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Abstract

We obtain a closed-form solution to rational expectations equilibrium with transaction costs in the framework of Grossman and Stiglitz (1980) [On the impossibility of informationally efficient markets. American Economic Review 70, 543-566]. Individual private information incorporated into prices is reduced due to suppressed trading activities by transaction costs. The equilibrium fraction of informed traders increases (decreases) with transaction costs when the costs are low (high). The informativeness of prices decreases with transaction costs.
Key words: rational expectations, transaction cost, information acquisition
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1. Introduction

In rational expectations models, prices convey information. When traders are asymmetrically informed in financial markets, the information is transmitted from informed traders to uninformed traders through prices, i.e. unin-

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formed traders learn from prices. Transaction costs suppress traders' trading activities, hence reduce the amount of individual private information incorporated into prices, which has consequences on information acquisition and transmission in rational expectations equilibrium.

As is remarked in Barron and Karpoff (2004), "it is difficult to incorporate transaction costs into rational expectations models". This remark is true if the transaction costs are fixed or proportional to the traded shares of the risky asset. We adopt the quadratic form of transaction costs, and solve the rational expectations equilibrium in closed form. Section 3.2 provides more discussions about the adoption of quadratic form of transaction cost.

The purpose of this letter is twofold. First, we study the problem of how the equilibrium fraction of informed traders depends on transaction costs. If transaction costs are prohibitively high, no trader is willing to trade, hence no one is interested in acquiring information. If the transaction costs are not very high, transaction costs suppress trading activities, hence reduce individual private information incorporated into prices. This helps informed traders to keep their relative informational advantages over the uninformed. Hence traders have motives to acquire private information, resulting in a higher fraction of informed traders than that without transaction costs. Consequently, we expect to see that the equilibrium fraction of informed traders is not a monotonic function of transaction costs. To be specific, we will see that when transaction costs are low (high), the equilibrium fraction of informed traders is an increasing (decreasing) function of transaction costs.

The second point of this letter is to study the informativeness of equilibrium prices in the presence of transaction costs, which depends on three factors. The first is how much individual private information is incorporated into prices, and the second is how many traders acquire information, i.e. the equilibrium fraction of informed traders. The third is the random supply of risky asset provided by the noise traders.[1] We will only focus on the first two factors. When the transaction costs are high, both the fraction of informed traders and the individual private information revealed through prices are low. So we expect to see the informativeness of equilibrium prices is low with high transaction costs. When the transaction costs are not very high, a larger fraction of traders acquire information, as was pointed out earlier, with each revealing a reduced amount of private information compared with the case without transaction costs. We will see that the net effect is reduced informativeness of equilibrium prices.

This letter is organized as follows. In Section 2, we solve the rational expectations equilibrium with quadratic transaction costs in the framework of Grossman and Stiglitz (1980). We discuss in Section 3 and conclude in Section 4.

2. Rational Expectations Equilibrium with Transaction Costs

In this section, we will focus on the effect of the suppressed optimal change in stock position due to quadratic transaction costs on information transmission and information acquisition in the framework of Grossman and Stiglitz (1980). Because there are no market makers in Grossman and Stiglitz (1980), we cannot include the behavior of the players who receive transaction

[1] We assume that this random supply is not affected by transaction costs.

costs within the framework of Grossman and Stiglitz (1980), and we have to take them as exogenous. Further discussions about the endogeneity of transaction costs is provided in Section 3.1.

