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Optimal Monetary Policy with Durable Services: User Cost versus Purchase Price *

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Abstract

This paper investigates the inflation rate that should be set as the target for the central bank. To this end, we develop a two-sector economy model in the existence of long-lived durables. In contrast to recent studies that have been conducted on how monetary policy can affect the role of durable goods, which examine only the production sector, we introduce a service market. Accordingly, we can endogenously derive the traditional user cost equation and the price-rent ratio. Our main findings are as follows: First, even in cases where both service and production sectors are equally sticky, the user cost is more important than the purchase price, from the perspective of welfare loss. Second, in contrast to the situation in the economy that includes only nondurables, a temporary shock persistently influences output fluctuations. However, this does not mean that welfare loss increases as the degree of durability increases. Third, welfare is found to be a strictly increasing function of durability.

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1 Introduction

What is the role of the central bank? New Keynesian literature focuses on characterizing the optimal monetary policy in environments where there are nominal rigidities and imperfect competition. A key finding of these studies is that a zero inflation rate in all periods should be a characteristic of any optimal monetary policy. The reason price stability has been identified as a key component of optimal monetary policy is straightforward: by the central bank keeping the price level constant, inflation costs are kept to a minimum under nominal rigidities.

However, in reality, there are numerous prices and inflations; hence, the central bank should carefully choose what inflation to target. Accordingly, this issue has been examined by many researchers. For example, Aoki (2001) considers a situation where there are different rigidities among sectors. He constructs a two-sector model in which prices are flexible in one sector and sticky in the other, and concludes that stabilizing sticky-price inflation is sufficient for ensuring optimum monetary policy. Erceg, Henderson, and Levin (2000) introduce distortions in the labor market as well as in the goods market. They state that wage inflation is a very important target of the central bank. Huang and Liu (2006) investigate the case where both CPI and PPI goods sectors are sticky. They find that to decide on an optimal inflation rate, the central bank should put weight on both CPI and PPI sectors. Carlstrom, Fuerst, and Ghironi (2006) find that the Taylor principle, when applied at the sectoral level, is not needed at the aggregate level. Some studies have examined the role of monetary policy in the existence of durable goods. For instance, Erceg and Levin (2006) introduce durable goods into the model of Erceg, Henderson, and Levin (2000) and obtain a similar result. Monacelli (2007, 2009) insists that durable goods act as collateral when there are heterogeneous households. However, these previous studies do not explicitly introduce the service market into their models. Accordingly, we infer that in the previous literature, the service market is implicitly assumed to be a frictionless shadow market.

In this paper, we build a general equilibrium model that has two different elements.

First, we explicitly introduce a service market so that we can differentiate the price of goods in the production sector and the rental price for the service flow in the service sector that the household actually consumes.[1][2] From this, we can derive a traditional user cost equation that determines the purchase of additional durable goods and the rent for the corresponding services. One of our goals is to verify the relationship between the rental price and the purchase price in this situation. In contrast to previous studies, which develop one type of pricing relationship between a representative household and production firms, our model features service firms, referred to as investors, who manage the durable stocks. At the beginning of each period, the investors purchase new output from the production sector, transform this output flow with the existing stocks into durable service flows, and rent them to the household. At the end of each period, the used durables are returned to the investors and become the initial stocks for the beginning of the next period.

Furthermore, we can derive a price-rent ratio by introducing a service market. The price-rent ratio of highly durable goods can offer another explanation of why durable goods generate larger responses to exogenous shocks. Most previous studies, such as Barsky, House, and Kimball (2007), have stressed that the decision to purchase durable goods depends on the household's perception of the relationship between the marginal utility of the good's nondurable consumption and the marginal gains that are derived from the durable services it offers. The high elasticity of substitution for the purchase of durable goods based on this relationship is a key mechanism that can explain why the response of durable goods to the macro shocks of the market is large. In our model, the investor in the service sector manages the total durable stocks and functions as the decision maker in terms of durable purchases. High durability implies that revenues from

[1] With a slight abuse of notation, I interchangeably use the terms "rental price," "service price," and "user cost" in the paper.

[2] Our endogenously derived user cost equation separates prices into user cost and purchase price. Previous studies do not include the service market, in their models, which means that the prices of durable goods and the corresponding service tend to be used in a confusing way. The purchase price is the price of the physical good itself while the rental price is the actual price of the corresponding service that follows consumption. In our model, which includes the service market, the price index in the goods market and utility-based price index are considered separately.

the ownership of durable goods are consistently obtained. This is because future demand for the services by households is related to the current purchase of durable goods by investors. Therefore, the investors' demand for durable goods is generally an increasing function of durability. Thus, the investor's decision can account for the larger response of durable goods to exogenous shocks, in comparison to nondurable goods.

The second way in which our approach departs from that adopted by previous studies on this topic is that, rather than basing our model on a standard complete-market framework with flexible prices, we introduce nominal rigidity in each sector and assess the extent to which the service market influence household welfare. We can consider two nominal rigidities because we have two durable prices. In particular, the nominal rigidity in the service sector restricts the efficient relationship between two prices, and hence, the user cost equation and the price-rent ratio are distorted.

The main findings of this paper are as follows: First, in the economy with long-lived durables, where nominal rigidity exists in both service and good markets, the movements of user cost inflation and service flows are more important than the movements of purchase price inflation and output flows, from the perspective of social welfare. We also find that the utility-based social loss function implies that service-inflation variability and the fluctuation of the service flows should be given more weight. Second, we find that welfare loss is not an increasing function of durability. In the economy with nondurables, a temporary shock influences only the current output. However, in the economy with long-lived durables, even a temporary shock persistently affects the sequence of output until the currently affected output depreciates entirely. However, our endogenously derived loss function with only output gaps reveals that welfare loss does not necessarily increase as durability increases. Third, household welfare is found to be an increasing function of durability. The first-ordered terms of the derived lifetime welfare are strictly increasing functions of durability. In a highly durable economy, the loss from the volatilities of macro variables may increase, but services to be consumed are abundant in comparison to the nondurable economy.

The remainder of the paper is organized as follows. Section 2 presents the theoretical

model. Section 3 defines the equilibrium. Section 4 presents a descriptive analysis of the flexible case. Section 5 discusses the distortion economy. Sections 6 and 7 present the optimal monetary policy and the optimal simple rules. Sections 8 and 9 compare the highly durable economy with the nondurable economy and analyze the relationship between durability and welfare. Section 10 concludes.

2 The model

In this section, we develop a model for an infinite-horizon economy with durable goods and services. The key feature of our model is that durable goods can be traded in two ways–as an ownership of a good or as a lease contract of the corresponding service flow in each period. To investigate the role of durable services and prices, we segment the durable market into product and service markets.

In the service sector, there are final and intermediate service firms called investors. The final investors are competitive and produce a single homogenous good by using a CES technology to combine the differentiated intermediate goods. The investors offer this durable service to the household. The intermediate investors are monopolistically competitive suppliers in the service market and price-takers in the input market. They purchase output flows from the production sector and lease services from their durable stock. The returned durable stocks then become their wealth. Furthermore, we assume that the intermediate-good firms set their prices on a staggered basis because price changes incur adjustment costs.

The other assumptions are standard as in the New Keynesian literature. Within any production sector, there are both final and intermediate-good firms. The final-good firms share the same features as the final investors in the service sector. The intermediate firms are monopolistically competitive producers who demand labor from the households. For simplicity, we assume linear technology in labor input. The other features of the model are the same as those in the service sector.

The economy is composed of a continuum of homogeneous households in the interval

$(0,1)$ who supply labor to the intermediate-good firms in the production sector. The households purchase durable services from the final investor by using their income and debts.

2.1 Service sector

In the service market, a perfectly competitive final investor purchases $D_t(i)$ units from the intermediate investor i. The final investor operates the production function

$$D_t \equiv \left(\int_0^1 D_t(i)^{\frac{\varepsilon_{r_D}-1}{\varepsilon_{r_D}}} di \right)^{\frac{\varepsilon_{r_D}}{\varepsilon_{r_D}-1}}, \tag{1}$$

where $D_t(i)$ is the quantity of the intermediate service i demanded by the final investor, and ε_{r_D} is the elasticity of substitution among the differentiated varieties. The maximization of profits yields the demand function for the intermediate service i for all $t \geq 0$:

$$D_t(i) = \left(\frac{r_{D,t}(i)}{r_{D,t}} \right)^{-\varepsilon_{r_D}} D_t, \tag{2}$$

where $r_{D,t} \equiv \left(\int_0^1 r_{D,t}(i)^{1-\varepsilon_{r_D}} di \right)^{\frac{1}{1-\varepsilon_{r_D}}}$ is the service price index.

There is a continuum of firms producing differentiated services indexed in the interval $[0,1]$. Each firm i is a monopolistic competitor in the service market and a price-taker in the input market. Following Rotemberg (1982), the firm is assumed to face a quadratic cost proportional to the total durable services in changing its price equal to $\frac{\theta_{r_D}}{2}(\frac{r_{D,t}(i)}{r_{D,t-1}(i)} - 1)^2 D_t$, measured by the finished service. θ_{r_D} governs the magnitude of the price adjustment cost, measuring the degree of sectoral nominal price rigidity.

