Simulation based estimation of threshold moving average models with contemporaneous shock asymmetry

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Simulation Based Estimation of Threshold Moving Average Models with Contemporaneous Shock Asymmetry

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Abstract

Persistence of shocks to macroeconomic time series may differ depending on the sign or on whether a threshold value is crossed. For example, positive shocks to gross domestic product may be more persistent than negative shocks. Threshold (or asymmetric) moving average (TMA) models, by explicitly taking into account threshold behavior, can help discriminate whether there exists persistence asymmetry. Recently, building on the works of Wecker (1981, *JASA*, 76(373)) and De Gooijer (1998, *JTSA*, 19(1)) among others, Guay and Scaillet (2003, *JBES*, 21(1)) proposed TMA model in which both contemporaneous and lagged asymmetric effects are present and provided indirect inference framework for estimation and testing. This paper builds on their work and examines the properties of efficient method of moments (EMM) estimation of TMA class of models using Monte Carlo simulation experiments. The model is also applied to analyze the persistence properties of shocks in Turkish business cycles.

Keywords: Threshold moving average models, contemporaneous asymmetry, persistence of shocks, Efficient Method of Moments

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1 Introduction

Recent years have witnessed a growing interest in nonlinear modeling of economic variables. Much effort has been devoted to nonlinear extensions of basic autoregressive integrated moving average (ARIMA) models. In general, most popular class of nonlinear models have an extension in the form thresholds or state-dependency on the AR part (e.g., Tsay (1989), Terasvirta (1994), Hamilton (1989)). These models have been relatively successful in describing asymmetries in macroeconomic variables such as real GNP growth rate. However, relatively little attention has been given to asymmetric/threshold moving average (MA) models. Asymmetric MA models can be useful when an economic variable responds to past and present shocks in a different manner based on the shock's properties.

Among the first contributions to the literature, Wecker (1981) proposed an asymmetric moving average (asMA(q)) model in which time series respond to positive and negative shocks with different rules. Brännäs & DeGooijer (1994) extended Wecker's model by including autoregressive dynamics in asMA(q) model (ARasMA(p,q) model) and examined the US GNP growth rate. Zakoian (1994) proposed a threshold conditional heteroskedasticity model by modifying Engle's ARCH model to reflect differing effects of lagged shocks on volatility. Motivated by the observation that conditional mean and conditional volatility dynamics of most financial return series respond to past shocks in an asymmetric way, Brännäs & DeGooijer (2004) proposed modeling daily stock returns using a model in which conditional mean dynamics follow an asMA(q) model and conditional variance follows an asymmetric quadratic generalized autoregressive conditional heteroskedasticity (asQGARCH) model. DeGooijer (1998) proposed self-exciting threshold moving average model (SETMA) in which threshold effect depends on an observed variable. El-Babsiri & Zakoian (2001) introduced contemporaneous asymmetry in conditional volatility models. Guay & Scaillet (2003) modified basic asymmetric MA(q) model of Wecker in two ways: they allowed for contemporaneous effects of shocks to differ and they did not impose a priori value for the threshold parameter. Since the existence of asymmetric contemporaneous effects prevents direct estimation using Maximum Likelihood (ML) they proposed using simulation based indirect inference (II) of Gourieroux, Monfort & Renault (1993) and Smith (1993) to estimate the parameters of threshold MA model and suggested a testing framework.

This paper follows Guay & Scaillet (2003) and considers estimation of threshold MA models in which contemporaneous shocks are also allowed to be asymmetric. The estimation of the parameters is carried out using Efficient Method of Moments (EMM) of Gallant & Tauchen (1996). In both II and EMM, it is assumed that endogenous variables of a structural model can be summarized by a so-called auxiliary or instrumental model whose estimation is relatively easier as compared to the structural model. In the II method, artificial observations are simulated from the structural model and the parameters of the auxiliary model are estimated. Then, parameter estimates from the simulated data are matched to their counterparts in the observed data using a quadratic, GMM-type objective function. Because the data is simulated for each trial parameter vector of structural model during the minimization algorithm, the II method can be very computationally burdensome. Instead of matching parameter estimates from artificial and observed data, EMM method relies on evaluating the scores of auxiliary model at the structural parameter vector. Auxiliary model is estimated once with the observed data and evaluated with the simulated data at a trial structural parameter vector. At the true structural parameter vector the expectations of the scores should be zero. Evidently, if the auxiliary model are the same II and EMM methods are asymptotically equivalent. One can also use simulated method of moments (SMM) suggested by McFadden (1989) and Lee & Ingram (1991). In fact, EMM and SMM are special cases of II (Fackler & Tastan (2009), Guay & Scaillet (2003)).

Under the null hypothesis of symmetric contemporaneous shock effects the model reduces to an asymmetric MA model in which only lagged shocks have differing effects (such as Wecker's asMA(q) model with threshold value of zero). If this test indicates that contemporaneous shock has symmetric effects then one can still use ML estimation and conduct classical testing procedures. In this paper we propose to carry out this test using LR-type tests within the EMM framework. If only asymmetric lagged effects are relevant then an appropriate asymmetric model can be chosen following the framework suggested by Brännäs & DeGooijer (1994).

General framework is applied to the problem of determining whether negative and positive shocks to measures of economic activity have different degrees of persistence. This issue has been investigated by Beaudry & Koop (1993), Brännäs & DeGooijer (1994) and Guay & Scaillet (2003) who found that shocks to US GNP growth series have asymmetric effects. Gonzalo & Martinez (2006) also provided evidence in favor of small shocks being transitory. Elwood (1998), on the other hand, argued that there is no difference in persistence between negative and positive shocks for both US GNP and industrial production index series. In this paper, we apply our framework to Turkish real GDP and industrial production index data for the post-1980 period.

The outline of this paper is as follows. Next section describes the basic properties of threshold MA models with contemporaneous shock asymmetry. Section 3 discusses the details of the EMM estimation. A set of Monte Carlo experiments designed to asses the properties of EMM estimation and testing strategy is discussed in Section 4. The framework is applied to the disentanglement of the persistence of negative and positive shocks to Turkish real GDP and industrial production index growth rates. Section 6 provides concluding remarks.

2 Threshold Moving Average Models

The simplest asymmetric moving average model is a first order MA model in which the effects of lagged positive and negative shocks are different. This model and its generalization to asMA(q) was proposed by Wecker (1981)

$$y_t = \epsilon_t + \beta_1^+ \epsilon_{t-1}^+ + \beta_1^- \epsilon_{t-1}^-, \quad \epsilon_t \sim \ iid\ N(0, \sigma^2) \tag{1}$$

with

$$\epsilon_{t-1}^+ = \epsilon_{t-1}\ \mathbb{1}_{(\epsilon_{t-1}>0)}$$

$$\epsilon_{t-1}^- = \epsilon_{t-1}\ \mathbb{1}_{(\epsilon_{t-}\leq 0)}$$

An asMA(q) model and its extension to ARasMA(p,q) (see Brännäs & DeGooijer (1994)) can easily be estimated using maximum likelihood method. The model nests the linear ARMA(p,q) specification enabling the application of classical tests in the ML framework (for example Wecker proposed a likelihood ratio (LR) test statistic). Since the model applies different filters to positive and negative shocks it can be especially useful to examine persistence properties of shocks. Lasting effects of positive and negative shocks will be given by the sums of respective parameter estimates.

The existence of asymmetric contemporaneous effects may effect the degree of the persistence of shocks. Therefore, one needs to take into account the effects of current shocks in computing persistence. This can be done in the framework of Guay & Scaillet (2003) who proposed the following threshold moving average model with contemporaneous shock asymmetry:

$$y_t = \boldsymbol{\beta}^+(L)\epsilon_t^+ + \boldsymbol{\beta}^-(L)\epsilon_t^- \tag{2}$$

where

$$\epsilon_t^+ = \epsilon_t \, \mathbb{1}_{(\epsilon_t > \gamma)}$$
$$\epsilon_t^- = \epsilon_t \, \mathbb{1}_{(\epsilon_t \leq \gamma)}$$

and

$$\epsilon_t \ \sim iid\ N(0, \sigma^2)$$

$\mathbb{1}_{(A)}$ is the indicator function that equals 1 if the event A is correct and 0 otherwise. γ is the threshold parameter. The polynomial terms include current as well as lagged asymmetric affects:

$$\boldsymbol{\beta}^+(L) = \beta_0^+ + \beta_1^+ L + \ldots + \beta_q^+ L^q$$
$$\boldsymbol{\beta}^-(L) = \beta_0^- + \beta_1^- L + \ldots + \beta_q^- L^q$$

For example a TMA(1) model is written as

$$y_t = \beta_0^+ \epsilon_t^+ + \beta_0^- \epsilon_t^- + \beta_1^+ \epsilon_{t-1}^+ + \beta_1^- \epsilon_{t-1}^- \tag{3}$$

Guay & Scaillet (2003) studied properties of TMA models in some detail. For the case where threshold is $\gamma = 0$ moments and cross-moments of random variables $\epsilon_t, \epsilon_t^+, \epsilon_t^-$ are given by

$$\mathrm{E}(\epsilon_t^+) = \frac{\sigma}{\sqrt{2\pi}}$$
$$\mathrm{E}(\epsilon_t^-) = -\frac{\sigma}{\sqrt{2\pi}}$$

$$\mathrm{E}(\epsilon_t^2) = \sigma^2$$

$$\mathrm{E}(\epsilon_{t-i}\epsilon_{t-j}^+) = \mathrm{E}(\epsilon_{t-i}\epsilon_{t-j}^-) = \begin{cases} \frac{\sigma^2}{2}, & \text{if } i = j; \\ 0, & \text{for } i \neq j. \end{cases}$$

$$\mathrm{E}(\epsilon_{t-i}^+\epsilon_{t-j}^+) = \mathrm{E}(\epsilon_{t-i}^-\epsilon_{t-j}^-) = \begin{cases} \frac{\sigma^2}{2}, & \text{if } i = j; \\ \frac{\sigma^2}{2\pi}, & \text{for } i \neq j. \end{cases}$$

$$\mathrm{E}(\epsilon_{t-i}^+\epsilon_{t-j}^-) = \mathrm{E}(\epsilon_{t-i}^-\epsilon_{t-j}^+) = \begin{cases} 0, & \text{if } i = j; \\ -\frac{\sigma^2}{2\pi}, & \text{for } i \neq j. \end{cases}$$

for $i, j \geq 0$. Using these one can find the unconditional moments of y_t. The mean of the process in general will not be zero.

$$\mu = \mathrm{E}(y_t) = \left[(\beta_0^+ + \beta_1^+) - (\beta_0^- + \beta_1^-)\right] \frac{\sigma}{\sqrt{2\pi}}$$

Second moment is given by

$$\mathrm{E}(y_t^2) = \left({\beta_0^+}^2 + {\beta_0^-}^2 + {\beta_1^+}^2 + {\beta_1^-}^2\right)\frac{\sigma^2}{2} + \left(\beta_0^+\beta_1^+ + \beta_0^-\beta_1^- - \beta_0^+\beta_1^- - \beta_0^-\beta_1^+\right)\frac{\sigma^2}{\pi}$$

Using this unconditional variance is written as

$$\begin{aligned}\gamma_0 &= \mathrm{E}(y_t^2) - \mu^2 \\ &= \left({\beta_0^+}^2 + {\beta_0^-}^2 + {\beta_1^+}^2 + {\beta_1^-}^2\right)\frac{(\pi-1)}{2\pi}\sigma^2 + \left(\beta_0^+\beta_0^- + \beta_1^+\beta_1^-\right)\frac{\sigma^2}{\pi}\end{aligned}$$

First autocovariance is given by

$$\begin{aligned}\mathrm{E}(y_t y_{t-1}) = &\left({\beta_0^+}^2 + {\beta_0^-}^2 + {\beta_1^+}^2 + {\beta_1^-}^2 + \beta_0^+\beta_1^+ + \beta_0^-\beta_1^- - \beta_0^+\beta_1^- - \beta_0^-\beta_1^+\right)\frac{\sigma^2}{2\pi} \\ &+ \left(\beta_0^+\beta_1^+ + \beta_0^-\beta_1^-\right)\frac{\sigma^2}{2} - \left(\beta_0^+\beta_0^- + \beta_1^+\beta_1^-\right)\frac{\sigma^2}{\pi}\end{aligned}$$

$$\begin{aligned}\gamma_1 &= \mathrm{E}(y_t y_{t-1}) - \mu^2 \\ &= \left(\beta_0^+\beta_1^+ + \beta_0^-\beta_1^-\right)\frac{(\pi-1)}{2\pi}\sigma^2 + (\beta_0^+\beta_1^- + \beta_0^-\beta_1^+)\frac{\sigma^2}{2\pi}\end{aligned}$$

Second and higher autocovariances will all be zero.

Under the hypothesis

$$\beta_0^+ = \beta_0^- = \beta_0, \quad \beta_1^+ = \beta_1^- = \beta_1$$

we obtain a symmetrical moving average model of order one:

$$y_t = \beta_0\epsilon_t + \beta_1\epsilon_{t-1}$$

The mean of the process will be zero. The variance and the first autocovariance reduce to

$$\gamma_0 = (\beta_0^2 + \beta_1^2)\sigma^2,$$

and

$$\gamma_1 = \beta_0\beta_1\sigma^2,$$

respectively. Notice that, under the null hypothesis, the model can be written as

$$y_t = u_t + \beta u_{t-1}, \quad u \sim iid\ N(0, \sigma_*^2)$$

where $u_t = \epsilon_t\beta_0$, $\beta = \beta_1/\beta_0$ and $\sigma_*^2 = \sigma^2\beta_0^2$. Thus, the variance and the first autocovariance can be written as

$$\gamma_0 = (1+\beta^2)\sigma_*^2,$$
$$\gamma_1 = \beta\sigma_*^2,$$

which are familiar formulas for an MA(1) process. The autocovariance structure for a TMA(q) model with threshold $\gamma = 0$ and iid standard normal errors has been derived by Guay & Scaillet (2003). For the general iid normal errors autocovariances are

$$\begin{aligned}\gamma_h &= \sum_{i=0}^{q-h}\left\{(\beta_{i+h}^+\beta_i^+ + \beta_{i+h}^-\beta_i^-)\frac{(\pi-1)\sigma^2}{2\pi} + (\beta_{i+h}^+\beta_i^- + \beta_{i+h}^-\beta_i^+)\frac{\sigma^2}{2\pi}\right\}, \quad \text{if } h \le q \\ &= 0, \quad \text{if } h > q\end{aligned}$$

Therefore autocorrelation function (γ_h/γ_0) of a TMA(q) process will be zero after lag q. In practice, this makes it particularly difficult to distinguish a TMA(q) model from a linear MA(q) model using sample correlogram alone. Since y_t is a combination of truncated normal errors its unconditional distribution can be skewed and leptokurtic or platykurtic. Figures (1) and ((2)) display distribution of a simulated TMA(1) process with 10^6 points. Figure (1) shows histogram and normal probability plot of TMA(1) process using parameters $\beta_0^+ = 0.5, \beta_0^- = 1, \beta_1^+ = 0.2, \beta_1^- = 0.8, \sigma^2 = 1$. In this case negative shocks have a greater impact than positive shocks. Unconditional skewness and kurtosis are -0.65 and 3.33, respectively. The opposite case in which positive shocks have a greater impact than negative shocks is shown in Figure (2).

