian matrix of system (6) evaluated at the fixed point E_3 determined by Eq. (8.2), that is:

$$J = \begin{pmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{pmatrix} = \begin{pmatrix} 1 + \alpha\left(1 - c + b - 5q^*\right) & -\alpha q^* \\ -\alpha q^* & 1 + \alpha\left(1 - c + b - 5q^*\right) \end{pmatrix}, \tag{11}$$

whose trace and determinant are given by:

$$T := Tr(J) = 2J_{11} = \frac{2}{3}[3 - 2\alpha(1 - c + b)], \tag{12}$$

$$D := Det(J) = J^2_{11} - J^2_{12} = \frac{1}{3}[1 - \alpha(1 - c - b)][3 - \alpha(1 - c + b)]. \tag{13}$$

Therefore, the characteristic polynomial of (11) can be written as follows:

$$G(\lambda) = \lambda^2 - T\lambda + D, \tag{14}$$

with its discriminant being determined by $Z := T^2 - 4D = \frac{4}{9}\alpha^2(1 - c + b)^2 > 0$. Since the discriminant is positive, the existence of complex eigenvalues of J is prevented. As a consequence the Cournot-Nash equilibrium E_3 of the two-dimensional system (6) cannot loose stability through a Neimark-Sacker bifurcation.

As is known, bifurcation theory describes the way according to which topological features of a dynamic system (such as the number of stationary points or their stability) vary as some parameter values change. In particular, for a system in two dimensions, the stability conditions ensuring that both eigenvalues remain within the unit circle[9] are:

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[8] It should be noted that assuming either that both players have bounded rational expectations or, alternatively, only one of them has bounded rational expectations and the other one has Cournot-naïve expectations, does not change the qualitative behaviour of the dynamic outcomes that emerge after the loss of stability of the unique interior equilibrium E_3. The proof is here omitted for economy of space, while being available upon request.

[9] If no eigenvalues of the linearised system around the fixed points of a first order discrete system lie on the unit circle, then such points are defined *hyperbolic*. Roughly speaking, at non-hyperbolic points topological features are not structurally stable.

$$\begin{cases} (i) \quad F := 1 + T + D > 0 \\ (ii) \quad TC := 1 - T + D > 0 \\ (iii) \quad H := 1 - D > 0 \end{cases}. \tag{15}$$

The violation of any single inequality in (15), with the other two being simultaneously fulfilled leads to: (*i*) a flip bifurcation (a real eigenvalue that passes through -1) when $F = 0$; (*ii*) a fold or transcritical bifurcation (a real eigenvalue that passes through $+1$) when $TC = 0$; (*iii*) a Neimark-Sacker bifurcation (i.e., the modulus of a complex eigenvalue pair that passes through 1) when $H = 0$, namely $Det(J) = 1$ and $|Tr(J)| < 2$.

While the dynamical role played by the speed of adjustment α on stability of a Nash equilibrium in duopoly models has widely been investigated in literature (see, e.g., Agiza and Elsadany, 2003, 2004; Zhang et al., 2007, 2009; Tramontana, 2010), and since this model shows similar results with respect to those found by the existing literature as regards the role played by such a parameter (namely, the higher α, the more likely the destabilisation of the Cournot-Nash equilibrium), in what follows we focus on the dynamical effects of the existence of managerial sales delegation and compare the results with respect to the case of profit-maximising firms. In particular, we let the parameter c, i.e. the average and marginal cost of production, vary and analyse the dynamic properties of a Cournot duopoly when $b = 0$ and when $0 < b < 1/2$.

As regards the particular case of the Jacobian matrix defined by (11), the stability conditions stated in (15) can be written as follows:

$$\begin{cases} (i) \quad F = \frac{1}{3}[2 - \alpha(1-c) - \alpha b][6 - \alpha(1-c) - \alpha b] > 0 \\ (ii) \quad TC = \frac{1}{3}\alpha^2(1-c+b)^2 > 0 \\ (iii) \quad H = \frac{1}{3}\alpha(1-c+b)[4 - \alpha(1-c) - \alpha b] > 0 \end{cases}. \tag{16}$$

Therefore, while condition (*ii*) is always satisfied, conditions (*i*) and (*iii*) can be violated. In particular, the flip bifurcation surface $F > 0$ is violated ($F = 0$) when

$$c = c^F{}_1 := \frac{\alpha(1+b) - 2}{\alpha}, \tag{17}$$

or when

$$c := c^F{}_2 := \frac{\alpha(1+b) - 6}{\alpha}, \tag{18}$$

where $c^F{}_1 > c^F{}_2$, while the bifurcation surface $H > 0$ is violated ($H = 0$) when

$$c := c^H := \frac{\alpha(1+b) - 4}{\alpha}, \tag{19}$$

where $c^F{}_1 > c^H > c^F{}_2$ for every $\alpha > 0$ and $0 < b < 1/2$. Since c is non-negative, then from Eqs. (17)-(19) it can easily be seen that $c^F{}_1 > 0$ if, and only if,

$$\alpha > \alpha^F{}_1 := \frac{2}{1+b} < 2, \tag{20}$$

$c^H > 0$ if, and only if,

$$\alpha > \alpha^H := \frac{4}{1+b} < 4, \tag{21}$$

and $c^F{}_2 > 0$ if, and only if,

$$\alpha < \alpha^{F}{}_{2} := \frac{6}{1+b} < 6, \tag{22}$$

where $\alpha^{F}{}_{1} < \alpha^{H} < \alpha^{F}{}_{2}$ for every $0 < b < 1/2$, $\alpha^{F}{}_{1}$, α^{H} and $\alpha^{F}{}_{2}$ are negative monotonic function of b,[10] and $\lim_{\alpha \to +\infty} c^{F}{}_{1} = 1+b$, $\lim_{\alpha \to +\infty} c^{H} = 1+b$ and $\lim_{\alpha \to +\infty} c^{F}{}_{2} = 1+b$.