Given the initial value, $D_{-1}(i)$, the intermediate service firm $i \in [0,1]$ chooses the

sequence $\{r_{D,t}(i), I_{D,t}(i)\}_{t=0}^{\infty}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \Lambda_t \left[r_{D,t}(i) D_t(i) - P_{D,t}(i) I_{D,t}(i) - \frac{\theta_{r_D}}{2} \left(\frac{r_{D,t}(i)}{r_{D,t-1}(i)} - 1 \right)^2 r_{D,t} D_t \right] \tag{3}$$

s.t. (2) and

$$D_t(i) = (1-\delta) D_{t-1}(i) + I_{D,t}(i), \tag{4}$$

where E is an expectation operator, and $\Lambda_{t,t+k} \equiv \frac{\Lambda_{t+k}}{\Lambda_t} = \beta^k \frac{U_{S_D,t+k}}{U_{S_D,t}} \frac{r_{D,t}}{r_{D,t+k}}$ is a stochastic discount factor, where $U_{S_D,t}$ measures the marginal utility value to the household of an additional unit of real profits during period t. δ is the depreciation rate of durable goods, and $I_{D,t}(i)$ is the newly purchased output flows from the production sector in period t. The "used" or second-hand $(1-\delta)D_{t-1}(i)$ stocks that are returned to the intermediate investor at the end of the previous period can be sold at $P_{D,t}(i)$ in the current period. Thus, intermediate investors make new demands for as much as $D_t(i) - (1-\delta)D_{t-1}(i)$ durable goods from the final-good firm in the production sector.

In the symmetric equilibrium, where $r_{D,t}(i) = r_{D,t}$ for all i, the first order condition is

$$\begin{aligned}(\pi_{r_D,t} - 1)\pi_{r_D,t} =& E_t \left[\Lambda_{t,t+1} \frac{r_{D,t+1}}{r_{D,t}} \frac{D_{t+1}}{D_t} (\pi_{r_D,t+1} - 1)\pi_{r_D,t+1} \right] \\ &+ \frac{\varepsilon_{r_D}}{\theta_{r_D}} \left\{ \Xi_{r_D,t} - \frac{\varepsilon_{r_D} - 1}{\varepsilon_{r_D}} - (1-\delta) E_t \left[\Lambda_{t,t+1} \frac{r_{D,t+1}}{r_{D,t}} \Xi_{r_D,t+1} \right] \right\},\end{aligned} \tag{5}$$

where $\pi_{r_D,t} \equiv \frac{r_{D,t}}{r_{D,t-1}}$ is the gross service inflation rate, and $\Xi_{r_D,t}$ is the real marginal cost in period t in the service sector. One distinctive feature of this sector is that the current inflation is a function of the expectation of the real marginal cost in the next period as well as the inflation in the next period and the current real marginal cost. This feature insulates the most important mechanism in our model.

2.2 Production sector

In the production sector, a perfectly competitive final-good producer purchases $Y_t(i)$ units of intermediate good j. The final-good producer operates the production function

$$Y_t \equiv \left(\int_0^1 Y_t(j)^{\frac{\varepsilon_{P_D}-1}{\varepsilon_{P_D}}} di \right)^{\frac{\varepsilon_{P_D}}{\varepsilon_{P_D}-1}}, \tag{6}$$

where $Y_t(j)$ is the quantity of the intermediate good j that is demanded by the final-good producer, and ε_{P_D} is the elasticity of substitution among the differentiated varieties. The maximization of profits yields the demand function for the intermediate good j for all t:

$$Y_t(j) = \left(\frac{P_{D,t}(j)}{P_{D,t}} \right)^{-\varepsilon_{P_D}} Y_t, \tag{7}$$

where the price index is $P_{D,t} \equiv \left(\int_0^1 P_{D,t}(j)^{1-\varepsilon_{P_D}} dj \right)^{\frac{1}{1-\varepsilon_{P_D}}}$.

A continuum of firms produces differentiated products indexed in the interval $[0, 1]$. A typical firm j hires $N_t(j)$ units of labor from the households in order to produce $Y_t(j)$ units of intermediate good j, using a linear production technology:

$$Y_t(j) = A_t N_t(j), \quad j \in [0, 1], \tag{8}$$

where A_t is a productivity shock. a_t, which is a logarithm of the t-period productivity shock in the production sector, follows

$$a_{t+1} = \rho_a a_t + u^a_{t+1}, \quad \rho_a \in [0, 1), \tag{9}$$

where $E_t u^a_{t+1} = 0$ and $E_t u^a_t u^{a\prime}_t = \sigma^{a2}$.

Each firm j is a monopolistic competitor in the product markets. Following Rotemberg (1982), we assume that the firm faces a quadratic cost proportional to output in changing its price equal to $\frac{\theta_{P_D}}{2}(\frac{P_{D,t}(j)}{P_{D,t-1}(j)} - 1)^2 Y_t$, measured by the finished good. θ_{P_D} governs the magnitude of the price adjustment cost, measuring the degree of sectoral nominal price

rigidity.

Subject to (7) and (8), the intermediate firm $j \in [0,1]$ in the good sector solves

$$\max_{P_{D,t}(j),N_t(j)} E_k \sum_{t=k}^{\infty} \Lambda_{t,k} \left[P_{D,t}(j)Y_t(j) - W_t N_t(j) - \frac{\theta_{P_D}}{2} \left(\frac{P_{D,t}(j)}{P_{D,t-1}(j)} - 1 \right)^2 P_{D,t} Y_t \right], \tag{10}$$

where W_t denotes a nominal wage rate.

In the symmetric equilibrium where $P_{D,t}(j) = P_{D,t}$ for all j, the first order condition becomes

$$\begin{aligned}(\pi_{P_D,t} - 1)\pi_{P_D,t} =& E_t \left[\Lambda_{t,t+1} \frac{P_{D,t+1}}{P_{D,t}} \frac{Y_{t+1}}{Y_t} (\pi_{P_D,t+1} - 1)\pi_{P_D,t+1} \right] \\ &+ \frac{\varepsilon_{P_D}}{\theta_{P_D}} \left[\Xi_{P_D,t} - \frac{\varepsilon_{P_D} - 1}{\varepsilon_{P_D}} \right],\end{aligned} \tag{11}$$

where $\pi_{P_D,t} \equiv \frac{P_{D,t}}{P_{D,t-1}}$ is the gross producer inflation rate, and $\Xi_{P_D,t}$ is the real marginal cost in the production sector.

2.3 Household

The utility function of the representative household is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[U(S_{D,t}) - V(N_t) \right], \tag{12}$$

where $\beta \in (0,1)$ is a subjective discount factor. $S_{D,t}$ denotes the total durable service flows to be consumed in period t, and N_t denotes the amount of labor supplied in the production sector. The period utility is assumed to be continuous and twice differentiable, with $U_{S_D,t} \equiv \frac{\partial U(S_{D,t})}{\partial S_{D,t}} > 0$, $U_{S_D S_D,t} \equiv \frac{\partial^2 U(S_{D,t})}{\partial S_{D,t}^2} \leq 0$, $V_{N,t} \equiv \frac{\partial V(N_t)}{\partial N_t} \leq 0$, and $V_{NN,t} \equiv \frac{\partial^2 V(N_t)}{\partial N_t^2} \leq 0$.

The purchase of a durable service is financed by the labor income, the ownership of the intermediate firms in the service and production sectors, government transfers, and assets. The nominal budget constraint for all t is given by

$$r_{D,t} S_{D,t} + R_{t-1} B_{t-1} = B_t + W_t N_t + \Gamma_t + T_t, \tag{13}$$

where $r_{D,t}$ is a service price, B_t denotes a nominal bond, R_t denotes a nominal return of the nominal bond, Γ_t denotes dividends from the ownership of firms in all sectors, and T_t denotes the lump-sum transfer from the government.

Dividing this by the service price, $r_{D,t}$, we obtain the real budget constraint:

$$S_{D,t} + R_{t-1}\frac{b_{t-1}}{\pi_{r_{D,t}}} = b_t + \frac{W_t}{r_{D,t}}N_t + \frac{\Gamma_t + T_t}{r_{D,t}}. \tag{14}$$

We also assume that no Ponzi scheme holds:

$$\lim_{T\to\infty} E_t B_T \leq 0 \tag{15}$$

for all t.

2.3.1 Optimal allocation and implication

Given the initial value, b_{-1}, the household chooses the labor, consumption, and asset profile $\{N_t, S_{D,t}, b_t,\}_{t=0}^{\infty}$ to maximize (12) subject to (14). The first-order-necessary conditions thus become

$$-\frac{V_{N,t}}{U_{S_D,t}} = \frac{W_t}{r_{D,t}}, \tag{16}$$

$$1 = \beta E_t\left[\frac{U_{S_D,t+1}}{U_{S_D,t}}\frac{R_t}{\pi_{r_D,t+1}}\right]. \tag{17}$$

Equation (16) is the intra-temporal decision condition between the labor supply and the consumption of durable service flows in period t. Equation (17) is a standard Euler condition with respect to the inter-temporal consumption decision of durable service flows for all t.

