On amending the Maskin's theorem by using complex numbers

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Abstract

The Maskin's theorem is a fundamental work in the theory of mechanism design. A recent work [Wu, Quantum mechanism helps agents combat "bad" social choice rules. *Intl. J. of Quantum Information* 9 (2011) 615-623] shows that when an additional condition is satisfied, the Maskin's theorem will not hold if agents use quantum strategies. Inspired by the quantum mechanism, in this paper, we will propose an algorithmic mechanism which uses complex numbers. We show by an example that a Pareto-efficient social choice rule that is not monotonic may be Nash implemented by using the algorithmic mechanism. This result is positive not only to the agents, but also to the designer if the designer wishes to maximize the total social surplus.

Key words: Algorithmic mechanism; Mechanism design; Nash implementation.

1 Introduction

Nash implementation is the cornerstone of the mechanism design theory. The Maskin's theorem provides an almost complete characterization of social choice rules (SCRs) that are Nash implementable. When the number of agents are at least three, the sufficient conditions for Nash implementation are monotonicity and no-veto, and the necessary condition is monotonicity [1]. Note that an SCR is specified by a designer. If the designer wants to maximize the total social surplus, he would like to implement a Pareto-efficient SCR. However, a Pareto-efficient SCR may not satisfy monotonicity (an example is given in Table 1, Section 4). According to the Maskin's theorem, it is impossible for the designer to implement such non-monotonic SCR in Nash equilibrium.

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In 2011, Wu [2] generalized the theory of mechanism design to a quantum domain. He proposed that the Maskin's theorem should be amended by virtue of quantum strategies. For n agents, the time and space complexity of the quantum mechanism are $O(n)$. Therefore the quantum mechanism is theoretically feasible. However, there exists an obstacle for the quantum mechanism to be practical: It needs a quantum equipment to work, but so far the experimental technologies for quantum information are not commercially available [3]. As a result, the quantum mechanism may be viewed only as a "toy". In this paper, we will go beyond this obstacle and propose an algorithmic mechanism which uses complex numbers. The algorithmic mechanism can work in the macro world immediately.

The rest of the paper is organized as follows: Section 2 recalls preliminaries of classical and quantum mechanisms published in Refs. [4,2] respectively; Section 3 is the main part of this paper, where we will propose an algorithmic mechanism; In Section 4, we show by an example that a Pareto-efficient social choice rule that is not monotonic can be Nash implemented by using the algorithmic mechanism when an additional condition is satisfied; Section 5 draws conclusions.

2 Preliminaries

2.1 The classical theory of mechanism design [4]

Let $N=\{1,\cdots,n\}$ be a finite set of *agents* with $n\geq 3$, $A=\{a_1,\cdots,a_k\}$ be a finite set of social *outcomes*. Let T_i be the finite set of agent i's types, and the *private information* possessed by agent i is denoted as $t_i\in T_i$. We refer to a profile of types $t=(t_1,\cdots,t_n)$ as a *state*. Let $\mathcal{T}=\prod_{i\in N}T_i$ be the set of states. At state $t\in\mathcal{T}$, each agent $i\in N$ is assumed to have a complete and transitive *preference relation* $\succeq_i^t$ over the set A. We denote by $\succeq^t=(\succeq_1^t,\cdots,\succeq_n^t)$ the profile of preferences in state t, and denote by $\succ_i^t$ the strict preference part of $\succeq_i^t$. Fix a state t, we refer to the collection $E=<N,A,(\succeq_i^t)_{i\in N}>$ as an *environment*. Let ε be the class of possible environments. A *social choice rule* (SCR) F is a mapping $F:\varepsilon\to 2^A\backslash\{\emptyset\}$. A *mechanism* $\Gamma=((M_i)_{i\in N},g)$ describes a message or strategy set M_i for agent i, and an outcome function $g:\prod_{i\in N}M_i\to A$. M_i is unlimited except that if a mechanism is direct, $M_i=T_i$.

An SCR F satisfies *no-veto* if, whenever $a\succeq_i^t b$ for all $b\in A$ and for all agents i but perhaps one j, then $a\in F(E)$. An SCR F is *monotonic* if for every pair of environments E and E', and for every $a\in F(E)$, whenever $a\succeq_i^t b$ implies that $a\succeq_i^{t'} b$, there holds $a\in F(E')$. We assume that there is *complete information* among the agents, i.e., the true state t is common knowledge among them. Given a mechanism $\Gamma = ((M_i)_{i\in N}, g)$ played in state t, a *Nash equilibrium* of Γ in state t is a strategy profile m^* such that: $\forall i \in N, g(m^*(t)) \succeq_i^t g(m_i, m^*_{-i}(t)), \forall m_i \in M_i$. Let $\mathcal{N}(\Gamma, t)$ denote the set of Nash equilibria of the game induced by Γ in state t, and $g(\mathcal{N}(\Gamma, t))$ denote the corresponding set of Nash equilibrium outcomes. An SCR F is *Nash implementable* if there exists a mechanism $\Gamma = ((M_i)_{i\in N}, g)$ such that for every $t \in \mathcal{T}$, $g(\mathcal{N}(\Gamma, t)) = F(t)$.

Maskin [1] provided an almost complete characterization of SCRs that were Nash implementable. The main results of Ref. [1] are two theorems: 1) (*Necessity*) If an SCR is Nash implementable, then it is monotonic. 2) (*Sufficiency*) Let $n \geq 3$, if an SCR is monotonic and satisfies no-veto, then it is Nash implementable. In order to facilitate the following investigation, we briefly recall the Maskin's mechanism published in Ref. [4] as follows:

Consider the following mechanism $\Gamma = ((M_i)_{i\in N}, g)$, where agent i's message set is $M_i = A \times \mathcal{T} \times \mathbb{Z}_+$, and $\mathbb{Z}_+$ is the set of non-negative integers. A typical message sent by agent i is described as $m_i = (a_i, t_i, z_i)$. The outcome function g is defined in the following three rules: (1) If for every agent $i \in N$, $m_i = (a, t, 0)$ and $a \in F(t)$, then $g(m) = a$. (2) If $(n-1)$ agents $i \neq j$ send $m_i = (a, t, 0)$ and $a \in F(t)$, but agent j sends $m_j = (a_j, t_j, z_j) \neq (a, t, 0)$, then $g(m) = a$ if $a_j \succ_j^t a$, and $g(m) = a_j$ otherwise. (3) In all other cases, $g(m) = a'$, where a' is the outcome chosen by the agent with the lowest index among those who announce the highest integer.

2.2 Quantum mechanisms [2]

In 2011, Wu [2] combined the theory of mechanism design with quantum mechanics. He found that when an additional condition was satisfied, monotonicity and no-veto were not sufficient conditions for Nash implementation in the context of a quantum domain. Following Section 4 in Ref. [2], two-parameter quantum strategies are drawn from the set:

$$\hat{\omega}(\theta, \phi) \equiv \begin{bmatrix} e^{i\phi}\cos(\theta/2) & i\sin(\theta/2) \\ i\sin(\theta/2) & e^{-i\phi}\cos(\theta/2) \end{bmatrix}, \tag{1}$$

$\hat{\Omega} \equiv \{\hat{\omega}(\theta, \phi) : \theta \in [0, \pi], \phi \in [0, \pi/2]\}$, $\hat{J} \equiv \cos(\gamma/2)\hat{I}^{\otimes n} + i\sin(\gamma/2)\hat{\sigma_x}^{\otimes n}$ (where $\gamma \in [0, \pi/2]$ is an entanglement measure, σ_x is Pauli matrix), $\hat{I} \equiv \hat{\omega}(0, 0)$, $\hat{D}_n \equiv \hat{\omega}(\pi, \pi/n)$, $\hat{C}_n \equiv \hat{\omega}(0, \pi/n)$.

Following the complete information assumption by Serrano (Page 392, the last paragraph, [4]), here we also assume there is complete information among

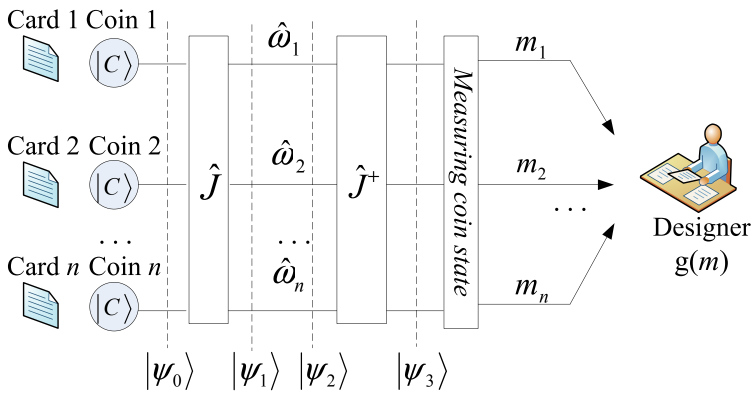

Fig. 1. The setup of a quantum mechanism. Each agent has a quantum coin and a card. Each agent independently performs a local unitary operation on his/her own quantum coin.

agents. Put differently, there is no private information for any agent. Without loss of generality, we assume that:

1) Each agent i has a quantum coin i (qubit) and a classical card i. The basis vectors $|C\rangle = [1,0]^T$, $|D\rangle = [0,1]^T$ of a quantum coin denote head up and tail up respectively.
2) Each agent i independently performs a local unitary operation on his/her own quantum coin. The set of agent i's operation is $\hat{\Omega}_i = \hat{\Omega}$. A strategic operation chosen by agent i is denoted as $\hat{\omega}_i \in \hat{\Omega}_i$. If $\hat{\omega}_i = \hat{I}$, then $\hat{\omega}_i(|C\rangle) = |C\rangle$, $\hat{\omega}_i(|D\rangle) = |D\rangle$; If $\hat{\omega}_i = \hat{D}_n$, then $\hat{\omega}_i(|C\rangle) = |D\rangle$, $\hat{\omega}_i(|D\rangle) = |C\rangle$. $\hat{I}$ denotes "*Not flip*", $\hat{D}_n$ denotes "*Flip*".
3) The two sides of a card are denoted as Side 0 and Side 1. The message written on the Side 0 (or Side 1) of card i is denoted as $card(i,0)$ (or $card(i,1)$). A typical card written by agent i is described as $c_i = (card(i,0), card(i,1))$. The set of c_i is denoted as C_i.
4) There is a device that can measure the state of n quantum coins and send messages to the designer.

A *quantum mechanism* $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$ describes a strategy set $\hat{S}_i = \hat{\Omega}_i \times C_i$ for each agent i and an outcome function $\hat{G} : \otimes_{i\in N}\hat{\Omega}_i \times \prod_{i\in N} C_i \to A$. We use $\hat{S}_{-i}$ to express $\otimes_{j\neq i}\hat{\Omega}_j \times \prod_{j\neq i} C_j$, and thus, a strategy profile is $\hat{s} = (\hat{s}_i, \hat{s}_{-i})$, where $\hat{s}_i \in \hat{S}_i$ and $\hat{s}_{-i} \in \hat{S}_{-i}$. The strategic behavior of each agent i is to strategically choose $\hat{\omega}_i$, $card(i,0)$ and $card(i,1)$.

A *Nash equilibrium* of a quantum mechanism Γ^Q played in state t is a strategy profile $\hat{s}^* = (\hat{s}_1^*, \cdots, \hat{s}_n^*)$ such that for any agent $i \in N$ and $\hat{s}_i \in \hat{S}_i$, $\hat{G}(\hat{s}_1^*, \cdots, \hat{s}_n^*) \succeq_i^t \hat{G}(\hat{s}_i, \hat{s}_{-i}^*)$. The setup of $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$ is depicted in Fig. 1. The working steps of Γ^Q are given as follows (with some differences in Step 2 from Ref. [2]):

Step 1: The state of every quantum coin is set as $|C\rangle$. The initial state of the

n quantum coins is $|\psi_0\rangle = \underbrace{|C \cdots CC\rangle}_{n}$.

