Exploitation of Labour and Exploitation of Commodities: a "New Interpretation"

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Exploitation of Labour and Exploitation of Commodities: a "New Interpretation"[1]

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Abstract: In the standard Okishio-Morishima approach, the existence of profits is proved to be equivalent to the exploitation of labour. Yet, it can also be proved that the existence of profits is equivalent to the 'exploitation' of any good. Labour and commodity exploitation are just different numerical representations of the productiveness of the economy. This paper presents an alternative approach to exploitation theory which is related to the New Interpretation (Duménil 1980; Foley 1982). In this approach, labour exploitation captures unequal social relations among producers. The equivalence between the existence of profits and labour exploitation holds, whereas it is proved that there is no relation between profits and commodity 'exploitation'.

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1 Introduction

A core insight of exploitation theory is that profits are one of the key determinants for the existence of exploitation: profits represent the way in which capitalists appropriate social surplus and social labour. In the standard Okishio-Morishima (henceforth, OM) approach to value theory and exploitation, this insight has been incorporated into the so-called "Fundamental Marxian Theorem" (Okishio 1963; Morishima 1973; henceforth, FMT). In the OM approach, the FMT proves that positive profits are synonymous with the exploitation of labour, and it is interpreted as showing that labour is the only source of surplus value and profits. Although the FMT is mathematically robust, its economic interpretation has been questioned.

One of the most devastating criticisms of the FMT highlights some conceptual issues with the standard definition of exploitation. In the OM approach, in fact, exploitation is essentially defined as the *technologically efficient use of labour as a productive factor*. The FMT itself can be interpreted as proving that the exploitation of labour is simply *one* numerical representation of the existence of surplus products in a *productive* economy using labour as the *numéraire*. The problem is that this property is not uniquely associated with labour, and whenever the (standard) FMT holds, the so-called, "Generalised Commodity Exploitation Theorem" (Bowles and Gintis 1981; Roemer 1982; henceforth, GCET) also holds, according to which *exploitation as the technologically efficient use of any commodity as a productive factor* is equivalent to positive profits. Thus, in the OM approach, there is no analytical basis for distinguishing labour exploitation from the 'exploitation' of any other commodity: they are just alternative representations of the existence of a surplus product by means of different numéraire.

This paper argues that the key shortcoming of the OM approach lies with the notion of exploitation as merely representing the existence of a surplus in a productive economy. The standard OM approach defines exploitation as a purely technological, and in this sense asocial phenomenon. Instead, exploitation should be seen as an inherently social phenomenon, which characterises social relations between producers. The relation between exploitation and profits, then, has not only to do with the properties of the existing technology and its efficient use by capitalists. It reflects social relations of production and distributions among individuals.

This paper analyses an alternative approach to exploitation theory related to the 'New Interpretation' (Dumènil 1980; Foley 1982; henceforth, NI-form), which has been recently proposed by Yoshihara and Veneziani (2009) and Yoshihara (2010). In the latter papers, a complete axiomatic characterisation of this new approach is provided, based on a small set of weak axioms which emphasise the relational nature of the concept of exploitation. Indeed, Veneziani and Yoshihara (2011) prove that, under the NI-form, the FMT characterises capitalist economies with positive profits as generating *exploitative social relations*, rather than as guaranteeing the existence of surplus products in a productive economy.

Given this interpretation of the FMT under the NI-form of exploitation, it is not obvious what the counterpart-definition of commodity exploitation should be, nor is it clear whether the counterpart GCET holds or not. This paper shows that while the notion of commodity exploitation is well-defined even in the NI-form, the counterpart GCET no longer holds. Therefore the approach analysed in this paper is arguably superior to the standard OM approach in that it characterises exploitation as a social relation between producers whereby the creation and distribution of social surplus is uniquely mediated by the exchange of human labour. The exploitation of labour and the 'exploitation' of goods are no longer equivalent.

The rest of the paper is organised as follows. Section 2 presents the basic economic model. Section 3 discusses the classical definitions of labour and commodity exploitation.