Now, define

$$\overline{\alpha} := \frac{2}{3b}, \tag{23}$$

$$\overline{\overline{\alpha}} = \frac{4}{3b}, \tag{24}$$

$$\overline{\overline{\overline{\alpha}}} = \frac{2}{b}, \tag{25}$$

three threshold values of the speed of adjustment α, where $\overline{\alpha} < \overline{\overline{\alpha}} < \overline{\overline{\overline{\alpha}}}$, which are the roots for α obtained by equating Eqs. (17), (18) and (19) to $\overline{c}$, respectively, i.e., the boundary of the economically meaningful domain of definition of c when b is positive. Moreover, the following inequalities hold depending on the size of the weight of the output volume in the manager's objective:

$$\alpha^{F}{}_{1} < \alpha^{H} < \alpha^{F}{}_{2} < \overline{\alpha} < \overline{\overline{\alpha}} < \overline{\overline{\overline{\alpha}}}, \tag{26}$$

for any $0 < b < 1/8$,

$$\alpha^{F}{}_{1} < \alpha^{H} < \overline{\alpha} < \alpha^{F}{}_{2} < \overline{\overline{\alpha}} < \overline{\overline{\overline{\alpha}}}, \tag{27}$$

for any $1/8 < b < 1/5$,

$$\alpha^{F}{}_{1} < \overline{\alpha} < \alpha^{H} < \alpha^{F}{}_{2} < \overline{\overline{\alpha}} < \overline{\overline{\overline{\alpha}}}, \tag{28}$$

for any $1/5 < b < 2/7$,

$$\alpha^{F}{}_{1} < \overline{\alpha} < \alpha^{H} < \overline{\overline{\alpha}} < \alpha^{F}{}_{2} < \overline{\overline{\overline{\alpha}}}, \tag{29}$$

for any $2/7 < b < 1/2$.

Therefore, the local stability properties of the fixed point E_3 can be summarised in the following proposition.

Proposition 1. *Let $0 < b < 1/2$ hold. Then, for every $0 < \alpha < \alpha^{F}{}_{1}$ the Cournot-Nash equilibrium E_3 of the two-dimensional system (6) is locally asymptotically stable for every $\overline{c} > c > 0$. For every $\alpha^{F}{}_{1} < \alpha < \overline{\alpha}$, the Cournot-Nash equilibrium E_3 is locally asymptotically stable for every $\overline{c} > c > c^{F}{}_{1}$; it looses stability through a flip or period-doubling bifurcation when c is reduced to $c = c^{F}{}_{1}$; it is locally unstable for every $c^{F}{}_{1} > c > 0$. Moreover, the Cournot-Nash equilibrium E_3 cannot undergo a re-switch towards stability when $c = c^{F}{}_{2}$. For every $\alpha > \overline{\alpha}$, the Cournot-Nash equilibrium E_3 is locally unstable for every $\overline{c} > c > 0$.*

Proof. See the Appendix.

Provided that the speed of adjustment α is not fairly low, Proposition 1 shows that the Cournot-Nash equilibrium E_3 can loose stability exclusively through a flip or

[10] To avoid confusion, we note that for symmetry purposes we chose to denote the threshold values of α ($\alpha^{F}{}_{1}, \alpha^{F}{}_{2}$ and α^{H}) in similar way as the corresponding bifurcation values of c ($c^{F}{}_{1}, c^{F}{}_{2}$ and c^{H}), but the local stability analysis is made by considering the average and marginal cost c as the main parameter of interest.

period-doubling bifurcation when the average and marginal cost of production is reduced to $c = c^F{}_1$.

Now, after having analytically shown the existence of a flip bifurcation in the model with managerial incentive contracts (see Proposition 1), we address the following question: is a duopoly with managerial firms, loosely speaking, more stable than that with profit-maximising firms? To answer this question, we state the following proposition.

Proposition 2. *An increase in the weight of sales in the manager's objective, b, monotonically increases (reduces) the flip bifurcation value $c^F{}_1$ (the threshold $\bar{c}$), and then acts as an economic de-stabiliser.*

Proof. Since $\partial c^F{}_1 / \partial b > 0$ ($\partial \bar{c} / \partial b > 0$) holds for every $0 < b < 1/2$, then Proposition 2 follows. **Q.E.D.**

Proposition 2 shows an unambiguous role played by the weight of sales in the manager's objective b on stability of the Cournot-Nash equilibrium E_3: when the owners of a firm in a Cournot duopoly hire a manager and delegate the output decision to him, the unique interior Nash equilibrium of the economy is more likely to be destabilised than when firms are profit-maximising.

We now compare the parametric stability regions of the Cournot-Nash equilibrium E_3 in the cases of profit-maximising firms (i.e., $b = 0$) and managerial sales delegation (i.e., $0 < b < 1/2$). Figure 1 depicts in a stylised way, for two different illustrative cases of the weight of sales in the manager's objective, i.e., $b = 0$ and $0 < b < 1/8$, the parametric regions of stability and instability in the (α, c) plane

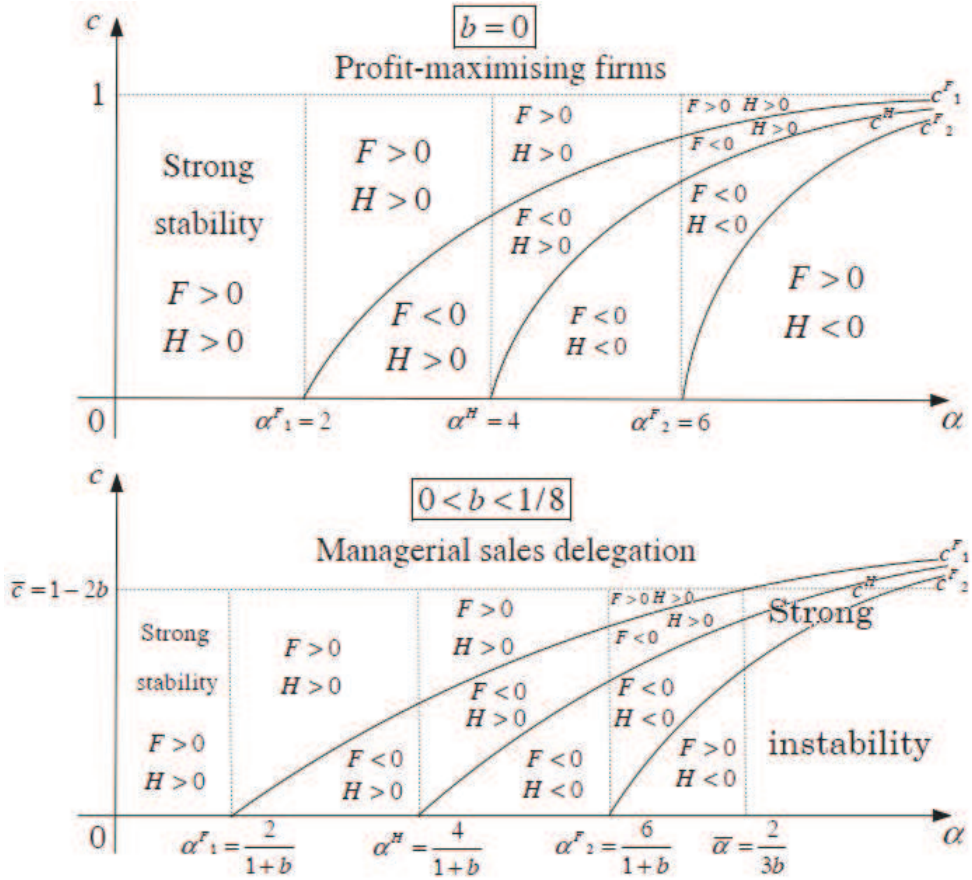

Figure 1. Stability-instability regions in the (α, c) plane.

