Response to a review of voting theory for democracy, in the light of the economic crisis and the role of mathematicians

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Abstract

Economic theory needs a stronger defence against unwise application of mathematics. Mathematicians are trained for abstract thought and not for empirical science. Their contribution can wreak havoc, for example in education with real life pupils and students, in finance by neglecting real world risks that contribute to a world crisis, or in voting theory where they don't understand democracy. In 1951 the mathematician Kenneth Arrow formulated his Impossibility Theorem in social welfare theory and since then mathematicians have been damaging democracy. My book Voting Theory for Democracy (VTFD) tries to save democracy and social welfare from such destruction. VTFD applies deontic logic to Arrow's Theorem and shows that Arrow's interpretation cannot hold. The editor of a journal in voting matters has VTFD reviewed by a mathematician instead of a researcher who is sensitive to economics, democracy and empirical issues. Guess what happens. The review neglects economics, democracy and empirical issues. Curiously it also neglects the argument in deontic logic, perhaps given the distinction between mathematics and logic. Given the importance of democracy it is advisable that economists study the situation and rethink how economics and mathematics interact in practice.

Introduction

This paper is a response to the review of *Voting Theory for Democracy* (2011a) (VTFD) by Markus Schulze (2011) (MSR). I have submitted this response to the editor and he has responded in a friendly and constructive manner, in the spirit of Leibniz: let's sit down and check the formulas. Nevertheless, the MSR has already been published. The MSR is unscientific. It contains errors, misrepresentations and fallacies. The text was not presented to me in draft so that there was no opportunity to suggest corrections. It is a "review" rather than a review. Hence, I agree with the editor and accept his invitation but in the mean time it is advisable to state my response.

It is useful to see this matter in the light of the current economic crisis and the role of mathematicians in science and society. The MSR is another case of mathematical blindness to social and empirical science. Schulze has studied mathematics and physics at TU Berlin and developed his own voting scheme but is not necessarily well positioned to understand the application of voting theory to democracy. It is curious that MSR does not really discuss VTFD chapter 9 on the mathematics of voting theory and the application of deontic logic (the logic of norms).

Economics relies on mathematics. In this dependence there is a problematic role of mathematicians who are trained for abstract theory and who are less disciplined by reality. A key example is the current economic crisis. Mathematicians and "rocket scientists" designed financial

products without sufficient attention for the risks in the real world. Banks collapsed when risks materialized that theory had neglected. Of course, politicians, misguided fellow economists and law makers, greedy bankers and lax overseeers were as important in the cause of the crisis. But the misapplication of mathematical abstraction to the real world was important too. This blindness plays an important role in voting theory as well. The misapplication of mathematics goes against democracy, always, in a fundamental sense. If you cannot calculate or reason then you are pretty defenceless in the market place or against rosy promises by political candidates. But it is more complicated. See *Elegance with Substance* (2009) how mathematicians, trained for abstract theory, wreak havoc in the education in mathematics when they meet real life pupils and students. In addition, a population that has learned to hate mathematics is less likely to adopt a spirit of democracy.

The context of this paper is my book *Definition & Reality in the General Theory of Political Economy* (2011b) (DRGTPE). The update to this 3rd edition concerns a chapter on the current economic crisis, that was expected in some form or other. The main thesis of DRGTPE is that our system of democracy of the Trias Politica with the separation of powers of the Executive, Legislative and Judiciary branches fails, and that we need an amendment for an Economic Supreme Court that is based in science. World War I and II already are sufficient arguments and hopefully the world can prevent World War III. One part in DRGTPE contains a discussion of Arrow's 1951 Impossibility Theorem in the theory of social welfare. This part finds a further development in the separate book *Voting Theory for Democracy*.

Common views in voting theory contain many errors. This is bad for democracy. It is difficult to have proper democracy when you misunderstand what democracy means. Some of those errors already occur at the most basic level, some are at a high level of abstract mathematics, and again a third kind exists in the verbal interpretation of abstract mathematics. A correction of these errors is urgently needed. In my experience these errors have become such a strong mixture at the academia that a refoundation of voting theory applied to democracy is required. *Voting Theory for Democracy* was written with this objective. It starts with a foundation for democracy and then develops the notions in voting theory while avoiding common errors and pointing out what those common errors are.