2.4 Transmission mechanism of the monetary policy

In this economy, the stance of the monetary authority has two direct effects. The first is a traditional effect exerted on the inter-temporal decision by the household. As the Euler

equation indicates, a high interest rate encourages households to save their wealth and postpone consumption. The new second channel is the service market. Iacoviello and Neri (2011) state that the nominal rigidity of wage is very important because housing investment becomes very sensitive with the introduction of wage rigidity. In our economy, the nominal rigidity of service price plays a similar role.

Rearranging the first order condition of the investors, we get the following no arbitrage condition between purchases of bonds and durable goods:

$$R_t = E_t(Z_t). \tag{18}$$

The right-hand side is a one-period holding return, $Z_t \equiv \frac{(1-\delta)P_{D,t+1}}{P_{D,t}-\Psi^{-1}_{r_D,t}r_{D,t}-\iota_t}$, which results from buying the durable goods in period t and selling them in period $t+1$. The term in the numerator is the capital gain. To obtain a rate of return, we divide capital gain by the net purchase price in the denominator. ι_t is a risk-premium purchasing durable good. Therefore, the households are indifferent between purchasing riskless bonds and investing in durable goods.[3] From this channel, the increase in the interest rate induces the investors to purchase more durable goods from the production sector. When durability is low, this channel effect is trivial. However, as durability increases, this channel exerts a bigger effect than the IS channel does.

2.5 Monetary policy

We assume that the monetary authority obeys the Taylor-type rule. We consider the following instrument:

$$\frac{R_t}{R} = \left(\frac{\pi_{r_D,t}}{\pi_{r_D}}\right)^{\rho_{\pi_{r_D}}} \left(\frac{\pi_{P_D,t}}{\pi_{P_D}}\right)^{\rho_{\pi_{P_D}}}, \tag{19}$$

where the variables with no subscript denote the steady state levels of corresponding variables.

[3] Calza, Monacelli, and Stracca (2007) analyze the difference between flexible and fixed rates in the housing market. However, we do not analyze these effects in this paper.

Sectoral inflation targeting occurs when the central bank targets only one of the Taylor rules. When $\rho_{\pi_{r_D}} > 1$ and the other coefficients are zero, the target becomes the aggregate service inflation targeting. When $\rho_{\pi_{P_D}} > 1$ and the other coefficients are zero, the target becomes the aggregate good inflation targeting.

2.6 Market clearing condition

The market clearing conditions in the service and good markets are

$$D_t = S_{D,t} + \frac{\theta_{r_D}}{2}(\pi_{r_D,t} - 1)^2 D_t \tag{20}$$

$$Y_t = I_{D,t} + \frac{\theta_{P_D}}{2}(\pi_{D,t} - 1)^2 Y_t, \tag{21}$$

where some proportions of the final service and good are allocated to the resource costs that originate from the price adjustment. Labor and bond markets also clear in the equilibrium.

3 Equilibrium

The equilibrium consists of the allocation $S_{D,t}$, b_t, N_t for the households; the allocations $D_t(i)$ and $I_{D,t}(i)$ and price $r_{D,t}(i)$ for the durable investor $i \in [0, 1]$; and the allocations $Y_t(i)$ and $N_t(i)$ and price $P_{D,t}(i)$ for the durable-goods producer $i \in [0, 1]$. Together with wages W_t, these satisfy the following: Taking prices and the wage as given, the household's allocations solve its utility maximizing problem; taking the wage and all prices but its own as given, the allocations and the price of each durable service investor solve its profit maximizing problem; taking all prices but its own as given, the allocations and the price of each durable-good producer solve its profit maximizing problem; taking the wage and all prices but its own as given, the market for bonds and labor clears.

4 Efficient allocation

In this section and the subsequent ones, we investigate the difference between the allocations of an efficient economy and those for one that is distorted. From the household perspective, the sequence of durable service flows and leisure is the most important factor. However, there exist two kinds of distortions−of price and mark-up−in this economy. In the economy with price stickiness, a good monetary policy is to minimize distortions because doing so indirectly supports the optimal path of durable service and leisure.

As a first step, we focus on household preferences, the technology for producing new durable output flows, and the price-rent ratio for managing durable stocks, to investigate the optimal allocation. The intra-temporal condition of the household can be written as

$$-\frac{V_{N,t}}{U_{S_D,t}} = \frac{W_t}{P_{D,t}} \frac{P_{D,t}}{r_{D,t}}, \tag{22}$$

where the left-hand side is the marginal rate of substitution between labor supply and durable consumption in period t and the right-hand side is composed of the multiplied sum of the labor income and the price-rent ratio, where the second term represents an asset effect managing durable stocks. If the household marginally increases the quantity of labor supply, the additional effect of these two factors would be exactly offset by the utility loss associated with the decrease in leisure time.

In a frictionless economy, the first term on the right-hand side equals the marginal product of labor in the production sector. The second term is the price-rent ratio that explains the relationship between the purchase price of durable goods and the rental price of the durable service flow. The efficient price allocation in the service market is directly related to the efficient allocation of the durable stocks. For the same disutility of labor supply, the marginal utility of durable consumption in period t decreases as durability increases. This is simply because the service flow to be consumed becomes abundant in each period. Therefore, the marginal rate of substitution on the left-hand side increases. This effect is absorbed into the price-rent ratio term on the right-hand

side. When durability is high, the price-rent ratio increases because this allows persistent enjoyment the marginal gains that result from possessing the good.

4.1 Frictionless price-rent ratio

To investigate the dynamics of the price-rent ratio in a frictionless economy, we derive this ratio by solving the investor's maximization problem. The real marginal cost of purchasing a new durable good should be equated to the total marginal gains from managing durable services until the newly purchased good depreciates entirely:

$$\Xi_{r_D,t} = E_t\left[\sum_{k=t}^{\infty}(1-\delta)^{(k-t)}\Lambda_{t,k}\frac{r_{D,k}}{r_{D,t}}\right], \tag{23}$$

where the price-rent ratio is a function of a rational forecast of the stochastic future discount rates and the expected growth rate of rental costs. Furthermore, the price-rent ratio is an increasing function of durability, $(1-\delta)$. High durability implies that the purchased good survives for a long period, and hence, the value of the good also increases. Note that when $\delta = 1$, the price-rent ratio equals one, which means that the good price and the service price are the same.

4.2 Labor supply and the demand for services

Plugging the frictionless price-rent ratio and marginal product of labor into the intra-temporal condition, we get

$$-V_{N,t} = A_t E_t\left\{\sum_{k=t}^{\infty}[(1-\delta)\beta]^{(k-t)}U_{S_D,k}\right\}. \tag{24}$$

The marginal disutility of labor on the left-hand side can be understood as the shadow value of current production, while the right-hand side shows the marginal gains that result from the consumption of services from period t onwards. As discussed by Barsky, House, and Kimball (2007), the steady state stock-flow ratio is $\frac{1}{\delta}$. For a highly durable good, this ratio is high. If β is high and δ is low, the shadow value of current production

is dominated by future terms.

5 Distortions

Previous New Keynesian studies focus on the distortion in only the production sector. However, the distortion in the service sector is also an important aspect of the real economy. The housing market is a very good example of a market where the rental price is rigid while the goods price is flexible and volatile. Therefore, in this section, we analyze how in each sector, these distortions differ.

5.1 Sticky purchase price

To investigate the transmission mechanism of distortions in each sector, we present the price equation in terms of distortions. When the purchase price is sticky, the relationship of the nominal wage rate with the purchase price becomes

$$\Xi_{P_D,t} = \Psi_{P_D,t}, \tag{25}$$

where $\Psi_{P_D,t} \equiv \frac{1}{\mu_{P_D}} + \psi_{P_D,t}$.[4] $\Psi_{P_D,t}$ is an efficiency parameter that consolidates all frictions in the production sector. The first term on the right-hand side represents the mark-up friction, and the second term, $\psi_{P_D,t}$, is a distortion that originates from the price stickiness in the goods sector. Note that when the production sector is perfectly competitive ($\mu_{P_D} = 1$) and the purchase price is flexible ($\theta_{P_D} = 0$ and $\psi_{P_D,t} = 1$), all distortions disappear ($\Psi_{P_D,t} = 1$). However, $\Psi_{P_D,t}$ deviates from its efficient level when the purchase price is sticky. For example, when the firms attempt to increase their current prices, a positive gap may occur between current inflation and the expected rate of future inflation in the production sector. In this case, the price adjustment cost depresses the firms' decision. Thus, they set the current purchase cost inefficiently at a lower level than the flexible case, and vice versa.

[4] $\psi_{P_D,t} \equiv \frac{\theta_{P_D}}{\varepsilon_{P_D}} \left\{ (\pi_{D,t} - 1)\pi_{D,t} - \beta E_t \left[\frac{U_{S_D,t+1}}{U_{S_D,t}} \frac{\Xi_{r_D,t+1}}{\Xi_{r_D,t}} \frac{Y_{t+1}}{Y_t} (\pi_{D,t+1} - 1)\pi_{D,t+1} \right] \right\}$.