In fact, the presence of contemporaneous asymmetry in the effects of shocks alone can create wide range of values for skewness and excess kurtosis in the observed series. To visualize this, we fixed the lagged asymmetry parameters at $\beta_1^+ = \beta_1^- = 0.5$ and created a grid over $[-1 : 0.1 : 1]$ for both parameters β_0^+ and β_0^-. Fixing the variance at $\sigma^2 = 1$ and threshold parameter $\gamma = 0$ we simulated samples of size 10^5 and calculated sample skewness and kurtosis for each parameter combination. Figure (3) shows the behavior of skewness and kurtosis when $\gamma = 0$. Also, Figure (4) displays skewness and kurtosis values over threshold grid set $[-1 : 0.1 : 1]$. Evidently, depending on parameter combinations and threshold value TMA process may have a symmetric, or skewed and/or leptokurtic or platykurtic distribution. Although in practice skewness and excess kurtosis can be a good indicator to consider a nonlinear model specification, they alone are not sufficient since a threshold MA model can also have a symmetric and bell-shaped distribution.

3 Efficient Method of Moments Estimation

As we mentioned before, moving average models with lagged asymmetric effects (asMA(q) model, see (Wecker 1981)) and its generalization to ARasMA(p,q) model (Brännäs & DeGooijer 1994) can be estimated efficiently using maximum likelihood method. However, the presence of contemporaneous asymmetry in the effects of shocks prevents direct estimation methods such as maximum likelihood. In this case, as suggested by Guay & Scaillet (2003), one needs to resort to indirect simulation-based methods. Indirect inference strategy is based on using moment conditions from an auxiliary model which provides information to identify the structural parameters and whose estimation is relatively easier compared to the structural model.

Let θ be $p \times 1$ vector of structural parameters of interest. Indirect inference estimator is defined as

$$\hat{\theta} = \underset{\theta \in \Theta}{\arg\min}\, \psi(\theta)^\top W \psi(\theta), \tag{4}$$

where $\psi(\theta)$ is $q \times 1$ vector of moment conditions whose expectation at the true θ is zero, and W is $q \times q$ positive-definite weighting matrix. Simulation based methods differ in how they define the moment vector and the optimal weighting matrix. Gourieroux et al. (1993) and Smith (1993) suggest matching the parameters of an auxiliary model in the moment vector. For each trial value of θ in the numerical minimization routine, auxiliary parameter vector must be estimated using simulated data and matched to the auxiliary parameter estimates obtained from observed data. If the estimation of the auxiliary model is trivial, such as least squares, the indirect inference is numerically fast and reliable. However, if the estimation of the auxiliary parameter vector is relatively involved, such as numerical maximization of a loglikelihood function then the method can be computationally very intensive. The efficient method of moments estimation strategy, suggested by Gallant & Tauchen (1996), can overcome this computational burden through bypassing the numerical optimization of the auxiliary criterion function. This is accomplished by using the scores of the auxiliary model likelihood function as the moment conditions. Let β be $p \times 1$ vector of auxiliary model parameters. Also let y_t be vector of observed data and $\tilde{y}_t(\theta)$ be simulated data. The estimation of β is based on the maximization of loglikelihood function of the auxiliary model:

$$Q(y_t, \beta) = \sum_{t=1}^{T} \ell(y_t, \beta)$$

whose solution is $\hat{\beta}$. EMM uses the following moment conditions

$$\psi(\theta) = \mathrm{E}\left[\frac{\partial \ell(\tilde{y}_t(\theta); \hat{\beta})}{\partial \beta}\right] = \frac{1}{N}\sum_{t=1}^{N} \frac{\partial \ell(\tilde{y}_t(\theta); \hat{\beta})}{\partial \beta} \tag{5}$$

For notational simplicity let the individual scores be given by

$$s_t\left(\tilde{y}(\theta)\right) \equiv \frac{\partial \ell(\tilde{y}_t(\theta); \hat{\beta})}{\partial \beta}$$

Notice that $\hat{\beta}$ is calculated only once using the observed data. The expectation of the score vector is then approximated using a long realization of simulated data from the structural model. Now let H be the simulation constant and T the sample size, then based on $N = TH$ simulated values of endogenous variables, the EMM estimator can be defined as

$$\hat{\theta} = \underset{\theta \in \Theta}{\arg\min} \left[N^{-1} \sum s_t\left(\tilde{y}(\theta)\right)\right]^{\top} W \left[N^{-1} \sum s_t\left(\tilde{y}(\theta)\right)\right], \tag{6}$$

Optimal weighting matrix can be calculated as the the inverse of the long-run covariance matrix of the scores based on observed data.

$$W = \mathcal{I}_0^{-1}$$

$$\mathcal{I}_0 = \mathrm{Var}\left(\frac{1}{\sqrt{T}} \sum_{t=1}^{T} s_t(y_t, \hat{\beta})\right)$$

HACC procedure suggested by Newey & West (1987) can be adopted to estimate the long-run covariance matrix. Gallant & Tauchen (1996) show that $\sqrt{T}(\hat{\theta} - \theta)$ is asymptotically normal with covariance matrix

$$\Omega = \left(D_\theta^{\top} \mathcal{I}_0^{-1} D_\theta\right)^{-1},$$

where D_θ is $q \times p$ matrix of derivatives of moment conditions

$$D_\theta = \frac{\partial \psi(\theta)}{\partial \theta} \equiv \mathrm{E}\left[\frac{\partial \psi(\theta)}{\partial \theta}\right]$$

which can be approximated using

$$D_\theta \approx \frac{1}{N} \sum \frac{\partial s_t\left(\tilde{y}(\theta)\right)}{\partial \theta}$$

To estimate TMA(q) model using EMM let $(2q+3)\times 1$ vector

$$\theta = [\beta_0^+,\ \beta_0^-, \beta_1^+, \ldots, \beta_q^+,\ \beta_1^-, \ldots, \beta_q^-,\ \sigma]^\top$$

be structural parameters of interest. The following models can be used as auxiliary models in the EMM estimation.

Auxiliary Model 1 The first auxiliary model is AR(p) model:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \cdots + \phi_p y_{t-p} + u_t$$

$$Q(y_t, \beta) = -\frac{(T-p)}{2}\ln(2\pi) - \frac{(T-p)}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=p+1}^{T}\hat{u}_t^2$$

$$\hat{u}_t = y_t - \hat{\phi}_0 - \hat{\phi}_1 y_{t-1} - \hat{\phi}_2 y_{t-2} - \cdots \hat{\phi}_p y_{t-p}$$

Auxiliary Model 2 Second auxiliary model was used by Michaelides & Ng (2000) in the simulation estimation of nonlinear models of speculative storage. This model contains polynomial autoregressive terms whose loglikelihood is given as follows

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-1}^2 + \phi_3 y_{t-1}^3 + \cdots + \phi_{3p} y_{t-p}^3 + u_t$$

$$Q(y_t, \beta) = -\frac{(T-p)}{2}\ln(2\pi) - \frac{(T-p)}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=p+1}^{T}\hat{u}_t^2$$

$$\hat{u}_t = y_t - \hat{\phi}_0 - \hat{\phi}_1 y_{t-1} - \hat{\phi}_2 y_{t-1}^2 - \hat{\phi}_3 y_{t-1}^3 - \cdots - \hat{\phi}_{3p} y_{t-p}^3$$

Auxiliary Model 3 Third auxiliary model was suggested by Guay & Scaillet (2003). This model is a mix of the previous models. For example, for $p = 4$ and third order polynomial terms the model can be written as

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-1}^2 + \phi_3 y_{t-1}^3 + \phi_4 y_{t-2} + \phi_5 y_{t-2}^2 + \phi_6 y_{t-2}^3 + \phi_7 y_{t-3} + \phi_8 y_{t-4} + u_t$$

$$Q(y_t, \beta) = -\frac{(T-p)}{2}\ln(2\pi) - \frac{(T-p)}{2}\ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{t=4}^{T}\hat{u}_t^2$$

$$\hat{u}_t = y_t - \hat{\phi}_0 - \hat{\phi}_1 y_{t-1} - \hat{\phi}_2 y_{t-1}^2 - \hat{\phi}_3 y_{t-1}^3 - \hat{\phi}_4 y_{t-2} - \hat{\phi}_5 y_{t-2}^2 - \hat{\phi}_6 y_{t-2}^3 - \hat{\phi}_7 y_{t-3} - \hat{\phi}_8 y_{t-4}$$

This model was found to be the most successful by Guay & Scaillet (2003) among the three models above.

Auxiliary Model 4 An asymmetric MA(q) model can also be used as an auxiliary model. For example with $q = 2$ an asMA model and its conditional loglikelihood can be written as

$$y_t = \epsilon_t + \beta_1^+ \epsilon_{t-1}^+ + \beta_2^+ \epsilon_{t-2}^+ + \beta_1^- \epsilon_{t-1}^- + \beta_2^- \epsilon_{t-2}^-,$$

$$Q(y_t, \beta) = -\frac{T}{2}\log(2\pi) - \frac{T}{2}\log(\sigma^2) - \sum_{t=1}^{T}\frac{\hat{\epsilon}_t^2}{2\sigma^2}$$

$$\hat{\epsilon}_t = y_t - \beta_1^+ \hat{\epsilon}_{t-1}^+ - \beta_2^+ \hat{\epsilon}_{t-2}^+ - \beta_1^- \hat{\epsilon}_{t-1}^- - \beta_2^- \hat{\epsilon}_{t-2}^-$$

The first three models can easily be estimated using OLS and the scores can be evaluated in a straightforward manner. Estimation of the last auxiliary model is relatively more involved as it requires numerical maximization of the loglikelihood function. Also, another difficulty with this models is that scores and derivatives need to be evaluated recursively and using numerical differentiation routines. This may add to the computational burden.

4 Simulation Study

This section provides a set of Monte Carlo experiments designed to assess properties of the EMM estimator for the TMA model. Data is generated from the following TMA(1) model

$$y_t = \beta_0^+ \epsilon_t^+ + \beta_0^- \epsilon_t^- + \beta_1^+ \epsilon_{t-1}^+ + \beta_1^- \epsilon_{t-1}^-$$

where the threshold is $\gamma = 0$. The true parameter values are $\beta_0^+ = 0.5, \beta_0^- = 1, \beta_1^+ = 0.2, \beta_1^- = 0.8, \sigma = 1$ which were also used by Guay & Scaillet (2003) in their simulation study. Sample sizes (T) and simulation lengths ($N = TH$) are $T = 100, 250, 500, 1000$ and $H = 10, 20, 50, 100$, respectively. For each combination of T and N we simulated data from the data generating process described above and estimated the parameter vector using EMM with auxiliary models described in the previous section. The first auxiliary model is a linear AR(4) model. The second auxiliary model is polynomial AR model with two lags and second and third powers. The third model, is a mix of the previous two and was found to be the most successful auxiliary model among the three. We also used auxiliary model 4. This is asymmetric MA model with two lags. Since the estimation is more computationally burdensome for this auxiliary model we only used $T = 250$ with $H = 10, 50$. The number of replications is set to 1000 for each experiment. In our calculations we used Simulated GMM toolbox written in Matlab programming language by P. L. Fackler and H. Tastan (see Fackler & Tastan (2009)). We calculated bias and root mean squared error (RMSE) to compare the performance of auxiliary models. Results are summarized in Tables 1 through 4. We also plot bias, RMSE and kernel density estimates for each auxiliary model in Figures 5 to 14.

Table 1 summarizes experiment results from the auxiliary model 1 - AR(4) model. In their experiments Guay & Scaillet (2003) also used AR(6), AR(8) and AR(10) but found that AR(4) has the smallest RMSE. Indeed increasing the AR order will reduce bias but at the same time increase the variance. Therefore we only used lag 4 in the linear AR specification. In Table 1 we see that for each combination of T and H, β_0^+, β_0^- and σ are underestimated whereas β_1^+ and β_1^- are overestimated. Our results cannot be directly compared to Guay and Scaillet's results because they did not estimate σ^2 and they only performed 500 Monte Carlo replications. Nevertheless, our results from EMM estimation of TMA model with AR(4) moments, $T = 250$ and $H = 10$ are very close to their results from the indirect inference estimation procedure (Table 6 last column in GS, 2003, p.6, note also that their sample size is 200). In general for a fixed T, increasing the simulation constant H does not much contribute to the bias and the precision of the estimates. The biggest reduction in bias and precision comes from increasing the sample size. This behavior can be observed in Figures 5 and 6. Kernel density estimates for the EMM estimators are given in Figure 7. Except for σ all densities are bimodal and obviously they are not distributed normally. Overall, linear AR(p) models perform poorly as the score generator in EMM context.

Results from auxiliary model 2 are summarized in Table 2. In general there is significant improvement in bias and RMSE compared to AR(4) model especially for samples sizes larger than 250. However, for $T = 100$ bias and RMSE are larger for the parameters corresponding to negative shocks (β_0^- and β_1^-) and σ. Increasing the sample size improves the bias and precision of the estimates. For example, for $T = 250$ and $H = 10$, biases for positive shock parameters are -0.0085 and -0.0035 with RMSE 0.1446 and 0.1458, respectively. This corresponds to about 44% improvement in the precision of the estimates.

However, for the negative shock parameters improvement in bias and RMSE is negligible. Again for a fixed sample size increasing the simulation length in the EMM estimation does not much contribute to the reduction in bias and RMSE. The biggest improvement comes from increasing the sample size. For example, for $T = 500$ and $H = 10$, bias and RMSE of β_0^+ are 0.0038 and 0.1158, respectively. For $T = 500$ and $H = 100$ the bias reduces to only 0.0036 with RMSE 0.1098. This behavior can also be observed in Figures 8 and Figure 9. Comparing our results to results from GS instrumental model II (Table 3, second column, p. 6) we see that EMM estimators have smaller bias and RMSE as compared to indirect inference estimators. We also note that the biggest improvement in RMSE for σ is attained with sample sizes larger than 500. Figure 10 plots kernel density estimates for the auxiliary model 2. Unlike AR(4) case these densities have one mode and more symmetrical.