Since Arrow's Theorem in 1951 – mind you, sixty years ago – researchers in voting theory have ample information for the design of better voting mechanisms. Society however still uses archaic methods that are rather undemocratic, also for the selection of the very leaders of our "democratic" nations. It is not just that law makers neglect the advice of scientists, but it are also researchers in voting theory who create the errors that VTFD is a cure for.

And now this MSR "review" blocks the cure. Now VTFD is misstated itself. This misstatement has been published on the internet, to be found by everyone with an interest in voting theory. This MSR misrepresentation may stop people from considering the corrections to common errors in voting theory. VTFD warns about the role of misguided mathematicians, but the editor has given the task of reviewing to a misguided mathematician.

It is a field of inquiry itself how people select their sources and whom they assign value to. The authority assigned to a mathematician with a degree from TU Berlin may cause people to disregard an analysis by someone with a degree in econometrics from the University of Groningen and a degree in the teaching of mathematics from the University of Leiden. According to MSR, my mathematical development in chapter 9 is only "mumbo-jumbo". This is unscientific nonsense and my impression is that he did not really study it.

The editor of *Voting Matters,* Nicolaus Tideman, has responded in a kind manner to an earlier version of my present response. Indeed, with Leibniz, let us sit down and reason. I will accept that invitation but it is important to note that there are limits. The problem is not just formulas but also the interpretation how to apply them to the world. There are limits to what one can achieve in getting a mathematician to see the world. See the next section on the MSR "core error". I have written VTFD precisely to allow people to understand the argument but if they stubbornly refuse to study it then there seems little that can be done.

Professor Tideman apparently did not ask: "Did you check with the author ?" Perhaps editors don't have to do so when they can rely on competence and scientific mores. In this case the "review" uses hard language like "mumbo-jumbo" that ought to have rung alarm bells. Will Schulze correct his text now that he has committed himself so unconditionally ? Since he did not present his hard language draft for comments and thus does not understand the basic notion of decency and truthfulness in science, we may fear the worst. Will *Voting Matters* publish a correction, withdraw this "review" and ask someone else to try again ? Will the McDougall Trust show itself an institute of integrity or will it allow itself to become a vehicle in the corruption of scientific inquiry ? Only time will tell. Professor Tideman's kind invitation to look at the formulas is a hopeful sign but for now science and democracy have been damaged again and it is hope only.

I invite scientists over the world to look at this. The PDF of VTFD has been available on the internet since 2001 and the analysis is of key importance for democracy. Check the analysis and help to resolve the situation.

To be sure: technical issues on voting schemes can be separated from issues on the current crisis, from issues on constitutional amendment, and from the role of mathematicians. But *Voting Theory for Democracy* is not a book on mere voting schemes. When scientists advise on what is a good voting scheme then values and considerations on real life people and real economies are relevant. Good mathematics is of value too, but it are also values what we might advise for a good society and a good life for all. That said, and also given that the reviewer is a mathematician, a main response with respect to MSR is that it neglects the mathematics and deontic logic in chapter 9.

The core error

Given that Schulze is a mathematician, his core error is his statement:

> "The author's criticism of Arrow's theorem (which covers about one fourth of this book) is just mumbo-jumbo."

If only he had looked more carefully at chapter 9 on my mathematical treatment. Only 9.1 and 9.2 ought to be sufficient to understand the importance of VTFD, with its new contribution to the theory of social welfare and voting.

VTFD accepts the mathematics of Arrow's theorem but rejects the interpretation that is suggested by Arrow and that has been adopted by many researchers in voting theory. Chapter 9 casts this interpretation into mathematics itself and it shows via deontic logic that Arrow's interpretation cannot be maintained consistently. VTFD is the only book in the world that properly explains what Arrow's theorem means. Everyone with middle school can already understand it but it now

has been put into formula's so that also mathematicians can understand it, provided that they look at it.