5.2 Distorted price-rent ratio

When the user cost is flexible, the price-rent ratio is a function of the marginal utility gaps. However, when the user cost is sticky, the price-rent ratio is distorted as follows:

$$\Xi_{r_D,t} = E_t\left[\sum_{k=t}^{\infty}(1-\delta)^{(k-t)}\Lambda_{t,k}\frac{r_{D,k}}{r_{D,t}}\Psi_{r_D,k}\right], \tag{26}$$

where $\Psi_{r_D,t} \equiv \frac{1}{\mu_{r_D}} + \psi_{r_D,t}$.[5] $\Psi_{r_D,t}$ is an efficiency parameter in the service sector. $\psi_{r_D,t}$ represents the distortion from the price stickiness.

When the user cost is sticky, the current level of the price-rent ratio is attributed to three factors: (a) the entire paths of future discount rates, (b) dividend growth rates, and (c) the entire sequence of $\{\Psi_{r_D,k}\}_{k=t}^{\infty}$. In the asset price literature, the paths of future discount rates and dividends are key factors in explaining the price-rent ratio.[6] However, in this economy, another factor, price distortion, severely distorts the price-rent ratio. The current price-rent ratio is influenced by future distortions when the service market is distorted. When nominal rigidity exists in the service sector, the current inflation of the user cost is distorted. Furthermore, the current purchase price is a function of the present value of the future user cost. Therefore, the price-rent ratio and the purchase price are heavily distorted when the rental market is sticky.

5.3 Descriptive explanation of welfare loss

Combining equations (25) and (26), the marginal rate of substitution between service flows and labor in period t can be written as a function of the total distortions in this economy:

$$-\frac{U_{N,t}}{U_{S_D,t}} = A_t\Psi_{P_D,t}E_t\left[\sum_{k=t}^{\infty}(1-\delta)^{(k-t)}\Lambda_{t,k}\frac{r_{D,k}}{r_{D,t}}\Psi_{r_D,k}\right]. \tag{27}$$

[5] $\psi_{r_D,t} \equiv \frac{\theta_{r_D}}{\varepsilon_{r_D}}\left\{(\pi_{r_D,t}-1)\pi_{r_D,t} - \beta E_t\left[\frac{U_{S_D,t+1}}{U_{S_D,t}}\frac{D_{t+1}}{D_t}(\pi_{r_D,t+1}-1)\pi_{r_D,t+1}\right]\right\}$.

[6] For example, Cochrane (1992).

Note that $\Psi_{P_D,t} = \{\Psi_{r_D,k}\}_{k=t}^{\infty} = 1$ when all prices are flexible and mark-up distortions disappear. The marginal rate of substitution between labor and durable service flow is a function of the current distortion in the production sector, and current and future distortions in the service sector. Price stickiness in the service market heavily distorts the current price-rent ratio, and the price stickiness of the good market amplifies this effect.

Which type of price stickiness quantitatively distorts the economy more? To answer this question, Fig. 1 shows the gap of the marginal rate of substitution between labor and service flows from the flexible-price economy.[7] To analyze the stickiness effect, we change the degree of stickiness. The black lines, dashed dark-gray lines, and dash-dotted light-grey lines indicate firm prices change per year, half year, and quarter, respectively. The left panel illustrates the case when the purchase price is sticky. When only prices in the good market are sticky, the price-rent ratio does not deviate too much from the flexible case. When prices in the service market are sticky, the price-rent ratio distorts heavily. This is mainly because the service firms are price-takers for the purchase price and monopolistic competitors for the user cost. When prices in the service market are flexible, the user cost can be adjusted efficiently even though the purchase price is sticky. However, when the user cost is sticky, its responding path to the exogenous shocks is highly distorted; hence, the price-rent ratio and consumption path also deviate considerably from the case of the flexible-price economy. In response to productivity shocks, the marginal rate of substitution gap consistently deviates around 0.01 percent; moreover, twelve quarters later, it still does not return to the steady state. This deviation is directly related to the welfare cost.

6 Optimal monetary policy

In this section, we derive a welfare-loss function when both service and purchase prices are sticky. Erceg and Levin (2006) and Petrella and Santoro (2010) derive the welfare-loss function in the durable-good economy. However, in contrast to their approach, we

[7] For calibration, see Appendix B.

analyze the welfare implication of the service flow and user cost and compare the relative importance of the service sector in the decision of monetary policy.

We can write a second-order approximation to the household's welfare losses resulting from the deviations from the efficient allocation as follows:[8]

$$\begin{aligned}\pounds &\equiv -E_0 \sum_{t=0}^{\infty} \beta^t \{[U(S_{D,t}) - V(N_t)] - [U(S^e_{D,t} - V(N^e_t)]\} \\ &= \frac{U_{S_D} S_D}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ (\sigma - 1)\tilde{s}^2_{D,t} + \frac{(1+\phi)\delta}{[1-(1-\delta)\beta]} \tilde{y}^2_t + \theta_{r_D} \tilde{\pi}^2_{r_D,t} + \frac{\delta \theta_{P_D}}{[1-(1-\delta)\beta]} \tilde{\pi}^2_{P_D,t} \right\},\end{aligned} \tag{28}$$

where $S^e_{D,t}$ and N^e_t are the efficient levels of durable service and labor in the frictionless economy. The variables with a tilde denote the log deviations from their efficient levels.

The coefficients in the brace on the right-hand side can be interpreted as the optimal weights that the central bank should minimize. This social loss function reveals that the central bank should balance not only the fluctuations in output flow gaps and good-price inflation, but also the variability of service flow gaps and service-price inflation. The coefficient of each loss term can be interpreted as the relative weight that the central bank should stabilize. What is the relationship between welfare loss and the physical depreciation rate? The social loss function equation (28) has at least two important implications. The coefficients of the output gap and producer inflation in the brace are the functions of the physical depreciation rate. Basically, higher durability quantitatively lowers the weights of the variables in the production sector. In other words, the central bank should stabilize the fluctuations of the service flow gap and service inflation variability in the highly durable economy.

[8]For further derivation, refer to Appendix E.

7 Optimal simple rule

In this section, we identify parameterizations of monetary rules following the Taylor rule. We find the optimized rule by selecting policy-rule coefficients within the set of implementable rules so as to minimize the level of welfare loss associated with the resulting competitive equilibrium.[9] In the interest-rate rule, the nominal interest rate depends linearly on the rates of user cost and purchase price inflations:

$$\frac{R_t}{R} = \left(\frac{\pi_{r_D,t}}{\pi_{r_D}}\right)^{\rho_{\pi_{r_D}}} \left(\frac{\pi_{P_D,t}}{\pi_{P_D}}\right)^{\rho_{\pi_{P_D}}}. \tag{29}$$

The target values R, π_{r_D}, and π_{P_D} are assumed to be the steady-state values of their associated endogenous variables, which are the same as those in the efficient allocation case. Table 1 presents the results. The optimized interest-rate rule turns out to respond actively to the user cost inflation and put less weight on the purchase price inflation. Fig. 2 displays the welfare cost with different coefficient parameters in the Taylor rule. We can observe that a more active response to the user cost inflation lowers the welfare cost more. The right panel shows the one inflation targeting case. In all cases, a high coefficient decreases the welfare cost. More importantly, user cost inflation targeting lowers the welfare cost more than purchase price targeting does.

8 Comparison with a nondurable good economy

How is the size of welfare losses affected by a change in durability? To answer this question, we characterize the relationship between durability and the loss function. For brevity, we consider the flexible-price economy case.

We analyze the durability effect on the variations of real variables, which are the sizes of service flow and output flow gaps with changing durability. However, service flow and output flow are different in the durable-good economy. Therefore, to compare the service flow term with the nondurable-good economy, we rewrite the service flow gap in terms of

[9] For our computations, we adopt the perturbation method, following Schmit-Grohe and Uribe (2004).

the output flow gaps as follows:[10]

$$\frac{U_{S_D}S_D}{2}E_0\sum_{t=0}^{\infty}\beta^t\ (\sigma-1)\hat{s}_{D,t}^2$$
$$=\frac{U_{S_D}S_D}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{\frac{(\sigma-1)\delta^2}{[1-\beta(1-\delta)^2]}\hat{y}_t^2+\frac{2(\sigma-1)\delta^2}{[1-\beta(1-\delta)^2]}\left[\hat{y}_t\left(\sum_{k=t+1}^{\infty}[\beta(1-\delta)]^k\hat{y}_k\right)\right]\right\}, \tag{30}$$

where $\hat{s}_{D,t}$ and $\hat{y}_t$ denote the log-deviation of service flow consumption and output flow respectively from the steady state level.[11] Note that the second term in the brace disappears when $\delta = 1$.

The first term in the brace can be ignored when δ approaches zero. In a highly durable-good economy, the steady-state level of consumption service from the durable stock is huge. As the durable stock-flow ratio increases, the relative impact of the current output flow gap decreases. Therefore, the welfare loss in the first term becomes negligible. The second term is a new term in the durable-good economy. With a high degree of durability, the newly produced current output flow influences the durable stock persistently. Therefore, the multiplied term in the bracket strictly increases as durability increases. Compared to the nondurable good economy, the output fluctuations are huge. However, the coefficient converges to zero and the effect on the household's welfare is limited.