Step 2: Given a state t, if two following conditions are satisfied, go to Step 4:
1) There exists $\hat{t} \in \mathcal{T}$, $\hat{t} \neq t$ such that $\hat{a} \succeq_i^t a$ (where $\hat{a} \in F(\hat{t})$, $a \in F(t)$) for every $i \in N$, with strict relation for some agent;
2) If there exists $\hat{t}' \in \mathcal{T}$, $\hat{t}' \neq \hat{t}$ that satisfies the former condition, then $\hat{a} \succeq_i^t \hat{a}'$ (where $\hat{a} \in F(\hat{t})$, $\hat{a}' \in F(\hat{t}'))$ for every $i \in N$, with strict relation for some agent.
Step 3: Each agent i sets $c_i = ((a_i, t_i, z_i), (a_i, t_i, z_i))$ (where $a_i \in A$, $t_i \in \mathcal{T}$, $z_i \in \mathbb{Z}_+$), $\hat{\omega}_i = \hat{I}$. Go to Step 7.
Step 4: Each agent i sets $c_i = ((\hat{a}, \hat{t}, 0), (a_i, t_i, z_i))$. Let n quantum coins be entangled by $\hat{J}$. $|\psi_1\rangle = \hat{J}|C \cdots CC\rangle$.
Step 5: Each agent i independently performs a local unitary operation $\hat{\omega}_i$ on his/her own quantum coin. $|\psi_2\rangle = [\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n]|\psi_1\rangle$.
Step 6: Let n quantum coins be disentangled by $\hat{J}^+$. $|\psi_3\rangle = \hat{J}^+|\psi_2\rangle$.
Step 7: The device measures the state of n quantum coins and sends $card(i, 0)$ (or $card(i, 1)$) as a message m_i to the designer if the state of quantum coin i is $|C\rangle$ (or $|D\rangle$).
Step 8: The designer receives the overall message $m = (m_1, \cdots, m_n)$ and let the final outcome be $g(m)$ using rules (1)-(3) of the Maskin's mechanism Γ.

3 An algorithmic mechanism

As specified by Ladd *et al* [3], nowadays the experimental technology for quantum information is still in its infancy. Thus, the quantum mechanism is valuable only from the theoretical aspect. In addition, it is hard to introduce the quantum notion into macro disciplines (such as economics, politics, sociology). In order to overcome these shortcomings, in this section we will first give mathematical representations of quantum states; then we will propose an algorithmic mechanism which can work in the macro world immediately.

3.1 Matrix representations of quantum states

In quantum mechanics, a quantum state can be described as a vector. For a two-level system, there are two basis vectors: $[1, 0]^T$ and $[0, 1]^T$. The matrix representations of quantum states $|\psi_0\rangle$, $|\psi_1\rangle$, $|\psi_2\rangle$ and $|\psi_3\rangle$ are given as follows.

$$|C\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \hat{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \hat{\sigma}_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, |\psi_0\rangle = \underbrace{|C \cdots CC\rangle}_{n} = \begin{bmatrix} 1 \\ 0 \\ \cdots \\ 0 \end{bmatrix}_{2^n \times 1} \tag{2}$$

$$\hat{J} = \cos(\gamma/2)\hat{I}^{\otimes n} + i\sin(\gamma/2)\hat{\sigma}_x^{\otimes n} \tag{3}$$

$$= \begin{bmatrix} \cos(\gamma/2) & & & & & i\sin(\gamma/2) \\ & \cdots & & & \cdots & \\ & & \cos(\gamma/2) & i\sin(\gamma/2) & & \\ & & i\sin(\gamma/2) & \cos(\gamma/2) & & \\ & \cdots & & & \cdots & \\ i\sin(\gamma/2) & & & & & \cos(\gamma/2) \end{bmatrix}_{2^n \times 2^n} \tag{4}$$

Here, the symbol i denotes an imaginary number. In what follows, we will not explicitly claim whether i is an imaginary number or an index. It is easy for the reader to know its exact meaning from the context. For $\gamma = \pi/2$,

$$\hat{J}_{\pi/2} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & & & & & i \\ & \cdots & & & \cdots & \\ & & 1 & i & & \\ & & i & 1 & & \\ & \cdots & & & \cdots & \\ i & & & & & 1 \end{bmatrix}_{2^n \times 2^n} \tag{5}$$

$$|\psi_1\rangle = \hat{J}|\psi_0\rangle = \begin{bmatrix} \cos(\gamma/2) \\ 0 \\ \cdots \\ 0 \\ i\sin(\gamma/2) \end{bmatrix}_{2^n \times 1} \tag{6}$$

Following formula (1), we define:

$$\hat{\omega}_1 = \begin{bmatrix} e^{i\phi_1}\cos(\theta_1/2) & i\sin(\theta_1/2) \\ i\sin(\theta_1/2) & e^{-i\phi_1}\cos(\theta_1/2) \end{bmatrix}, \cdots, \hat{\omega}_n = \begin{bmatrix} e^{i\phi_n}\cos(\theta_n/2) & i\sin(\theta_n/2) \\ i\sin(\theta_n/2) & e^{-i\phi_n}\cos(\theta_n/2) \end{bmatrix}, \tag{7}$$

The dimension of $\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n$ is $2^n \times 2^n$. Since only two values in $|\psi_1\rangle$ are non-zero, it is not necessary to calculate the whole $2^n \times 2^n$ matrix to obtain $|\psi_2\rangle$. Indeed, we only need to calculate the leftmost and rightmost column of $\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n$ to derive $|\psi_2\rangle = [\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n]|\psi_1\rangle$.

$$\hat{J}^+ = \begin{bmatrix} \cos(\gamma/2) & & & & & -i\sin(\gamma/2) \\ & \cdots & & & \cdots & \\ & & \cos(\gamma/2) & -i\sin(\gamma/2) & & \\ & & -i\sin(\gamma/2) & \cos(\gamma/2) & & \\ & \cdots & & & \cdots & \\ -i\sin(\gamma/2) & & & & & \cos(\gamma/2) \end{bmatrix}_{2^n \times 2^n} \tag{8}$$

$$|\psi_3\rangle = \hat{J}^+|\psi_2\rangle. \tag{9}$$

3.2 A simulating algorithm

Based on the matrix representations of quantum states, in the following we will propose a simulating algorithm that simulates the quantum operations and measurements specified in Steps 4-7 of the quantum mechanism. Since the entanglement measurement γ is a control factor, γ can be simply set as its maximum $\pi/2$. For n agents, the inputs and outputs of the simulating algorithm are illustrated in Fig. 2. The *Matlab* program is given in Fig. 3.

Inputs:
1) θ_i, ϕ_i, $i = 1, \cdots, n$: the parameters of agent i's local operation $\hat{\omega}_i$, $\theta_i \in [0, \pi], \phi_i \in [0, \pi/2]$.
2) $card(i,0), card(i,1)$, $i = 1, \cdots, n$: the information written on the two sides of agent i's card, where $card(i,0) = (a_i, t_i, z_i) \in A \times \mathcal{T} \times \mathbb{Z}_+$, $card(i,1) = (a'_i, t'_i, z'_i) \in A \times \mathcal{T} \times \mathbb{Z}_+$.

Outputs:
m_i, $i = 1, \cdots, n$: $m_i \in A \times \mathcal{T} \times \mathbb{Z}_+$.

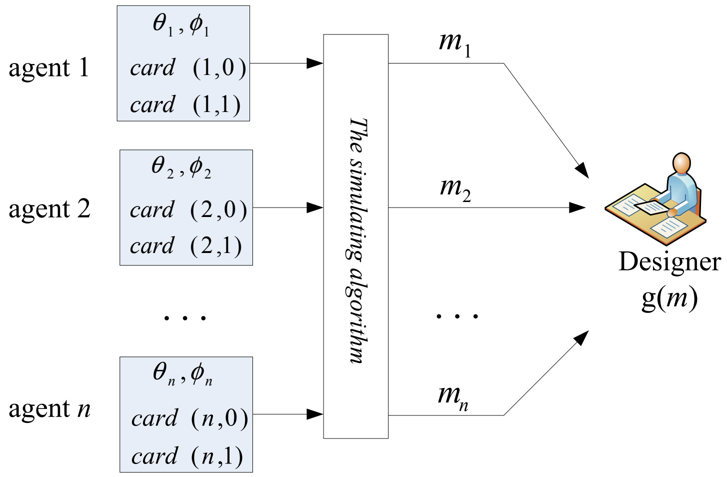

Fig. 2. The inputs and outputs of the simulating algorithm.

Procedures of the simulating algorithm:
Step 1: Reading two parameters θ_i and ϕ_i from each agent $i \in N$ (See Fig. 3(a)).
Step 2: Computing the leftmost and rightmost columns of $\hat{\omega}_1 \otimes \hat{\omega}_2 \otimes \cdots \otimes \hat{\omega}_n$ (See Fig. 3(b)).
Step 3: Computing the vector representation of $|\psi_2\rangle = [\hat{\omega}_1 \otimes \cdots \otimes \hat{\omega}_n]\hat{J}_{\pi/2}|\psi_0\rangle$.
Step 4: Computing the vector representation of $|\psi_3\rangle = \hat{J}^+_{\pi/2}|\psi_2\rangle$.
Step 5: Computing the probability distribution $\langle\psi_3|\psi_3\rangle$ (See Fig. 3(c)).
Step 6: Randomly choosing a state from the set of all 2^n possible states $\{|C\cdots CC\rangle, \cdots, |D\cdots DD\rangle\}$ according to the probability distribution $\langle\psi_3|\psi_3\rangle$.
Step 7: For each agent $i \in N$, the algorithm sends $card(i, 0)$ (or $card(i, 1)$) as m_i to the designer if the i-th basis vector of the chosen state is $|C\rangle$ (or $|D\rangle$) (See Fig. 3(d)).

Remark 1: In Step 6, the possible states $|C\cdots CC\rangle, \cdots, |D\cdots DD\rangle$ are simply mathematical notions, not physical entities.

Remark 2: Although the time and space complexity of the simulating algorithm are exponential, i.e., $O(2^n)$, it works well when the number of agents is not large. For example, the runtime of the simulating algorithm is about 0.5s for 15 agents, and about 12s for 20 agents (MATLAB 7.1, CPU: Intel (R) 2GHz, RAM: 3GB).

Remark 3: The problem of Nash implementation requires complete information among all agents. In the last paragraph of Page 392, Ref. [4], Serrano wrote: "*We assume that there is complete information among the agents... This assumption is especially justified when the implementation problem concerns a small number of agents that hold good information about one another*". Hence, the fact that the simulating algorithm is suitable for small-scale cases (e.g., less than 20 agents) is acceptable for Nash implementation.

3.3 An algorithmic mechanism that uses complex numbers

In the quantum mechanism $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$, the key parts are quantum operations and measurements. In Section 3.2, these parts are replaced by a simulating algorithm that uses complex numbers. Now we update the quantum mechanism $\Gamma^Q = ((\hat{S}_i)_{i\in N}, \hat{G})$ to an algorithmic mechanism.