By comparing the two cases depicted in Figure 1, the following remarks emerge.

Remark 1. *The strong stability region reduces when b increases. Indeed, while when $b=0$ such a region is included within $0<\alpha<2$ on the horizontal axis and $0<c<1$ on the vertical axis, when $0<b<1/8$ it reduces to $0<\alpha<2/(1+b)$ on the horizontal axis and $0<c<\bar{c}$ on the vertical axis.*

Remark 2. *Since the loss of stability of the Cournot-Nash equilibrium occurs only via a period-doubling bifurcation when $F=0$ in (16), and since $2/(1+b)<2$ for any $0<b<1/2$, then an increase in b monotonically acts as an economic de-stabiliser. This result can easily be ascertained by looking at the threshold $\bar{c}$, which reduces to zero as b is continuously raised.*

Remark 3. *While in the case of profit-maximising firms a fairly high value of c always exists, irrespective of the size of the speed of adjustment α,*[11] *to keep the Cournot-Nash equilibrium of system (6) locally asymptotically stable, in the case of managerial sales delegation $0<b<1/8$, the Cournot-Nash equilibrium of system (6) is always locally*

[11] This result can easily be ascertained since when $b=0$ we find that: $\bar{c}=1$, $c^F{}_1=\dfrac{\alpha-2}{\alpha}$, $c^H=\dfrac{\alpha-4}{\alpha}$, $\alpha^F{}_2=\dfrac{\alpha-6}{\alpha}$, $\alpha^F{}_1=2$, $\alpha^H=4$ and $\alpha^F{}_2=6$, $\lim_{\alpha\to+\infty} c^F{}_1=1$, $\lim_{\alpha\to+\infty} c^H=1$ and $\lim_{\alpha\to+\infty} c^F{}_2=1$. Therefore, we have the following proposition.

unstable when $\alpha > 2/3b$ (see the region of strong instability depicted in Figure 1), and the strong instability region monotonically increases as b raises.

Figures 2 shows a pictorial view of the content of Propositions 1-2 depicting the bifurcation diagrams for the equilibrium values of the variable q_1 (vertical axis) when the average and marginal cost c (horizontal axis) varies in the range $(0,1)$ (Figure 2.a) and $(0, \bar{c} = 0.8)$ (Figure 2.b), using the following parameter values (chosen only for illustrative purposes): $\alpha = 3.5$, $b = 0$ (Figure 2.a), $b = 0.1$ (Figure 2.b), $q_{1,0} = 0.05$ and $q_{2,0} = 0.03$ as initial conditions.

The case $\boxed{b=0}$. For any $1 > c > 0.428$ the Cournot-Nash equilibrium is locally asymptotically stable. When $c = 0.428$ a period-doubling bifurcation emerges, and for further reduction in c the equilibrium level of output q_1 shows 2-period cycle, 4-period cycle, highly periodicity and then a cascade of flip bifurcations which leads to unpredictable chaotic oscillations.

The case $\boxed{b=0.1}$. For any $0.8 > c > 0.528$ the Cournot-Nash equilibrium is locally asymptotically stable. When $c = 0.528$ a period-doubling bifurcation emerges, and for further reduction in c, the equilibrium level of output q_1 shows the same dynamic properties than when $b = 0$. Of course, when b raises the stability sharply reduces.

Therefore, it is clear that when firms hire a manager and delegate the output decisions to him, the Nash equilibrium is more likely to be destabilised than when firms maximise profits.

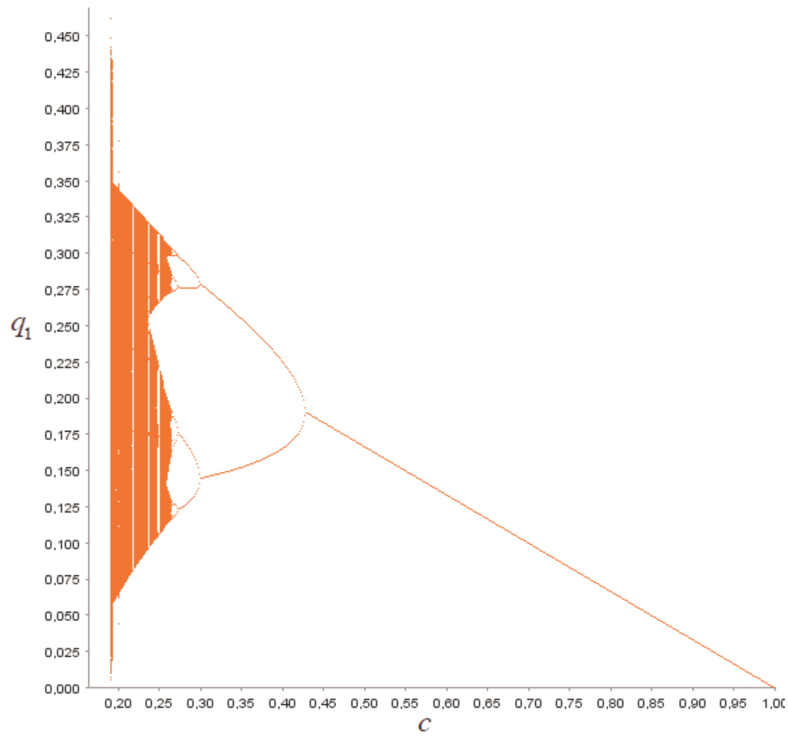

Figure 2.a. Bifurcation diagram for c ($b = 0$).

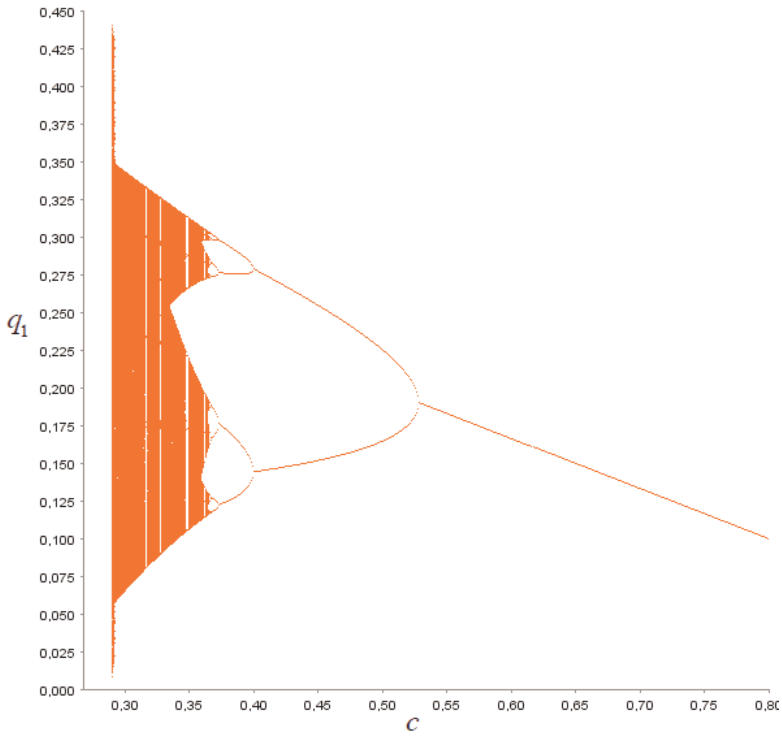

Figure 2.b. Bifurcation diagram for c ($b = 0.1$).