As a student in econometrics in Groningen I had courses in mathematics together with the students in math, physics and astronomy. My mathematics is quite decent. When I noted the errors in Arrow's interpretation of his theorem and noted how insensitive mathematicians are to notions of democracy when it comes down to their formulas, it was a deliberate act to translate that interpretation into mathematics, and to expose the inconsistencies, in the language that mathematicians ought to understand: logic.

However, Schulze apparently does not look deeply into chapter 9, dodges the issue, and refuses to face up to the problem. He does not do a proper review.

Details on the first section of the "review"

(1) MSR's abstract suggests that VTFD is a "primer". It is not. A primer is a book for teachers. VTFD is a textbook for first year students or other adult novices to voting theory, that also contains some advanced material. It explicitly asks students to first study the first chapters, then look into other books, and then proceed with the more complex parts of VTFD. There is no need that I copy what has been stated elsewhere. It is curious that MSR criticises me for not copying. The value of VTFD lies in its focus on democratic application and on the proper interpretation of Arrow's theorem. The News.

(2) On the Introduction in VTFD, MSR states: "Unfortunately, there are no formal definitions for preferences, orderings, single-seat elections, etc." No, of course not. VTFD is not a course in mathematics. It is intended for students in general, with some command of mathematics and logic. The Introduction develops notions that are relevant for understanding democracy. The formal treatment for advanced readers is in chapter 9. After reading some other books, the students will understand chapter 9 too.

The use of the environment of *Mathematica*, a system for doing mathematics on the computer, should not distract and suggest that VTFD is a book on mathematics. People can drive a car without being mechanics. The use of *Mathematica* is just to take away the tedium of all these voting calculations, and, indeed, to also provide support in "driving" through voting theory.

(3) VTFD holds that the paradoxes in voting theory are caused by the possibility that people can cheat ("strategic voting"). MSR's "objection" is a non-sequitur. Everyone is free of course to have another analysis but a non-sequitur is not convincing.

(4) In VTFD the term "budget" stands for the list of available items (candidates), as distinct from the whole domain of all potential possibilities. What is available depends upon rules. Often people have to declare themselves candidate but there are also cases where people are not allowed to dodge their responsibility. The discussion in VTFD on the budget is intended to clarify to students the importance of its creation. MSR does not seem to understand this and produces a convoluted statement that I actually don't understand. He then suggests that my terminology merely replaces Arrow's term of "irrelevant alternatives" with "availability". This is a misunderstanding. There is a cognitive distinction between dependence upon the budget and Arrow's specific axiom of pairwise decision making (that Arrow misleadingly calls "independence of irrelevant alternatives", see point 7 below). Perhaps in some particular instances it is hard to pinpoint the distinction, but in general there is a cognitive distinction.

One might think that Arrow's terms present a glass as half empty while my terms present it as half full. But this misrepresents the argument. When a President visits a kindergarten and reads a story about Santa Claus and tells the kids that Santa Claus lives in the North Pole, a newspaper might run the headline "President still believes in Santa Claus". This is fallacious.

The value of VTFD lies in both a distinction and a link between the mathematics and the choice of words. Arrow's interpretation causes statements on the impossibility of democracy and the unavoidability of some form of dictatorship. But his terms are misleading. The mathematics of Arrow's theorem is sound but the interpretation is wrong. And this is proven in deontic logic in chapter 9.

(5) About chapter 5, MSR states: "The author's use of some terms differs significantly from their use in the scientific literature. This leads to misunderstandings when, for example, the author concludes that "plurality voting can violate Pareto optimality" (page 70)."

MSR's statement is definitely wrong. The notion of Pareto optimality is well established in economic theory. VTFD presents the Pareto voting scheme where each voter can veto items that are worse than the Status Quo. Clearly Plurality can violate that when it neglects that veto.

One of Arrow's axioms departs from Pareto optimality. It is generally called "weak Pareto optimality". It is unclear how a student of VTFD could get confused since these two notions are well indicated in VTFD. Or how someone who knows the literature could get confused. Perhaps MSR himself gets confused. Perhaps he bases himself upon a limited and wrong section of the literature ?

