The river sharing problem: A review of the technical literature for policy economists

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Abstract

Water is essential for life. However, the basic problem of water resource allocation has been that water tends to be over-allocated. Demand for water exceeds the available supply. Essentially, the water economy is bankrupt. Bankruptcy problems have been almost exhaustively studied in the literature on economic theory-primarily from the perspective of cooperative game theory. The main concern of this literature has been how to fairly divide up the assets of a bankrupt entity. In water resource economics co-operative game theory has often been employed as a means of analyzing water resource allocation. It was only recently that the problem of directional flow was incorporated into such analyses. This has come to be known as the "river sharing problem" in the theoretical literature. Accounting for the direction of flow in water resource allocation problems has profound implications for policies that wish to facilitate both fair and efficient water allocations. This is the case whether proposed policies are interventionist or market based in nature. There is now a considerable literature on the allocation and distribution of water resources characterized by unidirectional flow. In this paper I critically review and appraise this literature with a view to making it more accessible to applied and policy economists. A key feature of the paper is that the connection between the bankruptcy literature, which has recently also realized the importance of flow, and the river sharing literature is discussed. The current state of the art in game theoretic models of water resource allocation with directional flow is discussed and implications and consequences for water resource policy highlighted.

1. Introduction

Water is essential for life[2]. However, the basic problem of water resource allocation has been that water tends to be over-allocated. Demand for water exceeds the available supply. Essentially, the water economy is bankrupt. Bankruptcy problems have been almost exhaustively studied in the literature on economic theory-primarily from the perspective of cooperative game theory. The main concern of this literature has been how to fairly divide up the assets of a bankrupt entity. In water resource economics co-operative game theory has often been employed as a means of analyzing water resource allocation (see Parrachino, Dinar and Patrone, (2006) for an extensive review), however it was not until the seminal paper by Ambec and Sprumont (2002) that the problem of directional flow was incorporated into such analyses. This has come to be known as the "river sharing problem" in the theoretical literature. Accounting for the direction of flow in water resource allocation problems has profound implications for policies that wish to facilitate both fair and efficient water allocations. This is the case whether proposed policies are interventionist or market based in nature. Interventionist policies based on legislative solutions need to seek fair and just solutions to water allocation. Market based solutions involving water trading need to account fro the fact that the market is a way of implementing in a decentralized manner a particular cooperative outcome and that different market rules and prices will lead to different outcomes. If one reverses this logica and begins with the cooperative outcome and asks what market rules will achieve this outcome? Then the market designer needs to understand something about the properties of alternative cooperative principles in terms of whether they are fair, just and equitable. In other words the market designer needs to be cognizant of the distributional

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[2] This statement is almost identical to the opening sentence in Ambec and Ehlers (2008a) however I actually read that article after I drafted this section.

implications of a particular market design. This knowledge is found in the literature on the fair division of water resources a subset of which is surveyed here.

The original paper by Ambec and Sprumont (2002), in which the importance of directional flow was pointed out has since led to a considerable literature on the allocation and distribution of water resources characterized by unidirectional flow (some examples include Ambec and Ehlers (2008a,b), Coram (2006, 2009), Beard and McDonald (2007) and Ni and Wang (2007) and Houba (2008)). In this paper I critically review and appraise this literature with a view to making it more accessible to applied and policy economists. A key feature of the paper is that the connection between the bankruptcy literature, which has recently also realized the importance of flow (see Branzei, et al., 2008), and the river sharing literature is discussed. The current state of the art in game theoretic models of water resource allocation with directional flow is discussed and implications and consequences for water resource policy highlighted.