Table 3 reports experiment results from auxiliary model 3. For $T = 100$ parameter estimates are not very precise and increasing the simulation length does not much improve the results. Increasing the sample size to 250 we observe that RMSE gets smaller. As compared to the previous two models, we can say that auxiliary model 3 has smaller RMSE in general. Similarly, increasing H does not much contribute to the reduction in bias and RMSE. This behavior can be observed in Figure 11 and Figure 12. Compared to the indirect inference estimation results given in Table 4 in GS (2003, p.7) RMSE is slightly larger for contemporaneous shock parameters whereas it is smaller for lagged shock parameters. We also note σ is not estimated very precisely for sample sizes 100 and 250 and it is always underestimated. Figure 13 displays kernel density estimates for the auxiliary model 3.

Finally, we estimated the parameters of the TMA(1) model using scores of the auxiliary model 4. We only used one sample size, 250, and two simulation lengths 2500 and 12500 corresponding to $H = 10$ and $H = 50$, respectively. Results are summarized in Table 4. For all parameter estimates RMSE is much smaller in this case. However, we note there are some convergence issues for this auxiliary model. Quasi-Newton optimization routine was interrupted several times because it produced NaNs and Infs. Therefore, we used Nelder-Mead simplex search method which does not rely on derivatives of the objective function. Although the results seem superior to other auxiliary models they should be interpreted with caution.

As we mentioned before, if the effect of contemporaneous shock is symmetric then the model reduces to an asymmetric moving average model suggested by Wecker (1981). An MA(q) model with only lagged asymmetries can easily be estimated using maximum likelihood method and classical testing procedures can be applied. In the EMM framework, the symmetry of contemporaneous shock can be tested using Likelihood Ratio type tests. Under the null hypothesis

$$\beta_0^+ = \beta_0^- = \beta_0$$

the restricted model is an MA(q) model with only lagged asymmetries. The EMM LR-type test statistic can be calculated by estimating both constrained and unconstrained models and calculating

$$LR = TQ(\tilde{\theta}) - TQ(\hat{\theta}) \sim \chi^2_\nu$$

where $Q(\tilde{\theta})$ is the value of the simulated GMM objective function value at the restricted parameter vector and $Q(\hat{\theta})$ is the objective function value at the unrestricted parameter vector. T is the number of observations used in the estimation and $\nu = \dim(\hat{\theta}) - \dim(\tilde{\theta}) = 2$ is the number of restrictions.

There is an established and efficient estimation and testing framework for MA models with only lagged asymmetric effects. However, to carry out the LR-type test we need to estimate the restricted model using EMM method. Thus, it would be useful to evaluate the properties of EMM estimators for the asymmetric MA(q) model and compare it to the ML estimator. For this purpose we conducted another Monte Carlo experiment in which data is generated from the following asMA(1) model

$$y_t = \epsilon_t + \beta_1^+ \epsilon_{t-1}^+ + \beta_1^- \epsilon_{t-1}^-, \quad \epsilon_t \sim \ iid\ N(0, \sigma^2)$$

in which Parameter values are chosen as $\beta_1^+ = 0.6$, $\beta_1^- = 0.9$, $\sigma = 1$ and $\gamma = 0$. As in the previous experiments, sample sizes (T) and simulation lengths ($N = TH$) are chosen as $T = 100, 250, 500, 1000$ and $H = 10, 20, 50, 100$, respectively. For each combination of T and N we simulated data from asMA(1) model and estimated the parameter vector using EMM with AR(4) chosen as the score generator. For comparison we also computed ML estimates. We conducted 2000 Monte Carlo replications and calculated bias and RMSE. The results are summarized in Table 5. As expected conditional ML estimator is much more efficient and accurate than EMM estimator. Although β_1^- is generally much more accurately estimated for sample sizes 100 and 250 the bias of the EMM estimator of β_1^+ becomes smaller at $T = 500$ and $T = 1000$. σ is always underestimated in both EMM and ML and its RMSE rapidly gets smaller especially for sample sizes greater than 250. Again, we note that the biggest improvements in both accuracy and precision of the estimates come from increasing the sample size. The contribution of H seems negligible. Figure 15 and Figure 16 display kernel density plots of EMM estimators for sample sizes $T = 250$ and $T = 1000$, respectively. These plots reveal that EMM sampling distributions are approximately normal and reasonably accurate. Although conditional ML estimators are more precise than EMM estimators RMSE values are acceptable for sample sizes greater than 250. Overall large sample sizes may be needed to produce reliable inference.

To assess the behavior of the LR-type test statistic we conducted a small Monte Carlo experiment. We generated data from the asymmetric MA(1) model using true parameter values $\beta_1^+ = 0.2$, $\beta_1^- = 0.8$ and $\sigma = 1$. Threshold value is set to $\gamma = 0$. We use auxiliary model 3 with five lags for the first order polynomial and two lags for both second and third order polynomials. We use the same instrumental model, hence the same moment conditions for both restricted and unrestricted models. Two sample sizes, $T = 250$ and $T = 500$ are considered and the simulation constant is set to $H = 20$. We used 1000 Monte Carlo replications to calculate rejection frequencies for nominal significance levels 0.01, 0.05 and 0.1. The results are summarized in Table 6. For the sample size 250 the LR-type test is slightly oversized: it tends to reject the true model 1.3% of the time when the nominal α-level is 0.01, 6.8% of the time when $\alpha = 0.05$ and 12.6% of the time when $\alpha = 0.1$. When we increase the sample size to 500 rejection frequencies are much closer to their nominal counterparts. In this case the LR-type test rejects the true model 5.6% of the time when $\alpha = 0.05$. The Type-I error behavior of LR-type test can also be followed from Figure 17 which plots rejection frequencies and nominal significance levels.

To assess empirical power of the LR-type test statistic we fixed the parameter corresponding to one-period lagged effects of positive shocks at $\beta_1^+ = 0.5$ and varied the the parameter corresponding to the effect of negative shocks as

$$\beta_1^- = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1\}$$

For each 11 value of β_1^- we computed LR-type test statistic and the rejection frequency of the null hypothesis at $\alpha = 0.05$ significance level for sample sizes 250, 500 and 1000.

The null hypothesis is incorrect for these values except for $\beta_1^- = 0.5$. Figure 18 displays size-unadjusted empirical power curves for each sample size. The first thing to notice is that the power curve is not symmetric. The test is more powerful when $\beta_1^- < \beta_1^+$. Although, as expected, the test becomes more powerful as the sample size increases its asymmetry remains. We also note that empirical size at these parameter values is greater than the nominal size. The smallest empirical size is achieved using $T = 1000$.

5 Application: Persistence of Shocks in Turkish Business Cycles

Asymmetric persistence of shocks have been investigated by several studies. Beaudry & Koop (1993) provided evidence that there is statistically significant difference in persistence between negative and positive shocks for the post-war US GNP data. They found that negative shocks are less persistent than positive shocks. Using modified critical values Hess & Iwata (1997) re-examined the results of Beaudry and Koop (1993) and concluded that their results are slightly weakened but still valid. Hess & Iwata (1997) also applied their model to G-7 countries and, except for France, did not find any asymmetric persistence. Using an autoregressive asymmetric moving average (ARasMA) model Brännäs & DeGooijer (1994) provided evidence on the asymmetric effects of shocks in US GNP series. In contrast, using asymmetric moving average and autoregressive models Elwood (1998) argued that there is no difference in persistence between negative and positive shocks for both US GNP and industrial production index series. Guay & Scaillet (2003) analyzed the same problem without a priori imposing a value for the threshold parameter and allowed for contemporaneous asymmetry in a threshold moving average model for the US GNP data. They found that a shock with a value greater than the threshold value of -0.85 is significantly more persistent than a smaller shock. Gonzalo & Martinez (2006) also analyzed US GNP data using a threshold integrated moving average (TIMA) framework in which asymmetry is produced by the size of the shock. They found that the size of the shock is significant in generating persistence asymmetry and that small shocks tend to be transitory.

In this section we investigate whether the persistence of positive shocks to output is significantly different from the persistence of negative shocks in Turkish economy. To see if the results are robust to the measure of economic activity we use both real GDP and industrial production index to estimate an appropriate threshold model.

A mentioned by Elwood (1998) the question of asymmetry in business cycle is different from the question of asymmetry in persistence. Business cycle asymmetry focuses on the behavior of an aggregate measure of economic activity over expansions and contractions (e.g., Neftci (1982), Hamilton (1989), Sichel (1993), among others). Persistence in asymmetry, on the other hand, focuses on the lasting influence of negative and positive shocks regardless of where they occur in the business cycle (Elwood 1998). Empirical studies using Turkish data have identified four contraction and five expansion phases for the post-1985 period. For example, using 2-state Markov-switching autoregression model Tastan & Yildirim (2008) found that expansionary and contractionary phases in industrial production index growth rate have different characteristics. They estimates intercept terms for the recession and expansion phases as -0.0419 and 0.0605, respectively . The expansion regime is found to be slightly more volatile than the recession regime. They also found that the duration of recessions is about 10 months while the duration of expansion is about 38 months (about 3.16 years). Their results reflect that recessions are relatively short-lived compared to other developing countries whereas the duration of expansions is relatively larger compared to EU enlargement countries. Also, their results from parametric asymmetry tests indicate that recessions are more deep and more steep than expansions and the probability of switching from recession to expansion is significantly larger than the probability of switching from expansion to recession. Yildirim & Tastan (2007) argued that there is a negative correlation between the deepness and duration of recessions. The deeper the recessions the shorter the recessions.

We first examine the persistency asymmetry in shocks to Turkish real GDP growth rate for the period 1987.Q1-2009.Q4. The quarterly data is taken from IMF-IFS database and seasonally adjusted using Census-X12 method. Natural logarithm of the seasonally adjusted RGDP series is subjected to Augmented Dickey-Fuller tests. ADF test with a constant and time trend indicated that the series contain a unit root.[1] The growth rate of the RGDP series is calculated as the first difference of logarithmic series, $\Delta y_t = \ln(RGDP_t) - \ln(RGDP_{t-1})$, and ADF test indicates that it is stationary. Then, a linear ARMA(p,q) specification for Δy_t is investigated. The correlogram of Δy_t indicates and ARMA(1,1), MA(4) or ARMA(1,4) model can possibly be fit. However, neither specification passes the RESET and BDS tests. For example, the BDS test applied to the residuals of ARMA(1,1) model is calculated as 2.03 with p-value of 0.04 when integrating dimension is 2, and 2.68 with p-value 0.007 when integrating dimension is 3. Similarly, the BDS test statistic for the residuals of MA(4) is 3.41 with p-value 0.0007. The RESET test for the MA(4) model is calculated as 9.57 with p-value 0.008. Linear models seem to be not able to produce iid residuals, thus a nonlinear alternative would be more adequate.

In search of an appropriate asymmetric model for the mean-adjusted growth series we estimated several TMA(q) models for $q = 1, 2, 3, 4$ using the EMM framework with auxiliary model III with five lags for the first-order polynomial and three lags for the second and third-order polynomials. Simulation constant is set to $H = 50$. Then, we tested whether contemporaneous shocks are asymmetric by allowing only lagged asymmetries in the model. To test this hypothesis we used the LR-type test discussed in the previous section. Since our purpose is to examine the persistence of negative and positive shocks, we imposed zero threshold in the model. We used the same auxiliary model for both restricted and unrestricted models in the LR-test. Results are summarized in Table 7. LR-type tests for each TMA(q) model indicates that the impact of contemporaneous shocks is symmetric. Thus, an MA model with asymmetries in only lagged shock effects would be more appropriate for the growth rate of Turkish Real GDP.

To find the most appropriate specification we estimated several asymmetric MA models. We also allowed for autoregressive dynamics in our search for best fit. Among several ARasMA(p,q) models with $p = 1, 2$ and $q = 1, 2, 3, 4$ an asymmetric MA(4) model provides the best fit using the Akaike Information Criterion. Minimum Residual Sum of Squares (RSS) and maximum loglikelihood values were also achieved with this asMA(4) model. The results are given below:

[1]The ADF test statistic is -2.99 with a p-value of 0.14. The lag length of the test regression is chosen SIC and is zero.

$$\Delta y_t = \underset{(0.1\times10^{-4})}{0.019} - \underset{(0.15)}{0.14}\,\hat{\epsilon}^{+}_{t-1} - \underset{(0.16)}{0.35}\,\hat{\epsilon}^{+}_{t-2} + \underset{(0.13)}{0.03}\,\hat{\epsilon}^{+}_{t-3} - \underset{(0.17)}{1.15}\,\hat{\epsilon}^{+}_{t-4}$$
$$- \underset{(0.12)}{0.27}\,\hat{\epsilon}^{-}_{t-1} - \underset{(0.13)}{0.05}\,\hat{\epsilon}^{-}_{t-2} - \underset{(0.21)}{0.18}\,\hat{\epsilon}^{-}_{t-3} - \underset{(0.15)}{0.12}\,\hat{\epsilon}^{-}_{t-4} + \hat{\epsilon}_t$$

$$\hat{\sigma} = 0.023(0.001) \quad RSS = 0.052 \quad AIC = -4.42 \quad \ln(L) = 210.504$$

Under the null hypothesis of $\beta_i^+ = \beta_i^-$, $i = 1, \ldots, q$ the model reduces to a symmetric MA(q) model which can easily be tested using likelihood ratio test statistic:

$$LR = 2[\ln L(\hat{\theta}) - \ln L(\tilde{\theta})] \sim \chi^2_q,$$

where $\ln L(\hat{\theta})$ and $\ln L(\tilde{\theta})$ are unrestricted and restricted values of loglikelihood. The LR test statistic is calculated as 19.97 which has a p-value 0.0005 at 4 degrees of freedom. Thus, there is significant asymmetric effects from past innovations in the growth rate of real GDP. We also note that the BDS test statistic for the residuals from this model is -0.26 with p-value 0.79. An LBQ test up to 12 lags has a p-value of 0.99. In contrast, a linear symmetric MA(4) model cannot produce iid residuals. Parameter estimates reflect that the extent of the response of RGDP growth to positive and negative shocks is quite different. We observe that all parameter estimates associated with negative shocks have negative sign. This is also true for positive shocks except for lag three. The biggest difference is observed at lag 4. A positive shock at lag four reduces the growth rate by 1.15 whereas the impact of a negative shock with the same magnitude is only -0.12. The LR test statistic for the null hypothesis that the sums of parameters associated with positive and negative shocks, i.e.,

$$H_0: \sum_{i=1}^{4} \beta_i^+ = \sum_{i=1}^{4} \beta_i^-$$

is calculated as 16.416 with p-value$<$ 0.0001. Thus, the null hypothesis of symmetry is decisively rejected in favor of positive shocks having a greater impact on the growth rate of real GDP. The sums of positive and negative parameters are estimated as -1.61 and -0.62, respectively. The impact of positive shocks is about 2.5 times larger than the impact of negative shocks.