The phase portraits depicted in Figure 3 show, for the above parameter values and under the case $b = 0.1$, the different scenarios that occurs when the cost of production reduces. Figure 3.a shows that a 2-period cycle exists when $c = 0.41$. When c reduces a 4-period cycle emerges when $c = 0.39$ (Figure 3.b), which becomes a 8-period cycle when $c = 0.37$ (Figure 3.c). Then, there is a change in the structure of the attractor when $c = 0.365$, as can be seen by looking at Figure 3.d which shows several chaotic bacterium-like pieces. Finally, for $c = 0.33$ Figure 3.e shows the chaotic attractor.

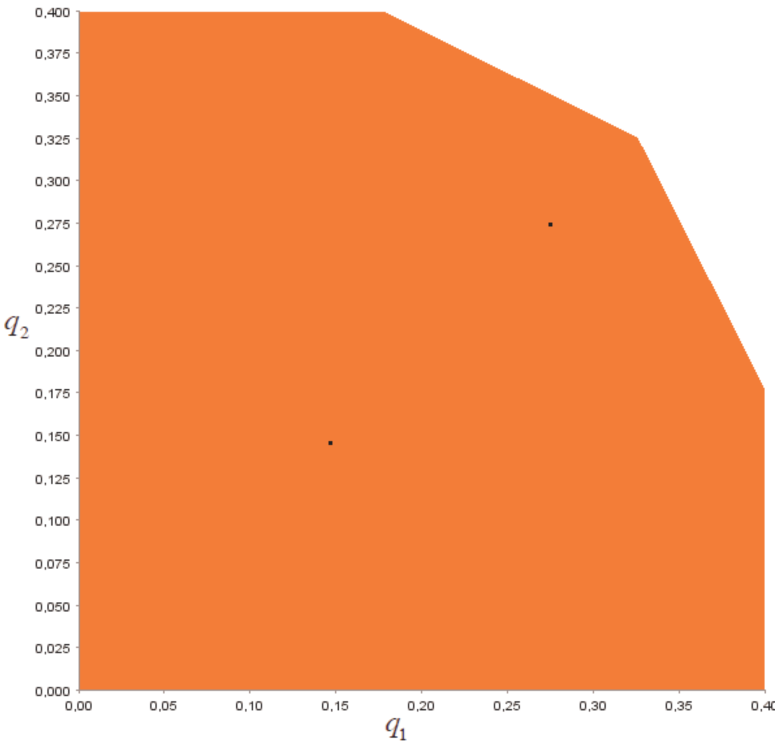

Figure 3.a. Phase portrait for $c = 0.41$.

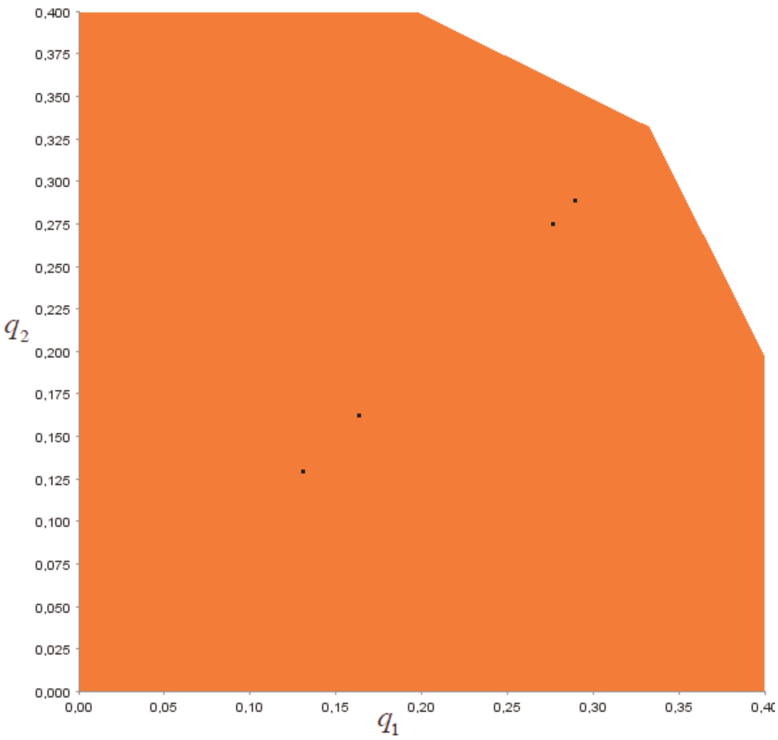

Figure 3.b. Phase portrait for $c = 0.39$.

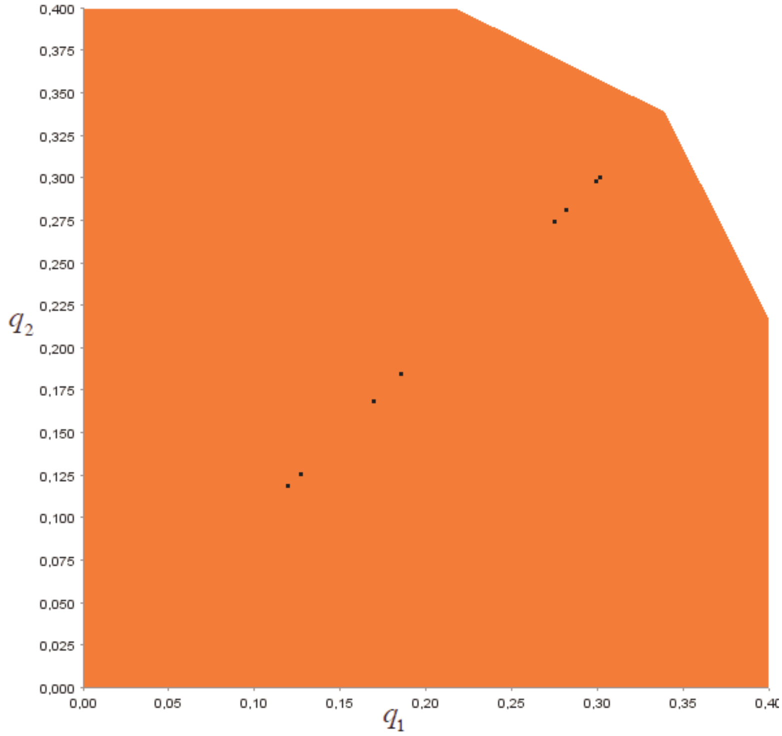

Figure 3.c. Phase portrait for $c = 0.37$.