Some of the terminology employed needs explanation. The core for example is the set of allocations (of water) that would not be blocked or objected to by any stakeholder in negotiations over how to divide up the total water allocation. It captures the idea of Pareto optimality of the water allocation as well as assuming stakeholders would have an incentive to participate in any agreement to share water. The nucleolus is a cooperative game theoretic solution concept based on the excess value of a coalition. Consider a group of farmers located along a river and a given allocation of water amongst the farmers. The excess gives the total welfare of the group of famers after distributing the

water. Another allocation would result in a different excess. The allocation which minimizes the maximum excess is the nucleolus of the game. It will be only mentioned briefly in what follows and is defined here for reasons of completeness. Finally the Shapley value should be mentioned. This is found by averaging the marginal contribution of a player across all possible coalitions of players it captures the essential properties of a fair allocation of resources. All these ideas assume that the allocations being considered are efficient. The remaining terminology will be discussed in the course of the paper, the terminology discussed in this paragraph provides the necessary background for the remainder of the paper.

The paper is structured as follows section 2 surveys the early on fair cost allocation using cooperative game theory before discussing the newer literature on the river sharing problem specifically. Section 3 discusses recent contribution that extend the Ambec and sprumont analysis of the river sharing problem in an inter-temporal dimension. Section 4 discusses applications of the downstream incremental distribution principle to other problems, for example machine scheduling in industrial production. Section 5 discusses the connection between network flow problems generally and the bankruptcy literature and discusses how the river sharing problem may be interpreted from a bankruptcy perspective in addition this section discusses the connection between bankruptcy problems and over-allocation of water rights. Finally, it examines how some solution principles from the bankruptcy principle could be considered to address the problem of fair allocation of water rights in a situation of over-allocated permits.

2. Early work on Fairness and the River Sharing problem

Early work on water resource management employed co-operative game theory. Parrachino et al. (2006) provides a comprehensive review of this literature that I will not reproduce here. However, some key papers are worth mentioning in order to set the background and to give the reader an indication of the long tradition of this type of analysis in the water resources area. Much of the early research focused on cost allocation issues. A seminal paper in this area was Ransmeier (1942) who studied the cost allocation problem of dams for the Tennessee valley authority predominantly from a non-cooperative perspective, this work was revisited the following year by Parker (1943) who studied a number of cooperative approaches to cost allocation for dams again with reference to the Tenessee valley authority. Later work applied cooperative game theory to transboundary river basins and international aspects of water resource allocation. Early examples of include Rogers (1969) who studied water resource allocation along the Ganges and Brahmaputra rivers using linear programming and game theory. According to Parrachino et al. (2006) which can be regarded as the most authoritative review of the literature to date, the next major contribution to the literature was by Straffin and Heaney (1981) who discussed the cooperative game theoretic content of the Tennessee valley authority literature. However, this overlooks the earlier work of Suzuki and Nakayama (1974, 1976) in Japan who apply the nucleolus (see Schmeidler (1969)) to examine sharing of water in a Japanese river basin. Their working paper appeared in 1974 and was published in the journal *Management Science* in 1976 which still predates Straffin and Heaney's work. The Straffin and Heaney paper studied solutions to the water resource allocation problem such as the core, a special case of the nucleolus and a cost allocation method known as the alternative cost avoided method which

minimizes the maximum propensity to disrupt[3]. This latter method had previously been proposed by Gately (1974) in the context of sharing the costs of electric power generation construction and by a consultant to the Tennessee valley authority, Martin Glaeser in 1948 (Parrachino et al. (2006): 4). This method was later placed on an axiomatic basis by Owen (1993). All of these various contributions have concentrated on the sharing of the costs of dam construction. Flow was therefore not relevant to the problem of interest because there was no cost externality between different dams.

A number of other papers go beyond cost-sharing rules for dam allocation to study cost sharing to agriculture. The first of these is Suzuki and Nakayama paper mentioned in the previous paragraph. Although it also considers dam construction this paper marks a transition from the cost-sharing literature on dam construction to the literature on farm irrigation by introducing water use for agricultural purposes specifically into the analysis.

Ambec and Sprumont (2002) is the seminal paper introducing directional flow into river basin models. In their paper they developed a compromise solution between two principles of international law that have been developed in the context of water resource sharing in transboundary river basins. These principles are that of absolute territorial sovereignty otherwise known as the Harmon doctrine and unlimited territorial integrity. Harmon was the US attorney general from 1895 to 1897 and was asked for an opinion on whether or not the US was in anyway accountable for downstream consequences of actions on US territory on the Rio Grande river which flows into Mexico. He answered in the negative, this became known in international law as the Harmon doctrine.