We now examine the asymmetry in persistence of shocks to growth rate of Turkish Industrial Production Index for the period from the first month of 1985 to the second month of 2010. The data is taken from IMF IFS database and the last four observations were updated from Turkish Statistical Institute. The skewness of the growth rate of industrial production index is -0.41 and its kurtosis is 4.74. Normality of the series is decisively rejected using Jarque-Bera test. We take the first difference of logarithmic series and then search for the best linear ARMA model. The ACF has peaks at first and second lags as well several peaks at more distant lags. The PACF also has significant peaks at first and second lags. LBQ tests are all significant for each lag lengths from 1 to 36. We first attempted to fit low-order ARMA(p,q) models to this series. AIC chooses ARMA(1,4) whereas SIC chooses ARMA(1,1). However, none of the models produce uncorrelated residuals as evidenced by significant LBQ statistics. Also the BDS statistics for each series have p-values less than 0.001. Although RESET tests applied to each

ARMA model did not indicate any functional form misspecification, linear models are not capable of producing uncorrelated residuals. We also augmented ARMA(p,q) models by adding autoregressive and moving average terms at higher lags but residuals were still correlated indicating that there may still be predictive content in the series.

To see if a threshold model is more appropriate for the industrial production growth rate series we followed the same methodology. We first tested for the significance of contemporaneous shocks and then proceeded according to the test results. We used the same moment generator with simulation constant $H = 50$. After adjustments for the auxiliary model the data set contains 296 observations. Table 8 summarizes estimation results from TMA(q), $q = 1, 2, 3, 4$, models. The first column presents results from TMA(1) model. The OID test statistic is calculated as 23.026 with p-value 0.003 indicating that TMA(1) model may not be appropriate. The EMM LR-type test statistic has a p-value of 0.0004 indicating that the null hypothesis of symmetric contemporaneous shock effects is rejected in favor of TMA(1) model. The LR-type test for TMA(2) model (second column) also rejects the null hypothesis of symmetric contemporaneous effects. But the OID test statistic is significant at 4.6% level or higher. OID test statistics for TMA(3) and TMA(4) models have p-values 0.2 and 0.78, respectively. Also, EMM LR-type test statistics are all insignificant for these models indicating that an MA model with only lagged asymmetries may be more appropriate. To choose the lag length for TMA model one can also use EMM LR-type test statistics. The test statistic for TMA(1) (null model) vs. TMA(2) (alternative model) is 10.203 which is significant at 5% level (critical value is $\chi^2_{2,0.05} = 5.99$). The test statistics for TMA(2) vs. TMA(3) is calculated as 6.833 indicating that TMA(3) is preferred to TMA(2). The test statistic between TMA(3) and TMA(4) is insignificant, thus, we prefer TMA(3) model to base our inference regarding the persistence of growth rate shocks. The sum of positive and negative shock parameters gives us information on the relative persistency of shocks. In TMA(3) model these are $\sum_{j=0}^{3} \beta_j^+ = 0.23$ and $\sum_{j=0}^{3} \beta_j^- = 0.197$ for positive and negative shocks, respectively. These results indicate that there is not much difference in the persistence of shocks based on the sign for the industrial production growth rate.

Brännäs & Ohlsson (1999) argued that detection of nonlinearities in time series may depend on sampling frequency. For example, asymmetric monthly time series may become symmetric when aggregated to quarterly or annual frequencies. To see if our results from monthly industrial production index growth are robust to sampling frequency, we also carried out the estimation and testing for asymmetries in quarterly growth rates. The quarterly growth rates series extend from 1980.Q1 to 2009.Q4 and contain 120 observations. We estimated TMA(q) models from lag one to four using the same auxiliary model as before. LR-type tests indicate that, unlike the monthly series, for each lag specification there are no contemporaneous asymmetric effects. For example, for TMA(4) model LR-type test statistics is 0.15 with p-value=0.93. Therefore, an asymmetric ARasMA(p,q) specification should be preferred. Preliminary analysis indicates that an ARMA(1,4) model could be fitted to the data which produces serially uncorrelated residuals. Thus, we estimated an ARasMA(1,4) model using conditional ML for the quarterly growth rate series. The log-likelihood value for the unrestricted model is 202.39 whereas the restricted model has a log-likelihood value of 200.21. The LR test statistic is 4.36 (p-value=0.36). Thus, we cannot reject the null hypothesis that the effects of past shocks to quarterly growth rates are symmetric. Apparently when aggregated to quarterly frequency, asymmetries in monthly growth rates of industrial production index disappears. There is no difference between negative and positive shocks in terms of persistence which is in line with the results obtained from TMA model with contemporaneous shock asymmetry for the monthly growth rates of industrial production.

6 Concluding Remarks

In this paper, we have examined properties of efficient method of moments (EMM) estimator for asymmetric (or threshold) moving average models with contemporaneous and lagged asymmetries in the effects of innovations. An LR-type test statistic in EMM framework is proposed to test for the null hypothesis of symmetric effects of contemporaneous shocks. Since under the null hypothesis the model reduces to an asymmetric MA model, direct estimation by maximum likelihood can be performed if the LR-type test statistic is insignificant. A set of Monte Carlo experiments indicates that EMM estimator has reasonable accuracy and precision for autoregressive auxiliary models with second and third order lag polynomials.

The framework is also applied to the disentanglement of the effects of positive and negative shocks to Turkish output growth rate. Results indicate that contemporaneous shocks to both real GDP and quarterly industrial production index growth rates are symmetric. Past shocks to real GDP growth rate tend to have asymmetric effects with positive shocks having greater impact than negative shocks. Although results from monthly industrial production growth rate indicates that the effects of contemporaneous negative and positive shocks are asymmetric, their persistence is very close to each other. However, when aggregated to quarterly frequency both contemporaneous and lagged asymmetries disappear.

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Table 1: Auxiliary Model 1: AR(4)

		β_0^+		β_0^-		β_1^+		β_1^-		σ	
T	H	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
100	10	-0.1036	0.2774	-0.0946	0.2590	0.0890	0.2861	0.0285	0.2362	-0.1000	0.2145
	20	-0.1138	0.2790	-0.0915	0.2526	0.0954	0.2929	0.0376	0.2367	-0.0925	0.1896
	50	-0.1111	0.2866	-0.0901	0.2528	0.1006	0.2840	0.0379	0.2321	-0.0928	0.1689
	100	-0.1017	0.2716	-0.0871	0.2447	0.0927	0.2832	0.0290	0.2299	-0.0945	0.1702
250	10	-0.1069	0.2459	-0.0615	0.2040	0.1169	0.2593	0.0581	0.1957	-0.0467	0.0711
	20	-0.1070	0.2461	-0.0630	0.2084	0.1143	0.2595	0.0569	0.1974	-0.0503	0.0735
	50	-0.1015	0.2405	-0.0652	0.2025	0.1116	0.2542	0.0630	0.1963	-0.0450	0.0680
	100	-0.1002	0.2415	-0.0640	0.2038	0.1068	0.2520	0.0579	0.1971	-0.0485	0.0768
500	10	-0.1006	0.2288	-0.0552	0.1933	0.1148	0.2474	0.0663	0.1902	-0.0310	0.0482
	20	-0.1028	0.2304	-0.0587	0.1938	0.1090	0.2476	0.0752	0.1927	-0.0295	0.0464
	50	-0.0960	0.2286	-0.0649	0.1995	0.1090	0.2435	0.0760	0.1937	-0.0320	0.0490
	100	-0.0948	0.2271	-0.0617	0.1986	0.1052	0.2404	0.0737	0.1961	-0.0307	0.0476
1000	10	-0.0908	0.2204	-0.0641	0.1915	0.0996	0.2312	0.0850	0.1910	-0.0226	0.0357
	20	-0.0940	0.2230	-0.0597	0.1931	0.1074	0.2356	0.0779	0.1903	-0.0240	0.0358
	50	-0.0990	0.2275	-0.0512	0.1883	0.1107	0.2405	0.0698	0.1843	-0.0244	0.0366
	100	-0.0849	0.2136	-0.0554	0.1921	0.0954	0.2245	0.0709	0.1867	-0.0230	0.0346

Table 2: Auxiliary Model 2

		β_0^+		β_0^-		β_1^+		β_1^-		σ	
T	H	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
100	10	-0.0679	0.1986	-0.1893	0.3487	-0.0486	0.1984	-0.1769	0.3115	-0.1801	0.5749
	20	-0.0722	0.2213	-0.2067	0.3640	-0.0497	0.2073	-0.1742	0.3249	-0.2019	0.5696
	50	-0.0654	0.1988	-0.2138	0.3602	-0.0494	0.1993	-0.1917	0.3150	-0.1277	0.4685
	100	-0.0602	0.1864	-0.2096	0.3562	-0.0536	0.1854	-0.1964	0.3146	-0.1430	0.4636
250	10	-0.0085	0.1446	-0.0559	0.1903	-0.0035	0.1458	-0.0611	0.1835	-0.0688	0.2078
	20	-0.0116	0.1434	-0.0548	0.1886	-0.0084	0.1426	-0.0640	0.1804	-0.0616	0.2022
	50	-0.0011	0.1410	-0.0637	0.1892	-0.0115	0.1379	-0.0650	0.1862	-0.0636	0.1862
	100	-0.0113	0.1363	-0.0559	0.1955	-0.0101	0.1422	-0.0723	0.1934	-0.0722	0.2241
500	10	0.0038	0.1158	-0.0321	0.1277	-0.0019	0.1120	-0.0189	0.1272	-0.0352	0.0743
	20	0.0003	0.1124	-0.0264	0.1228	-0.0010	0.1119	-0.0232	0.1262	-0.0369	0.0568
	50	-0.0004	0.1118	-0.0234	0.1217	-0.0025	0.1114	-0.0196	0.1279	-0.0350	0.0493
	100	0.0036	0.1098	-0.0297	0.1246	-0.0005	0.1121	-0.0174	0.1303	-0.0345	0.0554
1000	10	-0.0026	0.0921	-0.0150	0.0992	0.0025	0.0873	-0.0085	0.1025	-0.0204	0.0323
	20	0.0049	0.0829	-0.0197	0.0940	-0.0026	0.0812	-0.0034	0.0957	-0.0187	0.0310
	50	0.0005	0.0869	-0.0155	0.0966	-0.0017	0.0855	-0.0077	0.0988	-0.0198	0.0315
	100	0.0035	0.0904	-0.0189	0.0983	-0.0029	0.0864	-0.0048	0.0976	-0.0197	0.0313

Table 3: Auxiliary Model 3

		β_0^+		β_0^-		β_1^+		β_1^-		σ	
T	H	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
100	10	-0.0985	0.3586	-0.2167	0.3722	-0.0496	0.6347	-0.1721	0.3135	-0.1529	0.5139
	20	-0.0923	0.2659	-0.2258	0.3704	-0.0468	0.2202	-0.1960	0.3157	-0.1927	0.5491
	50	-0.0796	0.1999	-0.2475	0.3765	-0.0475	0.1901	-0.1943	0.3184	-0.1737	0.5186
	100	-0.0874	0.2355	-0.2189	0.3650	-0.0491	0.2216	-0.1980	0.3185	-0.1421	0.4639
250	10	-0.0152	0.1427	-0.0713	0.1882	-0.0021	0.1282	-0.0560	0.1744	-0.0760	0.2421
	20	-0.0160	0.1337	-0.0650	0.1821	-0.0005	0.1215	-0.0649	0.1710	-0.0651	0.2010
	50	-0.0059	0.1353	-0.0736	0.1883	-0.0058	0.1234	-0.0624	0.1718	-0.0702	0.2259
	100	-0.0083	0.1253	-0.0800	0.1978	-0.0010	0.1238	-0.0689	0.1794	-0.0766	0.2374
500	10	0.0032	0.0988	-0.0321	0.1162	-0.0007	0.0917	-0.0227	0.1130	-0.0417	0.0664
	20	0.0018	0.0974	-0.0330	0.1128	-0.0001	0.0941	-0.0182	0.1130	-0.0391	0.0603
	50	0.0032	0.1003	-0.0322	0.1113	0.0015	0.0941	-0.0193	0.1101	-0.0415	0.0621
	100	0.0017	0.0937	-0.0297	0.1105	-0.0008	0.0858	-0.0177	0.1082	-0.0393	0.0672
1000	10	0.0036	0.0680	-0.0157	0.0759	-0.0039	0.0588	-0.0104	0.0754	-0.0213	0.0339
	20	0.0105	0.0645	-0.0226	0.0774	-0.0016	0.0577	-0.0099	0.0766	-0.0241	0.0343
	50	0.0095	0.0663	-0.0232	0.0803	-0.0040	0.0608	-0.0038	0.0783	-0.0208	0.0320
	100	0.0067	0.0683	-0.0181	0.0759	-0.0013	0.0616	-0.0116	0.0758	-0.0227	0.0333

Table 4: Auxiliary Model 4

		β_0^+		β_0^-		β_1^+		β_1^-		σ	
T	H	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE	Bias	RMSE
250	10	-0.0168	0.1230	0.0333	0.1647	0.0019	0.0588	0.0258	0.1364	-0.0491	0.1284
250	50	-0.0133	0.1261	0.0356	0.1752	-0.0029	0.0562	0.0307	0.1350	-0.0501	0.1315

Table 5: EMM and ML Estimation of asMA(1) Model

EMM		β_1^+		β_1^-		σ	
T	H	Bias	RMSE	Bias	RMSE	Bias	RMSE
100	10	0.0161	0.2901	-0.0026	0.3619	-0.1152	0.1871
	20	0.0162	0.2804	-0.0022	0.3619	-0.1160	0.1891
	50	0.0153	0.2802	-0.0035	0.3575	-0.1147	0.1844
	100	0.0146	0.2790	-0.0057	0.3535	-0.1129	0.1817
250	10	0.0162	0.1574	0.0062	0.2304	-0.0430	0.0767
	20	0.0119	0.1535	0.0036	0.2211	-0.0424	0.0751
	50	0.0102	0.1483	0.0032	0.2180	-0.0416	0.0731
	100	0.0107	0.1485	0.0031	0.2178	-0.0419	0.0735
500	10	0.0020	0.1032	0.0081	0.1603	-0.0238	0.0496
	20	0.0022	0.1017	0.0069	0.1573	-0.0230	0.0479
	50	0.0021	0.0998	0.0070	0.1559	-0.0231	0.0479
	100	0.0017	0.0986	0.0056	0.1539	-0.0227	0.0473
1000	10	-0.0013	0.0704	0.0056	0.1092	-0.0109	0.0305
	20	-0.0013	0.0685	0.0049	0.1074	-0.0106	0.0298
	50	-0.0019	0.0681	0.0053	0.1054	-0.0105	0.0290
	100	-0.0020	0.0679	0.0045	0.1041	-0.0104	0.0288
ML							
100		0.0045	0.1340	-0.0006	0.1474	-0.0057	0.0738
250		0.0018	0.0736	-0.0002	0.0836	-0.0008	0.0457
500		0.0006	0.0512	-0.0001	0.0547	-0.0021	0.0317
1000		-0.0000	0.0355	-0.0007	0.0384	-0.0001	0.0220