9 Does durability increase welfare?

In the former section, the investigation of the loss function in only the form of the output flow gaps reveals that durability influences the persistence of output gap fluctuations but does not necessarily increase welfare loss. In this section, we attempt to determine the relationship between durability and household welfare.

[10] For further derivation, see Appendix F.
[11] We also use the production function: $\hat{y}_t = a_t + \hat{n}_t$.

9.1 First moment effect

We begin with second-order approximations of the period utility, $U(S_{D,t})$ and $V(N_t)$, around the steady state:

$$U(S_{D,t}) \simeq U(S_D) - U_{S_D} S_D \left(- \hat{s}_{D,t} + \frac{(\sigma - 1)}{2} \hat{s}_{D,t}^2 \right), \tag{31}$$

$$-V(N_t) \simeq -V(N) - V_N N \left[\hat{y}_t - a_t + \frac{1+\phi}{2} (\hat{y}_t - a_t)^2 \right]. \tag{32}$$

These two equations hold even when all prices are flexible. Two effects of durability on welfare are revealed. The first is the absolute level effect with increasing stocks. The second involves the coefficient parameters of loss terms in the utility-based loss function, which we discuss in Proposition 2. The following proposition formally establishes the result of the first effect.

Proposition 1. *Durability strictly increases the steady-state consumption level of the household.*

Proof. On the right-hand side of equation (31), $U(S_D)$ is a steady-state utility level from service flows, which is a monotonically increasing function of S_D, which is also monotonically increasing with durability, $(1 - \delta)$. On the other hand, the labor disutility term, $V(N)$, in equation (32) is a decreasing function of durability in the steady state because it should be adjusted to equate the intra-temporal equation under the stationary labor supply in the steady state. □

With the same input, the household can enjoy more consumption because higher durability implies more durable stocks. Therefore, durability strictly increases the first moment of the household utility. Fig. 5 depicts this first-moment effect of lifetime welfare, $\frac{[U(S_D)-V(N)]}{1-\beta}$. We find that the first term is increasing particularly in the region $\delta \in \{0, 0.1\}$.

9.2 Second moment effect

The coefficient terms of the brackets of equations (31) and (32) capture the size of welfare loss originating from the fluctuations.

Proposition 2. *Durability strictly decreases the coefficients of the deviation terms, $U_{S_D}S_D$ and $V_N N$, in the household's welfare function.*

Proof. $U_{S_D}S_D$, the coefficient term of consumption utility, is strictly increasing with δ unless the relative risk aversion parameter equals one. Plugging $S_D = \frac{Y}{\delta}$ and $U_{S_D} = S_D^{-\sigma}$, we get $(Y/\delta)^{1-\sigma}$, which is strictly decreasing as durability increases. When the durability is high, the total stocks to be consumed are abundant. Therefore, the marginal utility of consumption is lowered.

In the presence of production subsidies, from the equation of the steady state marginal rate of substitution between labor supply and consumption of durable services, we get

$$-V_N = \frac{1}{[1-(1-\delta)\beta]}U_{S_D}. \tag{33}$$

The fraction term on the right-hand side scales the relative size of the period disutility of labor compared to the period utility of consumption; it is a function of the depreciation rate. Newly made output is consumed until it totally depreciates, and hence, the marginal disutility of labor equals the sum of marginal utility of consuming its services. We call this effect the "*marginal gain effect.*" Second, from the measure of one-period marginal disutility of labor, it is clear that the marginal utility of consumption from service flows, U_{S_D}, decreases as durability increases. Overall, the second effect dominates the first one, so the coefficient terms strictly decrease as durability increases.[12] N is independent of durability.[13] □

[12]Even when there exists no subsidy, Proposition 2 holds.

[13]To the best of my knowledge, there is no empirical evidence to explain the relationship between labor hours and the durability of goods.

The two panels in the first row of Fig. 4 numerically exhibit both the marginal gain effects and $V_N N$, which is a coefficient term of output deviations in equation (32), which has different risk-aversion parameters. The marginal gains increase as durability increases. For example, when $\delta = 0.1$, the marginal gains are almost nine times as large as they are in the nondurable case. Note that if $\delta = 1$, when there is no durability of goods, there is no marginal gain. On the other hand, the disutility of labor and the coefficient term of output deviations decrease as durability increases. This is because the strong concavity of the utility function of service flows implies that the marginal utility of consumption does not increase even though consumption does. Therefore, the scale parameter, ν should be adjusted to equate the marginal disutility of labor and the marginal gains. Panel D displays the relative size of coefficient terms, $\frac{V_N N}{U_{S_D} S_D}$. In all cases, we observe that the coefficient terms of disutility of working decrease more as durability increases.

Proposition 2 reveals that the period welfare is inclined to increase as durability increases. The smaller the coefficient terms in Proposition 2, the higher the period welfare. This is because they are the scale parameters of the second-order loss terms.

10 Concluding remarks

In this paper, we have studied the role of durable goods and the service market in the New Keynesian model. In the presence of price stickiness, inflation variability is costly. Hence, a monetary policy will have to optimally balance the incentive to offset the price stickiness distortion. In this paper, we split the durable market into the service and production sectors, and derive the traditional user cost equation, the price-rent ratio, and the social loss function.

The fundamental contribution of this paper is as follows. First, the price-rent ratio critically depends on nominal rigidity in the service market. This is because when the user cost is sticky and the purchase price is flexible (as is the case in a housing market), the price-rent ratio is influenced by the future distortions of the user cost. Second, we find that the central bank should stabilize the service sector when goods are highly durable,

even in the case where the degrees of nominal rigidity in both service and goods markets are the same. Third, we find that in comparison with the nondurable economy, social welfare loss is not increasing, although a temporary shock persistently affects the output fluctuations. Fourth, high durability increases welfare.

However, there are several remaining issues that would be worthwhile areas for future research. First, we may need to study a large shock, a situation that was not discussed in the paper. Productivity shocks in the production sector may be not enough to explain a large recession such as the recent housing boom and bust in the U.S. economy. There may well be other factors that affect this, such as irrational exuberance. Second, we do not examine the situation where goods with different durability coexist. There exist many different characteristics, introducing many goods. Third, we need to introduce other features into the model so that we can identify more implications of the monetary policy for durable goods. Lumpiness of durables or news shocks are both good examples that would warrant investigation. Fourth, it may also be beneficial to use another approach, such as Ramsey problem approach, to identify the optimal monetary policy.

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Appendix A: Deterministic steady state

We consider a frictionless steady-state in which all the shocks are zero and in which monetary policymakers set their respective CPI inflation rates to zero:

$$\pi_{r_D} = 1; \pi_{P_D} = 1.$$

From the Euler equation, the nominal interest rate and the price of claims become:

$$R = \frac{1}{\beta}.$$

The real marginal costs are

$$\Xi_{r_D} = \frac{\mu_{r_D}^{-1}\tau_{r_D}}{1-(1-\delta)\beta}; \quad \Xi_{P_D} = \mu_{P_D}^{-1}\tau_{P_D},$$

where $\tau_{r_D} = \mu_{r_D}$ and $\tau_{P_D} = \mu_{P_D}$. We assume that the steady state is not distorted by the monopolistic competition. Compared to production sectors, the real marginal cost in the durable service sector is higher.

The real wage becomes:

$$w = \Xi_{P_D}.$$

We set the steady state labor level as one third. ($N = 1/3$) Then combining production functions, the good market conditions, and the law of motion of durable goods, we get

$$N = Y = I_D = \delta D$$
$$\therefore D = \frac{1}{3\delta}.$$

From the rental market clearing condition

$$S_D = D.$$

Appendix B: Calibration and numerical simulation results

Time is in quarters and we set the quarterly discount factor as $\beta = 0.99$. This implies that the annual real interest rate is pinned down by the household's patience rate and is equal to 4 %. The annual depreciation rate in the benchmark case is 5 % ($\delta = 0.05/4$) following previous studies on long-lived durables. Following Monacelli (2009), the elasticity of substitution between varieties in the non-durable and the durable sectors ε_{P_D} and ε_{P_C} are set equal to 6, which yields a steady state mark-up of 20 %. In the benchmark case, we set the degree of nominal rigidity in service and good prices to generate a frequency of price adjustment of about four quarters. Let κ be the probability of not resetting prices in the standard Calvo-Yun model. Log linearized Phillips curve in this model is $\frac{\varepsilon}{\theta}$, while it is $\frac{(1-\kappa)(1-\beta\kappa)}{\kappa}$ in the Calvo-Yun model. A price rigidity of four quarters is a standard in the recent literature so we take it as a benchmark parameter ($\kappa = 0.75$). The period utility function is assumed to be: $\frac{S_{D,t}^{\sigma-1}}{\sigma-1} - \nu\frac{N_t^{1+\phi}}{1+\phi}$. Following the existing literature on durable goods, we set $\sigma = 1$ and $\phi = 1$. In the analysis of optimal monetary policy, we change the value of σ and search for the implication of welfare. The elasticity parameter, ϕ, is set to one in all cases. Therefore, the scale parameter, ν, is adjusted for the intra-temporal condition to hold in equality with the change of durability.