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[3] Disruption here refers to disrupting a cooperative agreement between stakeholders in a region, e.g. a river basin.

Ambec and Sprumont developed a compromise solution between absolute territorial sovereignty and unlimited territorial integrity which amounted to allocating water amongst riparian users according to marginal value of the water to a coalition of users that the individual user contributed. So as one moves downriver additional users of water are added to a coalition of users for sharing water, the marginal contribution of each additional member of the coalition determines that users share of the value of the water: monetary transfers between users are then used to guarantee that coalition members will agree. Figure 1 depicts this graphically in terms of utilities.

Ambec and Sprumont's original paper has been extended in a number of directions one of the first extensions based directly on a suggestion in the original paper is by Ni and Wang (2007) who examine a river pollution problem with directional flow. Ni and Wang re-interpret absolute territorial sovereignty and unlimited territorial integrity from a Hohfeldian perspective arguing that rights and responsibilities are dual to each other and that in the context of pollution treating absolute territorial sovereignty and unlimited territorial integrity from a responsibility perspective makes more sense. Ni and Wang term the equivalent of absolute territorial sovereignty in terms of responsibility the principle of local responsibility. This principle advocates that it is the responsibility of those located in a river segment to keep the river in that segment clean. Whereas the responsibility analogue of unlimited territorial integrity which they call downstream responsibility, for reasons that will be shortly apparent, gives downstream inhabitants of the river the "right" to ask all upstream inhabitants of the river to keep the river clean.

Perhaps this is just semantics but it does appear that downstream responsibility confounds rights and responsibilities somewhat. This leads them to two new concepts for fairly dividing the responsibility of keeping the river clean. The first is *local responsibility sharing* which simply equates the cost share for cleaning a river segment to the cost that would be incurred by an individual to keep their segment of the river clean. The second principle is *upstream equal sharing* which allocates a cost-share to each individual which is the sum of that individuals cost for keeping a river segment clean plus a sum of cost proportions for keeping the river clean for all upstream inhabitants. This means that the clean-up cost for inhabitants further upstream has a lower weight in a given inhabitants cost-share. Ni and Wang demonstrate the equivalence between both local responsibility sharing and upstream equal sharing and the Shapley value. The Shapley value is often considered a benchmark for what is considered fair in the literature on cost-sharing because it captures the idea of "equal treatment of equals and unequal treatment of unequals", one of the few principles in political theories of justice that is widely accepted. As a side note for practitioners although the Shapley value is often presented in a somewhat intimidating mathematical form, it is easily calculated in tabular form using a spreadsheet.

Houba (2008) presents an interesting graphical representation of the Ambec and Sprumont solution concept. That is reproduced here to help illustrate their idea. This consists in reducing there model to one involving negotiation between users an upstream user and a downstream user. He then borrows from the literature on bankruptcy to apply