Table 6: EMM LR-type Test Rejection Frequencies

	T	
Nominal α	250	500
0.01	0.013	0.01
0.05	0.068	0.056
0.1	0.126	0.093

Table 7: Estimation Results for RGDP Growth Rate

Parameter	TMA(1)	TMA(2)	TMA(3)	TMA(4)
β_0^+	0.0530 (0.349)	0.349 (1.292)	0.407 (2.060)	-0.509 (0.353)
β_0^-	0.932 (0.125)	-0.202 (0.564)	-0.457 (1.092)	-0.106 (0.744)
β_1^+	0.679 (0.296)	0.148 (0.480)	-0.00002 (4.095)	0.028 (0.308)
β_2^+	–	0.294 (1.072)	0.200 (4.168)	-0.085 (0.261)
β_3^+	–	–	0.164 (0.991)	0.064 (0.118)
β_4^+	–	–	–	0.246 (0.416)
β_1^-	-0.505 (0.131)	0.976 (0.157)	0.082 (1.442)	-0.048 (0.559)
β_2^-	–	-0.285 (0.504)	-0.095 (1.511)	-0.010 (0.310)
β_3^-	–	–	0.935 (0.766)	-0.004 (0.388)
β_4^-	–	–	–	-0.139 (0.678)
σ	0.028 (0.0002)	0.028 (0.0004)	0.028 (0.001)	0.061 (0.004)
OID	6.184 [0.63]	5.389 [0.49]	3.926 [0.42]	2.224 [0.33]
LR	0.353 [0.84]	0.99 [0.61]	1.204 [0.55]	1.607 [0.45]

Notes: Table summarizes estimation and test results for Real GDP growth rate. Auxiliary model III is used as moment generator for both restricted and unrestricted models. The number of structural parameters is $2q+3$ and the number of moment conditions is 13. Simulation constant is $H = 50$. Asymptotic standard errors are shown in parentheses. OID is the overidentifying restrictions test which has a chi-squared distribution with $10-2q$ degrees of freedom. LR is the EMM LR-type test statistic discussed in the text which has a χ_2^2 distribution. The null hypothesis of the LR test is that the effect of shocks in the current period is symmetric but the lagged effects are asymmetric (asMA(q) model). P-values are shown in brackets.

Table 8: Estimation Results for IPI Growth Rate

Parameter	TMA(1)	TMA(2)	TMA(3)	TMA(4)
β_0^+	-0.609 (0.214)	0.014 (0.137)	0.129 (0.206)	-0.188 (0.209)
β_0^-	0.438 (0.171)	0.654 (0.242)	0.733 (0.262)	-0.092 (0.371)
β_1^+	0.406 (0.146)	-0.123 (0.153)	-0.110 (0.269)	-0.004 (0.247)
β_2^+	–	0.368 (0.199)	0.439 (0.246)	0.249 (0.496)
β_3^+	–	–	-0.228 (0.194)	-0.779 (0.303)
β_4^+	–	–	–	0.560 (0.214)
β_1^-	-0.657 (0.252)	-0.384 (0.231)	-0.477 (0.312)	-0.385 (0.203)
β_2^-	–	-0.026 (0.080)	-0.035 (0.167)	0.101 (0.684)
β_3^-	–	–	-0.024 (0.132)	0.140 (0.561)
β_4^-	–	–	–	0.075 (0.225)
σ	0.083 (0.028)	0.089 (0.025)	0.079 (0.014)	0.083 (0.011)
OID	23.026 [0.003]	12.823 [0.046]	5.99 [0.20]	0.780 [0.68]
LR	9.006 [0.0004]	6.679 [0.036]	0.55 [0.76]	-0.0097 [1.00]

Notes: Table summarizes estimation and test results for the growth rate industrial production index (IPI). Auxiliary model III is used as moment generator for both restricted and unrestricted models. The number of structural parameters is $2q+3$ and the number of moment conditions is 13. Simulation constant is $H = 50$. Asymptotic standard errors are shown in parentheses. OID is the overidentifying restrictions test which has a chi-squared distribution with $10-2q$ degrees of freedom. LR is the EMM LR-type test statistic discussed in the text which has a χ_2^2 distribution. The null hypothesis of the LR test is that the effect of shocks in the current period is symmetric but the lagged effects are asymmetric (asMA(q) model). P-values are shown in brackets.

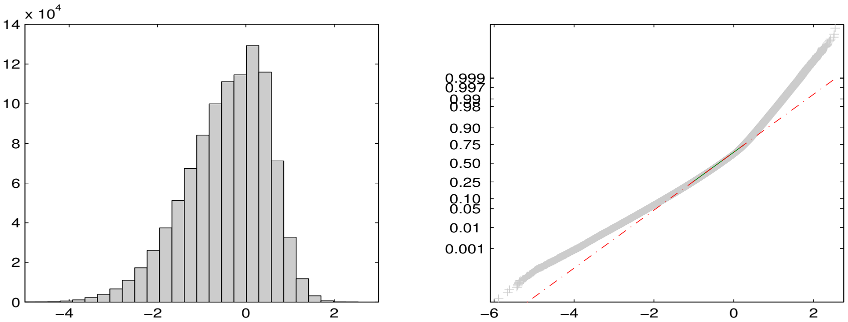

Figure 1: Histogram and Normal Probability Plot for TMA(1) Model with parameters $\beta_0^+ = 0.5, \beta_0^- = 1, \beta_1^+ = 0.2, \beta_1^- = 0.8, \sigma^2 = 1$

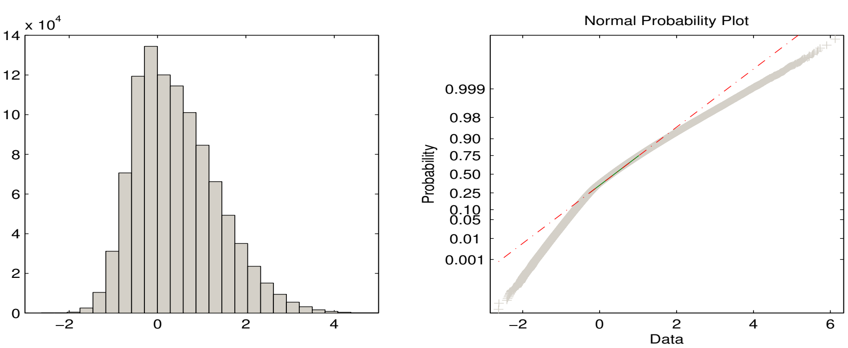

Figure 2: Histogram and Normal Probability Plot for TMA(1) Model with parameters $\beta_0^+ = 1, \beta_0^- = 0.5, \beta_1^+ = 0.8, \beta_1^- = 0.2, \sigma^2 = 1$