Definition 1: An *algorithmic mechanism* $\widetilde{\Gamma} = ((\widetilde{S}_i)_{i\in N}, \widetilde{G})$ describes a strategy set $\widetilde{S}_i = [0,\pi] \times [0,\pi/2] \times C_i$ for each agent i and an outcome function $\widetilde{G} : [0,\pi]^n \times [0,\pi/2]^n \times \prod_{i\in N} C_i \to A$, where $n \geq 3$, C_i is the set of agent i's card $c_i = (card(i,0), card(i,1))$. A typical strategy chosen by agent i is denoted by $\widetilde{s}_i = (\theta_i, \phi_i, c_i)$, where $c_i = (card(i,0), card(i,1)) = (a_i, t_i, z_i, a_i', t_i', z_i')$. We use $\widetilde{S}_{-i}$ to express $[0,\pi]^{n-1} \times [0,\pi/2]^{n-1} \times \prod_{j\neq i} C_j$, and thus, a strategy profile is $\widetilde{s} = (\widetilde{s}_i, \widetilde{s}_{-i})$, where $\widetilde{s}_i = (\theta_i, \phi_i, c_i) \in \widetilde{S}_i$ and $\widetilde{s}_{-i} = (\theta_{-i}, \phi_{-i}, c_{-i}) \in \widetilde{S}_{-i}$.

Definition 2: Suppose each agent i submits $\widetilde{s}_i = (\theta_i, \phi_i, c_i)$, where $c_i = (a_i, t_i, z_i, a_i', t_i', z_i')$, the rules for the designer to compute the outcome function $\widetilde{G}(\widetilde{s})$ are defined as follows:

Rule 1: If for each agent $i \in N$, $(\theta_i, \phi_i) = (0, 0)$, then the designer sets $m_i = (a_i, t_i, z_i)$ $(i \in N)$, $m = (m_1, \cdots, m_n)$. Go to Rule 5;

Rule 2: If there exists one agent j satisfying the following two conditions, then $\widetilde{G}(\widetilde{s}) = a_j$:

(2.1) Agent j submits non-zero (θ_j, ϕ_j) (i.e., $(\theta_j, \phi_j) \neq (0, 0)$), $a_j \in F(t_j)$; for each agent $i \neq j$, $(\theta_i, \phi_i) = (0, 0)$;

(2.2) For each agent $i \in N$, $a_j \succeq_i^{t_j} a_i$;

Rule 3: If there are at least two agents that submit non-zero (θ_j, ϕ_j), and $(a_i, t_i, z_i) = (a, t, 0)$ $(i \in N)$ with $a \in F(t)$, then the designer transfers $(\theta_i, \phi_i, a_i, t_i, z_i, a_i', t_i', z_i')$ $(i \in N)$ to the simulating algorithm and gets $m = (m_1, \cdots, m_n)$; Go to Rule 5;

Rule 4: Otherwise the designer sets $m_i = (a_i, t_i, z_i)$ $(i \in N)$, $m = (m_1, \cdots, m_n)$;

Rule 5: If for each agent $i \in N$, $m_i = (a, t, 0)$ and $a \in F(t)$, then $\widetilde{G}(\widetilde{s}) = a$.

Rule 6: If there exists one agent $j \in N$ such that $m_i = (a, t, 0)$ (for each agent $i \neq j$) with $a \in F(t)$, but $m_j = (a_j, t_j, z_j) \neq (a, t, 0)$, then:

(6.1) If $(\theta_j, \phi_j) = (0, 0)$, then [$\widetilde{G}(\widetilde{s}) = a_j$ if $a \succ_j^t a_j$; $\widetilde{G}(\widetilde{s}) = a$ otherwise];

(6.2) If $(\theta_j, \phi_j) \neq (0, 0)$, then [$\widetilde{G}(\widetilde{s}) = a_j$ if $a \succ_j^t a_j$ and $a_j \in F(t_j)$; $\widetilde{G}(\widetilde{s}) = a$ otherwise];

Rule 7: The designer claims $\widetilde{G}(\widetilde{s}) = a'$, where a' is the outcome specified in $m = (m_1, \cdots, m_n)$ and chosen by the agent with the lowest index among those who has the highest integer.

Definition 3: A *Nash equilibrium* of the algorithmic mechanism $\widetilde{\Gamma}$ played in state t is a strategy profile $\widetilde{s}^* = (\widetilde{s}_1^*, \cdots, \widetilde{s}_n^*)$ such that for any agent $i \in N$, $\widetilde{s}_i \in \widetilde{S}_i$, $\widetilde{G}(\widetilde{s}_1^*, \cdots, \widetilde{s}_n^*) \succeq_i^t \widetilde{G}(\widetilde{s}_i, \widetilde{s}_{-i}^*)$.

3.4 Condition $\lambda^{\pi/2}$

Following condition λ in Ref. [2], here we define a condition $\lambda^{\pi/2}$.

Definition 4: Condition $\lambda^{\pi/2}$ contains seven parts. The first four parts are defined as follows:
1) $\lambda_1^{\pi/2}$: Given an SCR F, there exist two states $\hat{t}$, $\bar{t} \in \mathcal{T}$, $\hat{t} \neq \bar{t}$ such that $\hat{a} \succeq_i^{\bar{t}} \bar{a}$ (for each $i \in N$, $\hat{a} \in F(\hat{t})$, $\bar{a} \in F(\bar{t})$) with strict relation for some agent; and the number of agents that encounter a preference change around $\hat{a}$ in going from state $\hat{t}$ to $\bar{t}$ is at least two. Denote by l the number of these agents. Without loss of generality, let these l agents be the last l agents among n agents, i.e., agent $(n-l+1), \cdots, n$.
2) $\lambda_2^{\pi/2}$: Consider the state $\bar{t}$ specified in condition $\lambda_1^{\pi/2}$, if there exists another $\hat{t}' \in \mathcal{T}$, $\hat{t}' \neq \hat{t}$ that satisfies $\lambda_1^{\pi/2}$, then $\hat{a} \succeq_i^{\bar{t}} \hat{a}'$ (for each $i \in N$, $\hat{a} \in F(\hat{t})$, $\hat{a}' \in F(\hat{t}'))$ with strict relation for some agent.
3) $\lambda_3^{\pi/2}$: Consider the states $\hat{t}, \bar{t}$ specified in condition $\lambda_1^{\pi/2}$, if there is an agent $i \in N$ such that some outcome $a' \in A$ satisfies $a' \succ_i^{\bar{t}} \hat{a}$ with $a' \in F(t')$, then $a' \succ_i^{\hat{t}} \hat{a}$.
4) $\lambda_4^{\pi/2}$: Consider the outcome $\hat{a}$ specified in condition $\lambda_1^{\pi/2}$, for any state $t \in \mathcal{T}$, $\hat{a}$ is top ranked for each agent i among the first $(n-l)$ agents.

Definition 5: Consider the state $\bar{t}$ specified in condition $\lambda_1^{\pi/2}$. Suppose $\lambda_1^{\pi/2}$ and $\lambda_2^{\pi/2}$ are satisfied, and $m = (m_1, \cdots, m_m)$ is computed by the simulating algorithm in Rule 3. $\$_{C\cdots CC}$, $\$_{C\cdots CD}$, $\$_{D\cdots DC}$ and $\$_{D\cdots DD}$ are defined as the payoffs to the n-th agent in state $\bar{t}$ when the chosen states in Step 6 of the simulating algorithm are $|C\cdots CC\rangle$, $|C\cdots CD\rangle$, $|D\cdots DC\rangle$ and $|D\cdots DD\rangle$ respectively.

Note: 1) Here $|C\cdots CC\rangle$, $|C\cdots CD\rangle$, $|D\cdots DC\rangle$ and $|D\cdots DD\rangle$ are simply mathematical notions, not quantum states.
2) When an agent faces a certain outcome, his payoff is the utility that he exactly obtains; when an agent faces an uncertain outcome among a set of outcomes, his payoff is the ex-ante expected utility before the final outcome is realized.

The rest parts of condition $\lambda^{\pi/2}$ are defined as:
5) $\lambda_5^{\pi/2}$: For each agent i, when he faces an uncertain outcome among a set of outcomes, his payoff is the utility corresponding to the worst outcome among the set of outcomes.
6) $\lambda_6^{\pi/2}$: $\$_{C\cdots CC} > \$_{D\cdots DD}$.
7) $\lambda_7^{\pi/2}$: $\$_{C\cdots CC} > \$_{C\cdots CD}\cos^2(\pi/l) + \$_{D\cdots DC}\sin^2(\pi/l)$.

Proposition 1: For $n \geq 3$, given an SCR F that is monotonic and satisfies no-veto, suppose the designer uses the algorithmic mechanism $\widetilde{\Gamma}$, then F will not be Nash implementable if condition $\lambda^{\pi/2}$ is satisfied.

Proof: Since the designer uses the algorithmic mechanism $\widetilde{\Gamma}$, each agent i submits $\widetilde{s}_i = (\theta_i, \phi_i, c_i)$ (where $c_i = (a_i, t_i, z_i, a'_i, t'_i, z'_i)$) to the designer. As specified in condition $\lambda_1^{\pi/2}$ and $\lambda_2^{\pi/2}$, there exists two states $\hat{t}$, $\bar{t} \in \mathcal{T}$, $\hat{t} \neq \bar{t}$ such that $\hat{a} \in F(\hat{t})$ is Pareto superior to $\bar{a} \in F(\bar{t})$ in state $\bar{t}$. Suppose the true state is $\bar{t}$. For each agent i among the first $(n-l)$ agents, let $c_i = (\hat{a}, \hat{t}, 0, \hat{a}, \hat{t}, 0)$; for each agent i among the last l agents, let $c_i = (\hat{a}, \hat{t}, 0, a'_i, *, *)$, where a'_i is top ranked for agent i in state $\bar{t}$, and $*$ represents any legal value.

Now consider the n agents choose $\widetilde{s} = (\theta, \phi, c)$, where $\theta = (\underbrace{0, \cdots, 0}_{n})$, $\phi = (\underbrace{0, \cdots, 0}_{n-l}, \underbrace{\pi/l, \cdots, \pi/l}_{l})$, $c = (c_1, \cdots, c_n)$. Then, $\widetilde{G}$ enters Rule 3. According to the proof of Proposition 2 in Ref. [2], in Step 6 of the simulating algorithm, the probability that the chosen "collapsed" state is $|C \cdots CC\rangle$ is equal to 1. In Step 7 of the simulating algorithm, $m_i = card(i, 0) = (\hat{a}, \hat{t}, 0)$ for each agent $i \in N$. Thus, by Rule 5 of $\widetilde{G}$, $\widetilde{G}(\widetilde{s}) = \hat{a} \notin F(\bar{t})$.

Let us check whether in state $\bar{t}$, the n-th agent has an incentive to unilaterally deviate from $\widetilde{s}_n = (0, \pi/l, \hat{a}, \hat{t}, 0, a'_n, *, *)$. There are four possible deviations:

Case 1: Consider a unilateral deviation $\bar{s}_n = (0, \pi/l, \bar{a}_n, \bar{t}_n, \bar{z}_n, *, *, *)$, where $(\bar{a}_n, \bar{t}_n, \bar{z}_n) \neq (\hat{a}, \hat{t}, 0)$. Because each agent $(n-l+1) \leq i \leq (n-1)$ still submits $(0, \pi/l, \hat{a}, \hat{t}, 0, a'_i, *, *)$, in Step 6 of the simulating algorithm, the probability that the chosen state is $|C \cdots CC\rangle$ is still equal to 1. After Rule 3, $m_i = (\hat{a}, \hat{t}, 0)$ for $i = 1, \cdots, (n-1)$, $m_n = (\bar{a}_n, \bar{t}_n, \bar{z}_n)$. Thus, Rule (6.2) is triggered. Note that agent n can obtain $\hat{a}$ in state $\bar{t}$ if he submits $(0, \pi/l, \hat{a}, \hat{t}, 0, a'_n, *, *)$, hence it is not profitable for him to deviate and submit another outcome $\bar{a}_n$ with $\bar{a}_n \preceq_n^{\bar{t}} \hat{a}$. Therefore, there must be $\bar{a}_n \succ_n^{\bar{t}} \hat{a}$.
- Suppose $\bar{a}_n \in F(\bar{t}_n)$. Since $\lambda_3^{\pi/2}$ is satisfied, then $\bar{a}_n \succ_n^{\hat{t}} \hat{a}$. By Rule (6.2), $\widetilde{G}(\widetilde{s}_{-n}, \bar{s}_n) = \hat{a}$;
- Suppose $\bar{a}_n \notin F(\bar{t}_n)$, then by Rule (6.2), $\widetilde{G}(\widetilde{s}_{-n}, \bar{s}_n) = \hat{a}$.