Appendix C: First-best economy

The social planner solves the following problem:

$$max E_0 \sum_{t=0}^{\infty} \beta^t U(S_{D,t}, N_t)$$

$$s.t. \quad D_t - (1-\delta)D_{t-1} = I_{D,t} = A_t N_{D,t},$$

where the constraint equation is the consolidated resource constraint.

The optimality conditions are given by

$$U_{S_{D,t}} = -\lambda_t + (1-\delta)\beta E_t\{\lambda_{t+1}\}.$$
$$U_{N_t} = A_t\lambda_t.$$

Combining this result we get

$$A_t = -\frac{U_{N_t}}{U_{S_{D,t}}} + (1-\delta)\beta E_t\left\{\frac{U_{S_{D,t+1}}}{U_{S_{D,t}}}\frac{A_t}{A_{t+1}}\frac{U_{N_{t+1}}}{U_{S_{D,t+1}}}\right\}.$$

The left-hand side is the marginal product of labor. The right-hand side is the net marginal rate of substitution between hours of work and consumption.

In the steady state,

$$-\frac{U_N}{U_{S_D}} = \frac{1}{[1-(1-\delta)\beta]}.$$

Appendix D: Sticky price economy

The Euler equation as a log-deviation form from the efficient allocation economy becomes

$$\tilde{s}_{D,t} = E_t\tilde{s}_{D,t+1} - \frac{1}{\sigma}(i_t - E_t\pi_{r_D,t+1} - r_t^e),$$

where $\tilde{s}_{D,t} \equiv s_{D,t} - s_{D,t}^e$ is the output gap.

Production function and marginal costs in the sticky price economy becomes

$$y_t = a_t + n_t$$
$$\xi_{r_D,t} = p_{D,t} - r_{D,t}$$
$$\xi_{P_D,t} = \phi\iota_t + \sigma s_{D,t} - \xi_{r_D,t} - (1+\phi)a_t.$$

Marginal cost gaps from the steady state are:

$$\hat{\xi}_{P_{D,t}} = \phi\hat{\iota}_t + \sigma\hat{s}_{D,t} - \hat{\xi}_{r_D,t} - (1+\phi)\hat{a}_t.$$

The new Keynesian Phillips curve is

$$\begin{aligned}
\hat{\pi}_{r_D,t} &= \beta E_t\hat{\pi}_{r_D,t+1} + \frac{\varepsilon_{r_D}-1}{\theta_{r_D}[1-(1-\delta)\beta]}\left[\hat{\xi}_{r_D,t} - (1-\delta)\beta E_t(\hat{U}_{S_D,t+1} - \hat{U}_{S_D,t} + \hat{\xi}_{r_D,t+1})\right] \\
\hat{\pi}_{P_D,t} &= \beta E_t\hat{\pi}_{P_D,t+1} + \frac{\varepsilon_{P_D}-1}{\theta_{P_D}}\hat{\xi}_{P_D,t},
\end{aligned}$$

where $\hat{\xi}_{r_D,t}$ is the price-rent ratio gap. The durability affects the slope of the Phillips curve and net real marginal cost gap in the service sector. The Phillips curve in the service sector exhibits a higher slope as durability increases. Furthermore, net real marginal cost decreases because two terms in the brace cancel out each other with high durability.

Plugging the Euler equation into the NKPC in the service sector,

$$\hat{\psi}_{r_D,t} = \left[\hat{P}_{D,t} - \hat{r}_{D,t} - (1-\delta)\beta E_t(\hat{U}_{S_D,t+1} - \hat{U}_{S_D,t} + \hat{P}_{D,t+1} - \hat{r}_{D,t+1})\right] \tag{34}$$

where $\hat{\psi}_{r_D,t} \equiv \frac{\theta_{r_D}[1-(1-\delta)\beta]}{\varepsilon_{r_D}-1}(\hat{\pi}_{r_D,t} - \beta E_t\hat{\pi}_{r_D,t+1})$.

We can also express the new Keynesian Phillips curve as the real marginal cost equation in terms of the price stickiness:

$$\begin{aligned}
\hat{\xi}_{r_D,t} &= \sum_{k=t}^{\infty}[(1-\delta)\beta]^{k-t}E_t\left\{\frac{\theta_{r_D}[1-(1-\delta)\beta]}{\varepsilon_{r_D}-1}(\hat{\pi}_{r_D,k} - \beta\hat{\pi}_{r_D,k+1}) + (1-\delta)\beta(\hat{U}_{S_D,k+1} - \hat{U}_{S_D,k})\right\} \\
&= \sum_{k=t}^{\infty}[(1-\delta)\beta]^{k-t}E_t[\hat{\psi}_{r_D,k} + (1-\delta)\beta(\hat{U}_{S_D,k+1} - \hat{U}_{S_D,k})] \equiv \hat{\hat{\psi}}_{r_D,t} \qquad (35) \\
\hat{\xi}_{P_D,t} &= \frac{\theta_{P_D}}{\varepsilon_{P_D}-1}(\hat{\pi}_{P_D,t} - \beta E_t\hat{\pi}_{P_D,t+1}) = \hat{\psi}_{P_D,t} \qquad (36)
\end{aligned}$$

Notice that the gap of price-dividend ratio is larger than that of real marginal cost in the production sector even in the case of the same rigidity ($\theta_{r_D} = \theta_{P_D}$).

Appendix E: Derivation of second-order approximation of welfare around the undistorted flexible price equilibrium allocation

Following Rotemberg and Woodford (1998), we derive a well-defined welfare function from the utility function of the representative household around the efficient equilibrium allocation:

$$W_t \equiv U(S_{D,t}) - V(N_t).$$

Under our assumptions the efficient equilibrium allocation corresponds to the flexible price equilibrium allocation with no mark-up distortion. For brevity, we define $U_t \equiv U(S_{D,t})$, $V_t \equiv V(N_t)$, $U_t^e \equiv U(S_{D,t}^e)$, and $V_t^e \equiv V(N_t^e)$. The second order approximation of the utility from consumption of durable services are:

$$\begin{aligned} U_t - U_t^e &\simeq U_{S_D,t}^e S_{D,t}^e \left(\frac{S_{D,t} - S_{D,t}^e}{S_{D,t}^e} \right) + \frac{1}{2} U_{S_D S_D,t}^e (S_{D,t}^e)^2 \left(\frac{S_{D,t} - S_{D,t}^e}{S_{D,t}^e} \right)^2 \\ &\simeq U_{S_D,t}^e S_{D,t}^e \left(\tilde{s}_{D,t} + \frac{1-\sigma}{2} \tilde{s}_{D,t}^2 \right) \end{aligned}$$

The disutility of labor in period t becomes:

$$\begin{aligned} V_t - V_t^e &\simeq V_{N_t}^e N_t^e \left(\frac{N_t - N_t^e}{N_t^e} \right) + \frac{1}{2} V_{NN,t}^e (N_t^e)^2 \left(\frac{N_t - N_t^e}{N_t^e} \right)^2 \\ &= V_{N,t}^e N_t^e \left(\tilde{n}_t + \frac{1+\phi}{2} \tilde{n}_t^2 \right) + o(||a||^2) \\ &\simeq V_{N,t}^e N_t^e \left(\tilde{y}_t + \frac{1+\phi}{2} (\tilde{y}_t)^2 \right), \end{aligned}$$

using the production function relationship, $\tilde{n}_t = \tilde{y}_t$, in the last equation.

Consider the linear terms in W_t:

$$L\tilde{W}_t \simeq U_{S_D,t}^e S_{D,t}^e \tilde{s}_{D,t} + V_{N,t}^e N_t^e \tilde{y}_t$$

Recalling that when the optimal subsidy is in place, the flexible price allocation is

efficient, we get

$$\frac{V^e_{N,t}}{U^e_{S_D,t}} = \frac{1}{[1-(1-\delta)\beta]}.$$

Therefore the linear term becomes

$$L\tilde{W}_t = U^e_{S_D,t} S^e_{D,t} \left\{ \tilde{s}_{D,t} - \frac{\delta}{[1-(1-\delta)\beta]} \tilde{y}_t \right\}.$$

By the way, from the market clearing conditions until the second-order approximation, we get:

$$\tilde{s}_{D,t} \simeq \tilde{d}_t - \frac{\theta_{r_D}}{2} \hat{\pi}^2_{r_D,t}; \quad \tilde{\iota}_t \simeq \tilde{y}_t - \frac{\theta_{P_D}}{2} \hat{\pi}^2_{P_D,t}.$$

Plugging these results into the linear term, we get

$$L\tilde{W}_t = U^e_{S_D,t} S^e_{D,t} \left\{ \tilde{d}_t - \frac{\theta_{r_D}}{2} \tilde{\pi}^2_{r_D,t} - \frac{\delta}{[1-(1-\delta)\beta]} \left(\tilde{\iota}_t + \frac{\theta_{P_D}}{2} \tilde{\pi}^2_{P_D,t} \right) \right\}.$$

We can drop the linear terms in LW_t after substituting the stock gap into the flow gap,[14] because:

$$\sum_{t=0}^{\infty} \beta^t U^e_{S_D,t} S^e_{D,t} \tilde{d}_t = \sum_{t=0}^{\infty} \beta^t U^e_{S_D,t} S^e_{D,t} \left\{ \frac{\delta \tilde{\iota}_t}{[1-\beta(1-\delta)]} \right\}.$$

We are left only with second-order real terms as well as inflation volatility:

$$\begin{aligned} & E_0 \sum_{t=0}^{\infty} \beta^t [(U_t - V_t) - (U^e_t - V^e_t)] \\ & \simeq -E_0 \sum_{t=0}^{\infty} \beta^t \frac{U^e_{S_D,t} S^e_{D,t}}{2} \left\{ (\sigma-1)\tilde{s}^2_{D,t} + \frac{(1+\phi)\delta}{[1-(1-\delta)\beta]} \tilde{y}^2_t + \theta_{r_D} \tilde{\pi}^2_{r_D,t} + \frac{\delta \theta_{P_D}}{[1-(1-\delta)\beta]} \tilde{\pi}^2_{P_D,t} \right\}. \end{aligned}$$

It is clear that the gaps of durable stocks and service inflation become important as durability increases. Notice that the coefficients of these terms become larger as the

[14] For further derivation, refer to the appendix G.

depreciation rate approaches to zero.