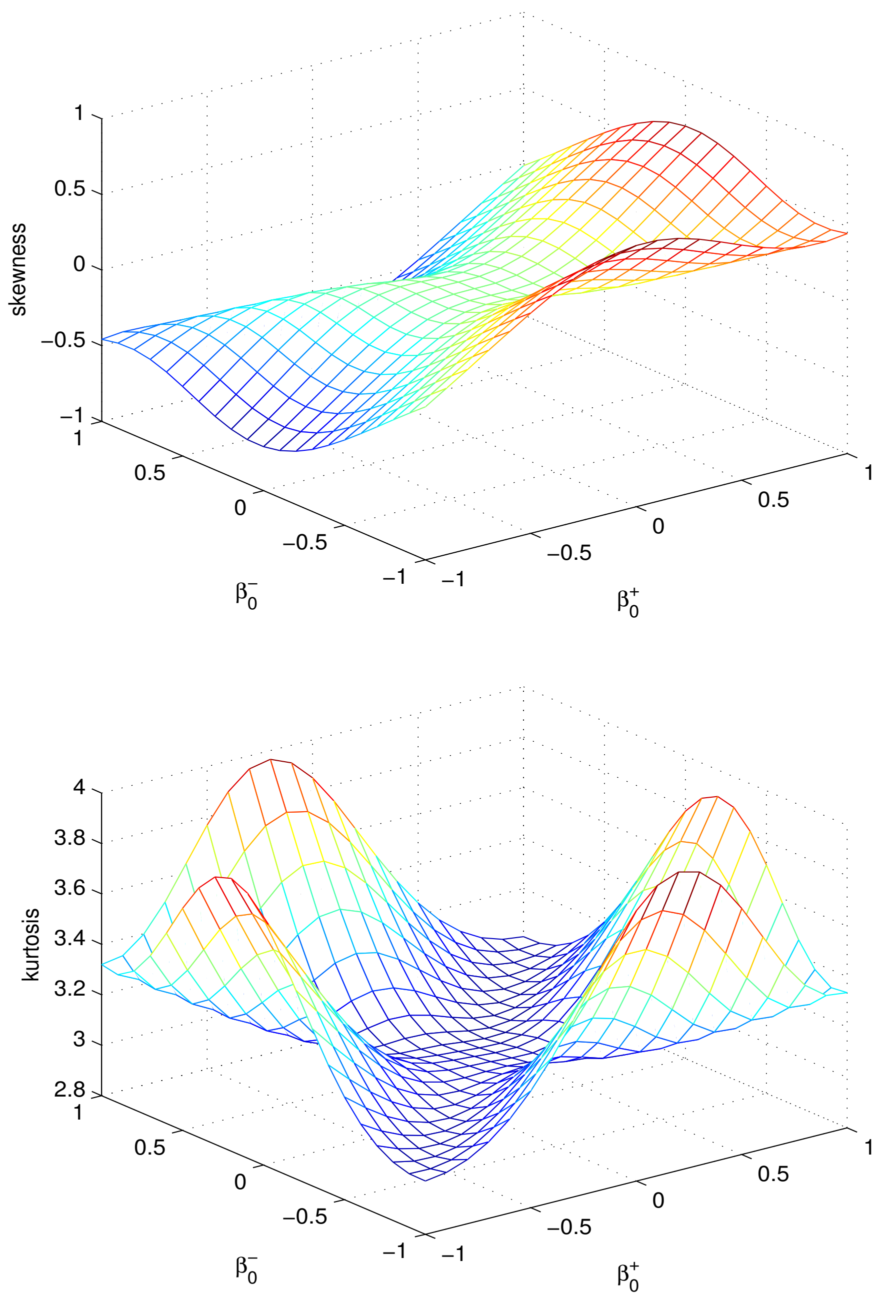

Figure 3: Skewness and kurtosis

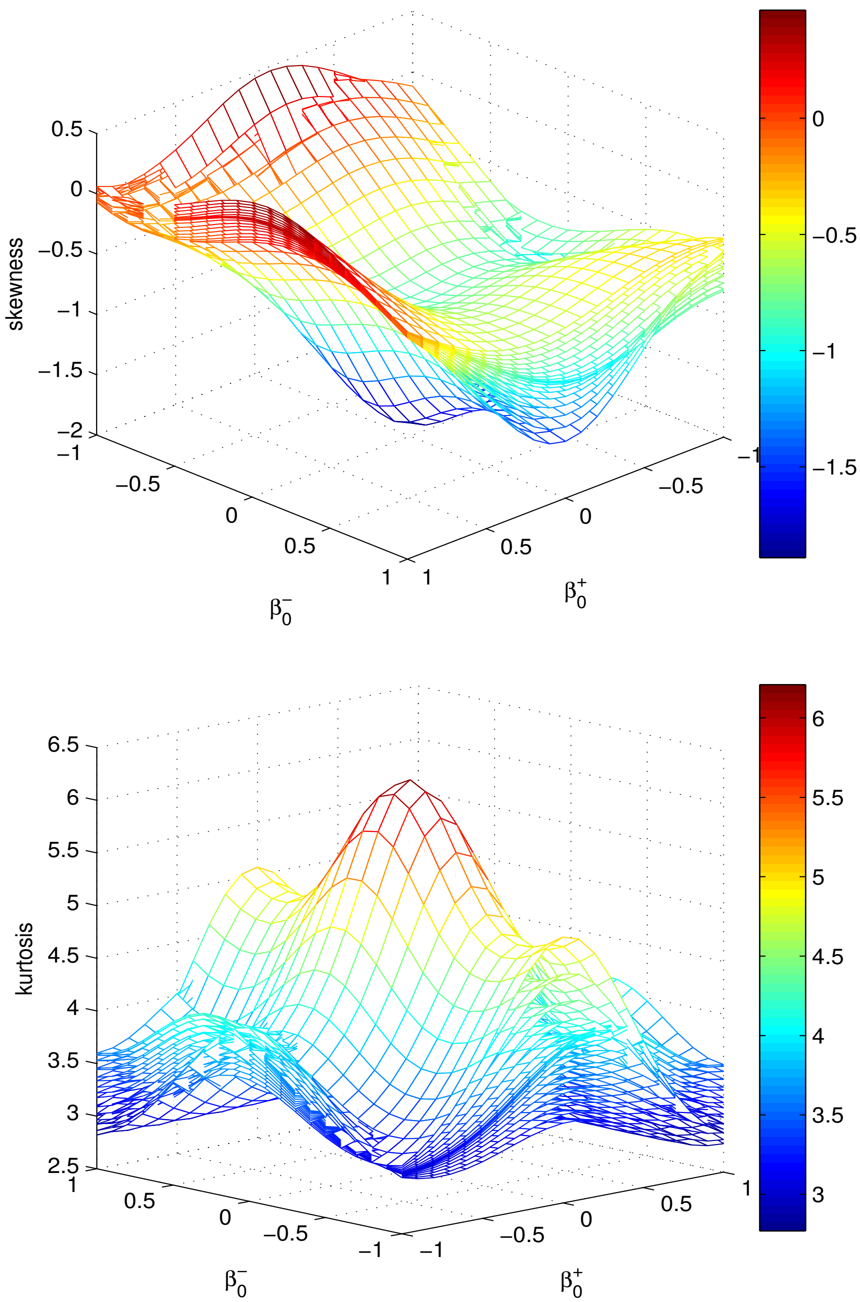

Figure 4: Skewness and kurtosis over $\gamma = [-1 : 0.1 : 1]$

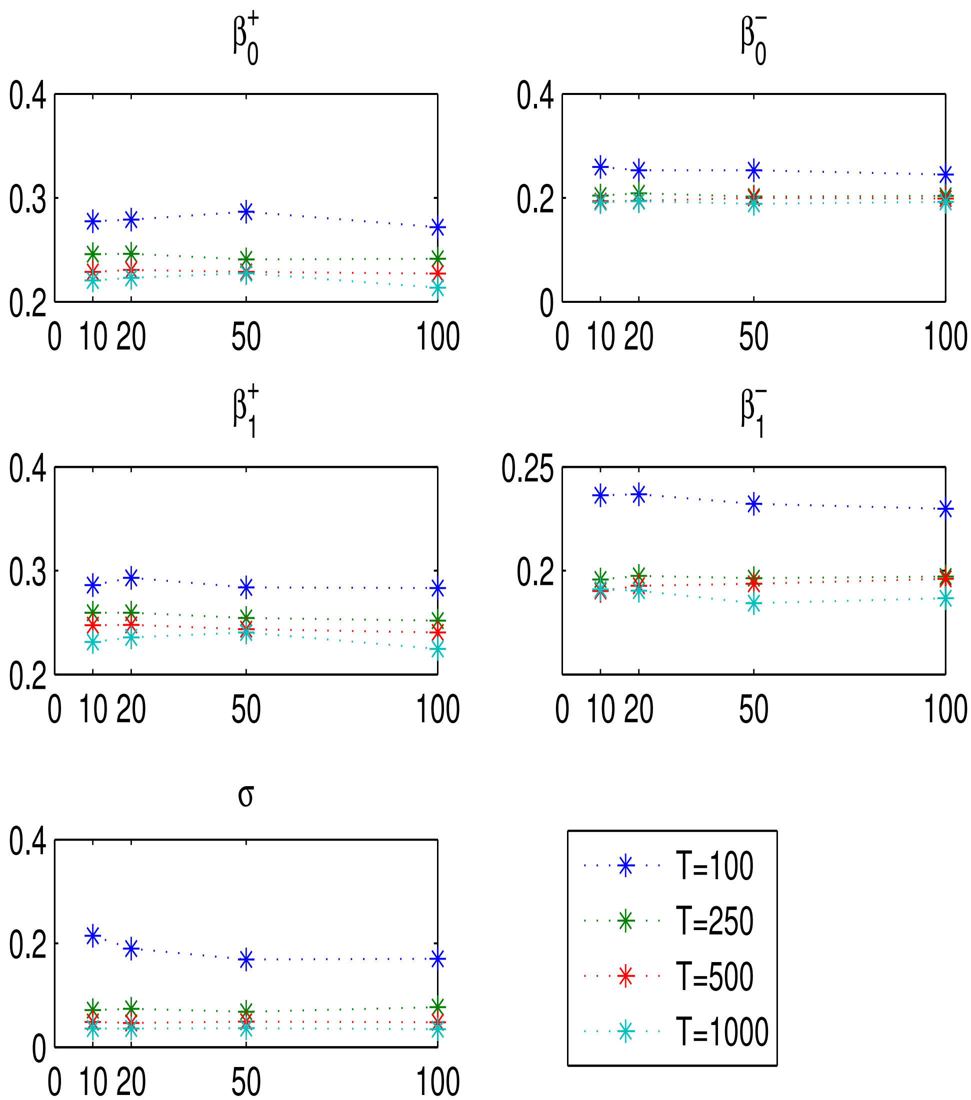

Figure 5: Auxiliary Model 1: RMSE and H

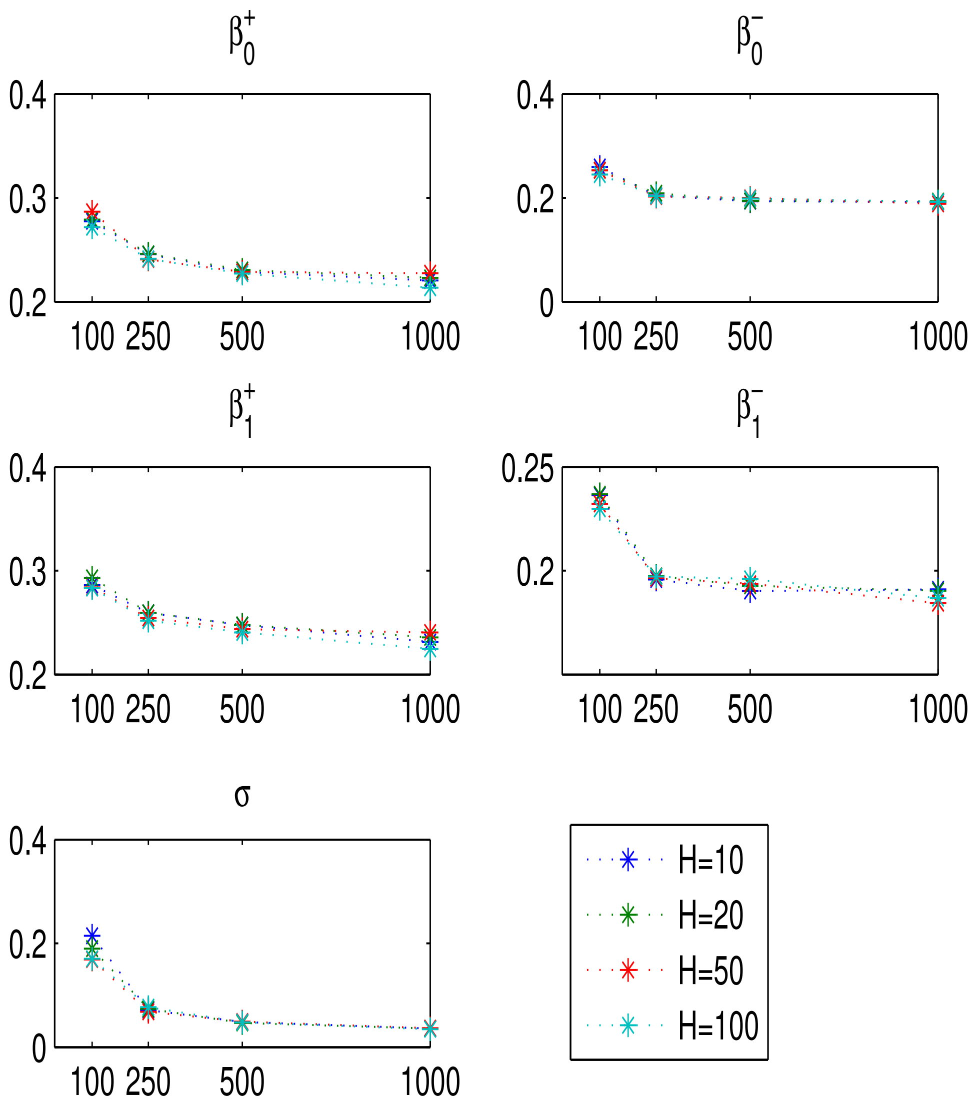

Figure 6: Auxiliary Model 1: RMSE and T

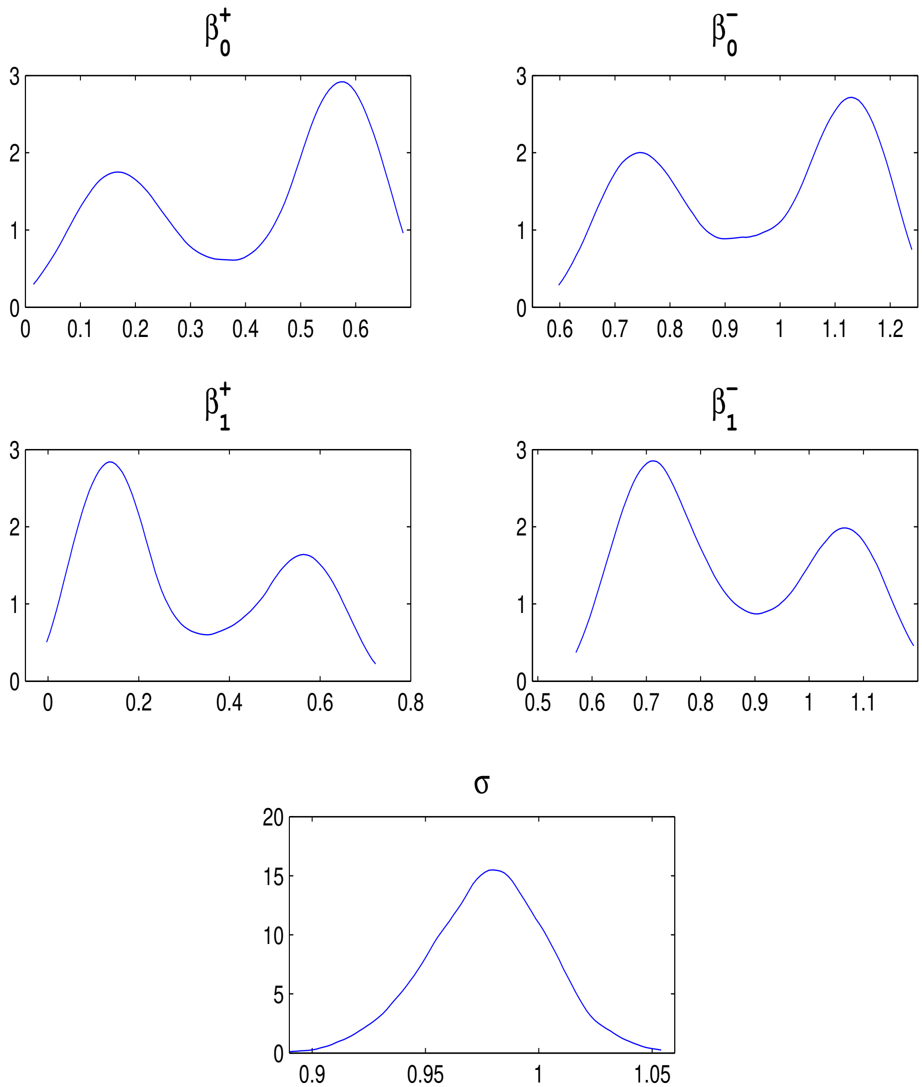

Figure 7: Auxiliary Model 1: Kernel density estimates

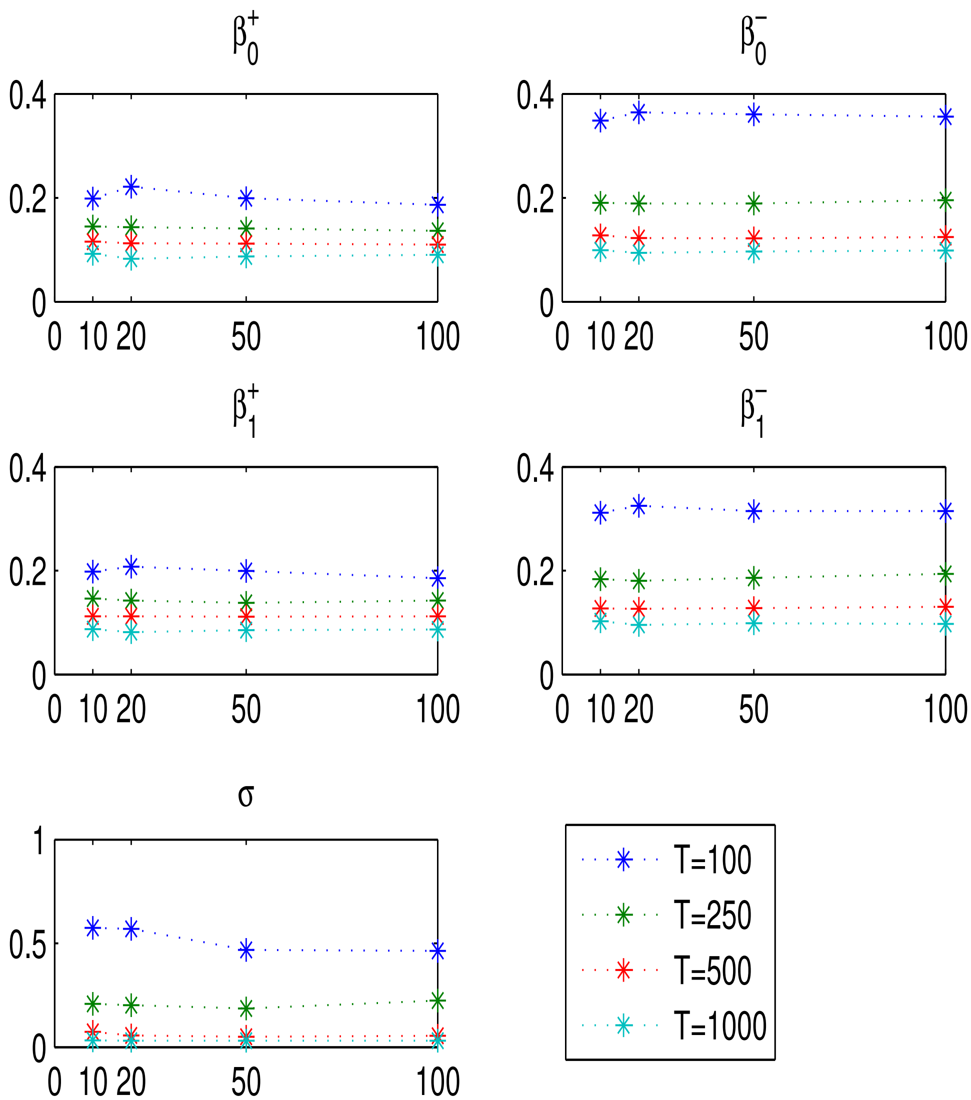

Figure 8: Auxiliary Model 2: RMSE and H