Therefore, this unilateral deviation $\bar{s}_n$ is not profitable for agent n. By symmetry, in state $\bar{t}$, no agent i among the last l agents can profit by unilaterally submitting $\bar{s}_i = (0, \pi/l, \bar{a}_i, \bar{t}_i, \bar{z}_i, *, *, *)$, where $(\bar{a}_i, \bar{t}_i, \bar{z}_i) \neq (\hat{a}, \hat{t}, 0)$.

Case 2: Consider a unilateral deviation $\bar{s}_n = (\bar{\theta}_n, \bar{\phi}_n, \hat{a}, \hat{t}, 0, a'_n, *, *)$, where $(\bar{\theta}_n, \bar{\phi}_n) \neq (0, \pi/l)$. If $l = 2$ and $(\bar{\theta}_n, \bar{\phi}_n) = (0, 0)$, let $j = n - 1$, then Rules 2 is triggered. $\widetilde{G}(\widetilde{s}_{-n}, \bar{s}_n) = \hat{a}$. Thus, this deviation is not profitable for agent

n. Otherwise, Rule 3 is triggered. According to the proof of Proposition 2 in Ref. [2], the payoff $\$_n$ to agent n is given as follows:

$$\$_n = \$_{C\cdots CC} P_{C\cdots CC} + \$_{C\cdots CD} P_{C\cdots CD} + \$_{D\cdots DC} P_{D\cdots DC} + \$_{D\cdots DD} P_{D\cdots DD} \quad (10)$$

where $P_{C\cdots CC}$, $P_{C\cdots CD}$, $P_{D\cdots DC}$ and $P_{D\cdots DD}$ are the probabilities that the chosen state is $|C\cdots CC\rangle$, $|C\cdots CD\rangle$, $|D\cdots DC\rangle$, $|D\cdots DD\rangle$ respectively:

$$\begin{aligned}
P_{C\cdots CC} &= \cos^2(\bar{\theta}_n/2)[1-\sin^2(\bar{\phi}_n-\pi/l)];\\
P_{C\cdots CD} &= \sin^2(\bar{\theta}_n/2)\cos^2(\pi/l);\\
P_{D\cdots DC} &= \sin^2(\bar{\theta}_n/2)\sin^2(\pi/l);\\
P_{D\cdots DD} &= \cos^2(\bar{\theta}_n/2)\sin^2(\bar{\phi}_n-\pi/l)
\end{aligned}$$

Note that conditions $\lambda_6^{\pi/2}$, $\lambda_7^{\pi/2}$ are satisfied. Since $(\theta_i,\phi_i)=(0,0)$ for $i=1,\cdots,(n-l)$, and $(\theta_i,\phi_i)=(0,\pi/l)$ for $i=(n-l+1),\cdots,(n-1)$, then the optimal parameters $(\bar{\theta}_n,\bar{\phi}_n)$ for the n-th agent are $(0,\pi/l)$, and the maximum value of $\$_n$ is $\$_{C\cdots CC}=u_n(\hat{a},\bar{t})$ (see the proof of Proposition 2 in Ref. [2]).
Note that $(\bar{\theta}_n,\bar{\phi}_n)\neq(0,\pi/l)$, therefore $\$_n<\$_{C\cdots CC}=u_n(\hat{a},\bar{t})$. By symmetry, in state $\bar{t}$, no agent i among the last l agents can profit by unilaterally setting $\bar{s}_i=(\bar{\theta}_i,\bar{\phi}_i,\hat{a},\hat{t},0,a'_i,*,*)$, where $(\bar{\theta}_i,\bar{\phi}_i)\neq(0,\pi/l)$.

Case 3: Consider a unilateral deviation $\bar{s}'_n=(0,0,\bar{a}_n,\bar{t}_n,\bar{z}_n,\bar{a}'_n,*,*)$, where $l=2$, $(\bar{a}_n,\bar{t}_n,\bar{z}_n)\neq(\hat{a},\hat{t},0)$. Thus, $\tilde{s}_i=(0,0,\hat{a},\hat{t},0,\hat{a},\hat{t},0)$ for $i=1,\cdots,(n-2)$; $\tilde{s}_{n-1}=(0,\pi/2,\hat{a},\hat{t},0,a'_{n-1},*,*)$. Let $j=n-1$, then condition (2.1) is satisfied. If $\hat{a}\succeq_n^{\hat{t}}\bar{a}_n$, then Rules 2 is triggered and $\widetilde{G}(\tilde{s}_{-n},\bar{s}'_n)=\hat{a}$. Otherwise $\hat{a}\prec_n^{\hat{t}}\bar{a}_n$, hence Rule 2 is not triggered. Note that Rule 3 cannot be triggered either, $\widetilde{G}$ enters Rule 4. Let $j=n$, then Rule (6.1) is triggered. Since $\hat{a}\prec_n^{\hat{t}}\bar{a}_n$, $\widetilde{G}(\tilde{s}_{-n},\bar{s}'_n)=\hat{a}$. Hence, this deviation is not profitable for agent n.
By symmetry, in state $\bar{t}$, no agent i among the last l agents can profit by unilaterally setting $\bar{s}'_i=(0,0,\bar{a}_i,\bar{t}_i,\bar{z}_i,\bar{a}'_i,*,*)$, where $l=2$, $(\bar{a}_i,\bar{t}_i,\bar{z}_i)\neq(\hat{a},\hat{t},0)$.

Case 4: Consider a unilateral deviation $\bar{s}'_n=(\bar{\theta}_n,\bar{\phi}_n,\bar{a}_n,\bar{t}_n,\bar{z}_n,\bar{a}'_n,*,*)$, where $[l\geq 3$ or $(\bar{\theta}_n,\bar{\phi}_n)\neq(0,0)]$, $(\bar{\theta}_n,\bar{\phi}_n)\neq(0,\pi/l)$. Thus, Rule 3 is triggered. In Step 6 of the simulating algorithm, when the chosen state is $|C\cdots CC\rangle$, $|C\cdots CD\rangle$, $|D\cdots DC\rangle$ or $|D\cdots DD\rangle$, we denote the corresponding probability is $P'_{C\cdots CC}$, $P'_{C\cdots CD}$, $P'_{D\cdots DC}$, $P'_{D\cdots DD}$ respectively. In Step 7 of the simulating algorithm, we denote the corresponding message as m'_i. In the end, we denote the payoff to the n-th agent is $\$'_{C\cdots CC}$, $\$'_{C\cdots CD}$, $\$'_{D\cdots DC}$, $\$'_{D\cdots DD}$ respectively.

As a comparison, we construct a temporal deviation $\bar{s}_n=(\bar{\theta}_n,\bar{\phi}_n,\hat{a},\hat{t},0,a'_n,*,*)$, where $(\bar{\theta}_n,\bar{\phi}_n)$ is the same as above. In Step 6 of the simulating algorithm, when the chosen state is $|C\cdots CC\rangle$, $|C\cdots CD\rangle$, $|D\cdots DC\rangle$ or $|D\cdots DD\rangle$, we denote the corresponding probability is $P_{C\cdots CC}$, $P_{C\cdots CD}$, $P_{D\cdots DC}$, $P_{D\cdots DD}$ respectively. In Step 7 of the simulating algorithm, we denote the corresponding message as m_i. In the end, we denote the payoff to the n-th agent is $\$_{C\cdots CC}$, $\$_{C\cdots CD}$, $\$_{D\cdots DC}$, $\$_{D\cdots DD}$ respectively.

Since the two deviations $\bar{s}'_n$ and $\bar{s}_n$ have the same parameters $(\bar{\theta}_n, \bar{\phi}_n)$, the probabilities that the chosen state is $|C\cdots CC\rangle$, $|C\cdots CD\rangle$, $|D\cdots DC\rangle$ or $|D\cdots DD\rangle$ are the same, i.e., $P'_{C\cdots CC} = P_{C\cdots CC}$, $P'_{C\cdots CD} = P_{C\cdots CD}$, $P'_{D\cdots DC} = P_{D\cdots DC}$, and $P'_{D\cdots DD} = P_{D\cdots DD}$. Therefore, the payoff $\$'_n$ to agent n for the deviation $\bar{s}'_n$ can be written as follows:

$$\$'_n = \$'_{C\cdots CC}P_{C\cdots CC} + \$'_{C\cdots CD}P_{C\cdots CD} + \$'_{D\cdots DC}P_{D\cdots DC} + \$'_{D\cdots DD}P_{D\cdots DD} \quad (11)$$

Now we compare the payoffs to the n-th agent for two deviations $\bar{s}'_n$, $\bar{s}_n$:
1) Suppose the chosen state is $|C\cdots CC\rangle$. Then $m'_i = m_i = (\hat{a}, \hat{t}, 0)$ for each $i = 1, \cdots, n-1$; $m'_n = (\bar{a}_n, \bar{t}_n, \bar{z}_n)$, $m_n = (\hat{a}, \hat{t}, 0)$. For deviation $\bar{s}'_n$, the outcome cannot be better than $\hat{a}$ by Rule 6; for deviation $\bar{s}_n$, the outcome is $\hat{a}$ by Rule 5. Thus, $\$'_{C\cdots CC} \leq \$_{C\cdots CC}$.
2) Suppose the chosen state is $|C\cdots CD\rangle$. Then $m'_i = m_i = (\hat{a}, \hat{t}, 0)$ for each $i = 1, \cdots, n-1$; $m'_n = (\bar{a}'_n, *, *)$, $m_n = (a'_n, *, *)$. Since a'_n is top ranked for agent n in state $\bar{t}$, then $\bar{a}'_n \preceq^{\bar{t}}_n a'_n$ for any $\bar{a}'_n \in A$. Thus, by Rule 6, $\$'_{C\cdots CD} \leq \$_{C\cdots CD}$.
3) Suppose the chosen state is $|D\cdots DC\rangle$. Then $m'_i = m_i = (\hat{a}, \hat{t}, 0)$ for each $i = 1, \cdots, (n-l)$; $m'_i = m_i = (a'_i, *, *)$ for each $i = (n-l+1), \cdots, (n-1)$; $m'_n = (\bar{a}_n, \bar{t}_n, \bar{z}_n)$, $m_n = (\hat{a}, \hat{t}, 0)$.
For deviation $\bar{s}'_n$, the outcome is uncertain among $\{\hat{a}, a'_{n-l+1}, \cdots, a'_{n-1}, \bar{a}_n\}$; for deviation $\bar{s}_n$, the outcome is uncertain among $\{\hat{a}, a'_{n-l+1}, \cdots, a'_{n-1}\}$. Since $\lambda_5^{\pi/2}$ is satisfied, $\$'_{D\cdots DC} \leq \$_{D\cdots DC}$.
4) Suppose the chosen state is $|D\cdots DD\rangle$. Then $m'_i = m_i = (\hat{a}, \hat{t}, 0)$ for each $i = 1, \cdots, (n-l)$; $m'_i = m_i = (a'_i, *, *)$ for each $i = (n-l+1), \cdots, (n-1)$; $m'_n = (\bar{a}'_n, *, *)$, $m_n = (a'_n, *, *)$.
For deviation $\bar{s}'_n$, the outcome is uncertain among $\{\hat{a}, a'_{n-l+1}, \cdots, a'_{n-1}, \bar{a}'_n\}$; for deviation $\bar{s}_n$, the outcome is uncertain among $\{\hat{a}, a'_{n-l+1}, \cdots, a'_{n-1}, a'_n\}$. Since a'_n is top ranked for agent n in state $\bar{t}$, then $\bar{a}'_n \preceq^{\bar{t}}_n a'_n$ for any $\bar{a}'_n \in A$. Thus, by Rule (6.2), $\$'_{D\cdots DD} \leq \$_{D\cdots DD}$.