Section 4 defines the NI-forms of labour and commodity exploitation, and shows that in this approach, the existence of positive profits and the 'exploitation' of goods are not equivalent.

2 The Basic Model

The model analysed in this paper is standard in the literature on the FMT (see, for example, Roemer 1981; Veneziani and Yoshihara 2011). An economy consists of a set H of agents, or households, who trade n commodities. Let $\mathbf{R}$ be the set of real numbers, and let $\mathbf{R}_+$ (respectively, $\mathbf{R}_{++}$) be the set of non-negative (respectively, strictly positive) real numbers. Production technology is freely available to all agents, who can operate any activity in the production set $P \subseteq \mathbf{R}_- \times \mathbf{R}_-^n \times \mathbf{R}_+^n$, which has elements of the form $\boldsymbol{\alpha} \equiv \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \in P$, where $\alpha_0 \in \mathbf{R}_+$ is the direct labour input, $\underline{\boldsymbol{\alpha}} \in \mathbf{R}_+^n$ are the inputs of the produced goods, and $\overline{\boldsymbol{\alpha}} \in \mathbf{R}_+^n$ are the outputs of the n goods. The net output vector arising from $\boldsymbol{\alpha}$ is denoted as $\hat{\boldsymbol{\alpha}} \equiv \overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}} \in \mathbf{R}^n$. Let the vector with all components equal to zero be denoted as $\mathbf{0}$. The following assumptions on P hold throughout the paper.[4]

A0. $P \subseteq \mathbf{R}_- \times \mathbf{R}_-^n \times \mathbf{R}_+^n$ is a closed convex cone with $\mathbf{0} \in P$.

A1. $\forall \boldsymbol{\alpha} = \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \in P$, [$\overline{\boldsymbol{\alpha}} > \mathbf{0} \Rightarrow \alpha_0 > 0$].

A2. $\forall \mathbf{c} \in \mathbf{R}_+^n$, $\exists \boldsymbol{\alpha} = \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \in P$ *s.t.* $\hat{\boldsymbol{\alpha}} \geq \mathbf{c}$.

A3. $\forall \boldsymbol{\alpha} = \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \in P$, $\forall \left(-\underline{\boldsymbol{\alpha}}', \overline{\boldsymbol{\alpha}}'\right) \in \mathbf{R}_-^n \times \mathbf{R}_+^n$,

$$\left[\left(-\underline{\boldsymbol{\alpha}}', \overline{\boldsymbol{\alpha}}'\right) \leq \left(-\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \Rightarrow \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}', \overline{\boldsymbol{\alpha}}'\right) \in P\right].$$

A1 implies that labour is indispensable to produce a positive amount of some good. **A2** states

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[4] For all vectors $\mathbf{x} = \left(x_1, \ldots, x_p\right)$, $\mathbf{y} = \left(y_1, \ldots, y_p\right) \in \mathbf{R}^p$, $\mathbf{x} \geq \mathbf{y} \Leftrightarrow x_i \geq y_i$ $(\forall i = 1, \ldots, p)$; $\mathbf{x} > \mathbf{y} \Leftrightarrow \mathbf{x} \geq \mathbf{y}$ & $\mathbf{x} \neq \mathbf{y}$; $\mathbf{x} >> \mathbf{y} \Leftrightarrow x_i > y$ $(\forall i = 1, \ldots, p)$. Vectors are columns unless otherwise specified.

that any non-negative commodity vector is producible as net output. **A3** is a standard free disposal assumption.

The standard Leontief production technology is a special case of the production sets satisfying **A0 ~ A3**. Let A denote a $n \times n$ non-negative input matrix, and let L denote a $1 \times n$ positive vector of direct labour inputs. Then,

$$P_{(A,L)} \equiv \left\{ \boldsymbol{\alpha} \in \mathbf{R}_+^{2n+1} \middle| \exists \mathbf{x} \in \mathbf{R}_+^n : \left(-L\mathbf{x}, -A\mathbf{x}, \mathbf{x}\right) \geq \boldsymbol{\alpha} \right\}$$

is the production set corresponding to (A, L) and $P_{(A,L)}$ satisfies **A0 ~ A3** whenever A is a productive matrix.