As is in Grossman and Stiglitz (1980), the random payoff for the stock is $\tilde{u} = \tilde{\theta} + \tilde{\epsilon}$, where $\tilde{\theta}$ and $\tilde{\epsilon}$ are normal random variables with expectations $\overline{\theta}$ and 0, and variances σ_θ^2 and σ_ϵ^2, respectively. Moreover, $\tilde{\theta}$ and $\tilde{\epsilon}$ are not correlated. There is a continuum of informed and uninformed traders in the market with a total number of 1. A fraction λ of the traders is informed indexed by $i \in [0, \lambda]$, and the rest $1 - \lambda$ is uninformed indexed by $j \in (\lambda, 1]$. Informed traders can observe $\tilde{\theta}$ directly by paying a fixed cost c, and uninformed traders can only observe the price P_λ, where the subscript denotes that the price is for a given value of λ. Both types of traders have the same initial wealth W_0 composed of the same cash amount $\overline{M}$ and the same $\overline{X}$ shares of stock. Besides these two types of traders, there are noise traders who trade for reasons not modeled in this paper. Their role is to provide a random supply $\tilde{x}$ of risky asset to the market, which is a normal variable with mean zero and variance σ_x^2. Moreover, the random variable $\tilde{x}$ is independent from $\tilde{\theta}$ and $\tilde{\epsilon}$. We also assume that transaction costs have no effect on noise traders' trading behavior.

In the case with quadratic transaction costs, when one trader changes her stock holding from $\overline{X}$ to a new position X, she has to pay transaction costs $\frac{1}{2}\, t\, (X - \overline{X})^2$, where t is a positive constant. We have the end-of-day wealth for the i^{th} informed trader and the j^{th} uninformed trader

$$
\begin{aligned}
\widetilde{W_{Ii}} &= W_0 + (\tilde{u} - P_\lambda)\, X_{Ii} - \frac{1}{2}\, t\, (X_{Ii} - \overline{X})^2 - c, \quad i \in [0, \lambda], \\
\widetilde{W_{Uj}} &= W_0 + (\tilde{u} - P_\lambda)\, X_{Uj} - \frac{1}{2}\, t\, (X_{Uj} - \overline{X})^2, \quad j \in (\lambda, 1],
\end{aligned}
$$

where X_{Ii} and X_{Uj} are the i^{th} informed trader's and the j^{th} uninformed trader's stock position, respectively. The informed traders base their decisions on their direct observation of $\tilde{\theta}$, and the uninformed traders can only base their decisions on the observation of the price P_λ. The maximization problems for the informed and uninformed traders are

$$\max_{X_{Ii}} \mathbb{E}[-\exp(-a\,\widetilde{W_{Ii}})\,|\theta\,], \qquad \max_{X_{Uj}} \mathbb{E}[-\exp(-a\,\widetilde{W_{Uj}})\,|P_\lambda\,].$$

We solve for the optimal stock positions X_{Ii} and X_{Uj}

$$X_{Ii} = \frac{\theta - P_\lambda + t\,\overline{X}}{a\,\sigma_\epsilon^2 + t}, \qquad X_{Uj} = \frac{\mathbb{E}[\tilde{u}|P_\lambda] - P_\lambda + t\,\overline{X}}{a\,\mathrm{Var}(\tilde{u}|P_\lambda) + t}. \tag{1}$$

It becomes transparent to rewrite (1) as

$$X_{Ii} - \overline{X} = (\frac{\theta - P_\lambda}{a\,\sigma_\epsilon^2} - \overline{X})/(1 + \frac{t}{a\,\sigma_\epsilon^2}), \tag{2}$$

$$X_{Uj} - \overline{X} = (\frac{\mathbb{E}[\tilde{u}|P_\lambda] - P_\lambda}{a\,\mathrm{Var}(\tilde{u}|P_\lambda)} - \overline{X})/(1 + \frac{t}{a\,\mathrm{Var}(\tilde{u}|P_\lambda)}). \tag{3}$$

We immediately see that the numerators in (2) and (3) are optimal changes in stock positions in absence of transaction costs, and the denominators are greater than 1 due to transaction costs.

We also obtain the maximal certainty equivalents for informed and uninformed traders

$$CE_I^* = W_0 - \frac{1}{2}\,t\,\overline{X}^2 + \frac{1}{2}\,\frac{(\theta - P_\lambda + t\,\overline{X})^2}{t + a\,\sigma_\epsilon^2} - c, \tag{4}$$

$$CE_U^* = W_0 - \frac{1}{2}\,t\,\overline{X}^2 + \frac{1}{2}\,\frac{(\mathbb{E}[\tilde{u}|P_\lambda] - P_\lambda + t\,\overline{X})^2}{t + a\,\mathrm{Var}(\tilde{u}|P_\lambda)}. \tag{5}$$

The market clearing condition is

$$\int_0^\lambda X_{Ii}\,di + \int_\lambda^1 X_{Uj}\,dj = \overline{X} + \tilde{x}. \tag{6}$$

This enables us to find the equilibrium price and strategies for informed and uninformed traders for a *given* value of λ, and we have the following result.