Furthermore, we derive a first order approximation to $U_{S^e_D,t}S^e_{D,t}$ around the steady state:

$$\begin{aligned} U^e_{S_D,t}S^e_{D,t} &= U_{S_D}S_D + (U_{S_DS_D}S_D + U_{S_D})\left(\frac{S^e_{D,t} - S_D}{S_D}\right) \\ &= U_{S_D}S_D + U_{S_D}S_D(1-\sigma)\hat{s}^e_{D,t}. \end{aligned}$$

Accordingly, we can write a second order approximation to the household's welfare losses resulting from deviations from the efficient allocation as:

$$\begin{aligned} \pounds &\equiv -E_0\sum_{t=0}^{\infty}\beta^t[(U_t - V_t) - (U^e_t - V^e_t)] \\ &= \frac{U_{S_D}S_D}{2}E_0\sum_{t=0}^{\infty}\beta^t\left\{(\sigma-1)\tilde{s}^2_{D,t} + \frac{(1+\phi)\delta}{[1-(1-\delta)\beta]}\tilde{y}^2_t + \theta_{r_D}\tilde{\pi}^2_{r_D,t} + \frac{\delta\theta_{P_D}}{[1-(1-\delta)\beta]}\tilde{\pi}^2_{P_D,t}\right\}. \end{aligned} \tag{37}$$

Appendix F: Welfare loss in terms of output flow

We express the first term in the social loss function in terms of output flow. The first term in the brace on the right hand side of equation (37) becomes:

$$
\begin{aligned}
E_0 \sum_{t=0}^{\infty} \beta^t \hat{s}_{D,t}^2 \simeq & E_0 \sum_{t=0}^{\infty} \beta^t \hat{d}_t^2 \\
= & \delta^2 E_0 \Bigg\{ \hat{\iota}_{D,0}^2 \\
& + \beta \Big[\Big((1-\delta)^2 \hat{\iota}_{D,0}^2 + \hat{\iota}_{D,1}^2 \Big) + 2(1-\delta) \hat{\iota}_{D,1} \hat{\iota}_{D,0} \Big] \\
& + \beta^2 \Big[\Big((1-\delta)^4 \hat{\iota}_{D,0}^2 + (1-\delta)^2 \hat{\iota}_{D,1}^2 + \hat{\iota}_{D,2}^2 \Big) \\
& \quad + 2 \Big((1-\delta) \hat{\iota}_{D,2} \hat{\iota}_{D,1} + (1-\delta)^2 \hat{\iota}_{D,2} \hat{\iota}_{D,0} + (1-\delta)^3 \hat{\iota}_{D,1} \hat{\iota}_{D,0} \Big) \Big] \\
& + \beta^3 \Big[\Big((1-\delta)^6 \hat{\iota}_{D,0}^2 + (1-\delta)^4 \hat{\iota}_{D,1}^2 + (1-\delta)^2 \hat{\iota}_{D,2}^2 + \hat{\iota}_{D,3}^2 \Big) \\
& \quad + 2 \Big((1-\delta) \hat{\iota}_{D,3} \hat{\iota}_{D,2} + (1-\delta)^2 \hat{\iota}_{D,3} \hat{\iota}_{D,1} + (1-\delta)^3 \hat{\iota}_{D,3} \hat{\iota}_{D,0} \\
& \qquad + (1-\delta)^3 \hat{\iota}_{D,2} \hat{\iota}_{D,1} + (1-\delta)^4 \hat{\iota}_{D,2} \hat{\iota}_{D,0} + (1-\delta)^5 \hat{\iota}_{D,1} \hat{\iota}_{D,0} \Big) \Big] + \cdots \Bigg\}.
\end{aligned}
$$

Therefore, we can divide it into the square-term part and the cross-term part. The square-term part becomes

$$
\begin{aligned}
& \delta^2 E_0 \left[\frac{\hat{\iota}_{D,0}^2}{1-\beta(1-\delta)^2} + \frac{\beta \hat{\iota}_{D,1}^2}{1-\beta(1-\delta)^2} + \frac{\beta^2 \hat{\iota}_{D,2}^2}{1-\beta(1-\delta)^2} + \cdots \right] \\
= & \frac{\delta^2}{1-\beta(1-\delta)^2} E_0 \sum_{t=0}^{\infty} \beta^t \hat{\iota}_{D,t}^2.
\end{aligned}
$$

On the other hands, the cross-term part becomes

$$\frac{2\delta^2}{1-\beta(1-\delta)^2}E_0\Big\{\beta(1-\delta)\hat{\iota}_{D,0}\hat{d}_{D,1}+[\beta(1-\delta)]^2\hat{\iota}_{D,0}\hat{\iota}_{D,2}+[\beta(1-\delta)]^3\hat{\iota}_{D,0}\hat{\iota}_{D,3}+\cdots$$
$$+\beta\Big[\beta(1-\delta)\hat{\iota}_{D,1}\hat{\iota}_{D,2}+[\beta(1-\delta)]^2\hat{\iota}_{D,1}\hat{\iota}_{D,3}+[\beta(1-\delta)]^3\hat{\iota}_{D,1}\hat{\iota}_{D,4}+\cdots$$
$$\vdots$$
$$+\beta^{t-1}\Big[\beta(1-\delta)\hat{\iota}_{D,t-1}\hat{\iota}_{D,t}+[\beta(1-\delta)]^2\hat{\iota}_{D,t-1}\hat{\iota}_{D,t+1}+[\beta(1-\delta)]^3\hat{\iota}_{D,t-1}\hat{\iota}_{D,t+2}+\cdots$$
$$\vdots\Big\}.$$

In the compact form, it becomes:

$$\frac{2\delta^2}{1-\beta(1-\delta)^2}E_0\sum_{t=0}^{\infty}\beta^t\hat{\iota}_{D,t}\Big\{\sum_{s=t+1}^{\infty}[\beta(1-\delta)]^s\hat{\iota}_{D,s}\Big\}.$$

Therefore, ignoring third and fourth ordered terms we get

$$E_0\sum_{t=0}^{\infty}\beta^t\hat{s}_{D,t}^2\simeq\frac{\delta^2}{1-\beta(1-\delta)^2}E_0\sum_{t=0}^{\infty}\beta^t\Big\{\hat{y}_t^2+2\hat{y}_t\Big[\sum_{s=t+1}^{\infty}[\beta(1-\delta)]^s\hat{y}_s\Big]\Big\},\tag{38}$$

which is the sum of the squared terms and cross-product terms.

Therefore the life-time welfare loss is

$$\sum_{t=0}^{\infty}(W_t-W)\simeq-\frac{U_SY}{2}\sum_{t=0}^{\infty}\beta^t\Big\{\frac{(\sigma-1)\delta}{[1-\beta(1-\delta)^2]}\Big[\hat{y}_t^2+2\hat{y}_t\Big(\sum_{s=t+1}^{\infty}[\beta(1-\delta)]^s\hat{y}_s\Big)\Big]$$
$$+\frac{(1+\phi)}{[1-(1-\delta)\beta]}(\hat{y}_t-a_t)^2$$
$$+\frac{\theta_{r_D}}{\delta}\hat{\pi}_{r_D,t}^2+\frac{\theta_{P_D}}{[1-(1-\delta)\beta]}\hat{\pi}_{P_D,t}^2\Big\}.\tag{39}$$

Appendix G: Relationship between life-time gaps of durable stock and flow

The derivations are explained by Petrella and Emiliano (2010). Thus, we only show a brief explaination. When the future gap is discounted,

$$
\begin{aligned}
E_0 \sum_{t=0}^{\infty} \beta^t \hat{d}_t = \delta E_0 \Big\{ & \hat{\iota}_{D,0} \\
& + \beta \Big[(1-\delta)\hat{\iota}_{D,0} + \hat{\iota}_{D,1} \Big] \\
& + \beta^2 \Big[(1-\delta)^2 \hat{\iota}_{D,0} + (1-\delta)\hat{\iota}_{D,1} + \hat{\iota}_{D,2} \Big] + \cdots \Big\} \\
= & \delta E_0 \Big\{ \Big[1 + \beta(1-\delta) + [\beta(1-\delta)]^2 + [\beta(1-\delta)]^3 + \cdots \Big] \hat{\iota}_{D,0} \\
& + \beta \Big[1 + \beta(1-\delta) + [\beta(1-\delta)]^2 + [\beta(1-\delta)]^3 + \cdots \Big] \hat{\iota}_{D,1} \\
& + \beta^2 \Big[1 + \beta(1-\delta) + [\beta(1-\delta)]^2 + [\beta(1-\delta)]^3 + \cdots \Big] \hat{\iota}_{D,2} + \cdots \Big\} \\
= & E_0 \sum_{t=0}^{\infty} \beta^t \frac{\delta \hat{\iota}_{D,t}}{[1-\beta(1-\delta)]}. \qquad (40)
\end{aligned}
$$

Minimizing the current output gap means minimizing its influence on the durable future stock gap.