(6) About chapter 7 on Elo-Rasch rating, MSR states: "There is no analysis of these schemes." This is a curious statement since the chapter itself is such analysis.

Mathematically inclined researchers in voting theory nowadays tend to focus on classifying voting methods in all kinds of properties. Example properties are "monotonicity" and "independence of clones". Nobody knows why such properties would be important since there is no way to compare or balance them. This process of classification is primarily an academic activity, but it is something that mathematicians can do to keep themselves busy. In this way they avoid looking at reality and application to democracy. Now MSR has the criticism that this would be lacking in a book directed at application for democracy. He has been so trained and focussed on something of lesser relevance that he is incapable to consider what the book is about.

He misrepresents what VTFD is intended for. He tries to fit a big square block into a small round hole. Mathematical developments of Elo-Rasch rating abound, see any decent volume on psychometrics. The discussion in VTFD is intended to help clarify cardinal utility, in relation to cheating, and to establish the generality of the dependence on the budget, also in soccer matches, and even in cricket. Soit.

(7) About chapter 9-10, MSR questions my solution to Arrow's paradox. Everyone is free to question my proposed solution. Indeed, criticism is even welcome, I know, big surprise, that I am not perfect. But this ought to be reasoned, and not a mere sneer. MSR: "At one point, the author "solves" Arrow's theorem by rejecting *independence of irrelevant alternatives* as unreasonable." Well, clearly, Arrow's axioms are inconsistent, so everyone will have to reject one axiom or other. My choice to reject IIA is reasoned. And its name is misleading. We better call it "axiom of pairwise decision making", which combines two aspects: the reduction of the budget to pairs and

the confusion of *voting* vs *deciding*. Unfortunately, MSR has not properly studied VTFD and cannot appreciate these fine points.

MSR: "At another point, the author "solves" Arrow's theorem by keeping the election method undefined in the case of circular ties." This is a "Santa Claus" misrepresentation. A cycle in pairwise *voting* actually is an indifference in terms of *decision* making. Much of Arrow's theorem depends upon the phenomenon that there is no universal method to resolve indifference or deadlocks. We may flip a coin or leave it to the chairperson or whatever. In deterministic mathematical theorems and proofs it is not standard to introduce such non-deterministic coin flipping. VTFD gives a very clear explanation of this phenomenon. I invite readers to check how MSR's sneering wholly misrepresents this insight.

Borda Fixed Point method

(1) MSR in section 2: "A serious problem of this book is that the author spends too much time introducing his own pet method: the "Borda Fixed Point" (BFP) method. This method has neither been published nor adopted somewhere. Even this book doesn't contain a proper analysis of this method. So why should we be interested in software to calculate the winner of the BFP method?"

(a) I did not know that I had a pet. I created the BordaFP since I consider it somewhat weak to merely reject Arrow's interpretation of his theorem without offering an alternative. It is better to constructively forward a scheme instead of leaving students helpless in the woods. VTFD discusses various standard schemes such as Pareto, Plurality, Borda, Condorcet, and, based upon this, I developed my own suggestion of BordaFP. (b) The method was published in the first edition of VTFD in 2001. As far as I know it is not adopted by others yet but it has not gotten much circulation. (c) Again MSR abuses the words "proper analysis". Perhaps the addiction to classification again, as if that would be so very useful ? (d) The method is fully encoded in the software. *Mathematica* is a high level programming language that makes no real distinction between formulas and programming. So why try to make a distinction that isn't there ? (e) Below we meet the Kemeny-Young and Tideman methods, and MSR indicates that they use a similar criterion as the BordaFP method. So he is inconsistent.

(2) MSR gives a complex statement of the "basic idea" of my design of the BordaFP method, which I however do not agree with. I don't think that VTFD explains the method unclearly, but MSR now invents some procedure that seems to based upon some misunderstanding. When I submitted VTFD to *Voting Matters* for a review I also offered a licence for the reviewer to run the software. This option was not used.