Given a market economy, a (row) vector $\mathbf{p} \in \mathbf{R}_+^n$ describes the price of each of the n commodities in the economy. For any agent $\nu \in H$, let $\boldsymbol{\omega}^\nu \in \mathbf{R}_+^n$ denote her initial endowments. In the literature on the FMT, it is assumed that the set of agents H can be partitioned into two disjoint subsets, namely the working class, denoted as W, which comprises agents with no initial endowments, and the set N of capitalists, who own at least some productive assets. Formally, $W = \left\{ \nu \in H \middle| \boldsymbol{\omega}^\nu = \mathbf{0} \right\}$ and $N = \left\{ \nu \in H \middle| \boldsymbol{\omega}^\nu > \mathbf{0} \right\}$. Further, it is assumed that workers are endowed with one unit of (homogeneous) labour.

For a given price vector $\mathbf{p}$ and wage rate $w > 0$, capitalists are assumed to maximise profits subject to their wealth constraint. Formally, each $\nu \in N$ solves the following (P1):[5]

$$\max_{\boldsymbol{\alpha}^\nu = \left(-\alpha_0^\nu, -\underline{\boldsymbol{\alpha}}^\nu, \overline{\boldsymbol{\alpha}}^\nu\right) \in P} \quad \mathbf{p}\overline{\boldsymbol{\alpha}}^\nu - \left(\mathbf{p}\underline{\boldsymbol{\alpha}}^\nu + w\alpha_0^\nu\right) \tag{P1}$$

$$\text{subject to } \mathbf{p}\underline{\boldsymbol{\alpha}}^\nu \leq \mathbf{p}\boldsymbol{\omega}^\nu .$$

In line with classical political economy, it is assumed that capitalists do not work and

5 Because inputs are traded at the beginning of the period and outputs at the end, the optimisation programme (P1) can be interpreted as incorporating an assumption of *stationary expectations* on prices (see Roemer 1981, Chapter 2).

do not consume: they use their revenues to accumulate for production in the next period. Moreover, workers are assumed to supply a fixed amount of labour, equal to their labour endowment, and to be abundant relative to social productive assets. This assumption reflects the Marxian view that involuntary unemployment is a structural feature of capitalist economies. Finally, workers are assumed to consume a fixed subsistence bundle of commodities, $\mathbf{b} \in \mathbf{R}_{+}^{n} \setminus \{\mathbf{0}\}$.

An economy is a list $E \equiv \left\langle H; (P, \mathbf{b}); (\boldsymbol{\omega}^{\nu})_{\nu \in N} \right\rangle$ with $H = W \cup N$. The definition of equilibrium for E can then be provided:

Definition 1 (Roemer 1981, Definition 2.5, p.41)**:** A *reproducible solution* (**RS**) *for the economy* E is a pair $\left((\mathbf{p}, w), (\boldsymbol{\alpha}^{\nu})_{\nu \in N}\right) \in \mathbf{R}_{+}^{n+1} \times P^{N}$, where $\mathbf{p} \in \mathbf{R}_{+}^{n} \setminus \{\mathbf{0}\}$, such that:

(a) $\forall \nu \in N$, $\boldsymbol{\alpha}^{\nu} \in P$ is a solution of (P1) (profit maximisation);

(b) $\hat{\boldsymbol{\alpha}} \geq \alpha_0 \mathbf{b}$, where $\boldsymbol{\alpha} \equiv \sum_{\nu \in N} \boldsymbol{\alpha}^{\nu}$ & $\hat{\boldsymbol{\alpha}} = \overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}}$ (reproducibility);

(c) $\mathbf{pb} = w$ (subsistence wage);

(d) $\underline{\boldsymbol{\alpha}} \leq \boldsymbol{\omega}$, where $\boldsymbol{\omega} \equiv \sum_{\nu \in N} \boldsymbol{\omega}^{\nu}$ (social feasibility).