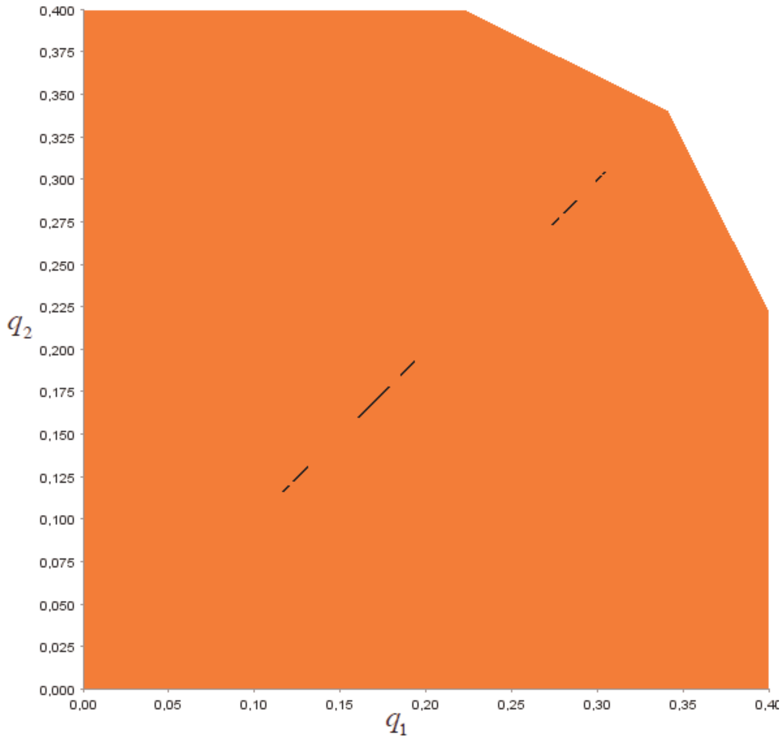

Figure 3.d. Phase portrait for $c = 0.365$.

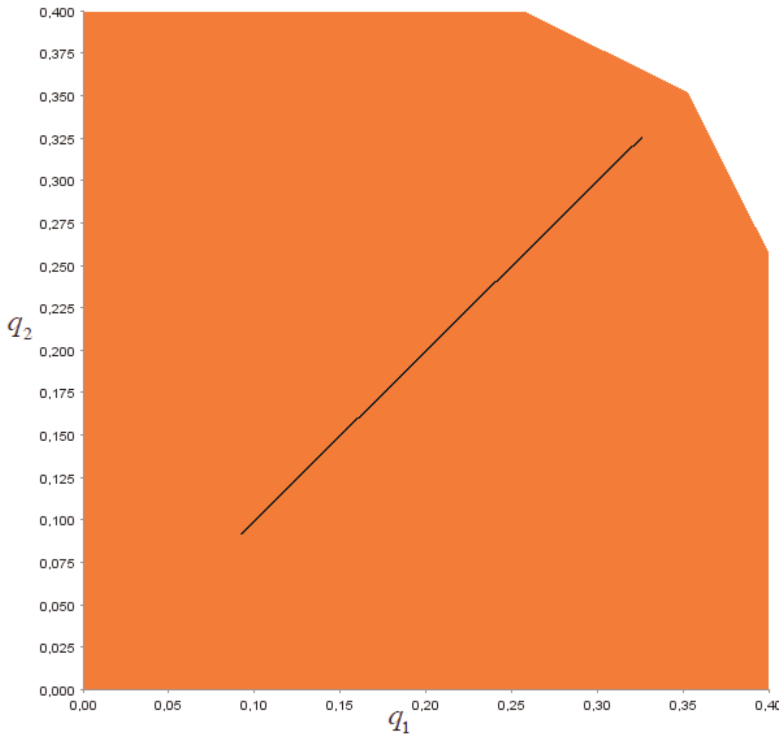

Figure 3.e. Phase portrait for $c = 0.33$.

Another numerical tool useful in order to determine the parameter sets for which the system (6) converges to cycles, quasi-periodic and chaotic attractors, is the study of the largest Lyapunov exponent, as a function of either the parameter of interest or time. As is known, there exists evidence for quasi periodic behaviour (chaos) when the largest Lyapunov exponent is zero (positive) or, alternatively, chaotic motions can be detected when the largest Lyapunov exponent is steadily positive when plotted against time.

Denoting $Le1$ as the largest Lyapunov exponent of our system and choosing the above parameter constellation (that is, $\alpha = 3.5$, $b = 0.1$, and $q_{1,0} = 0.05$ and $q_{2,0} = 0.03$ as initial conditions), we plot $Le1$ against time, t (see, e.g., Fanti and Manfredi, 2007). In order to better characterise the largest exponent from a quantitative point of view, and take account for the fact that there may be very long (periodic or aperiodic) transients, the dynamical system is left to evolve for $t = 10^5$ time units and then the Lyapunov exponents are calculated during a time of order $t = 10^5$. This allows to unambiguously detect the existence of chaotic motions as $Le1$ is steadily positive when $c = 0.365$ (Figure 4.a) and $c = 0.33$ (Figure 4.b). Moreover, the Lyapunov dimension evaluated according to the well-known Kaplan-Yorke conjecture (see Kaplan and Yorke, 1979), corresponding to the value of average and marginal costs used before, i.e. $c = 0.365$ and $c = 0.33$, is $DL = 1.095$ and $DL = 1.45$, respectively.[12] This confirms that the attractor are fractal objects.

[12] The Lyapunov dimension is computed as $DL \leq s + \frac{\sum_{k=1}^{s} \lambda_k}{|\lambda_{s+1}|}$, where λ_k is the kth Lyapunov exponent, s is the largest number for which $\sum_{k=1}^{s} \lambda_k > 0$ and $\lambda_1 + \lambda_2 + \ldots + \lambda_{s+1} < 0$ (see Medio, 1992).