the Nash solution concept to rationing over-allocated water resources. Assume that country 1 is located upstream and country 2 downstream.
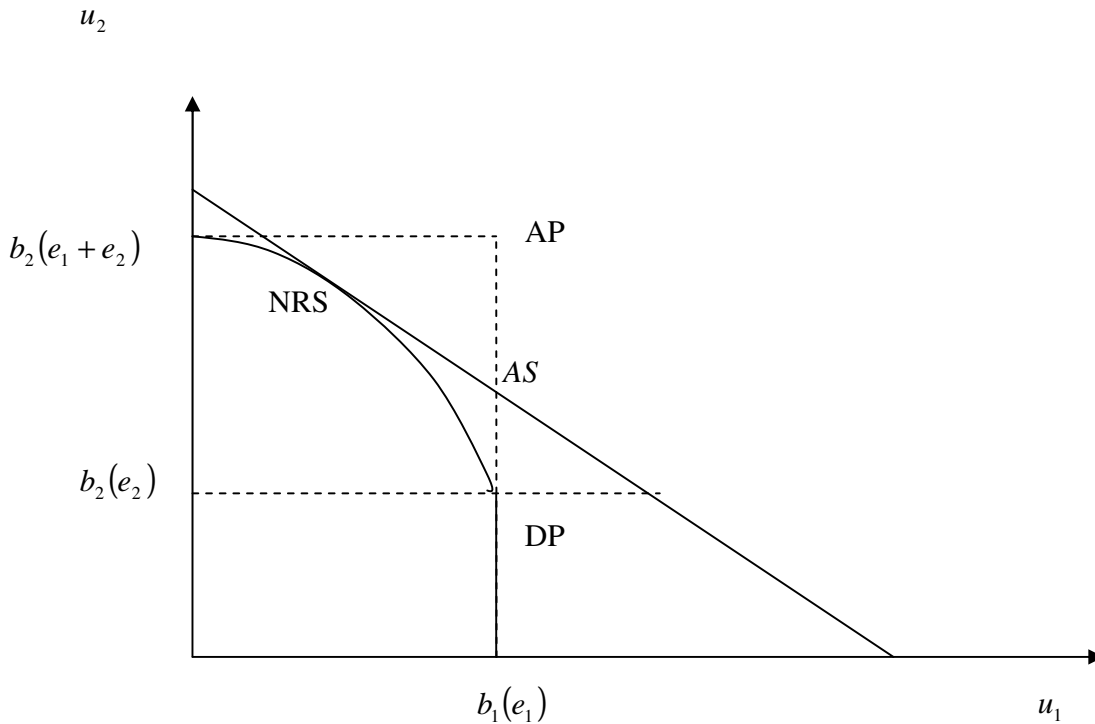
u_2

Figure 1: The Ambec and Sprumont solution in utility space. Source: adapted from Houba (2008)

Absolute territorial sovereignty awards each country a quantity of water e_i. Which would award each country utility located at the point DP. This is the disagreement point if one applies the Nash solution concept from bargaining theory. Unlimited territorial integrity would award the upstream country e_1 and the downstream country $e_1 + e_2$ this would give both countries utilities located at the aspiration point AP which is not feasible. Ambec and Sprumont's solution is located at the point AS. The point of tangency between the frontier of the bargaining or utility possibility set (the curved frontier) and the linear feasibility constraint is the Nash rationing solution (NRS). The Nash rationing solution si the solution to water rationing that would be dictated by the Nash bargaining

solution based on bargaining between upstream and downstream users. This figure clearly depicts the compromise nature of the Ambec and Sprumont solution to the river sharing problem.

Ambec and Ehlers (2008a) is primarily a review paper of the river sharing literature, it is however not comprehensive in this and overlooks a number of important papers. It is also more technical than the present paper and covers the broader literature on cooperative game models of cost sharing and fairness as they apply to water resources. A key point of their paper is that non-cooperative extraction leads to overexploitation of water resources and therefore cooperative solutions are necessary this is sued as a basis for arguing in favour of a number of cooperative solution concepts drawn from the cost-sharing literature. They survey a number of fairness principles all predicated on the fact that an efficient allocation ahs been achieved through some cooperative agreement. The first of these principles is *equal sharing individual rationality*. This states that "any agent should get at least as much as equal division." Essentially this says that users of the river under a fair division should have an allocation at least as good as a proportional division rule would give them (a commonly applied principle in the bankruptcy literature). A second criteria they examine is the *no envy* principle which states essentially that any two users of the river would not wish to swap there allocations because this would make them worse off. The full details of their discussed of the application of the no envy principle to water resource allocation will not be discussed here instead the reader is referred to their paper. A third criteria they examine is a solidarity axiom which is essentially the drop-out monotonicity condition discussed in section 4 of this paper and in Hendrickx (2004).

They then go on to discuss implementation of cooperative rules and decentralized water resource allocation, here they touch on issues related to the literature on cooperative implementation and mechanism design that are of key importance to anyone trying to design markets to trade water.