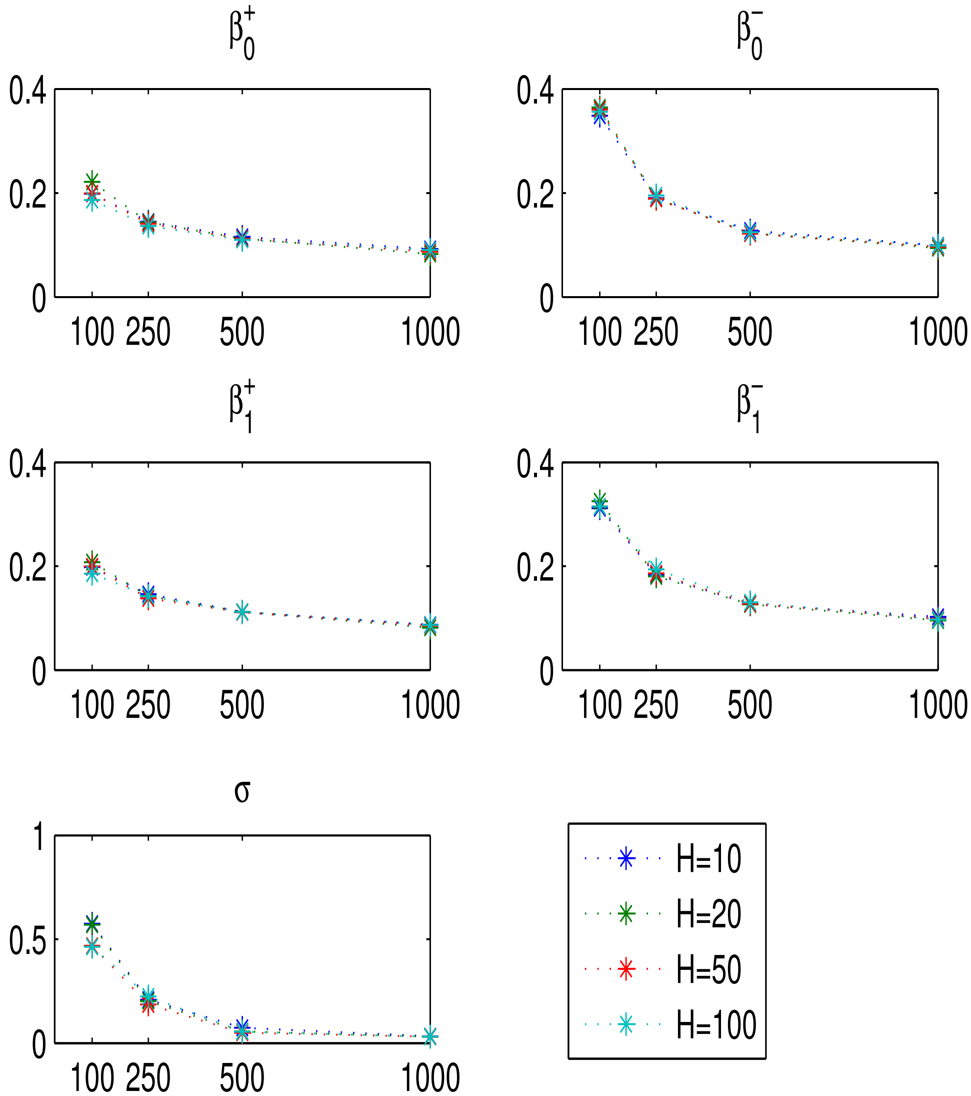

Figure 9: Auxiliary Model 2: RMSE and T

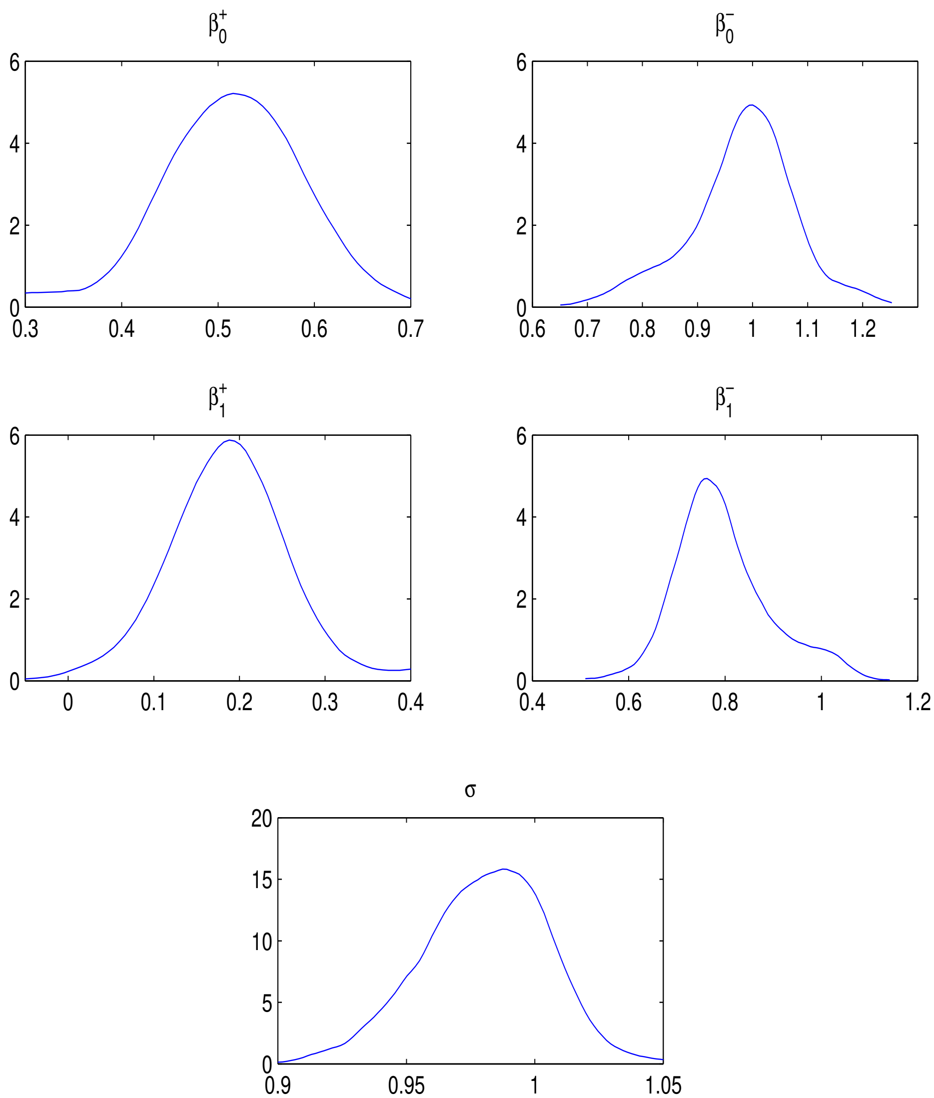

Figure 10: Auxiliary Model 2: Kernel density estimates

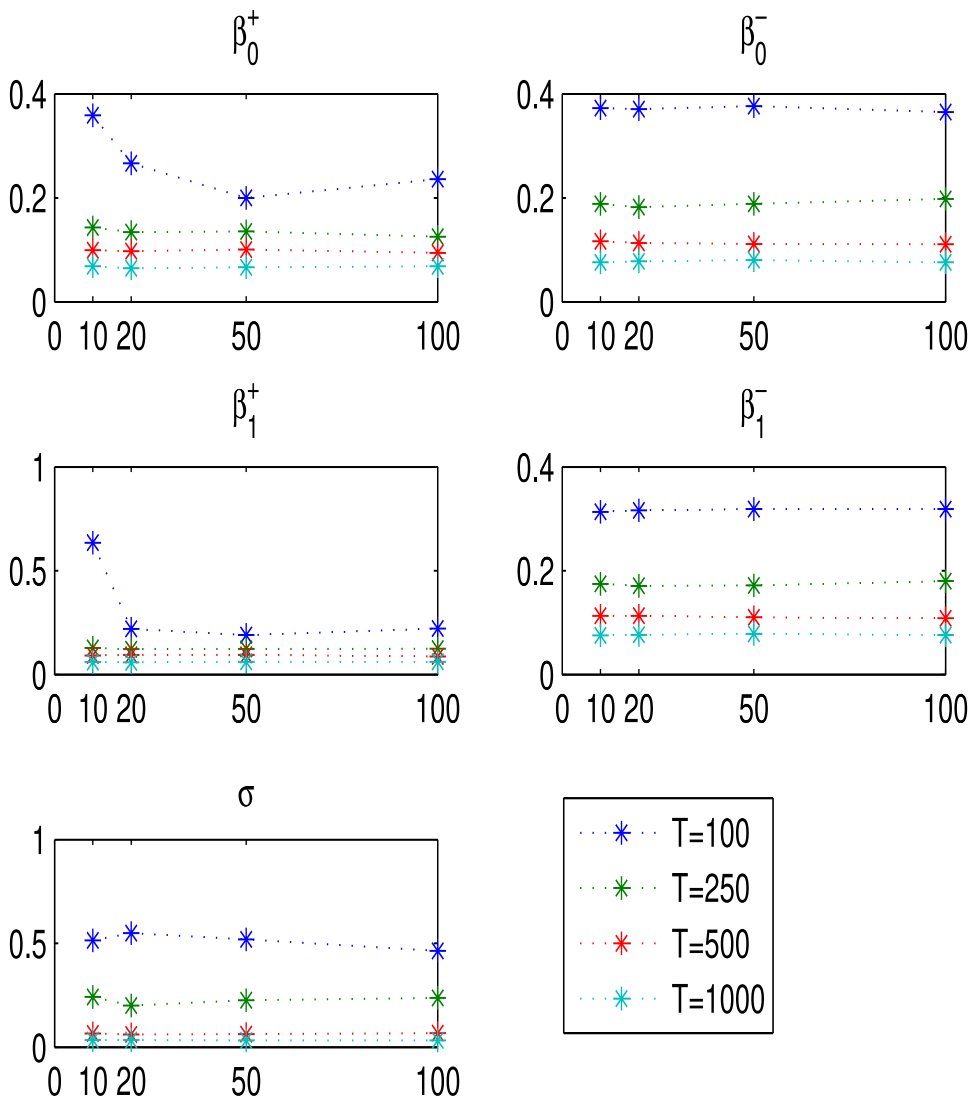

Figure 11: Auxiliary Model 3: RMSE and H

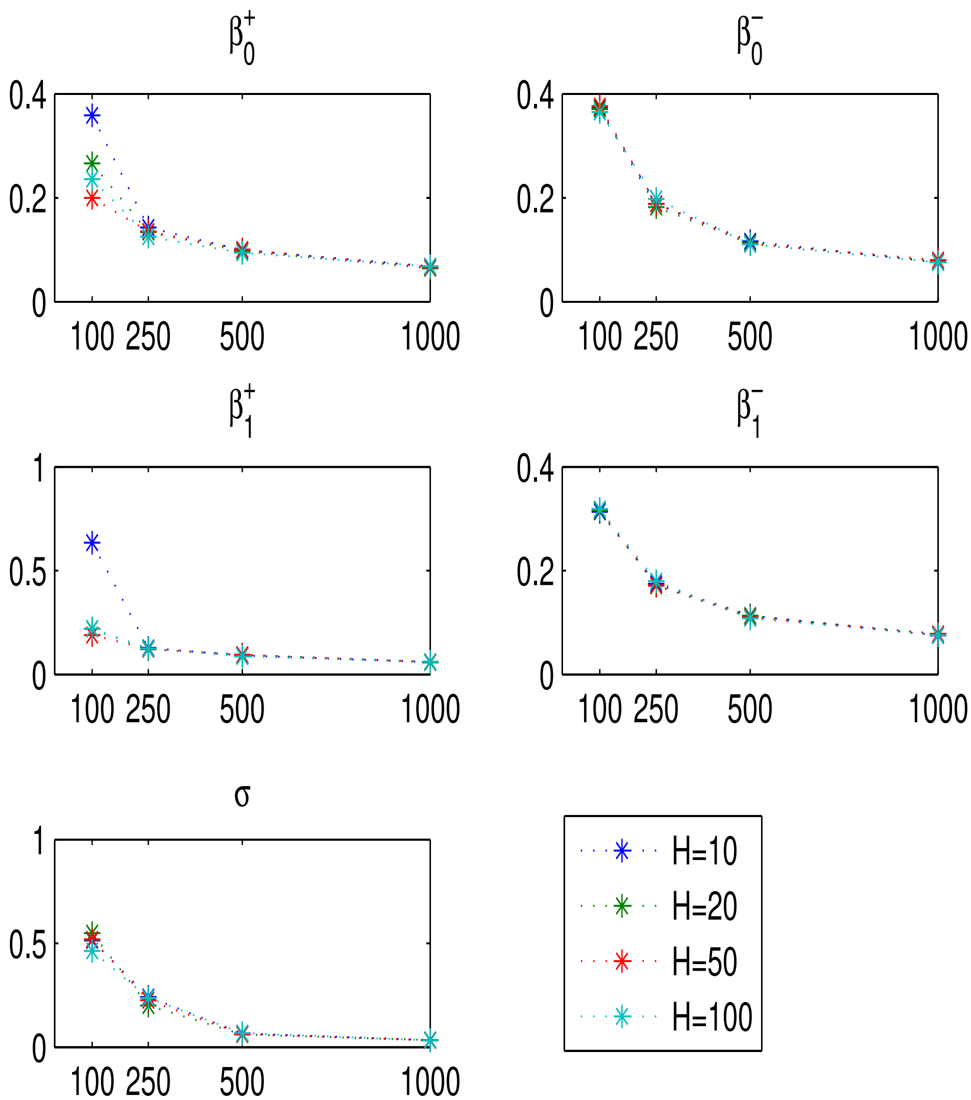

Figure 12: Auxiliary Model 3: RMSE and T

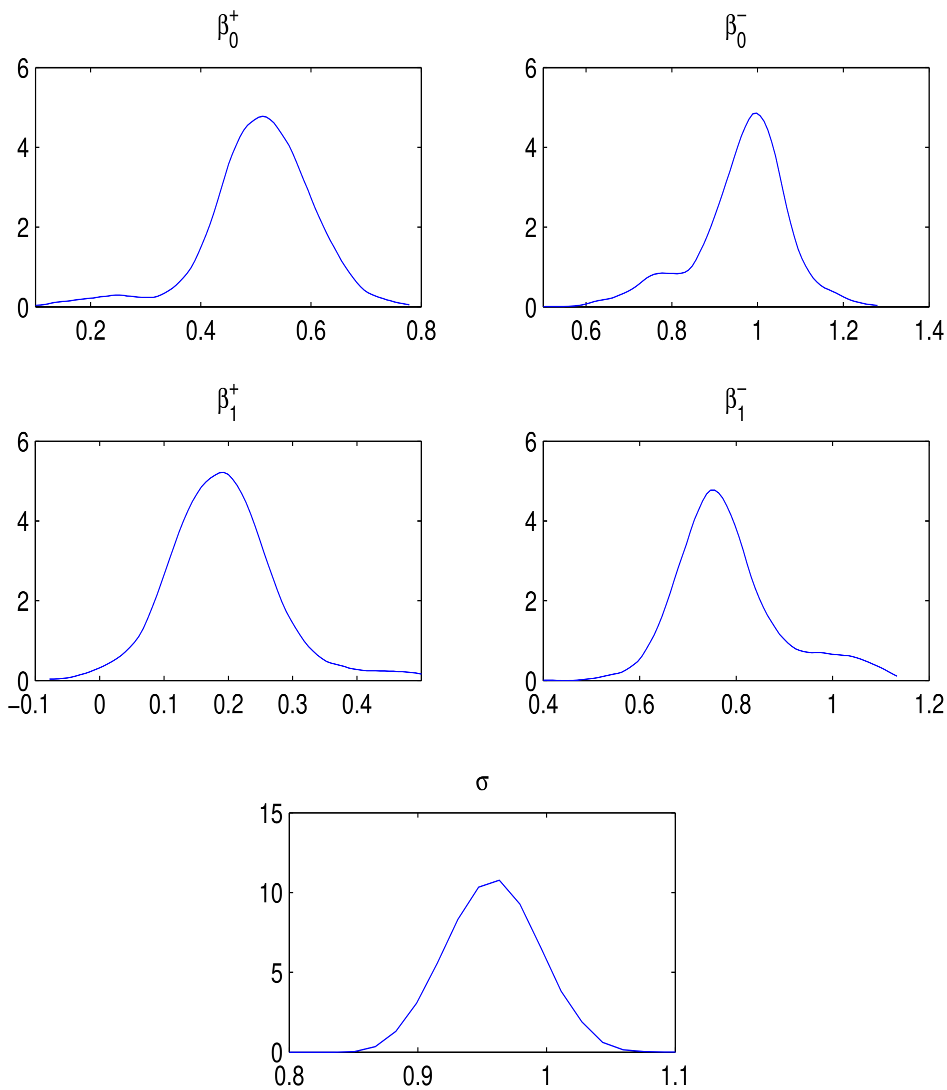

Figure 13: Auxiliary Model 3: Kernel density estimates

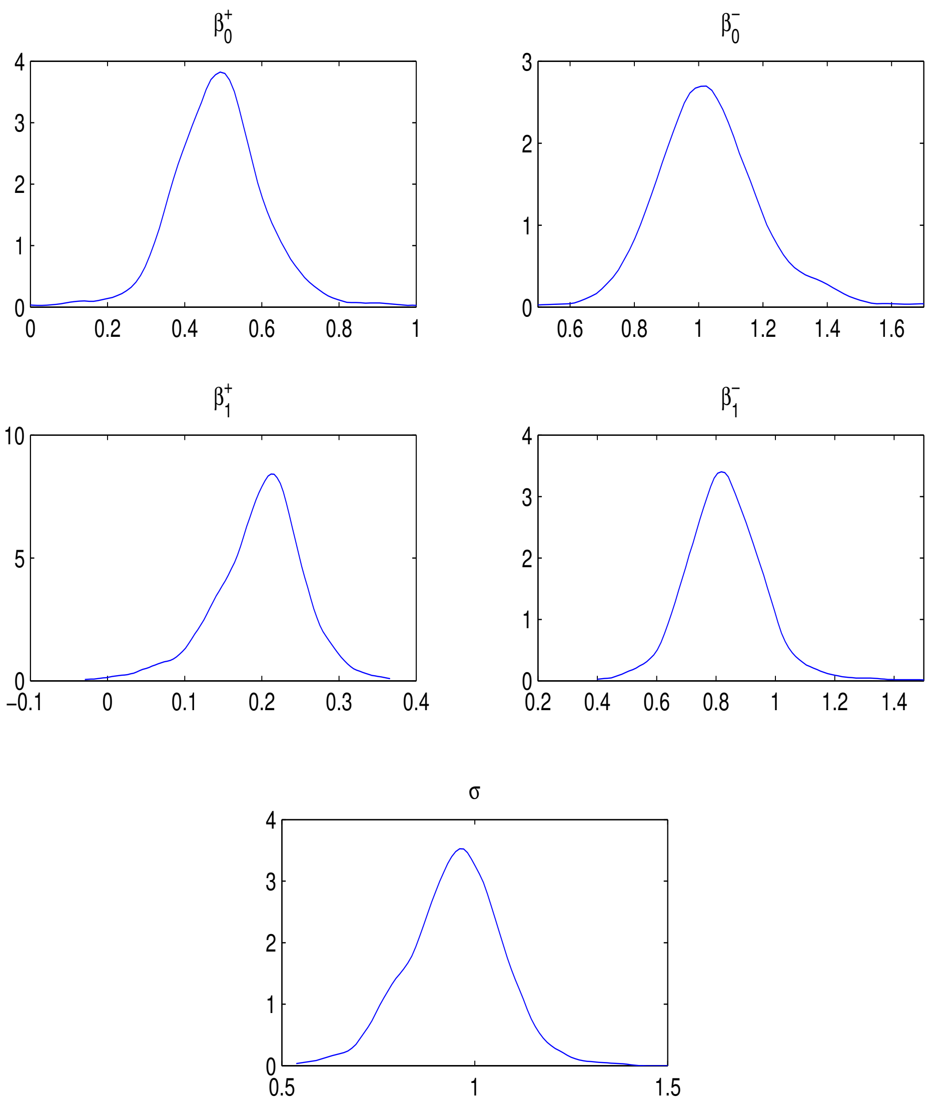