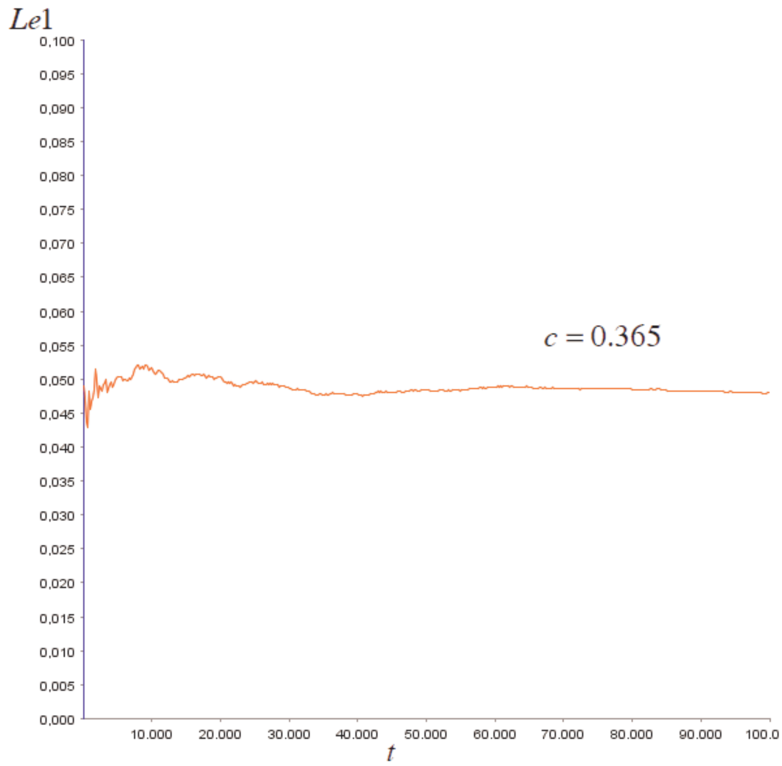

Figure 4.a. Time path of the largest Lyapunov exponent $Le1$ when $c = 0.365$).

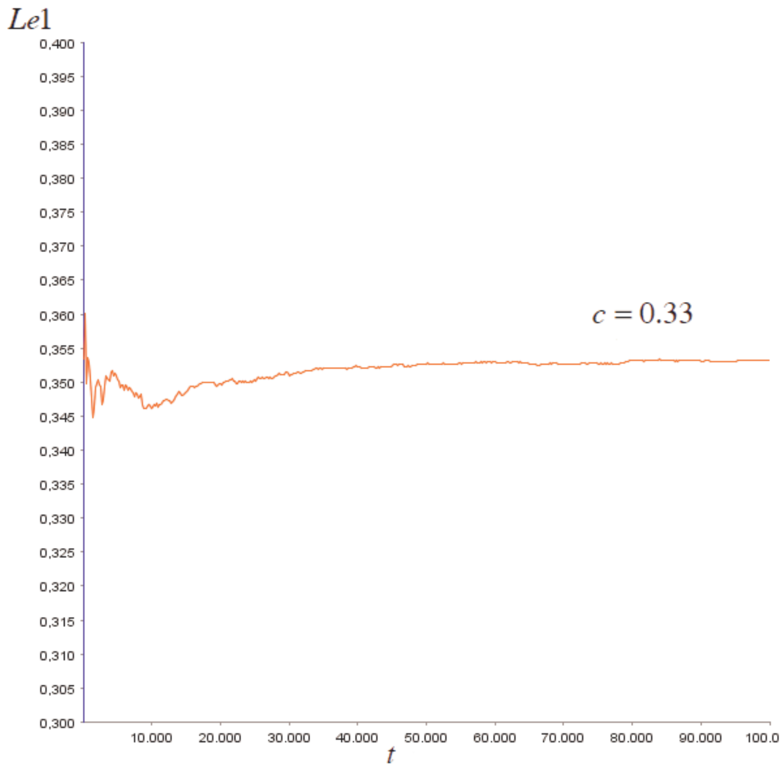

Figure 4.b. Time path of the largest Lyapunov exponent $Le1$ when $c = 0.33$.

Since the focus of the present paper is the study of the stability properties of a duopoly game with quantity competition when firms maximise profits ($b = 0$) and when the firms' owners, instead, decide to hire a manager and delegate the output decisions to him ($0 < b < 1/2$), in addition to the preceding numerical simulations which used c as the parameter of interest (for a given value of b), we now present numerical simulations on the role on stability of the parameter b for a given value of c.[13] In particular, Figure 5 represents, for $\alpha = 3.5$ and $c = 0.6$, a pictorial view of the bifurcation diagram when b increases in the range $0 < b < 1/2$. The figure clearly shows that the equilibrium output is stable for any $0 < b < 0.1714$. Then a flip bifurcation emerges when $b = 0.1714$. Further increases in b give rise to two-period cycles, four-period cycles, highly periodicity and a cascade of flip bifurcations that ultimately lead to chaotic behaviours, interspersed by windows of period motions. The existence of chaotic motions can be ascertained by looking at Figure 6, which plots the largest Lyapunov exponent against b. The figure displays the intervals of the positive values of the parameter b (namely, values of b larger than 0.33).

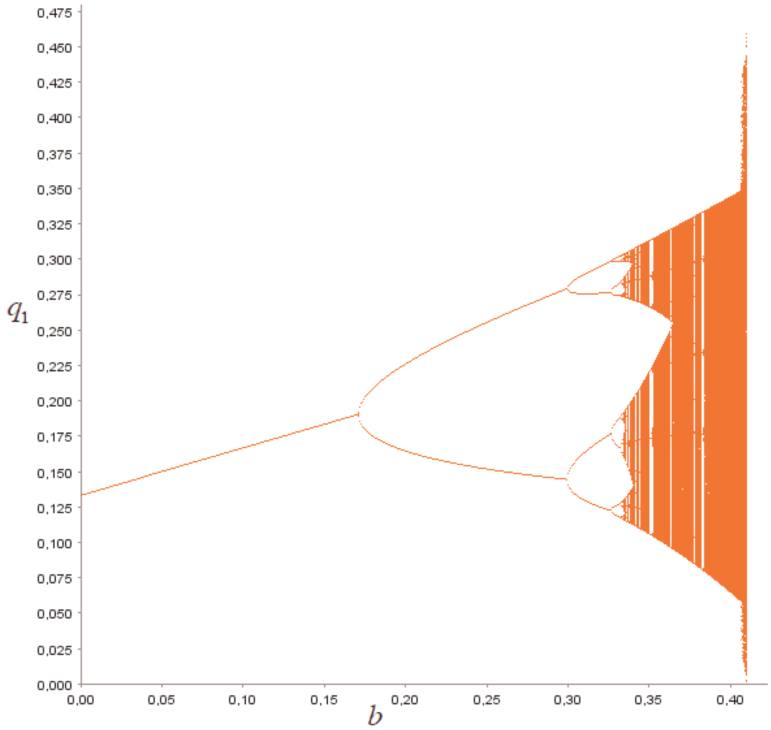

Figure 5. Bifurcation diagram for b. (Parameter values: $\alpha = 3.5$ and $c = 0.6$).