As a result, $\$'_n \leq \$_n$. Following Case 2, there holds $\$'_n < u_n(\hat{a}, \bar{t})$. Consequently, this deviation $\bar{s}'_n$ is not profitable for agent n. By symmetry, in state $\bar{t}$, no agent i among the last l agents can profit by a unilateral deviation $\bar{s}'_i = (\bar{\theta}'_i, \bar{\phi}'_i, \bar{a}_i, \bar{t}_i, \bar{z}_i, \bar{a}'_i, *, *)$, where $[l \geq 3$ or $(\bar{\theta}_i, \bar{\phi}_i) \neq (0, 0)]$, $(\bar{\theta}_i, \bar{\phi}_i) \neq (0, \pi/l)$.

Since $\lambda_4^{\pi/2}$ is satisfied, $\hat{a}$ is top ranked for each agent i among the first $(n-l)$ agents in state $\bar{t}$. Thus, no agent i among the first $(n-l)$ agents can profit by unilaterally changing his $\tilde{s}_i$. To sum up, $\tilde{s} \in \mathcal{N}(\widetilde{\Gamma}, \bar{t})$. Since $\widetilde{G}(\tilde{s}) = \hat{a} \notin F(\bar{t})$, F is not Nash implementable. $\square$

Remark 4: In Ref. [2], the setup of quantum mechanism is constructed by the agents to combat the designer who claims a bad social choice rule (from the agents' perspectives). In this paper, it is up to the designer to choose which mechanism to work. The designer can freely choose his favorite mechanism between the traditional Maskin's mechanism Γ and the algorithmic mechanism $\widetilde{\Gamma}$. Given an SCR F that is monotonic and satisfies no-veto, even if F would not be Nash implemented by virtue of the algorithmic mechanism $\widetilde{\Gamma}$, the designer can insist on choosing the Maskin's mechanism Γ as long as he is willing to implement F in Nash equilibrium. In this sense, the algorithmic mechanism has no effect to the designer.

An interesting question arises naturally: *by using the algorithmic mechanism $\widetilde{\Gamma}$, can the designer Nash implement an SCR F which is not Nash implementable according to the Maskin's theorem?* In the next Section, we will show a positive answer by constructing an example.

4 Example

Let $N = \{Apple, Lily, Cindy\}$, $\mathcal{T} = \{t^1, t^2, t^3\}$, $A = \{a^1, a^2, a^3, a^4, a^5\}$. In each state $t \in \mathcal{T}$, the preference relations $(\succeq_i^t)_{i\in N}$ over the outcome set A and two SCRs F, F' are given in Table 1. Obviously, F is monotonic and satisfies no-veto. Thus, by the Maskin's theorem, F is Nash implementable. Note that $F'(t^2) = \{a^1\}$ and in going from state t^2 to t^3, no preference change around a^1 is occurred, but $a^1 \notin F'(t^3) = \{a^5\}$. Therefore, F' does not satisfy monotonicity. According to the Maskin's theorem, F' cannot be Nash implemented.

Table 1: Consider two SCRs F and F' as follows. Although F' is Pareto-efficient to F, F' cannot be Nash implemented because it does not satisfy monotonicity; whereas F can be Nash implemented since it satisfies monotonicity and no-veto.

State t^1			State t^2			State t^3		
Apple	*Lily*	*Cindy*	*Apple*	*Lily*	*Cindy*	*Apple*	*Lily*	*Cindy*
a^3	a^2	a^1	a^4	a^3	a^1	a^4	a^3	a^1
a^5	a^5	a^3	a^5	a^5	a^2	a^5	a^5	a^5
a^1	a^1	a^2	a^1	a^1	a^3	a^1	a^1	a^2
a^2	a^4	a^5	a^2	a^2	a^4	a^2	a^2	a^3
a^4	a^3	a^4	a^3	a^4	a^5	a^3	a^4	a^4
$F'(t^1) = \{a^1\}$			$F'(t^2) = \{a^1\}$			$F'(t^3) = \{a^5\}$		
$F(t^1) = \{a^1\}$			$F(t^2) = \{a^2\}$			$F(t^3) = \{a^5\}$		

Note that the difference between F and F' occurs in state t^2, and every agent prefers $a^1 \in F'(t^2)$ to $a^2 \in F(t^2)$. Consequently, F' is Pareto superior to F

from the agents' viewpoints. Suppose the designer wishes to maximize the total social surplus, then he also prefers F' to F. However, *the Maskin's theorem says that it is F instead of F' that can be Nash implemented.* Can the designer find a way to break through the Maskin's theorem and let the Pareto-efficient SCR F' be Nash implementable? Interestingly, we will show that the answer may be "yes".

Let $\hat{t} = t^1$, $\hat{a} = a^1 \in F(\hat{t})$, $\bar{t} = t^2$, $\bar{a} = a^2 \in F(\bar{t})$, then $\hat{a} \succ_i^{\bar{t}} \bar{a}$ for every $i \in N$. Both *Apple* and *Lily* encounter a preference change around $\hat{a}$ in going from state $\hat{t}$ to $\bar{t}$. It can be easily checked that $\lambda_1^{\pi/2}$, $\lambda_2^{\pi/2}$, $\lambda_3^{\pi/2}$ and $\lambda_4^{\pi/2}$ are satisfied, and $l = 2$. Suppose $\lambda_5^{\pi/2}$ is satisfied.

To implement F' in Nash equilibrium, the designer can announce an algorithmic mechanism $\widetilde{\Gamma} = ((\widetilde{S}_i)_{i\in N}, \widetilde{G})$ as follows: The strategy set of each agent i is $S_i = [0,\pi] \times [0,\pi/2] \times A \times \mathcal{T} \times \mathbb{Z}_+ \times A \times \mathcal{T} \times \mathbb{Z}_+$; A typical strategy chosen by agent i is described as $\widetilde{s}_i = (\theta_i, \phi_i, a_i, t_i, z_i, a_i', t_i', z_i')$; The outcome function $\widetilde{G}(\widetilde{s})$ is just specified in Section 3.3. At first sight, the algorithmic mechanism $\widetilde{\Gamma}$ seems to be intended to implement F rather than F'. However, as we will show soon, in the end it is F' that will be Nash implemented.

Let *Cindy* be the first agent. Consider the strategy profile $\widetilde{s} = (\theta, \phi, c)$, where $\theta = (0,0,0)$, $\phi = (0, \pi/2, \pi/2)$, $c = (c_{Cindy}, c_{Apple}, c_{Lily})$, $c_{Cindy} = ((\hat{a}, \hat{t}, 0), (\hat{a}, \hat{t}, 0))$, $c_{Apple} = ((\hat{a}, \hat{t}, 0), (a^4, *, *))$, $c_{Lily} = ((\hat{a}, \hat{t}, 0), (a^3, *, *))$. Suppose $u_{Apple}(a^4, t^2) = 5$, $u_{Apple}(a^1, t^2) = 3$, $u_{Apple}(a^3, t^2) = 0$; $u_{Lily}(a^3, t^2) = 5$, $u_{Lily}(a^1, t^2) = 3$, $u_{Lily}(a^4, t^2) = 0$.

When the true state is t^2, for any agent $i \in \{Apple, Lily\}$, let her be the last agent and consider her payoff. Then, $\$_{CCC} = 3$ (the final outcome is a^1), $\$_{CCD} = 5$ (the final outcome is a^4 if $i = Apple$, and a^3 if $i = Lily$), $\$_{DDC} = 0$ (the final outcome is a^3 if $i = Apple$, and a^4 if $i = Lily$). By $\lambda_5^{\pi/2}$, $\$_{DDD} = 0$ (the final outcome is uncertain between a^3 and a^4). Hence, condition $\lambda_6^{\pi/2}$ and $\lambda_7^{\pi/2}$ are satisfied. Similar to the proof of Proposition 7 in Ref. [4], we need to show by two steps that for all $t \in \mathcal{T}$, the set of Nash equilibrium outcomes of the mechanism $\widetilde{\Gamma}$ coincides with $F'(t)$, i.e., $\widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t)) = F'(t)$.

4.1 Step 1: For all $t \in \mathcal{T}$, $F'(t) \subseteq \widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t))$

4.1.1 The true state is t^1

Consider the strategy profile used by the agents, $\widetilde{s}_i^* = (0, 0, a^1, t^1, 0, *, *, *)$ for every $i \in N$. First, note that this profile falls under Rule 1 of $\widetilde{G}$; then after Rule 5, $\widetilde{G}(\widetilde{s}^*) = a^1$ would be implemented. Now let us check whether in state t^1, there exists some agent j that has an incentive to unilaterally deviate from

$\widetilde{s}_j^*$:

(i) Suppose there exists some agent $j \in \{Apple, Lily\}$ that unilaterally submits $\widetilde{s}_j = (\theta_j, \phi_j, a^1, t^1, 0, *, *, *)$, where $(\theta_j, \phi_j) \neq (0, 0)$. Then, Rule 2 is triggered, and $\widetilde{G}(\widetilde{s}_{-j}^*, \widetilde{s}_j) = a^1$. Therefore, this deviation is not profitable for any agent $j \in \{Apple, Lily\}$.

(ii) Suppose there exists some agent $j \in \{Apple, Lily\}$ that unilaterally submits $\widetilde{s}_j = (0, 0, a_j, t_j, z_j, *, *, *)$, where $(a_j, t_j, z_j) \neq (a^1, t^1, 0)$. Note that after Rule 1, agent j could only hope to induce Rule (6.1). But since $\widetilde{s}^* = (\widetilde{s}_i^*)_{i \in N}$ includes a unanimous report of the true state t^1, agent j could only change the outcome if he chose an outcome that he does not prefer to a^1. Therefore, this deviation is not profitable for any agent $j \in \{Apple, Lily\}$.

(iii) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (\theta_{Apple}, \phi_{Apple}, a^3, *, *, *, *, *)$, where $(\theta_{Apple}, \phi_{Apple}) \neq (0, 0)$. Since $a^3 \notin F(t)$ for any $t \in \mathcal{T}$, Rule 2 cannot be triggered. On the other hand, Rule 3 cannot be triggered either. After Rule 4, Rule (6.2) is triggered, $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}_{Lily}^*, \widetilde{s}_{Cindy}^*) = a^1$ since $a^3 \notin F(t)$ for any $t \in \mathcal{T}$. Therefore, this deviation is not profitable for *Apple*.
Suppose *Lily* unilaterally submits $\widetilde{s}_{Lily} = (\theta_{Lily}, \phi_{Lily}, a^2, t_{Lily}, *, *, *, *)$, where $(\theta_{Lily}, \phi_{Lily}) \neq (0, 0)$. If $t_{Lily} = t^2$, since $a^2 \not\precsim_{Apple}^{t^2} a^1$, condition (2.2) is not satisfied; if $t_{Lily} \neq t^2$, then condition (2.1) is not satisfied. Hence, Rule 2 cannot be triggered. Note that Rule 3 cannot be triggered either. After Rule 4, Rule (6.2) is triggered. Since $a^1 \not\succ_{Lily}^{t^1} a^2$, $\widetilde{G}(\widetilde{s}_{Apple}^*, \widetilde{s}_{Lily}, \widetilde{s}_{Cindy}^*) = a^1$. Therefore, this deviation is not profitable for *Lily*.