Figure 14: Auxiliary Model 4: Kernel Density Estimates

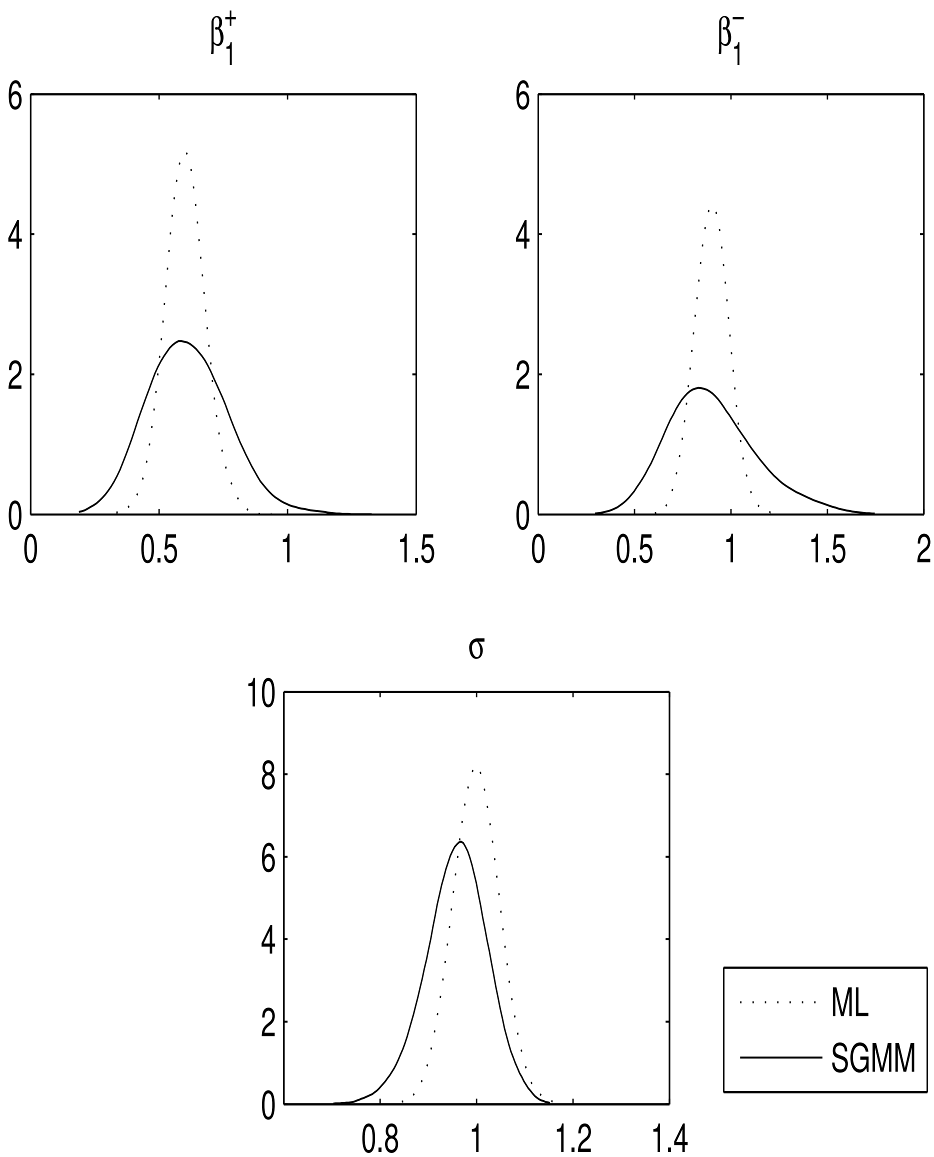

Figure 15: ML and EMM Estimation of asMA(1) Model: Kernel Density Estimates, $T = 250$

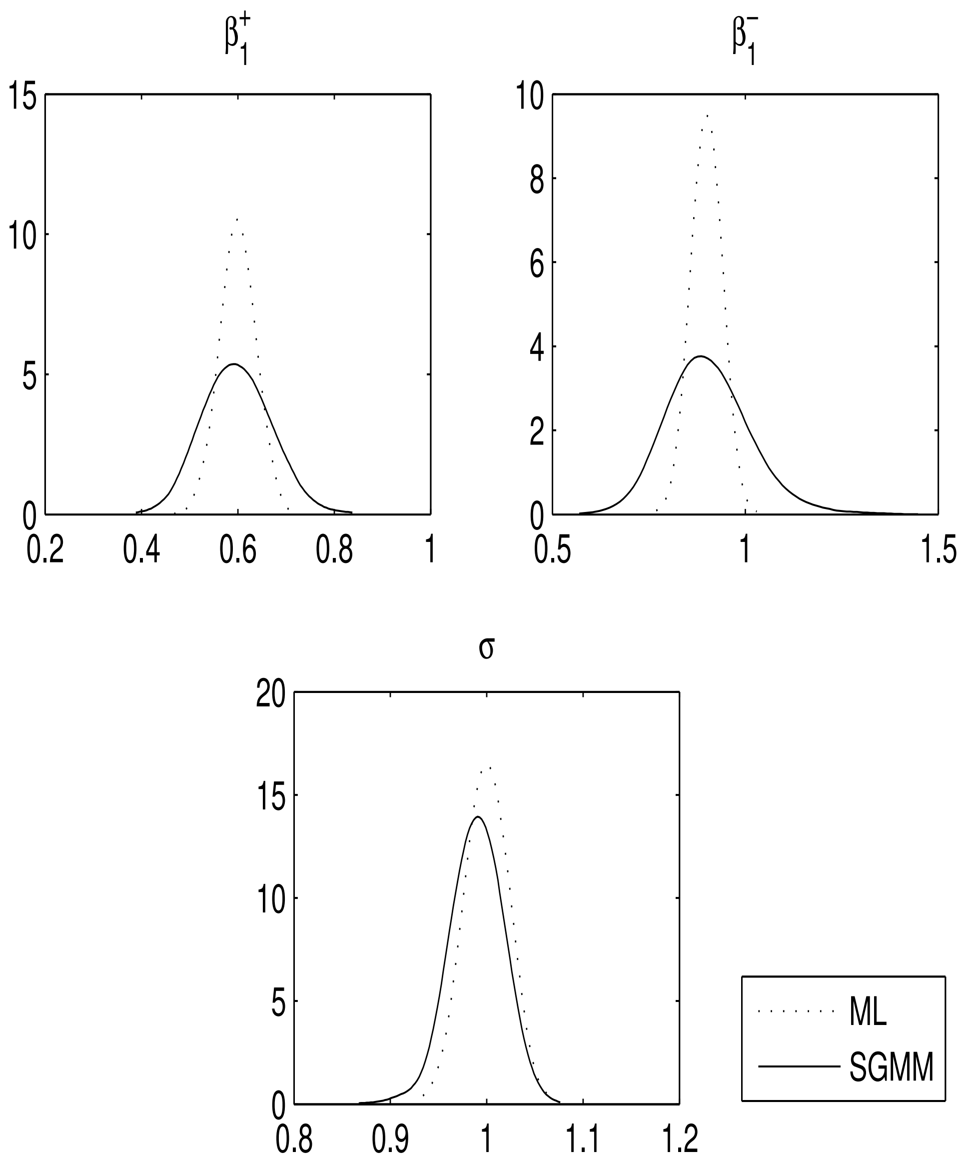

Figure 16: ML and EMM Estimation of asMA(1) Model: Kernel Density Estimates, $T = 1000$

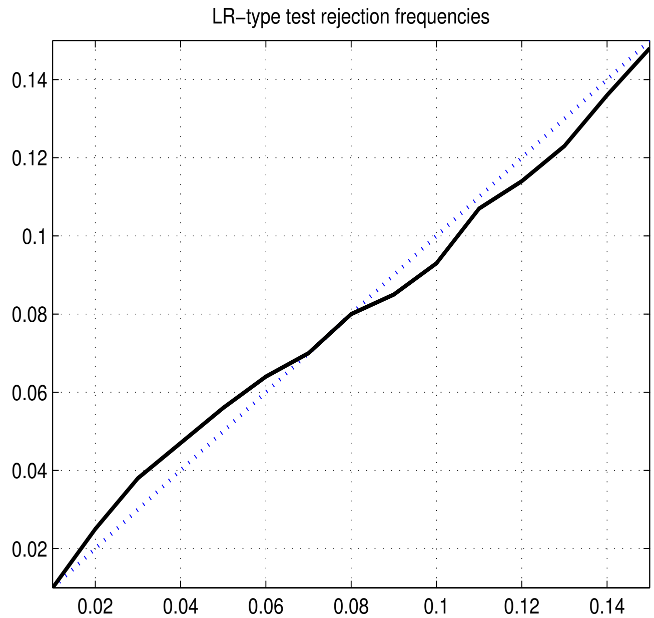

Figure 17: EMM LR-type test empirical rejection frequencies

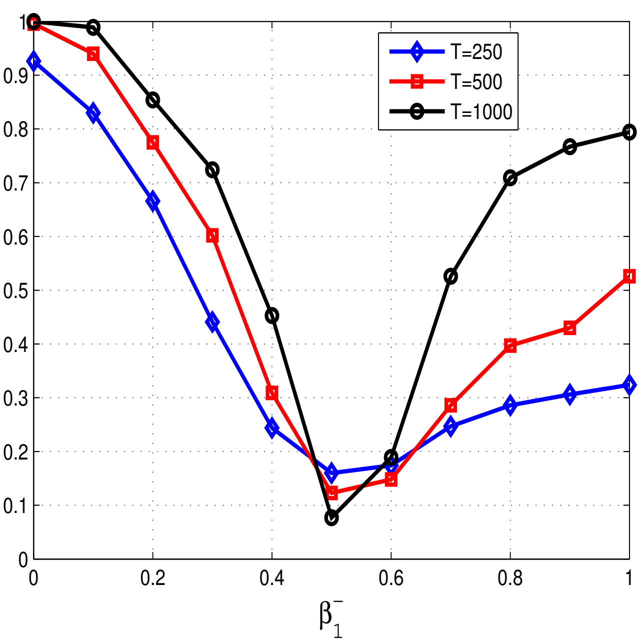

Figure 18: EMM LR-type test empirical power