[13] Note that the condition on the existence of positive values of both the market price and firms' profits expressed in terms of b is the following: $b < (1-c)/2 := \bar{b}$, where $0 < \bar{b} < 1/2$ for any $0 < c < 1$.

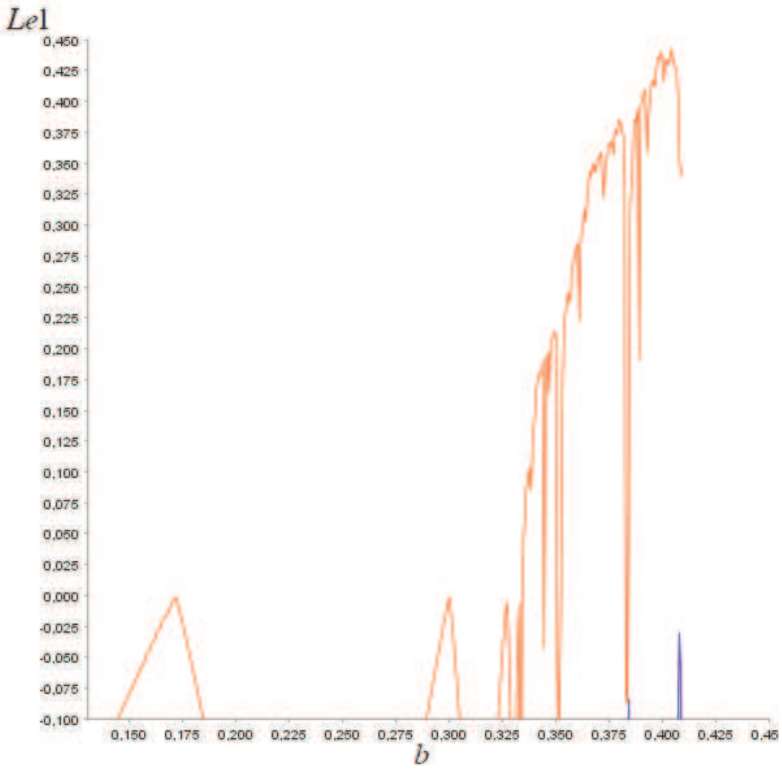

Figure 6. Largest Lyapunov exponent $Le1$ when b varies (one million iterations).

4. Conclusions

This paper has been originated from a twofold motivation: (*i*) the use, in several actual economies, of manager compensation practices, which has raised an established literature that extended the standard oligopoly model with managerial incentive contracts, and (*ii*) the increasing interest for a refined dynamic analysis in the nonlinear oligopoly literature (see. e.g., Bischi et al., 2010).

The novelty of the present paper is the analysis of the effects of the managerial economics captured by the weight of sales in the contracts that the owners of firms choose to compensate their managers, on stability of the Nash equilibrium in a Cournot duopoly with bounded rationality. We show that the higher the weight on sales in the manager's bonus, the smaller is the parametric stability region. Then under managerial sales delegation the Cournot duopoly with bounded rational players is unambiguously more prone to instability than the standard model with profit-maximising firms. In particular, the loss of stability can only occur through a flip bifurcation, which triggers cycles of high periodicity and a cascade of flip bifurcations which ultimately leads to deterministic chaos.

Therefore the contribution of this paper to the oligopoly literature lies in the following remark: while VFJS has clarified the ranking between the equilibrium outcomes, we have shown the ranking as regards the stability issue. In conclusion we argue that an economy with managerial sales delegation not only presents higher values of equilibrium outputs and lower profits, but also makes more prone the equilibrium to be destabilised through a flip bifurcation.

Finally, in our future research agenda we acknowledge to investigate the robustness of this result either in different competitive settings (e.g., for instance, the Bertrand's price competition) or when asymmetries between players exist (i.e., players are heterogeneous as regards the existence of managerial incentives or the type of expectations formation). Moreover, we note that the VFJS formulation of the manager's bonus used in this paper has subsequently been modified by a contract based on (*i*) a weighted sum of profits of the own firm and profits of the rival firm, i.e., the case of "relative profit delegation" or "relative performance delegation" (see Salas Fumas, 1992; Miller and Pazgal, 2002), and (*ii*) the market share, besides profit, as a natural part of managers' incentives (see Jansen et al. 2007, 2009). This topics will be discussed in future works. Moreover, other extensions can be the study of dynamic nonlinear duopoly models linking managerial incentive contract with union bargaining and entry deterrence in models with limit pricing, as studied by Pal and Saha (2008) in a static context à la Milgrom and Roberts (1982).

Appendix

Proof of Proposition 1

Let $0<b<1/8$ hold. Since (26) holds, then: (*i*) for any $0<\alpha<\alpha^F{}_1$, $F>0$ and $H>0$ for any $\bar{c}>c>0$, (*ii*) for any $\alpha^F{}_1<\alpha<\alpha^H$, $F>0$ and $H>0$ for any $\bar{c}>c>c^F{}_1$, $F=0$ and $H>0$ if, and only if, $c=c^F{}_1$, $F<0$ and $H>0$ for any $c^F{}_1>c>0$, (*iii*) for any $\alpha^H<\alpha<\alpha^F{}_2$, $F>0$ and $H>0$ for any $\bar{c}>c>c^F{}_1$, $F=0$ and $H>0$ if, and only if, $c=c^F{}_1$, $F<0$ and $H>0$ for any $c^F{}_1>c>c^H$, $F<0$ and $H=0$ if, and only if, $c=c^H$, $F<0$ and $H<0$ for any $c^H>c>0$, (*iv*) for any $\alpha^F{}_2<\alpha<\bar{\alpha}$, $F>0$ and $H>0$ for any $\bar{c}>c>c^F{}_1$, $F=0$ and $H>0$ if, and only if, $c=c^F{}_1$, $F<0$ and $H>0$ for any $c^F{}_1>c>c^H$, $F<0$ and $H=0$ if, and only if, $c=c^H$, $F<0$ and $H<0$ for any $c^H>c>c^F{}_2$, $F=0$ and $H<0$ if, and only if, $c=c^F{}_2$, $F>0$ and $H<0$ for any $c^F{}_2>c>0$, (*v*) for any $\bar{\alpha}<\alpha<\bar{\bar{\alpha}}$, $F<0$ and $H>0$ for any $\bar{c}>c>c^H$, $F<0$ and $H=0$ if, and only if, $c=c^H$, $F<0$ and $H<0$ for any $c^H>c>c^F{}_2$, $F=0$ and $H<0$ if, and only if, $c=c^F{}_2$, $F>0$ and $H<0$ for any $c^F{}_2>c>0$, (*vi*) for any $\bar{\bar{\alpha}}<\alpha<\bar{\bar{\bar{\alpha}}}$, $F<0$ and $H<0$ for any $\bar{c}>c>c^F{}_2$, $F=0$ and $H<0$ if, and only if, $c=c^F{}_2$, $F>0$ and $H<0$ for any $c^F{}_2>c>0$, (*vii*) for any $\alpha>\bar{\bar{\bar{\alpha}}}$, $F>0$ and $H<0$ for any $\bar{c}>c>0$.