(iv) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (\theta_{Apple}, \phi_{Apple}, a^5, t_{Apple}, *, *, *, *)$, where $(\theta_{Apple}, \phi_{Apple}) \neq (0, 0)$. If $t_{Apple} = t^3$, since $a^5 \not\precsim_{Cindy}^{t^3} a^1$, condition (2.2) is not satisfied; if $t_{Apple} \neq t^3$, then condition (2.1) is not satisfied. Hence, Rule 2 cannot be triggered. Note that Rule 3 cannot be triggered either. After Rule 4, Rule (6.2) is triggered. Since $a^1 \not\succ_{Apple}^{t^1} a^5$, $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}_{Lily}^*, \widetilde{s}_{Cindy}^*) = a^1$. Therefore, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally submit non-zero $(\theta_{Lily}, \phi_{Lily})$ and $c_{Lily} = (a^5, *, *, *, *, *)$.

It can be easily seen that for any agent $j \in \{Apple, Lily\}$, any other unilateral deviation from $\widetilde{s}_j^*$ in state t^1 will invoke Rule (6.2) and the outcome cannot be better than a^1. Since a^1 is already top ranked for *Cindy* in state t^1, it is not profitable for *Cindy* to unilaterally deviate from $\widetilde{s}_{Cindy}^*$.

As a result, $\widetilde{s}^* = (\widetilde{s}_i^*)_{i \in N} \in \mathcal{N}(\widetilde{\Gamma}, t^1)$, and $F'(t^1) = \{a^1\} \subseteq \widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t^1))$.

4.1.2 The true state is t^2

Consider the following strategy profile used by the agents,

$$\widetilde{s}^*_{Apple} = (0, \pi/2, a^1, t^1, 0, a^4, *, *),$$
$$\widetilde{s}^*_{Lily} = (0, \pi/2, a^1, t^1, 0, a^3, *, *),$$
$$\widetilde{s}^*_{Cindy} = (0, 0, a^1, t^1, 0, a^1, t^1, 0)$$

First, note that this profile falls under Rule 3 of $\widetilde{G}$. Since condition $\lambda^{\pi/2}$ is satisfied, after Rule 3 the designer will get $m_i = (a^1, t^1, 0)$ for each $i \in N$; and $\widetilde{G}(\widetilde{s}^*) = a^1$ would be implemented after Rule 5. Now let us check whether in state t^2, there exists some agent j that has an incentive to unilaterally deviate from $\widetilde{s}^*_j$:

(i) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (0, 0, a^1, t^1, 0, a^4, *, *)$. Let $j = Lily$, $a_j = a^1$, $t_j = t^1$. Note that for each $i \neq j$, $(\theta_i, \phi_i) = (0, 0)$; for each $i \in N$, $a_i = a^1$, so $a^1 \succeq^{t^1}_i a_i$. Hence, Rule 2 is triggered, and $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}^*_{Cindy}) = a^1$. Obviously, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally submit $(0, 0, a^1, t^1, 0, a^3, *, *)$.

(ii) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (0, 0, a^4, *, *, *, *, *)$. Let $j = Lily$, $a_j = a^1$, $t_j = t^1$. Note that for each $i \neq j$, $(\theta_i, \phi_i) = (0, 0)$; $a^1 \succeq^{t^1}_{Apple} a^4$, $a^1 \succeq^{t^1}_{Lily} a^1$, $a^1 \succeq^{t^1}_{Cindy} a^1$. Hence, Rule 2 is triggered and $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}^*_{Cindy}) = a^1$. Thus, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally submit $(0, 0, a^3, *, *, *, *, *)$.

(iii) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (0, 0, a^5, *, *, *, *, *)$. Let $j = Lily$, $a_j = a^1$, $t_j = t^1$. Since $a^1 \not\succeq^{t^1}_{Apple} a^5$, condition (2.2) is not satisfied. Note that Rule 3 cannot be triggered either. After Rule 4, Rule 6 is triggered. Now $j = Apple$, $a_j = a^5$, $a = a^1$, $t = t^1$. Since $(\theta_{Apple}, \phi_{Apple}) = (0, 0)$, Rule (6.1) is triggered. Since $a^1 \not\succ^{t^1}_{Apple} a^5$, $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}^*_{Cindy}) = a^1$. Thus, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally submit $(0, 0, a^5, *, *, *, *, *)$.

(iv) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (0, \pi/2, a^4, *, *, *, *, *)$. Then, Rule 4 is triggered. After Rule 4, $m_{Apple} = (a^4, *, *)$, $m_{Lily} = m_{Cindy} = (a^1, t^1, 0)$. Since $(\theta_{Apple}, \phi_{Apple}) = (0, \pi/2)$, Rule (6.2) is triggered. Since $a^4 \notin F(t)$ for any $t \in \mathcal{T}$, $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}^*_{Cindy}) = a^1$. Thus, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally submit $(0, \pi/2, a^3, *, *, *, *, *)$.

(v) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (0, \pi/2, a^5, *, *, *, *, *)$. Then, Rule 4 is triggered. After Rule 4, $m_{Apple} = (a^5, *, *)$, $m_{Lily} = m_{Cindy} = (a^1, t^1, 0)$. Since $(\theta_{Apple}, \phi_{Apple}) = (0, \pi/2)$, Rule (6.2) is triggered. Since $a^1 \not\succ^{t^1}_{Apple} a^5$, by Rule (6.2) $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}^*_{Cindy}) = a^1$. Thus, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally submit $(0, \pi/2, a^5, *, *, *, *, *)$.

(vi) Suppose *Apple* unilaterally sets $\widetilde{s}_{Apple} = (\theta_{Apple}, \phi_{Apple}, a^1, t^1, 0, a^4, *, *)$, where $(\theta_{Apple}, \phi_{Apple}) \neq (0,0)$, $(\theta_{Apple}, \phi_{Apple}) \neq (0, \pi/2)$. Thus, Rule 3 is triggered. Since c_{Apple} is not changed, then the payoffs $\$_{CCC}$, $\$_{CCD}$, $\$_{DDC}$ and $\$_{DDD}$ to *Apple* remain unchanged. According to the proof of Proposition 1, the optimal value of $(\theta_{Apple}, \phi_{Apple})$ is $(0, \pi/2)$. Put differently, for any arbitrary $(\theta_{Apple}, \phi_{Apple}) \neq (0, \pi/2)$, $\$_{Apple} < \$_{CCC} = u_{Apple}(a^1, t^2)$. Thus, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally set $\widetilde{s}_{Lily} = (\theta_{Lily}, \phi_{Lily}, a^1, t^1, 0, a^3, *, *)$, where $(\theta_{Lily}, \phi_{Lily}) \neq (0,0)$, $(\theta_{Lily}, \phi_{Lily}) \neq (0, \pi/2)$.

(vii) Suppose *Apple* unilaterally sets $\widetilde{s}_{Apple} = (\theta_{Apple}, \phi_{Apple}, a^1, t^1, 0, a^5, *, *)$, where $(\theta_{Apple}, \phi_{Apple}) \neq (0,0)$, $(\theta_{Apple}, \phi_{Apple}) \neq (0, \pi/2)$. Thus, Rule 3 is triggered. It can be seen easily that $\$_{CCC}$ and $\$_{DDC}$ to *Apple* remain unchanged by this deviation. Consider the payoff $\$_{CCD}$ to *Apple*, then the corresponding chosen state is $|CCD\rangle$. Thus, $m_{Apple} = (a^5, *, *)$, $m_{Lily} = m_{Cindy} = (a^1, t^1, 0)$. Since $(\theta_{Apple}, \phi_{Apple}) \neq (0,0)$, Rule (6.2) is triggered. Because $a^1 \not\succ^{t^1}_{Apple} a^5$, the outcome is still a^1. Hence, the payoff $\$_{CCD}$ to *Apple* is not changed by this deviation.

Consider the payoff $\$_{DDD}$ to *Apple*, then the corresponding chosen state is $|DDD\rangle$. Thus, $m_{Apple} = (a^5, *, *)$, $m_{Lily} = (a^3, *, *)$, $m_{Cindy} = (a^1, t^1, 0)$. The outcome is uncertain among a set of outcomes $\{a^1, a^3, a^5\}$. Note that for $\widetilde{s}^* = (\widetilde{s}^*_i)_{i \in N}$, the outcome is uncertain among a set of outcomes $\{a^1, a^3, a^4\}$. By condition $\lambda_5^{\pi/2}$, the payoff $\$_{DDD}$ to *Apple* remains unchanged.

Thus, conditions $\lambda_6^{\pi/2}$ and $\lambda_7^{\pi/2}$ still hold. According to the proof of Proposition 1, the optimal parameters $(\theta_{Apple}, \phi_{Apple})$ is $(0, \pi/2)$. Put differently, for any arbitrary $(\theta_{Apple}, \phi_{Apple}) \neq (0, \pi/2)$, $\$_{Apple} < \$_{CCC} = u_{Apple}(a^1, t^2)$. Therefore, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally set $\widetilde{s}_{Lily} = (\theta_{Lily}, \phi_{Lily}, a^1, t^1, 0, a^5, *, *)$, where $(\theta_{Lily}, \phi_{Lily}) \neq (0,0)$, $(\theta_{Lily}, \phi_{Lily}) \neq (0, \pi/2)$.

(viii) Suppose *Apple* unilaterally sets $\widetilde{s}_{Apple} = (\theta_{Apple}, \phi_{Apple}, a_j, t_j, z_j, *, *, *)$, where $(\theta_{Apple}, \phi_{Apple}) \neq (0,0)$, $(a_j, t_j, z_j) \neq (a^1, t^1, 0)$. Thus, Rule 4 is triggered. After then, Rule (6.2) is triggered. Consider the payoff $\$_{CCC}$ and $\$_{CCD}$ to *Apple*, since *Apple* can only obtain an outcome not better than a^1 by Rule (6.2), the payoff $\$_{CCC}$ and $\$_{CCD}$ to *Apple* cannot be increased by this deviation.

Consider the payoff $\$_{DDC}$ and $\$_{DDD}$ to *Apple*, no matter whether the chosen state is $|DDC\rangle$ or $|DDD\rangle$, $m_{Lily} = (a^3, *, *)$, $m_{Cindy} = (a^1, *, *)$. Since a^3 is the worst outcome for *Apple* in state t^2, by $\lambda_5^{\pi/2}$, the payoff $\$_{DDC}$ and $\$_{DDD}$ to

Apple remain unchanged by this deviation. According to Case 3 in the proof of Proposition 1, this deviation is not profitable for *Apple*. Similarly, it is not profitable for *Lily* to unilaterally set $\widetilde{s}_{Lily} = (\theta_{Lily}, \phi_{Lily}, a_j, t_j, z_j, *, *, *)$, where $(\theta_{Lily}, \phi_{Lily}) \neq (0, 0)$, $(a_j, t_j, z_j) \neq (a^1, t^1, 0)$.

It can be seen that for agent *Apple*, any other unilateral deviation from $\widetilde{s}^*_{Apple}$ in state t^2 cannot make *Apple* obtain more payoffs. Similarly, for agent *Lily*, there is no unilateral profitable deviation from $\widetilde{s}^*_{Lily}$ in state t^2. Since a^1 is already top ranked for *Cindy*, it is not profitable for *Cindy* to unilaterally deviate from $\widetilde{s}^*_{Cindy}$ in state t^2. As a result, $\widetilde{s}^* = (\widetilde{s}^*_i)_{i \in N} \in \mathcal{N}(\widetilde{\Gamma}, t^2)$, and $F'(t^2) = \{a^1\} \subseteq \widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t^2))$.