I have run the software. Case 1 is 51 *abcde*, 49 *cdeba*, with the first most preferred, and the BordaFP winner is also the Borda winner *c* (and not *a* as MSR holds). Case 2 is 51 *afbcde*, 49 *cdefba*, and the BordaFP winner is *f* while Borda gives *c*. In the competition at the top, *f* also wins from *c* in the pairwise comparison, while *c* is the winner when *f* does not partake.

(3) MSR: "The author claims that the BFP method satisfies the majority criterion. But example #2 shows that it doesn't." This appears to concern p77 of VTFD where I thought to have proven that BordaFP also gives the Majority Plurality winner. Schulze shows that this is not true, indeed. This observation is something to be grateful for since it is only by this kind of criticism that we make advances. I will include an erratum on the website and adapt the software so that the ParetoMajority scheme includes a check on finding the Majority Plurality winner.

NB. VTFD is very careful about the use of the term “majority”. It appears that the term is very vaguely used in everyday language and that interpretations can change upon circumstances. VTFD takes the position that the voting literature increases the confusion by introducing some apparent strictness while it is absent in general conversation. For example, President Sarkozy of France was elected by “majority”, as the general public, press and politicians hold, and he likely thinks himself too. But the system had two rounds. The BordaFP and even Borda and Condorcet winner was Bayrou, who would have beaten both Sarkozy and Royal also in pairwise contests, but who dropped out in the first round that used plurality. My impression is that voting theory needs to communicate with the general public, press and politicians, and grant that President Sarkozy indeed was elected by a majority in some sense. The voting literature tends to restrict “majority” to plurality with more than 50% with respect to all alternatives. I prefer Majority Plurality as a better technical term for this case. Hence the term “majority”in VTFD is used a bit more flexible, such that the user is always encouraged to be aware what it actually means in that particular instance.

(4) MSR refers to methods by Kemeny-Young and Nicolaus Tideman and states that these also satisfy: “where a newly added candidate *x* can win only if he pairwise beats that candidate who would be elected if candidate *x* didn’t run. So the problem that Colignatus addresses has already been solved in the scientific literature.”

(a) Well, this contradicts the earlier criticism “why should we be interested?” (point (1) in this section above). Or MSR wants to have textbooks that only present new material ?

(b) Subsequently, MSR rejects the criterion and proposes another. First of all this is a value judgement. Secondly, using the internet I see that Schulze has developed a method himself, known under the “Schulze method’, that apparently is used in organizations like “Wikimedia”. But his list of references does not refer to his own work, so perhaps the criterion he suggests does not relate to his own work, and he intends to revise his method ? Anyhow, this “review” episode is the first time that I hear about this “Schulze method”, and I am not in the position to say anything on it. I neither understand his quick comment upon which he bases his value judgement, and why it would be an important criterion.

(c) For the sake of history, allow me to clarify that I stated my criterion in Colignatus (1990), a paper at the Dutch Central Planning Bureau while oblivious of Kemeny-Young or Tideman. My paper in 1990 was directed at debunking misunderstandings about Arrow’s Theorem and not at developing a specific voting method. I mentioned the criterion then only as a rather obvious one, since while writing the notion popped up and it always remains useful to record an idea. The criterion is not exactly for a “newly added candidate” but more accurately: that the supposed winner *W* should also be able to win directly from the alternative supposed winner that comes about when *W* would not participate. Only when composing the first edition of VTFD in 2001 I decided to apply that notion to Borda, to create the BordaFP procedure, as a constructive contribution next to the debunking of misunderstandings. The method allows students to see Borda and Condorcet in perspective. It forms some sort of compromise, though it was not developed as a compromise. Given the intentions of VTFD there has never been the intention to provide a compendium of all of voting theory. MSR’s comment is the first time that I hear that others have attached much value to the criterion as well (or a similar criterion). Fine. The beauty of internet nowadays is that there is fast access to references, though it remains difficult to understand what authors intend precisely. At this moment I cannot state how BordaFP differs from Kemeny-Young or Tideman (or, indeed, where those methods differ themselves).