Proposition 1. *For a give fraction λ of informed traders, the trading strategies for informed and uninformed traders are*

$$\begin{aligned} X_{Ii} &= \alpha^{-1}(\theta - P_\lambda + t\,\overline{X}), \qquad (7) \\ X_{Uj} &= A\,(\overline{\theta} - P_\lambda + t\,\overline{X} + \frac{\lambda}{\alpha}\frac{\sigma_\theta^2}{\sigma_x^2}\,\overline{X}), \\ A^{-1} &= \alpha + a\,\sigma_\theta^2 + \frac{\lambda}{\alpha}\frac{\sigma_\theta^2}{\sigma_x^2}, \qquad \alpha \equiv a\,\sigma_\epsilon^2 + t. \\ P_\lambda &= \Lambda\,\frac{\lambda}{\alpha}[\tilde{z} + (t - \frac{\alpha}{\lambda})\overline{X} + \frac{1-\lambda}{\lambda}\,\alpha\,B], \quad \tilde{z} \equiv \tilde{\theta} - \frac{\tilde{x}}{\lambda/\alpha}, \qquad (8) \\ \Lambda^{-1} &= \frac{\lambda}{\alpha} + (1-\lambda)\,A, \qquad B = A\,(\overline{\theta} + t\,\overline{X} + \frac{\lambda}{\alpha}\frac{\sigma_\theta^2}{\sigma_x^2}\,\overline{X}). \end{aligned}$$

Proof. The proof is given in Appendix A. □

The quantity that is of interest to our problem is $\tilde{z} = \tilde{\theta} - \frac{\tilde{x}}{\lambda/\alpha}$, which is informationally equivalent to the price P_λ. The ratio λ/α can be regarded as a measure of how much total private information is aggregated into prices. The larger this ratio is, the more private information is aggregated into prices, hence prices are less affected by the noise traders. From (7) we can see that the value of α measures an informed trader's responsiveness to her private information. The more responsive the trading behavior is, the more private information is incorporated into prices. This quantity reflects how much an *individual's* private information is revealed to the uninformed traders through prices. The value of α is larger in the presence of transaction costs than that without transaction costs, which means that the existence of transaction costs suppresses an informed trader's activity, resulting in a reduced amount of *individual* private information incorporated into prices.

The informativeness of prices also depends on the value of λ. Clearly, the larger the fraction of informed traders is, the more private information is aggregated into prices.

To study the informativeness of equilibrium prices in the presence of transaction costs, we need to know how the equilibrium fraction of informed traders λ depends on t. For this purpose, we need the following result which will be used to determine the equilibrium fraction of informed traders in Proposition 3.

Proposition 2. *For a given value of λ, the ratio $\gamma(\lambda)$ of ex ante expected utilities between the informed and uninformed traders is*

$$\gamma(\lambda) = \frac{\mathbb{E}[U(CE_I^*)]}{\mathbb{E}[U(CE_U^*)]} = e^{a\,c}\sqrt{\frac{\alpha}{\alpha + (1-\xi)\,a\,\sigma_\theta^2}}, \quad \xi = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \frac{\alpha^2}{\lambda^2}\sigma_x^2}, \tag{9}$$

which is a monotone increasing function of λ.

Proof. The proof is given in Appendix B. □

We note that the utility is a negative value, so $\gamma(\lambda) < 1$ means that the expected utility for the informed is higher than that for the uninformed. The smaller this ratio $\gamma(\lambda)$ is, the higher the informed traders' ex ante expected utility is. The ratio $\gamma(\lambda)$ is a measure of informed traders' informational advantages over the uninformed. We are interested in how transaction costs affect this informational advantage and we have the following corollary.