Ambec and Ehlers (2008b) consider the case where preferences are single peaked that is to say the utility functions of water consumers are assumed to possess a maximum therefore there exists an optimal level of water consumption. This has important consequences. The most important of which is that the core of the game may be empty and that therefore no efficient allocation can be achieved. This paper essentially extends the earlier work of Ambec and Sprumont (2002) by allowing for externalities between agents this means that the behavior of members of any water sharing agreement will be affected by the behavior of non-members. This results in the need to modify the set of utility possibilities over which agents can bargain. Essentially the disagreement point in figure 1 now depends on the behavior of agents who are not a party to the negotiations. They manage to show that the Ambec and Sprumont solution is the unique compromise solution between the aspiration point implied by unlimited territorial integrity and the disagreement point (core lower bound) implied absolute territorial sovereignty if one assumes that non-signatories to any cooperative agreement do no co-operate with each-other. If outside agents are able to cooperate with each-other and there are more than three agents then there may be no fair way of allocating water that is a compromise between the two principles of absolute territorial sovereignty and unlimited territorial

integrity. An important feature of this paper is that it also draws a connection between their research and work in the area of international agreements on climate change.

Houba (2008) is an attempt to respond to a point raised by Dinar (1992) regarding the application relevance of cooperative game theoretic models of water resource allocation. Dinars point consists of two parts the first part is that stakeholders are loathe to accept solutions requiring monetary transfers not based on prices. In part this point explains the recent popularity of approaches based on market based instruments. Dinar's second point relates to the computational complexity of some game theoretic solution concepts. Houba responds to both points but begins by addressing the second. He argues that Rubinstein's bilateral bargaining model has lower computational complexity than cooperative game theoretic models; although he offers no formal proof of this claim. The claim may or may not be true. However there is a large body of literature on the computational complexity of game theoretic solution concepts many of which have different computational requirements. That they are al more complex in computational terms than Rubinsteins bilateral bargaining model is a strong claim. The second point regarding pricing is more interesting. The recent market based instruments fashion in agricultural economics has placed legislative solutions in the background as far as current practice in water resources policy is concerned. Cooperative game theoretic models have more to say about questions of distributive justice and legal principles whereas market based instruments in particular in the form of specific trading regimes have more to offer in terms of efficiency. Cooperative game theory assumes the issue of efficiency to have already been resolved. Houba demonstrates that the Ambec and Sprumont model can be given a

bargaining interpretation via the Rubinstein alternating offers model and that this can be supported by market prices. He argues via the second fundamental theorem of welfare economics that alternating offers provides a way of supporting a bargaining outcome via prices. However, as the figure 1 shows the bargaining outcome would almost certainly differ from the Ambec and Sprumont solution. Houba is therefore skeptical of their solution concept. Houba's paper is interesting for a number of reasons not the least of which is that it explicitly couches the river sharing problem within the framework of a bankruptcy problem. I will deal more with this idea in section 5 below.

The next section deals with dynamic extensions of Ambec and Sprumont's river sharing problem.

3. Dynamic aspects of river basin management

The literature on dynamic aspects of river basin management using cooperative game theory is relatively underdeveloped. To the best of my knowledge three papers have been written on this topic beginning with Coram (2006). Coram directly extends the Ambec and Sprumont model to a dynamic setting using optimal control theory in order to address the question as to the efficiency of water trading in the presence of directional flows. As such it is of particular interest to Australian water resources policy due to policy emphasis that is being placed on water trading in that country. Although Coram cites considerable precursor literature on water resources only Ambec and Sprumont (2002) is cited from the literature on fair sharing and Weber (2001) as an example of an attempt at designing a trading mechanism that takes into account directional flows. Corams model differs however from Weber's not least in that it uses a continuous-time rather than a discrete-time approach. Coram tests a proposition of Weber namely that only adjacent

agents will trade with each-other because if an upstream agent offers to buy an allocation from someone further downstream they would be outbid by agents in between, i.e. intermediate agents will block trades between agents upstream and downstream of themselves. Coram examines the credibility of this strategy by testing whether or not is sub-game perfect[4]. He finds that such blocking strategies are not credible. The argument here is slightly different to that of d'Albis and Ambec (see below). Coram then examines a sequential auction in which the agent furthest downstream bids first by nominating a quantity and price, then the next upstream agent bids and so-on. Coram demonstrates that this does indeed produce an appropriate equilibrium at socially optimal prices. However he points out this is once-off auction and that things such as payoff functions- not to mention availability of water change over time. He makes some suggestions about repeating the procedure at regular intervals to make allocations time dependent. Essentialy the issue is one of the equivalence between permanent and temporary water allocations and how to trade them. This issue is dealt with in more detail in the paper by Beard and McDonald (2007) which will be discussed below.