Part (a) is standard and needs no further comment. Part (b) states that net output in every sector should at least be sufficient for employed workers' total consumption. This amounts to requiring that social endowments do not decrease, because (b) is equivalent to $\boldsymbol{\omega} + \overline{\boldsymbol{\alpha}} - \underline{\boldsymbol{\alpha}} - \alpha_0 \mathbf{b} \geq \boldsymbol{\omega}$, where the right hand side is the social stock at the beginning of the period, and the left hand side is the stock at the beginning of next period. Given that workers are abundant relative to productive assets, part (c) states that unemployment drives the equilibrium real wage rate down to the subsistence level. Finally, part (d) requires that intermediate inputs can be anticipated from current stocks, while wages are assumed to be paid after production.

3. Definitions of Exploitation under the Okishio-Morishima Tradition

Consider a worker $\mu \in W$: exploitation is characterised by systematic differences between the labour contributed by μ to the economy and the labour 'received' by μ, which is given by the amount of labour contained, or embodied, in some relevant consumption bundle(s). Therefore, for any bundle $\mathbf{c} \in \mathbf{R}_+^n$, it is necessary to define the labour value (or labour content) of $\mathbf{c}$.

First, consider the standard OM approach. Let the set of activities that produce at least $\mathbf{c}$ as net output be denoted as:

$$\phi(\mathbf{c}) \equiv \left\{ \boldsymbol{\alpha} = \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \in P \middle| \hat{\boldsymbol{\alpha}} \geq \mathbf{c} \right\}.$$

Then, according to Morishima (1974)**,** the *labour value of a bundle* $\mathbf{c}$ is

$$l.v.(\mathbf{c}) \equiv \min\left\{ \alpha_0 \middle| \boldsymbol{\alpha} = \left(-\alpha_0, -\underline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\alpha}}\right) \in \phi(\mathbf{c}) \right\}.$$

Given that the subsistence consumption vector $\mathbf{b}$ is a commodity bundle necessary to 'produce' *one unit* of labour, labour exploitation is defined as follows.

Definition 2 (Morishima 1974)**:** *At a consumption bundle* $\mathbf{b} \in \mathbf{R}_+^n \setminus \{\mathbf{0}\}$, *labour exploitation* exists if and only if $l.v.(\mathbf{b}) < 1$.

Analogously, for any good k, one can define the commodity k-exploitation at a commodity bundle $\mathbf{c}$. As a first step, the k*-value of a bundle* $\mathbf{c}$ is defined as follows:

$$k.v.(\mathbf{c}) \equiv \min\left\{ \underline{\alpha}_k \in \mathbf{R}_+ \middle| \boldsymbol{\alpha} = \left(-\alpha_0, -\left(\underline{\boldsymbol{\alpha}}_{-k}, \underline{\alpha}_k\right), \overline{\boldsymbol{\alpha}}\right) \in \phi(\mathbf{c}) \right\}.$$

In order to define the commodity k-exploitation, it is necessary to find a commodity bundle, which is used to produce *one unit* of commodity k, just like the bundle $\mathbf{b}$ can be interpreted as relevant to produce one unit of labour. Let $\left(\underline{\boldsymbol{\alpha}}^{(k)}, \alpha_0^{(k)}\right) \in \mathbf{R}_+^{n+1}$ be a profile of input goods and labour which can be used in the production of one unit of commodity k. Let $\mathbf{c}^{(k)} \equiv \underline{\boldsymbol{\alpha}}^{(k)} + \alpha_0^{(k)}\mathbf{b}$: this can be interpreted as *a commodity vector necessary to produce one unit of commodity* k.[6]

Commodity k-exploitation can then be defined as follows.

Definition 3 (Bowles & Gintis 1981; Roemer 1982)**:** *At* $\mathbf{c}^{(k)} \in \mathbf{R}_+^n \setminus \{\mathbf{0}\}$, *commodity* k *-exploitation* exists if and only if $k.v.\left(\mathbf{c}^{(k)}\right) < 1$.