Corollary 1. *Given informed fraction λ, when $\sigma_\theta^2 > (\frac{a\,\sigma_\epsilon^2}{\lambda})^2\,\sigma_x^2$, $\gamma(\lambda)$ decreases (resp. increases) with t for $t <$ (resp. $>$) $t_1 \equiv \lambda\frac{\sigma_\theta}{\sigma_x} - a\,\sigma_\epsilon^2$. When $\sigma_\theta^2 \le (\frac{a\,\sigma_\epsilon^2}{\lambda})^2\,\sigma_x^2$, $\gamma(\lambda)$ is a monotone increasing function of t.*

Remark 1. Note that $(\frac{a\,\sigma_\epsilon^2}{\lambda})^2\,\sigma_x^2$ is a measure of informativeness of prices in absence of transaction costs as can be seen from $\tilde{z}$ in (8). When $\sigma_\theta^2 \leq (\frac{a\,\sigma_\epsilon^2}{\lambda})^2\,\sigma_x^2$, i.e. the informativeness of prices in absence of transaction costs is poor, no matter how small transaction costs are, they always hurt informed traders' informational advantages over the uninformed.

Remark 2. When $\sigma_\theta^2 > (\frac{a\,\sigma_\epsilon^2}{\lambda})^2\,\sigma_x^2$, i.e. the informativeness of prices in absence of transaction costs is good, transaction costs enhance (hurt) informed traders' informational advantages over uninformed traders when t is low (high). This has important implications to the dependence of the equilibrium fraction of informed traders on transaction costs in Corollary 2.

This can be understood intuitively as follows. On the one hand, informed traders trade less aggressively due to transaction costs, resulting in less private information incorporated into prices and less informativeness of prices to the uninformed. Uninformed traders suffer from reduced informativeness of prices in addition to transaction costs. Corollary 1 shows that uninformed traders suffer more than the informed when transaction costs are low, which implies that informed traders' informational advantages over the uninformed are enhanced by low transaction costs. On the other hand, when transaction costs are high, informed traders cannot effectively exploit their informational advantages. In other words, the gain from informational advantages cannot compensate for transaction costs paid, resulting in informed traders' informational advantages being hurt by transaction costs.

Proposition 1 characterizes the equilibrium price P_λ in (8) for given values of λ. Following Grossman and Stiglitz (1980), we define *overall* equilibrium to be a pair (λ, P_λ), where P_λ is given in (8), such that the expected utility of the informed is equal to that of the uninformed if $0 < \lambda < 1$; $\lambda = 0$ (resp. $\lambda = 1$) if the expected utility of the informed is less (resp. greater) than that of the uninformed at P_0 (resp. P_1).

Proposition 3. *If* $\gamma(1) < 1$ *(resp.* $\gamma(0) > 1$*), then* $(1, P_1)$ *(resp.* $(0, P_0)$*) is an overall equilibrium. If* $0 \le \lambda \le 1, \gamma(\lambda) = 1$*, then* (λ, P_λ) *is an overall equilibrium with* P_λ *given in (8). Moreover,* $\gamma(\lambda) = 1$ *determines the equilibrium value of* λ

$$\lambda^2 \;=\; \frac{\sigma_x^2}{\sigma_\theta^2}\,\alpha\,(\frac{a\,\sigma_\theta^2}{\exp(2\,a\,c)\;-\;1}\;-\;\alpha). \tag{10}$$

Proof. The first two sentences follow from the definition of overall equilibrium. Equation (10) is proved in Appendix B. □

Remark 3. Note that $\frac{a\,\sigma_\theta^2}{\exp(2\,a\,c)-1} \le \alpha$ is equivalent to $\gamma(0) \ge 1$, hence when the right hand side of (10) is negative due to large transaction costs, low quality of private information or high information cost, the overall equilibrium is $(0, P_0)$. Similarly, when the right hand side of (10) is greater than 1, the overall equilibrium is $(1, P_1)$