Coram considers a number of other trading mechanisms in the appendix to his paper, although they were in the main body in the unpublished working paper version. In any case this is where one finds the most important results of his paper. The first of these is a result showing that a direct auction of the water allocation would fail to price water appropriately. The second is a result that shows that a Walrasian exchange economy for water rights has an empty core and that therefore no efficient allocation of water via a

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[4] Sub-game perfect equilibrium is the usual Nash equilibrium refinement in game theory associated with eliminating threats that are not credible. Consequently a threat strategy that is not sub-game perfect is not credible.

market mechanism is possible. Given that most practitioners would likely choose either a (Walrasian) market mechanism or an auction mechanism in any attempt to develop a market for trading water. These results are significant impossibility results. It is also interesting that Corams result on the emptiness of the core is in accord with at least some of the results in Ambec and Ehlers (2008b) who come to similar conclusions.

Coram has recently produced another paper (Coram (2009)) on the river sharing problem. In Ambec and Sprumont (2002) they suggested a possible extension would be to consider branching river networks. To the best of my knowledge the first paper to seriously attempt this in the river sharing literature is Coram (2009). Coram solves the problem by partitioning the river system into subsystems based on nodes where streams flow together. This paper has I think little to offer practitioners beyond his first paper. It is mentioned here for completeness and to point out that many of the results discussed here could be extended without too much difficulty to the case of branching river networks.

Beard and McDonald (2007) extend the original "sharing a river" model by embedding the Ammbec and Sprumont's game within a multi-period dynamic cooperative game. The motivation behind this paper was two-fold on the one hand the authors intended to develop a multi-period analogue of the Ambec and Sprumont model and to examine whether or not the solution concept proposed by them was time consistent. A second motivation was to show that the concept of time consistency provided a natural way of capturing the controversy over temporary versus perpetual riparian rights. A time consistent solution of a multi-period cooperative game would provide for the same fair

allocation of water with both temporary and permanent transfers of water rights. In other words the division of the allocated rights would be the same and recipients of a water allocation would have no incentive to deviate from either a sequence of temporary agreements or from a permanent allocation. In their paper they also examined a second compromise solution concept the τ-value which had similar properties.

In a recent working paper D'Albis and Ambec (2009) extend the original Ambec and Sprumont paper in an intertemporal direction by interpreting the river as a timeline. This is similar in some respects to Corams approach however it differs in that Coram uses a continuous-time model and D'Albis and Ambec employ a discrete time framework. In their model they abstract somewhat and consider a natural resource that may be either renewable or non-renewable and then apply the Ambec and Sprumont solution concept to examine the question of fair sharing of the natural resource between generations. The model has an overlapping generations structure in order to allow for neighbouring generations to compensate each-other for forgoing consumption. Because they interpret the sequence of resource extraction dynamically rather than spatially as in the original river sharing paper, they introduce time discounting, this results in a time consistency problem such that compensatory transfers between generations grow the to the point that future generations will have insufficient resources unless technological progress is sufficient to compensate them. Although this paper is couched in more general terms its structure comes from the papers on the river sharing problem and it seems clear that there are lessons in this paper for river basin management. Firstly, environmental flows such as water flows can occur on very slow time scales. Long and slow flowing rivers may well

be characterized by discounting effects between upstream and downstream users. In which case, the Ambec and Sprumont solution would be problematic unless there was a sufficient degree of difference between the technological level of upstream and downstream users of the river. This is indeed sometimes the case. Upstream farms tend to operate on more marginal land and employ less sophisticated irrigation technology than famers in flood plains who are able to make use of much more extensive irrigation technology. The next section briefly examines the wider influences of this literature .