This relationship also holds around the undistorted flexible price equilibrium allocation. In this case, $\hat{s}_{D,t} = \hat{d}_t$ and $\hat{\iota}_{D,t} = \hat{y}_t$ so Proposition 3 holds.

Because $\tilde{d}_t = (1-\delta)\tilde{d}_{t-1} + \delta \tilde{\iota}_{D,t}$ holds, the following equality also holds:

$$
E_0 \sum_{t=0}^{\infty} \beta^t \tilde{d}_t = E_0 \sum_{t=0}^{\infty} \beta^t \frac{\delta \tilde{\iota}_{D,t}}{[1-\beta(1-\delta)]}. \qquad (41)
$$

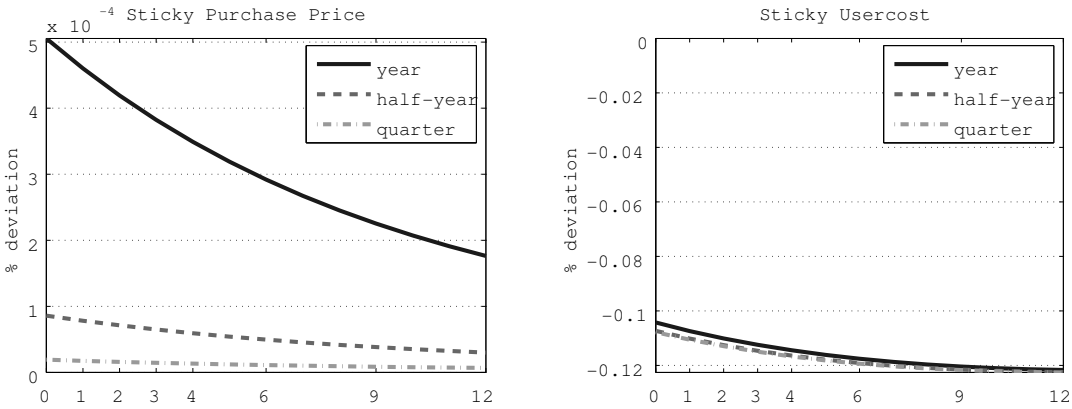
Table 1: Optimal monetary policy

	$\rho_{\pi_{r_D}}$	ρ_{π_D}	Welfare cost
Optimized Rule	4	1.9	-0.0086
User cost target rule	1.5	—	-0.0082
Purchase price targe rule	—	1.5	-0.0059

Note: (second-order) welfare cost computation of the policy rule

$$\frac{R_t}{R} = \left(\frac{\pi_{r_D,t}}{\pi_{r_D}}\right)^{\rho_{\pi_{r_D}}} \left(\frac{\pi_{P_D,t}}{\pi_{P_D}}\right)^{\rho_{\pi_{P_D}}}$$

Fig. 1: Marginal rate of substitution between labor and consumption



Note: The left panel is the case when the purchase price is sticky, while the right panel is the case when the user cost is sticky.

Fig. 2: Welfare Cost

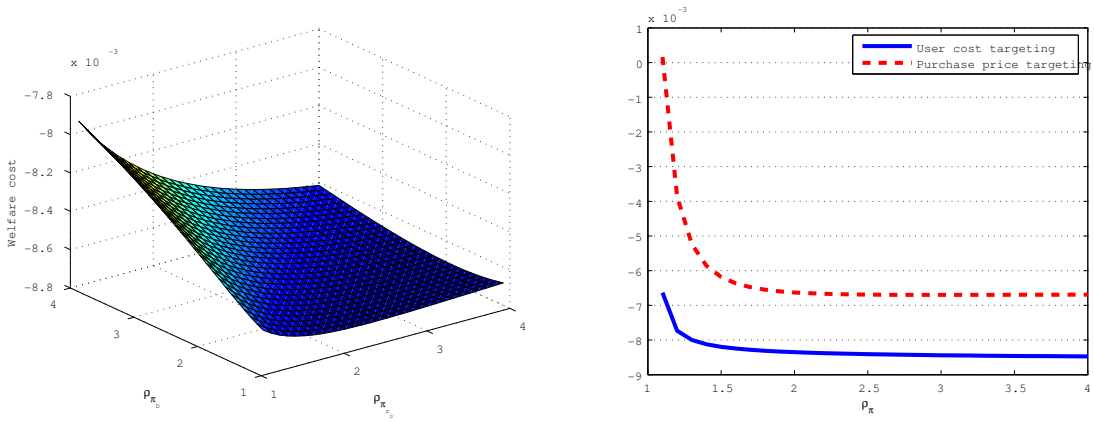


Fig. 3: First-moment welfare gain effect from durability

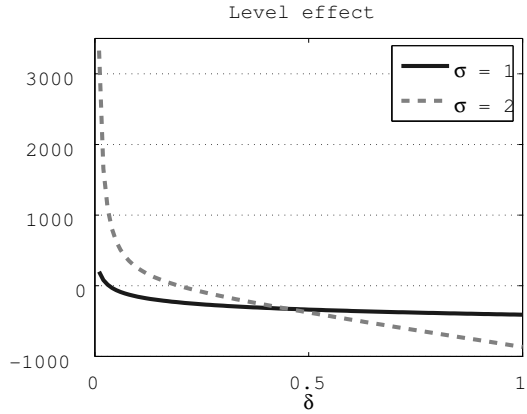
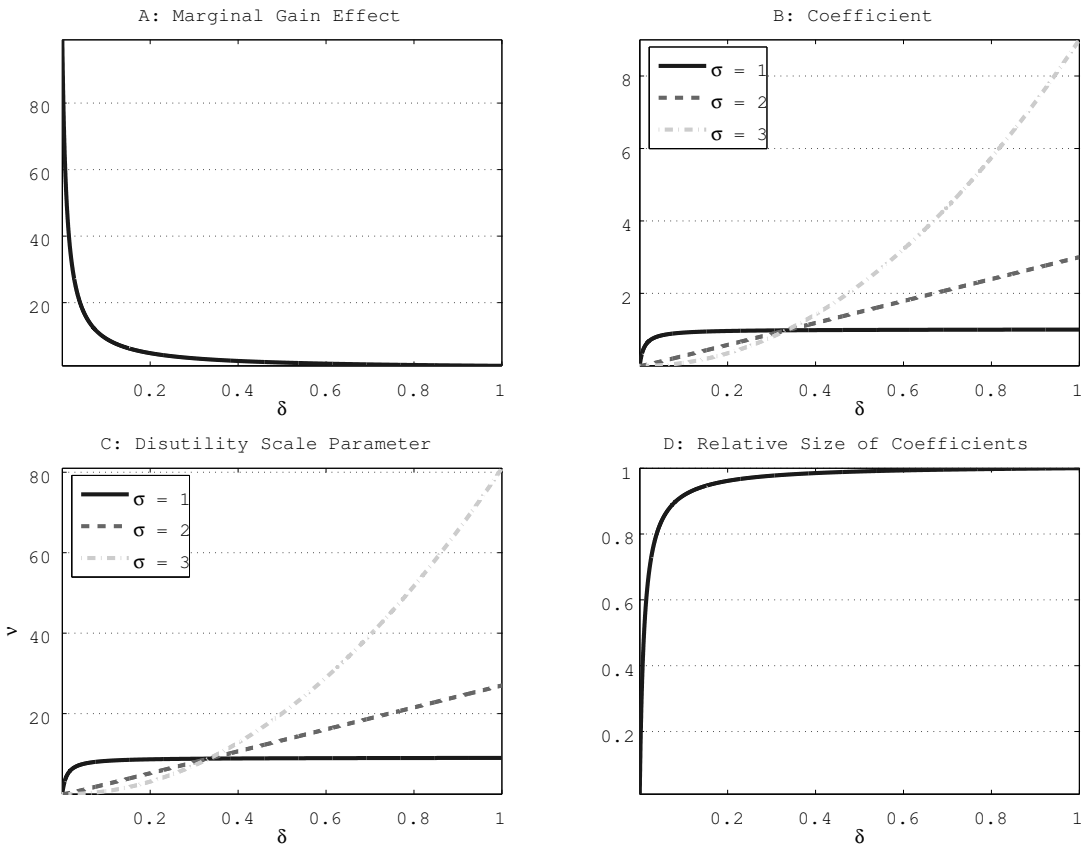



Fig. 4: Marginal gains and loss function coefficient term