In the following, we focus our attention on the case $0 < \lambda < 1$. Now we are ready to answer the question of how transaction costs affect the equilibrium fraction of informed traders. From (10) we immediately see that the equilibrium value of λ^2 is a quadratic function of α, hence it is a quadratic function of transaction costs. The value of λ^2 increases with transaction costs when t is below the critical value $t_c \equiv a\,(\frac{\sigma_\theta^2/2}{\exp(2\,a\,c)-1}\;-\;\sigma_\epsilon^2)$. This can be understood from Corollary 1 and Remark 2. Suppressed trading activities due to transaction costs by informed traders reduce their private information incorporated into prices. This helps the informed traders hide their private information, and thence keep their informational advantages over the uninformed traders. This encourages more traders to acquire private information, which results in an increased fraction of informed traders.

When the transaction cost is larger than the critical value t_c, the equilibrium value of λ^2 is a decreasing function of transaction costs. This can also be understood from Corollary 1 and Remark 2. With a large value of transaction cost, the gain from informational advantages cannot compensate for transaction costs. Hence the traders are discouraged to acquire private information, which results in a reduced fraction of informed traders. In the extreme case when the transaction costs is forbiddingly high, the traders will not trade at all no matter how good information they have, hence they have no interest in acquiring private information. We summarize the above discussion in the following corollary.

Corollary 2. *The equilibrium fraction of informed traders λ increases (decreases) with transaction costs when $t < t_c$ ($t > t_c$).*

We also want to answer the question of how the informativeness of equilibrium prices depends on transaction costs. From (8), we see that the random variable $\tilde{z} = \tilde{\theta} - \frac{\tilde{x}}{\lambda/\alpha}$ is informationally equivalent to the price P_λ. The informativeness of the price P_λ is measured by the variance of the noise term in $\tilde{z}$

$$\frac{\sigma_x^2}{\lambda^2/\alpha^2} = \frac{\alpha}{\frac{a\,\sigma_\theta^2}{\exp(2\,a\,c)-1} - \alpha}\sigma_\theta^2, \tag{11}$$

which is an increasing function of α, hence an increasing function of transaction costs. This says that the equilibrium price becomes less informative with the presence of transaction costs than that in the absence of transaction costs. This is consistent with our intuition. We summarize this in the following corollary.

Corollary 3. *The informativeness of the equilibrium price is a decreasing function of transaction costs.*

3. Discussions

3.1. Endogeneity of Transaction Costs

In this letter, the transaction cost parameter t is exogenous as is treated in most papers on transaction costs in finance literature like Constantinides (1986), Lo et al. (2004), Liu (2004) and Liu and Loewenstein (2002). Because Grossman and Stiglitz (1980) does not include market maker, if we want to limit our problem with transaction costs within the framework of Grossman and Stiglitz (1980), we have to take transaction costs as exogenously given.

However, whenever one studies a problem with transaction costs, there always arises the natural question where the paid transaction costs go in the economy. Hence the full description of a problem with transaction costs must model the behavior of players who receive the transaction costs. Recently Liu and Wang (2011) endogenizes transaction costs in rational expectations equilibrium, which motivates us to endogenize our transaction cost parameter t in a similar manner in a future work. Along this line, the noise traders' behavior might also need to be modeled, as is done in Liu and Wang (2011).

Because of the information cost in our model, an even fuller description of our problem should model the behavior of information providers, who can either trade on their own information or sell information to other investors.

3.2. Quadratic Specification of Transaction Costs

Fixed and Proportional transaction costs are widely studied in the literature, and we only list a few of them, Constantinides (1986), Liu and Loewenstein (2002), Liu (2004) and Lo et al. (2004). Garleanu and Pedersen (2011) offers micro-foundations for the quadratic form of transaction costs. They derive in closed form the optimal dynamic portfolio policy when trading is costly and security returns are predictable. Heaton and Lucas (1996) also use quadratic transaction costs to study the effect of transaction costs on equity premium in incomplete markets. They numerically compare the results calculated with quadratic and proportional transaction costs, and they find that the results are qualitatively similar.