4.1.3 The true state is t^3

Consider the strategy profile used by the agents, $\widetilde{s}^*_i = (0, 0, a^5, t^3, 0, *, *, *)$ for every $i \in N$. First, note that this profile falls under Rule 1 of $\widetilde{G}$; then after Rule 5, $\widetilde{G}(\widetilde{s}^*) = a^5$ would be implemented. Now let us check whether in state t^3, there exists some agent j that has an incentive to unilaterally deviate from $\widetilde{s}^*_j$:

(i) Suppose there exists some agent $j \in N$ that unilaterally submits $\widetilde{s}_j = (\theta_j, \phi_j, a^5, t^3, 0, *, *, *)$, where $(\theta_j, \phi_j) \neq (0, 0)$. Then, Rule 2 is triggered, and $\widetilde{G}(\widetilde{s}^*_{-j}, \widetilde{s}_j) = a^5$. Therefore, this deviation is not profitable for any agent $j \in N$.

(ii) Suppose there exists some agent $j \in N$ that unilaterally submits $\widetilde{s}_j = (0, 0, a_j, t_j, z_j, *, *, *)$, where $(a_j, t_j, z_j) \neq (a^5, t^3, 0)$. Note that after Rule 1, agent j could only hope to induce Rule (6.1). But since $\widetilde{s}^* = (\widetilde{s}^*_i)_{i \in N}$ includes a unanimous report of the true state t^3, agent j could only change the outcome if he chose an outcome that he does not prefer to a^5. Therefore, this deviation is not profitable for any agent $j \in N$.

(iii) Suppose *Apple* unilaterally submits $\widetilde{s}_{Apple} = (\theta_{Apple}, \phi_{Apple}, a^4, *, *, *, *, *)$, where $(\theta_{Apple}, \phi_{Apple}) \neq (0, 0)$. Since $a^4 \notin F(t)$ for any $t \in \mathcal{T}$, Rule 2 cannot be triggered. Note that Rule 3 cannot be triggered either. After Rule 4, Rule (6.2) is triggered. Since $a^4 \notin F(t)$ for any $t \in \mathcal{T}$, $\widetilde{G}(\widetilde{s}_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}^*_{Cindy}) = a^5$. Therefore, this deviation is not profitable for *Apple*.

Suppose *Lily* unilaterally submits $\widetilde{s}_{Lily} = (\theta_{Lily}, \phi_{Lily}, a^3, *, *, *, *, *)$, where $(\theta_{Lily}, \phi_{Lily}) \neq (0, 0)$. Since $a^3 \notin F(t)$ for any $t \in \mathcal{T}$, Rule 2 cannot be triggered. Note that Rule 3 cannot be triggered either. After Rule 4, Rule (6.2) is triggered. Since $a^3 \notin F(t)$ for any $t \in \mathcal{T}$, $\widetilde{G}(\widetilde{s}^*_{Apple}, \widetilde{s}_{Lily}, \widetilde{s}^*_{Cindy}) = a^5$. Therefore, this deviation is not profitable for *Lily*.

Suppose *Cindy* unilaterally submits $\widetilde{s}_{Cindy} = (\theta_{Cindy}, \phi_{Cindy}, a^1, t_{Cindy}, *, *, *, *)$, where $(\theta_{Cindy}, \phi_{Cindy}) \neq (0, 0)$. If $t_{Cindy} = t^1$, since $a^1 \not\succsim^{t^1}_{Apple} a^5$, condition (2.2) is not satisfied; if $t_{Cindy} \neq t^1$, then condition (2.1) is not satisfied. Hence, Rule 2 is not triggered. Note that Rule 3 cannot be triggered either. After Rule 4, Rule (6.2) is triggered. Since $a^5 \not\succeq^{t^3}_{Cindy} a^1$, $\widetilde{G}(\widetilde{s}^*_{Apple}, \widetilde{s}^*_{Lily}, \widetilde{s}_{Cindy}) = a^5$. Hence, this deviation is not profitable for $Cindy$.

It can be easily seen that for any agent $j \in N$, any other unilateral deviation from $\widetilde{s}^*_j$ in state t^3 will invoke Rule (6.2) and the outcome cannot be better than a^5. As a result, $\widetilde{s}^* = (\widetilde{s}^*_i)_{i\in N} \in \mathcal{N}(\widetilde{\Gamma}, t^3)$, and $F'(t^3) = \{a^5\} \subseteq \widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t^3))$.

4.2 Step 2: For all $t \in \mathcal{T}$, $\widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t)) \subseteq F'(t)$

Given any state $t \in \mathcal{T}$, let $\widetilde{s} \in \mathcal{N}(\widetilde{\Gamma}, t)$ and let a be the corresponding outcome according to $\widetilde{G}$. Suppose that a is a result of either Rule 6 or 7, then there exists $j \in N$ such that every $k \neq j$ can increase his payoff by choosing two high enough integers z_k and z'_k. Therefore, a must be top ranked for at least $(n-1)$ agents. However, it can be seen from Table 1 that no outcome is top ranked for at least $(n-1)$ agents in any state $t \in \mathcal{T}$. Therefore, the Nash equilibrium outcome of $\widetilde{\Gamma}$ cannot be yielded by Rule 6 or 7.

Lemma 1: a is not a result of Rule 2.
Proof: Suppose a is a result of Rule 2, then there exists one agent j that submits non-zero (θ_j, ϕ_j) with $a_j \in F(t_j)$; for each agent $i \neq j$, $(\theta_i, \phi_i) = (0, 0)$; for each agent $i \in N$, $a_j \succeq^{t_j}_i a_i$. Now let us check whether there exists some agent $k \neq j$ who can profit by unilaterally setting $(\theta_k, \phi_k) \neq (0, 0)$ and two large enough integers $z_k > 0$, $z'_k > 0$.

First, note that by this deviation, Rule 2 cannot be triggered because $(\theta_j, \phi_j) \neq (0, 0)$, $(\theta_k, \phi_k) \neq (0, 0)$; and Rule 3 cannot be triggered because $z_k > 0$. Hence, Rule 4 is triggered. After Rule 4, suppose Rule 6 can be triggered when some agent $k \neq j$ deviates. Since $z_k > 0$, then for every agent $i \neq k$, there must be $m_i = (a_j, t_j, 0)$ and $a_j \in F(t_j)$. Consequently, a_j should be top ranked for every agent $i \neq k$. Otherwise, by using Rule 7, any agent i' ($i' \neq k$, $i' \neq j$) can invoke his top ranked outcome by submitting two high enough integers $z_{i'}$ and $z'_{i'}$. However, for the case of Table 1, there is no outcome that is top ranked for $(n-1)$ agents in any state. Therefore, Rule 6 cannot be triggered when some agent $k \neq j$ deviates.

We are left with Rule 7. Note that when any agent $k \neq j$ deviates, according to Rule 7, he can invoke his top ranked outcome by submitting two large enough integers z_k and z'_k. Hence, in order to let a be a Nash equilibrium outcome of Rule 2, a must be top ranked for at least $(n-1)$ agents. However, for the case of Table 1, no outcome can be top ranked for at least $(n-1)$ agents in any state. As a result, a cannot be a result of Rule 2. $\square$

Finally we are left with a being a result of Rule 5, then there is a unanimous $m = (a, t, 0)$ with $a \in F(t)$. Consider the following three cases:

4.2.1 The true state is t^1

As we have seen in Section 4.1.1, one Nash equilibrium strategy for every agent i is the unanimous report $\widetilde{s}_i^* = (0, 0, a^1, t^1, 0, *, *, *)$, and the Nash equilibrium outcome is a^1.

Suppose there is another Nash equilibrium strategy $\bar{s} = (\bar{s}_i)_{i \in N}$ which yields a unanimous $\bar{m} = (\bar{a}, \bar{t}, 0)$ with $\bar{a} \in F(\bar{t})$, $\bar{a} \neq a^1$. Then for each agent $i \in N$, there are two cases about $\bar{s}_i$: $\bar{s}_i = (\bar{\theta}_i, \bar{\phi}_i, \bar{a}, \bar{t}, 0, \bar{a}'_i, \bar{t}'_i, \bar{z}'_i)$, or $\bar{s}_i = (\bar{\theta}_i, \bar{\phi}_i, \bar{a}_i, \bar{t}_i, \bar{z}_i, \bar{a}, \bar{t}, 0)$.

Because F is monotonic, in going from state $\bar{t}$ to t^1, a preference change around $\bar{a}$ must have occurred, i.e., there exists $i \in N$ and $b \in A$, such that $\bar{a} \succeq_i^{\bar{t}} b$, and $b \succ_i^{t^1} \bar{a}$. Therefore, agent i can profit by the following unilateral deviation from $\bar{s}_i$: for the former case of $\bar{s}_i$, agent i sends $(\bar{\theta}_i, \bar{\phi}_i, b, *, *, \bar{a}'_i, \bar{t}'_i, \bar{z}'_i)$; for the latter case of $\bar{s}_i$, agent i sends $(\bar{\theta}_i, \bar{\phi}_i, \bar{a}_i, \bar{t}_i, \bar{z}_i, b, *, *)$. By doing so, the outcome b would be implemented by Rule (6.1) and agent i profits from this deviation. Thereby contradicting $\bar{s} = (\bar{s}_i)_{i \in N}$ is a Nash equilibrium.

As a result, $\widetilde{s}^* = (\widetilde{s}_i^*)_{i \in N}$, where $\widetilde{s}_i^* = (0, 0, a^1, t^1, 0, *, *, *)$, is the unique Nash equilibrium in state t^1. $\widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t^1)) = \{a^1\} \subseteq F'(t^1)$.

4.2.2 The true state is t^2

As we have seen in Section 4.1.2, there is one Nash equilibrium yielded by Rules 3 and 5. Note that in Rule 5, a unanimous $m = (a, t, 0)$ with $a \in F(t)$ may come from Rule 1, 3 or 4.

Suppose a comes from Rules 4 and 5, then for each $i \in N$, $\bar{s}_i = (\bar{\theta}_i, \bar{\phi}_i, a, t, 0, *, *, *)$. Note that Rule 3 should not be triggered before Rule 4. Since $card(i, 0) = (a, t, 0)$ for each $i \in N$, then there exists only one agent j such that $(\bar{\theta}_j, \bar{\phi}_j) \neq (0, 0)$, and $(\bar{\theta}_i, \bar{\phi}_i) = (0, 0)$ for every $i \neq j$. Since there is a unanimous $m = (a, t, 0)$ with $a \in F(t)$, condition (2.1) is satisfied. Because Rule 2 should not be triggered before Rule 3, condition (2.2) should not be satisfied. Therefore, there must exist some $k \in N$ such that $a_j \not\succeq_k^{t_j} a_k$. Note that $a_i = a$, $t_i = t$ for every $i \in N$. Then, $a \not\succeq_k^t a$. This contradiction shows that a cannot come from Rules 4 and 5.

Let us check whether in state t^2, there is another unanimous $m = (a, t, 0)$ with $a \in F(t)$ come from Rules 1 and 5. There are three possible options:

(i) Suppose every agent i submits a unanimous report $(0,0,a^2,t^2,0,*,*,*)$. Thus, Rule 1 would be triggered and $m_i=(a,t,0)=(a^2,t^2,0)$ $(i\in N)$. The outcome would be a^2 yielded by Rule 5. However, *Apple* has an incentive to unilaterally deviate and submit $(0,\pi/2,a^5,t^3,0,*,*,*)$. Let $j=Apple$, $a_j=a^5$, $t_j=t^3$. Note that for each $i\neq Apple$, $(\theta_i,\phi_i)=(0,0)$, $a_i=a^2$; for each $i\in N$, $a^5\succeq_i^{t^3}a_i$. Thus, Rule 2 would be triggered and a profitable outcome a^5 for *Apple* would be implemented. Similarly, *Lily* has an incentive to unilaterally deviate and submit $(0,\pi/2,a^5,t^3,0,*,*,*)$. Consequently, in state t^2, the unanimous report $(0,0,a^2,t^2,0,*,*,*)$ cannot be a Nash equilibrium.