4. Influences on other problems

In a PhD thesis completed at CenTeR at Tilburg University in the Netherlands in 2004 Ruud Hendrickx studied the Ambec and Sprumonts downstream incremental distribution solution in considerable depth and applied it to a new area of application-production scheduling in factories. In his thesis Hendrickx termed Ambec and Sprumont's solution concept the μ-rule and this terminology was later also employed by Beard and McDonald (2007). Hendrickx extends this rule to the case where he allow a player to drop-out he argues that this should not lead to the other players being worse off and calls this drop-out monotonicity. He then show that the μ-rule is drop-out monotonic and is the unique rule which satisfies this. The next section examines how the importance of flow and the direction of flow is being recognized in the literature on bankruptcy.

5. Bankruptcy and Water Allocation

There is a huge literature on the economics of bankruptcy beginning with O'Neill (1982) and then Aumann and Maschler (1985) . Bankruptcy problems are situation in which an estate for example the assets of a firm or an individual are subject to individual claims

which sum in value to at least the value of the state which is to be divided up. It is easy to see how the over-allocation of riparian rights falls into this category. Little research has pursued this direction however some papers are noteworthy. Tijs et al. (2008) inspired by Kaminski (2000) point out a connection between bankruptcy problems and some flow problems, e.g. minimum cost flow problems in operations research. For a general treatment of flow problems see Ahuja et al. (1993). They show how various rules for the solution of simple bankruptcy problems can be represented as standard network flow problems. A standard network flow problem consists of network with two special nodes: a source and sink node. With arcs between the nodes that are endowed with a capacity called the flow. The network is essentially represented by a weighted directed graph. A bankruptcy problem may be represented as a special case of a flow problem in which the source node is the estate to be divided and the sink node represents the all the claimants and the flow represent monetary payments. It is also clear that Ambec and Sprumont's river sharing problem is a special case of a flow problem because they studied a linear river with a single source and a single sink it also easy to see that that the Ambec and Sprumont model can be represented as a bankruptcy problem. This is easily done by equating the root node in Ambec and Sprumont's linear river model to the source node in the network representation of a flow problem with the sink node in the flow problem representation of the bankruptcy problem consisting of all users of the river and the river-mouth as a dummy user.

A recent paper by Ansink and Weikard (2011) develops a means to explicitly couch the river-sharing problem in terms of a bankruptcy problem. They do this by transforming the

original n-agent problem to one involving pairs of agents bargaining with each-other. They then show that the resultant problem is equivalent to a bankruptcy problem and that rules for resolving asset allocation in the case of bankruptcy may be applied to the problem of water resource allocation between upstream and downstream users. This is an important paper because it applies to the case where water rights/claims are overallocated which is common in many real-world river systems. Australia's Murray-Darling system springs to mind as an example. Ansink and Weikard essentially propose a family of solutions water resource allocation problems on rivers that have a number of desirable properties including: the independence of upstream users of waters claims from downstream users of water, the inability of downstream water users to collude and a mononicity property which states that water allocations should remain unchanged when a claim is dropped. However, the family of solutions studied by Ansink and Weikard do not satisfy the property of "equal treatment of equals" a basic property of distributive justice due originally to Aristotle. A number of solution principles in bankruptcy problems suggest themselves. Ansink and Weigard consider four sequential sharing rules; the proportional rule,

These will not be discussed in detail here instead the reader is referred to survey articles such as Thomson (2003).