3.3. Property due to Quadratic Specification

We believe the following property of our solution is due to the quadratic specification of transaction costs. In absence of transaction costs, when σ_ϵ^2 vanishes, the equilibrium fraction λ of informed traders vanishes. Meanwhile, λ does not vanish in the presence of quadratic transaction costs which can be seen from (10). The following is the reason. When σ_ϵ^2 vanishes, i.e. informed traders obtain perfect information about the return of stock, informed traders will buy (sell) as much as possible as long as the expected profit is positive (negative). Then the private information is fully incorporated into prices, and uninformed traders can learn as well from prices about the return of the stock as informed traders. This discourages informed traders to acquire costly information resulting in a vanishing value of λ.

However, this vanishing value of λ will not happen in the presence of quadratic transaction costs. Quadratic transaction costs prevent informed traders from buying or selling an infinite amount of stock, which prevents prices from being fully revealing. The acquisition of private information remains to be valuable even when the private information is perfect. Consequently, a non-vanishing equilibrium value of λ results when σ_{ϵ}^{2} vanishes, as can be seen from (10).

4. Conclusions

We study the problem of how transaction costs affect information transmission and acquisition in rational expectations equilibrium. We find that in the presence of quadratic transaction costs, the equilibrium fraction of informed traders is not a monotonic function of transaction costs. To be specific, for small (large) transaction costs, the informed fraction increases (decreases) with transaction costs. The informativeness of the equilibrium price is a monotone decreasing function of transaction costs.

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A. Proof to Proposition 1

We postulate linear strategies for uninformed traders

$$X_{Uj} \;=\; -A\,P_\lambda \,+\, B, \quad j \in [0, \lambda], \tag{12}$$

where the constants A and B are to be determined. Plugging trading strategies for informed traders (1) and the postulated linear strategies (12) for uninformed traders into the market clearing condition (6), we solve for the price

$$\begin{aligned} P_\lambda \;&=\; \Lambda\,\frac{\lambda}{\alpha}[\tilde{z} + (t \,-\, \frac{\alpha}{\lambda})\overline{X} + \frac{1-\lambda}{\lambda}\,\alpha\,B], \qquad (13)\\ \Lambda^{-1} \;&\equiv\; \frac{\lambda}{\alpha} \,+\, (1 \,-\, \lambda)\,A,\\ \alpha \;&\equiv\; a\,\sigma_\epsilon^2 \,+\, t, \qquad \tilde{z} \equiv \tilde{\theta} \,-\, \frac{\tilde{x}}{\lambda/\alpha}. \end{aligned}$$

From this we see that the price P_λ is informationally equivalent to the random variable $\tilde{z}$. With price P_λ and its informational equivalent $\tilde{z}$, we obtain

$$\mathbb{E}[\tilde{u}|P_\lambda] \;=\; \mathbb{E}[\tilde{u}|z] \,=\, \xi\,z \,+\, (1 \,-\, \xi)\overline{\theta}, \tag{14}$$

$$\mathrm{Var}(\tilde{u}|P_\lambda) \;=\; \sigma_\epsilon^2 \,+\, (1-\xi)\sigma_\theta^2, \qquad \xi \;=\; \frac{\sigma_\theta^2}{\sigma_\theta^2 \,+\, \frac{\alpha^2}{\lambda^2}\sigma_x^2}. \tag{15}$$

Express $\tilde{z}$ in terms of P_λ from (13), and substitute (14) and (15) into (1), we can obtain X_{Uj} expressed in terms of the price P_λ. Now we are ready to solve the constants A and B by comparison with the postulated strategy (12)

$$B \,=\, A\,(\overline{\theta} \,+\, t\,\overline{X} \,+\, \frac{\lambda}{\alpha}\frac{\sigma_\theta^2}{\sigma_x^2}\,\overline{X}), \qquad A^{-1} \,=\, \alpha + a\,\sigma_\theta^2 \,+\, \frac{\lambda}{\alpha}\frac{\sigma_\theta^2}{\sigma_x^2} \tag{16}$$