(ii) Suppose every agent i submits a unanimous report $(0,0,a^1,t^1,0,*,*,*)$. Thus, Rule 1 would be triggered and $m_i=(a,t,0)=(a^1,t^1,0)$ $(i\in N)$. The outcome would be a^1 yielded by Rule 5. However, in going from state t^1 to t^2, a preference change around a^1 occurs for *Apple*, i.e., $a^1\succ_{Apple}^{t^1}a^4$, and $a^4\succ_{Apple}^{t^2}a^1$. Thus, *Apple* has an incentive to unilaterally deviate and submit $(0,0,a^4,*,*,*,*,*)$. After Rule 1, let $j=Apple$, $a_j=a^4$, $a=a^1$, $t=t^1$. Since $a^1\succ_{Apple}^{t^1}a^4$, a profitable outcome a^4 for *Apple* would be implemented by Rule (6.1). Similarly, *Lily* has an incentive to unilaterally deviate and submit $(0,0,a^3,*,*,*,*,*)$. Thus, in state t^2, the unanimous report $(0,0,a^1,t^1,0,*,*,*)$ cannot be a Nash equilibrium.

(iii) Suppose every agent i submits a unanimous report $(0,0,a^5,t^3,0,*,*,*)$. Thus, Rule 1 would be triggered and $m_i=(a,t,0)=(a^5,t^3,0)$ $(i\in N)$. The outcome would be a^5 yielded by Rule 5. However, in going from state t^3 to t^2, a preference change around a^5 occurs for *Cindy*, i.e., $a^5\succ_{Cindy}^{t^3}a^2$, and $a^2\succ_{Cindy}^{t^2}a^5$. Thus, *Cindy* has an incentive to unilaterally deviate and submit $(0,0,a^2,*,*,*,*,*)$. After Rule 1, let $j=Cindy$, $a_j=a^2$, $a=a^5$, $t=t^3$. Since $a^5\succ_{Cindy}^{t^3}a^2$, a profitable outcome a^2 for *Cindy* would be implemented by Rule (6.1). Thus, in state t^2, the unanimous report $(0,0,a^5,t^3,0,*,*,*)$ cannot be a Nash equilibrium.

As a result, in state t^2, there is no Nash equilibrium come from Rules 1 and 5. Put differently, any Nash equilibrium must come from Rules 3 and 5 in state t^2. According to Section 4.1.2, in state t^2, the unique Nash equilibrium is $\widetilde{s}^*=(\widetilde{s}^*_{Apple},\widetilde{s}^*_{Lily},\widetilde{s}^*_{Cindy})$, where $\widetilde{s}^*_{Apple}=(0,\pi/2,a^1,t^1,0,a^4,*,*)$, $\widetilde{s}^*_{Lily}=(0,\pi/2,a^1,t^1,0,a^3,*,*)$, $\widetilde{s}^*_{Cindy}=(0,0,a^1,t^1,0,a^1,t^1,0)$. Consequently, $\widetilde{G}(\mathcal{N}(\widetilde{\Gamma},t^2))=\{a^1\}\subseteq F'(t^2)$.

4.2.3 The true state is t^3

As we have seen in Section 4.1.3, one Nash equilibrium strategy for every agent i is the unanimous report $\widetilde{s}^*_i=(0,0,a^5,t^3,0,*,*,*)$, and the Nash equilibrium outcome is a^5.

Suppose there is another Nash equilibrium strategy $\bar{s} = (\bar{s}_i)_{i\in N}$ which yields a unanimous $\bar{m} = (\bar{a}, \bar{t}, 0)$ with $\bar{a} \in F(\bar{t})$, $\bar{a} \neq a^5$. Then for each $i \in N$, there are two cases about $\bar{s}_i$: $\bar{s}_i = (\bar{\theta}_i, \bar{\phi}_i, \bar{a}, \bar{t}, 0, \bar{a}'_i, \bar{t}'_i, \bar{z}'_i)$, or $\bar{s}_i = (\bar{\theta}_i, \bar{\phi}_i, \bar{a}_i, \bar{t}_i, \bar{z}_i, \bar{a}, \bar{t}, 0)$.

Because F is monotonic, in going from state $\bar{t}$ to t^3, a preference change around $\bar{a}$ must have occurred, i.e., there exists $i \in N$ and $b \in A$, such that $\bar{a} \succeq_i^{\bar{t}} b$, and $b \succ_i^{t^3} \bar{a}$. Therefore, agent i can profit by the following unilateral deviation from $\bar{s}_i$: for the former case of $\bar{s}_i$, agent i sends $(\bar{\theta}_i, \bar{\phi}_i, b, *, *, \bar{a}'_i, \bar{t}'_i, \bar{z}'_i)$; for the latter case of $\bar{s}_i$, agent i sends $(\bar{\theta}_i, \bar{\phi}_i, \bar{a}_i, \bar{t}_i, \bar{z}_i, b, *, *)$. By doing so, the outcome b would be implemented by Rule (6.1) and agent i profits from this deviation. Thereby contradicting $\bar{s} = (\bar{s}_i)_{i\in N}$ is a Nash equilibrium.

As a result, $\widetilde{s}^* = (\widetilde{s}_i^*)_{i\in N}$, $\widetilde{s}_i^* = (0, 0, a^5, t^3, 0, *, *, *)$, is the unique Nash equilibrium in state t^3. $\widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t^3)) = \{a^5\} \subseteq F'(t^3)$.

To sum up, for all $t \in \mathcal{T}$, $\widetilde{G}(\mathcal{N}(\widetilde{\Gamma}, t)) = F'(t)$. Although the Pareto-efficient SCR F' is not monotonic, the designer can implement it in Nash equilibrium by using the algorithmic mechanism $\widetilde{\Gamma}$ if condition $\lambda^{\pi/2}$ is satisfied.

5 Conclusions

In this paper, we propose an algorithmic mechanism to go beyond the obstacle of how to realize the quantum mechanism. It should be noted that the introduction of complex numbers is a novel idea to the theory of mechanism design. To the best of our knowledge, up to now there is no similar work before. Since the algorithmic mechanism works in the macro world, the Maskin's theorem are amended immediately in the macro world.

Furthermore, we propose that by using the algorithmic mechanism, a Pareto-efficient social choice rule that is not monotonic may be Nash implemented. This result is positive not only to the agents, but also to the designer if the designer wishes to maximize the total social surplus. Since the Maskin's mechanism has been widely applied to many disciplines, there are many works to do in the future to generalize the algorithmic mechanism further.

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```
start_time = cputime

% n: the number of agents. In Table 1, there are 3 agents: Apple, Lily, Cindy
n=3;

% γ : the coefficient of entanglement. It is simply set to its maximum π/2.
gamma=pi/2;

% Defining the array of θ_i and φ_i, i=1,···,n.
theta=zeros(n,1);
phi=zeros(n,1);

% Reading Apple's parameters. For example, ω̂_1 = Ĉ_2 = ω̂(0,π/2)
theta(1)=0;
phi(1)=pi/2;

% Reading Lily's parameters. For example, ω̂_2 = Ĉ_2 = ω̂(0,π/2)
theta(2)=0;
phi(2)=pi/2;

% Reading Cindy's parameters. For example, ω̂_3 = Î = ω̂(0,0)
theta(3)=0;
phi(3)=0;
```

Fig. 3 (a). Reading each agent *i*'s parameters θ_i and $\phi_i, i=1,\cdots,n.$

```
% Defining two 2*2 matrices
A=zeros(2,2);
B=zeros(2,2);

% In the beginning, A represents the local operation ω̂_1 of agent 1. (See Eq 7)
A(1,1)=exp(i*phi(1))*cos(theta(1)/2);
A(1,2)=i*sin(theta(1)/2);
A(2,1)=A(1,2);
A(2,2)=exp(-i*phi(1))*cos(theta(1)/2);
row_A=2;

% Computing ω̂_1 ⊗ ω̂_2 ⊗ ··· ⊗ ω̂_n
for agent=2 : n
        % B varies from ω̂_2 to ω̂_n
        B(1,1)=exp(i*phi(agent))*cos(theta(agent)/2);
        B(1,2)=i*sin(theta(agent)/2);
        B(2,1)=B(1,2);
        B(2,2)=exp(-i*phi(agent))*cos(theta(agent)/2);

        % Computing the leftmost and rightmost columns of C= A ⊗ B
        C=zeros(row_A*2, 2);
        for row=1 : row_A
                C((row-1)*2+1, 1) = A(row,1) * B(1,1);
                C((row-1)*2+2, 1) = A(row,1) * B(2,1);
                C((row-1)*2+1, 2) = A(row,2) * B(1,2);
                C((row-1)*2+2, 2) = A(row,2) * B(2,2);
        end
        A=C;
        row_A = 2 * row_A;
end
% Now the matrix A contains the leftmost and rightmost columns of ω̂_1 ⊗ ω̂_2 ⊗ ··· ⊗ ω̂_n
```

Fig. 3 (b). Computing the leftmost and rightmost columns of $\hat{\omega}_1 \otimes \hat{\omega}_2 \otimes \cdots \otimes \hat{\omega}_n$

```
% Computing |ψ2⟩ = [ω̂1 ⊗ ω̂2 ⊗ ··· ⊗ ω̂n]Ĵ|C···CC⟩
psi2=zeros(power(2,n),1);
for row=1 : power(2,n)
        psi2(row)=A(row,1)*cos(gamma/2)+A(row,2)*i*sin(gamma/2);
end

% Computing |ψ3⟩ = Ĵ+|ψ2⟩
psi3=zeros(power(2,n),1);
for row=1 : power(2,n)
        psi3(row)=cos(gamma/2)*psi2(row) - i*sin(gamma/2)*psi2(power(2,n)-row+1);
end

% Computing the probability distribution ⟨ψ3|ψ3⟩
distribution=psi3.*conj(psi3);
distribution=distribution./sum(distribution);
```

Fig. 3 (c). Computing $|\psi_2\rangle, |\psi_3\rangle, \langle\psi_3|\psi_3\rangle$.

```
temp=0;
for index=1: power(2,n)
   temp = temp + distribution(index);
   if temp >= random_number
      break;
   end
end

% indexstr: a binary representation of the index of the collapsed state
%   '0' stands for |C⟩, '1' stands for |D⟩
indexstr=dec2bin(index-1);
sizeofindexstr=size(indexstr);

% Defining an array of messages for all agents
message=cell(n,1);

% For each agent i ∈ N, the algorithm generates the message m_i
for index=1 : n - sizeofindexstr(2)
   message{index,1}=strcat('card(',int2str(index),',0)');
end
for index=1 : sizeofindexstr(2)
   if indexstr(index)=='0'      % Note: '0' stands for |C⟩
      message{n-sizeofindexstr(2)+index,1}=strcat('card(',int2str(n-sizeofindexstr(2)+index),',0)');
   else
      message{n-sizeofindexstr(2)+index,1}=strcat('card(',int2str(n-sizeofindexstr(2)+index),',1)');
   end
end

% The algorithm sends messages m1, m2, ···, mn to the designer
for index=1:n
   disp(message(index));
end

end_time = cputime;
runtime=end_time – start_time
```

Fig. 3 (d). Computing all messages $m_1, m_2, \cdots, m_n$.