Given Definitions 2 and 3, the following proposition can be proved:

Proposition 1 (Bowles & Gintis 1981; Roemer 1982)**:** *Let an economy* $E_{(A,L)} = \left\langle H; \left(P_{(A,L)}, \mathbf{b}\right); \left(\boldsymbol{\omega}^\nu\right)_{\nu \in N} \right\rangle$ *satisfy* **A0~A3**. *Then, for any* **RS** $\left(\left(\mathbf{p}, w\right), \left(\boldsymbol{\alpha}^\nu\right)_{\nu \in N}\right)$ *at* $E_{(A,L)}$, *the following statements are equivalent for* any commodity k :

(a) $\mathbf{p}\hat{\boldsymbol{\alpha}} - w\alpha_0 > 0$; (b) $l.v.\left(\mathbf{b}\right) < 1$; and (c) $k.v.\left(\mathbf{c}^{(k)}\right) < 1$.

Thus, FMT holds if and only if GCET holds. As Fujmoto and Opocher (2010) and Veneziani and Yoshihara (2010) forcefully argue, proposition 1 essentially implies the equivalence between positive profits and the productiveness of the economy. In other words, both labour exploitation and commodity k exploitation, as defined in Definitions 2 and 3, are just numerical representations of the productiveness of the economy. The standard OM approach does not properly capture the inherently social and relational aspect of exploitation as the

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[6] This vector need not be unique, given that there may be multiple techniques to produce one unit of good k in economies with a general convex cone production possibility set. In the standard Leontief model, however, the vector is unique.

unequal exchange of labour between agents that is central in Marxian theory.

4. Definitions of Exploitation à la New Interpretation

In this section, a new definition is discussed, which has been recently proposed by Yoshihara and Veneziani (2009) and Yoshihara (2010). For any $\mathbf{p} \in \mathbf{R}_{+}^{n} \setminus \{\mathbf{0}\}$ and $\mathbf{c} \in \mathbf{R}_{+}^{n}$, let the set of commodity bundles that cost exactly as much as $\mathbf{c}$ at prices $\mathbf{p}$ be denoted by $\mathbf{B}(\mathbf{p},\mathbf{c}) \equiv \{\mathbf{x} \in \mathbf{R}_{+}^{n} | \mathbf{p} \cdot \mathbf{x} = \mathbf{p} \cdot \mathbf{c}\}$. Then:

Definition 4: Given an economy $E = \left\langle H; (P, \mathbf{b}); (\boldsymbol{\omega}^{\nu})_{\nu \in N} \right\rangle$, let $\left((\mathbf{p}, w), (\boldsymbol{\alpha}^{\nu})_{\nu \in N}\right) \in \mathbf{R}_{+}^{n+1} \times P^{N}$ be an **RS** for E. For each $\mathbf{c} \in \mathbf{R}_{+}^{n}$ with $\mathbf{p} \cdot \mathbf{c} \leq \mathbf{p} \cdot \hat{\boldsymbol{\alpha}}$, let $\tau^{\mathbf{c}} \in [0,1]$ be such that $\tau^{\mathbf{c}} \hat{\boldsymbol{\alpha}} \in \mathbf{B}(\mathbf{p},\mathbf{c})$. The *labour embodied in* $\mathbf{c}$ *at the social reproduction point* $\boldsymbol{\alpha}$ is $\tau^{\mathbf{c}} \hat{\boldsymbol{\alpha}}$.

In Definition 4, the labour content of a bundle can be identified only if the price vector is known. Moreover, social relations play a central role, because the definition of labour content requires a prior knowledge of the social reproduction point and labour content is explicitly linked to the redistribution of total social labour (total labour employed), which corresponds to the total labour content of national income. The exploitation of labour can be defined as follows.

Definition 5: Given an economy $E = \left\langle H; (P, \mathbf{b}); (\boldsymbol{\omega}^{\nu})_{\nu \in N} \right\rangle$, let $\left((\mathbf{p}, w), (\boldsymbol{\alpha}^{\nu})_{\nu \in N}\right) \in \mathbf{R}_{+}^{n+1} \times P^{N}$ be an **RS** for E. For any $\mu \in W$, who supplies one unit of labour and consumes $\mathbf{b}$, let $\tau^{\mathbf{b}} \in [0,1]$ be defined as in Definition 5. Then, μ *is exploited* if and only if $1 > \tau^{\mathbf{b}} \alpha_0$.