6. Policy implications

What have we learnt from this literature about river basin management? I think that there is a considerable amount to be learnt by water policy economists from this literature. A key point that comes out of a number of papers namely Ambec and Ehlers (2008a), Beard

and McDonald (2007), Coram (2006), and Houba (2008) is that the problem of developing more efficient means of water allocation for example the design of trading mechanisms is not so easily separated from the problem of fair allocation: the two issues are intertwined. As Coram, but also Anbec and Ehlers suggest the problems of water resources and river basin management cannot simply be solved by adopting simplistic market solutions as these will in general not lead to efficient allocations in the presence of directional flow of water. What a number of the papers do suggest is that a better understanding of the economics of fair sharing of water in river basins will lead to improved legislative and market based solutions to water resource allocation. While the original legal concepts of absolute territorial sovereignty – the Harmon doctrine and unlimited territorial integrity originally applied to international law these concepts are clearly of relevance to inter-jurisdictional disputes within national boundaries. To that extent this literature is of wider applicability than to issues involving international and transboundary river systems. To a large extent these issues are political and the literature reflects this in couching the debate in terms of justice and fairness- yet economics remains at the core even of these questions. A number of papers suggest key criteria that are important for the practical implementation of both legislative and trading solutions. The solidarity principle or drop-out monotonicity is one such key criterion if this is not fulfilled then it would be difficult for individual agents to freely choose to cooperate or not without agreements becoming coercive. Clearly this is undesirable feature of any cooperative agreement. The question of permanent versus temporary allocations of water is touched on by this literature particularly Beard and McDonald (2007) and Coram (2006) address this point. The paper by Beard and McDonald has interesting implications

for Corams sequential auction mechanism. Corams sequential auction requires, in the form in which he presents it, a permanent sale of the allocation. A key point of the Beard and McDonald paper is that any rule that fairly divides the water allocation needs also to be time consistent. If the initial distribution of permits in Coram's sequential auction were time consistent, a condition which is satisfied by Ambec and Sprumont's solution, then whether one repeats the auction or not should make little difference as the result should be the same either way. This would however require that the initial allocation auction off the future water allocation. So each user of water would purchase in a single auction a sequence of permits to use water through time.

The work of Houba (2008) is more positive about the possibilities of water trading but is skeptical as to whether the Ambec and Sprumont solution would arise through bargaining. This raises questions more about the political realities of bargaining over the principles that might be used for allocating water. This applies whether or not legislative or market based solutions are used. This is because market based solutions still rely on an initial allocation or endowment of permits or water and that the final allocation will be sensitive to this.

In some real world applications the issue is not so much efficient and fair allocation of water per se but efficient and fair allocation of clean water. Conversely the problem can be formulated as one of efficiently and fairly allocating the costs of keeping a river clean or cleaning it up. An example here from Canada is the impact of tailings ponds in the Albertan oil sands on downstream water quality on the Athabasca river. While there are

other issues, such as the risk of leakage from tailing ponds, that would need to be addressed in any policy application. The solution principles suggested by Ni and Wang have some merit at least as a starting point for discussing possible solutions to a fair and equitable sharing of clean-up costs for this river.

Overallocation of water rights is another problem common in arid and semi-arid regions, examples include Australia's Murray-Darling system, the use of bankruptcy solutions as proposed by Ansink and Weikard suggests a number of possibilities for resolution of water resource allocation problems involving upstream and downstream users on overallocated rivers. An interesting feature of almost all these models for practitioners is that they can be formulated for the most part as linear programming models a technique familiar to most agricultural economists and widely employed with readily available software. Although it is true that sometimes multiple linear programming problems need to be computed and this is the point made by Houba (2008) when he argues that some of other techniques may be computationally more efficient. Nevertheless given a appropriate data the computational solution of these problems is possible given sufficient accounting data on the benefits and costs of water use.

7. Conclusion

In this paper I have surveyed the literature of the "river sharing problem" where possible identifying the links between the various contributions and their implications for policy. Much of this literature is quite technical yet it has a lot to offer water resource policy economists looking for criteria to use in adjudicating in disputes between upstream and downstream user of water in river basins. The approaches discussed have a long tradition

in the literature on water resource economics yet the addition of directional flow and the conception of water resource allocation in terms of bankruptcy problems and the links to the wider literature on bankruptcy are new. The problem of designing market based instruments for trading water also needs to take into account much of this literature. The design and implementation of market based instruments is always relative to some cooperative solution concept. The future of water resource economics in the 21st century is likely to involve further work on the implementation of cooperative solutions to sharing water via market based instruments.

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