We rewrite (12) as

$$X_{Uj} \,=\, A\,(\overline{\theta} \,-\, P_\lambda \,+\, t\,\overline{X} \,+\, \frac{\lambda}{\alpha}\frac{\sigma_\theta^2}{\sigma_x^2}\,\overline{X}) \tag{17}$$

B. Proof to Proposition 2 and 3

Following Grossman and Stiglitz (1980), we first calculate

$$\frac{\mathbb{E}[-\exp(-a\,CE_I^*)|P_\lambda]}{-\exp[-a\,(W_0\,-\,\frac{1}{2}\,t\,\overline{X}^2\,-c)]} \quad = \quad \mathbb{E}[\exp(-\frac{1}{2}\,a\,\frac{(\tilde{\theta}\,-\,P_\lambda\,+\,t\,\overline{X})^2}{t\,+\,a\,\sigma_\epsilon^2})|P_\lambda]$$

Using formula (A21) in Grossman and Stiglitz (1980)

$$\mathbb{E}[\exp(-\beta\,\tilde{v}^2)|w]\;=\;\frac{1}{\sqrt{1\,+\,2\,\beta}}\exp(-\frac{\beta}{1\,+\,2\,\beta}\mathbb{E}[\tilde{v}|w]^2),$$

and defining

$$\beta\;=\;\frac{1}{2}\,a\,\frac{\mathrm{Var}(\tilde{\theta}|P_\lambda)}{t\,+\,a\,\sigma_\epsilon^2},\qquad \tilde{v}\;=\;\frac{\tilde{\theta}\,-\,P_\lambda\,+\,t\,\overline{X}}{\sqrt{\mathrm{Var}(\tilde{\theta}|P_\lambda)}},$$

we have

$$\begin{aligned}\frac{\mathbb{E}[-\exp(-a\,CE_I^*)|P_\lambda]}{-\exp[-a(W_0-\frac{1}{2}\,t\overline{X}^2-c)]} \quad &= \quad \sqrt{\frac{t+a\,\sigma_\epsilon^2}{t+a\,\sigma_\epsilon^2+a\,\mathrm{Var}(\tilde{\theta}|P_\lambda)}}\\ &\quad\times\exp(-\frac{1}{2}\,a\,\frac{(\mathbb{E}[\tilde{u}|P_\lambda]-P_\lambda+t\,\overline{X})^2}{t+a\,\mathrm{Var}(\tilde{u}|P_\lambda)}).\end{aligned}$$

Recall (5) we obtain

$$\mathbb{E}[-\exp(-a\,CE_I^*)|P_\lambda] = e^{a\,c}\sqrt{\frac{t+a\,\sigma_\epsilon^2}{t+a\,\sigma_\epsilon^2+a\,\mathrm{Var}(\tilde{\theta}|P_\lambda)}}\,(-\exp(-a\,CE_U^*)).$$

Taking expectations on both sides gives

$$\begin{aligned}\gamma(\lambda) \quad &\equiv \quad \frac{\mathbb{E}[-\exp(-a\,CE_I^*)]}{\mathbb{E}[-\exp(-a\,CE_U^*)]} = e^{a\,c}\sqrt{\frac{t+a\,\sigma_\epsilon^2}{t+a\,\sigma_\epsilon^2+a\,\mathrm{Var}(\tilde{\theta}|P_\lambda)}}\\ &= \quad e^{a\,c}\sqrt{\frac{\alpha}{\alpha\,+\,(1\,-\,\xi)a\,\sigma_\theta^2}}\end{aligned}$$

This gives (9) in Proposition 2. The equilibrium fraction λ of informed traders is determined by the equality between the *ex ante* expected utilities of informed and uninformed traders

$$e^{a\,c}\sqrt{\frac{\alpha}{\alpha\,+\,(1\,-\,\xi)a\,\sigma_\theta^2}}\,=\,1.$$

The equilibrium fraction of informed traders λ is then given in (10) in Proposition 3.

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