With respect to the MSR summary

Given the above, the MSR summary has no base. The suggestion that there is no "formal definition of election methods" is absolutely untrue, see chapter 9. I also protest against the slanderous use of language. The MSR is unscientific, wrong and apparently incompetent as well.

A communication gap

Economists partake in a social science and seem to attach great value to Arrow's mathematics and his Nobel Prize. Mathematicians seem to want formulas only and get nervous when voting theory is discussed for social application. Econometrics deals with both worlds. It optimally meets with the benefits of both but it can also meet with the drawbacks of both. In the latter case, there appears to be no tolerance and no rational discussion. The innocent person who suggests another point of view instantly meets with canon fire from all directions. In practice, voting theory for democracy has fallen into that gap since 1951, and for me personally since 1990, and the world suffers from it.

In informal discussion the directorate of the Dutch Central Planning Bureau referred to Arrow's Impossibility Theorem to block research into social welfare. This caused me to write the paper (1990) to clarify many misunderstandings about the theorem, but this analysis was blocked from discussion and publication, with unclear reasons.

When discussing voting theory and Arrow's theorem since 1990 this problem in communication occurs over and over again. Next to CPB and MSR:

(a) a Dutch research group on social choice who blocked a discussion and did not study an earlier edition of VTFD, http://www.dataweb.nl/~cool/Thomas/English/Science/Letters/SCT-working-group.html
(b) a European network of researchers advising the European Union to use Penrose square root weights, http://mpra.ub.uni-muenchen.de/3885/
(c) Donald Saari and his suggestion that Borda would be the best mechanism because of mathematical symmetry, see (a)
(d) Steven Brams and his suggestion that Approval Voting would be the best mechanism because it is Approval Voting, http://econwpa.wustl.edu/eprints/get/papers/0503/0503014.abs
(e) wikipedia's entries on social choice, http://www.dataweb.nl/~cool/Thomas/English/Science/Letters/2006-03-20-Comments-RfC.pdf
(f) the curious 2011 UK referendum on AV, http://mpra.ub.uni-muenchen.de/22782/
(g) the Dutch journal for teachers of mathematics that rejected the article "With time no morality" in Dutch that was published in the Dutch journal for teachers of economics, http://www.dataweb.nl/~cool/Thomas/Nederlands/Wetenschap/Artikelen/2008-2011-Euclides-over-ZonderTijdGeenMoraliteit.html
(h) a Dutch mathematician who "explains" Arrow's theorem on the network for Dutch highschools and who claims that ideal democracy is impossible. After two years mulling it over he recently seems willing to correct but it is not entirely sure that he will do so correctly.

As an example, the mathematicians in (h) "explains" to Dutch school kids that "perfect democracy → Arrow's axioms → impossibility", while the theory of morals suggests "perfect democracy → the system should function → not Arrow's axioms". It is a world of difference.

I seem to have developed some toughness over time but for other scientists this CPB – Schulze – (a) – (h) treatment would be both painful and deathly for their career. It is a stable of Augias. My suggestion is that true scientists with roots in both empirics and mathematics clean it.

Conclusion

In responding to the Schulze "review" it is tempting to focus on voting issues only. The text appeared in a journal devoted to voting matters and the issue might be settled there. However, it is not guaranteed that it will be settled there and given the unscientific treatment in first instance I have reason to doubt that it will. Trust comes on feet and leaves on horse-back. The kind invitation by the editor to look at the formulas is reason to give this the benefit of the doubt, and there we stand. Where we should have been in the first place, but damaged. Given the role of mathematicians in other areas like finance and education there is a good argument to raise the discussion towards a higher and more general level. Economic theory needs a stronger defence against unwise application of mathematics.

This voting issue may be a good place to start and grow aware of the problem. Again: I invite scientists over the world to look at this. The PDF of VTFD has been available on the internet since 2001 and the analysis is of key importance for democracy. Check the analysis and help to resolve the situation.

Of course, and in particular on the economic crisis in which our notions of democracy play such an important role: it are economists who provide our governments with economic advice, and the overall responsibility about our advice remains with the profession.

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