Let $1/8<b<1/5$ hold. Since (27) holds, then: (*i*) for any $0<\alpha<\alpha^F{}_1$, $F>0$ and $H>0$ for any $\bar{c}>c>0$, (*ii*) for any $\alpha^F{}_1<\alpha<\alpha^H$, $F>0$ and $H>0$ for any $\bar{c}>c>c^F{}_1$, $F=0$ and $H>0$ if, and only if, $c=c^F{}_1$, $F<0$ and $H>0$ for any $c^F{}_1>c>0$, (*iii*) for any $\alpha^H<\alpha<\bar{\alpha}$, $F>0$ and $H>0$ for any $\bar{c}>c>c^F{}_1$, $F=0$ and $H>0$ if, and only if, $c=c^F{}_1$, $F<0$ and $H>0$ for any $c^F{}_1>c>c^H$, $F<0$ and $H=0$ if, and only if, $c=c^H$, $F<0$ and $H<0$ for any $c^H>c>0$, (*iv*) for any $\bar{\alpha}<\alpha<\alpha^F{}_2$, $F<0$ and $H>0$ for any $\bar{c}>c>c^H$, $F<0$ and $H=0$ if, and only if, $c=c^H$, $F<0$ and $H<0$ for any $c^H>c>0$, (*v*) for any $\alpha^F{}_2<\alpha<\bar{\bar{\alpha}}$, $F<0$ and $H>0$ for any $\bar{c}>c>c^H$, $F<0$ and $H=0$ if, and only if, $c=c^H$, $F<0$ and $H<0$ for any $c^H>c>c^F{}_2$, $F=0$ and $H<0$ if, and only if, $c=c^F{}_2$, $F>0$ and $H<0$ for any $c^F{}_2>c>0$, (*vi*) for any $\bar{\bar{\alpha}}<\alpha<\bar{\bar{\bar{\alpha}}}$, $F<0$ and $H<0$ for any $\bar{c}>c>c^F{}_2$, $F=0$ and $H<0$ if, and only if, $c=c^F{}_2$, $F>0$ and $H<0$ for any $c^F{}_2>c>0$, (*vii*) for any $\alpha>\bar{\bar{\bar{\alpha}}}$, $F>0$ and $H<0$ for any $\bar{c}>c>0$.

Let $1/5<b<2/7$ hold. Since (28) holds, then: (*i*) for any $0<\alpha<\alpha^F{}_1$, $F>0$ and $H>0$ for any $\bar{c}>c>0$, (*ii*) for any $\alpha^F{}_1<\alpha<\bar{\alpha}$, $F>0$ and $H>0$ for any $\bar{c}>c>c^F{}_1$, $F=0$ and $H>0$ if, and only if, $c=c^F{}_1$, $F<0$ and $H>0$ for any $c^F{}_1>c>0$, (*iii*) for any

$\overline{\alpha} < \alpha < \alpha^H$, $F < 0$ and $H > 0$ for any $\overline{c} > c > 0$, (*iv*) for any $\alpha^H < \alpha < \alpha^F{}_2$, $F < 0$ and $H > 0$ for any $\overline{c} > c > c^H$, $F < 0$ and $H = 0$ if, and only if, $c = c^H$, $F < 0$ and $H < 0$ for any $c^H > c > 0$, (*v*) for any $\alpha^F{}_2 < \alpha < \overline{\overline{\alpha}}$, $F < 0$ and $H > 0$ for any $\overline{c} > c > c^H$, $F < 0$ and $H = 0$ if, and only if, $c = c^H$, $F < 0$ and $H < 0$ for any $c^H > c > c^F{}_2$, $F = 0$ and $H < 0$ if, and only if, $c = c^F{}_2$, $F > 0$ and $H < 0$ for any $c^F{}_2 > c > 0$, (*vi*) for any $\overline{\overline{\alpha}} < \alpha < \overline{\overline{\overline{\alpha}}}$, $F < 0$ and $H < 0$ for any $\overline{c} > c > c^F{}_2$, $F = 0$ and $H < 0$ if, and only if, $c = c^F{}_2$, $F > 0$ and $H < 0$ for any $c^F{}_2 > c > 0$, (*vii*) for any $\alpha > \overline{\overline{\overline{\alpha}}}$, $F > 0$ and $H < 0$ for any $\overline{c} > c > 0$.

Let $2/7 < b < 1/2$ hold. Since (29) holds, then: (*i*) for any $0 < \alpha < \alpha^F{}_1$, $F > 0$ and $H > 0$ for any $\overline{c} > c > 0$, (*ii*) for any $\alpha^F{}_1 < \alpha < \overline{\alpha}$, $F > 0$ and $H > 0$ for any $\overline{c} > c > c^F{}_1$, $F = 0$ and $H > 0$ if, and only if, $c = c^F{}_1$, $F < 0$ and $H > 0$ for any $c^F{}_1 > c > 0$, (*iii*) for any $\overline{\alpha} < \alpha < \alpha^H$, $F < 0$ and $H > 0$ for any $\overline{c} > c > 0$, (*iv*) for any $\alpha^H < \alpha < \overline{\overline{\alpha}}$, $F < 0$ and $H > 0$ for any $\overline{c} > c > c^H$, $F < 0$ and $H = 0$ if, and only if, $c = c^H$, $F < 0$ and $H < 0$ for any $c^H > c > 0$, (*v*) for any $\overline{\overline{\alpha}} < \alpha < \alpha^F{}_2$, $F < 0$ and $H < 0$ for any $\overline{c} > c > 0$, (*vi*) for any $\alpha^F{}_2 < \alpha < \overline{\overline{\overline{\alpha}}}$, $F < 0$ and $H < 0$ for any $\overline{c} > c > c^F{}_2$, $F = 0$ and $H < 0$ if, and only if, $c = c^F{}_2$, $F > 0$ and $H < 0$ for any $c^F{}_2 > c > 0$, (*vii*) for any $\alpha > \overline{\overline{\overline{\alpha}}}$, $F > 0$ and $H < 0$ for any $\overline{c} > c > 0$. Q.E.D.

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