Definition 5 is conceptually related to the 'New Interpretation' developed by Duménil (1980) and Foley (1982). For any $\mu \in W$, $\tau^{\mathbf{b}}$ represents μ's share of national income, and so

$\tau^{\mathbf{b}}\alpha_0$ represents the share of social labour which μ receives by earning income barely sufficient to buy $\mathbf{pb}$. Then, as in the New Interpretation, the notion of exploitation is related to the production and distribution of national income and social labour among producers. In this sense, Definition 5 formulates the notion of exploitation as representing *social relations among producers* with respect to the *unequal exchange of labour*. Noting that the total labour embodied in social net product is equal to α_0, it follows that if Definition 5 is adopted, there exist exploited agents if and only if there are some agents exploiting them. As Yoshihara and Veneziani (2009) show, quite surprisingly the NI is the only approach that satisfies this property in general.

Similarly to Definitions 4 and 5, for any good k, one can also define the k-value of a bundle $\mathbf{c}$ and commodity k-exploitation at a consumption bundle $\mathbf{c}$ as follows:

Definition 6: Given $E = \left\langle H; (P, \mathbf{b}); (\boldsymbol{\omega}^\nu)_{\nu \in N} \right\rangle$, let $\left((\mathbf{p}, w), (\boldsymbol{\alpha}^\nu)_{\nu \in N}\right) \in \mathbf{R}_+^{n+1} \times P^N$ be an **RS** for E. For each $\mathbf{c} \in \mathbf{R}_+^n$ with $\mathbf{p} \cdot \mathbf{c} \leq \mathbf{p} \cdot \hat{\boldsymbol{\alpha}}$, let $\tau^{\mathbf{c}} \in [0,1]$ be such that $\tau^{\mathbf{c}} \hat{\boldsymbol{\alpha}} \in \mathbf{B}(\mathbf{p}, \mathbf{c})$. The *commodity* k *content of* $\mathbf{c}$ *at the social reproduction point* $\boldsymbol{\alpha}$ is $\tau^{\mathbf{c}} \underline{\alpha}_k$.

In Definition 6, the commodity k content of $\hat{\boldsymbol{\alpha}}$, at the social reproduction point $\boldsymbol{\alpha}$, is precisely $\underline{\alpha}_k$. Therefore, as for the definition of labour content, in equilibrium there will be a redistribution of the total commodity k content of $\hat{\boldsymbol{\alpha}}$ - namely $\underline{\alpha}_k$ - to all agents.

Next, let $\underline{\alpha}_k^\nu$ denote the amount of good k that agent $\nu \in H$ contributes to the economy in equilibrium. The notion of commodity k-exploitation can be defined as follows:

Definition 7: Given an economy $E = \left\langle H; (P, \mathbf{b}); (\boldsymbol{\omega}^\nu)_{\nu \in N} \right\rangle$, let $\left((\mathbf{p}, w), (\boldsymbol{\alpha}^\nu)_{\nu \in N}\right) \in \mathbf{R}_+^{n+1} \times P^N$ be an **RS** for E. For any $\nu \in H$, who supplies $\underline{\alpha}_k^\nu$ and consumes $\mathbf{c}^\nu \in \mathbf{R}_+^n$, let $\tau^{\mathbf{c}^\nu} \in [0,1]$

be defined as in Definition 6. Agent ν is *commodity* k *-exploited* if and only if $\underline{\alpha}_k^\nu > \tau^{\mathbf{c}^\nu} \underline{\alpha}_k$.

The notion of commodity k -exploitation in Definition 7 is therefore related to the production and distribution of national income and of the aggregate capital good k among producers. In this sense, as for Definition 5, Definition 7 also represents exploitative social relations, using commodity k as the value numéraire.

Concerning labour exploitation, Veneziani and Yoshihara (2011) show that at the equilibrium of any convex economy, every employed $\mu \in W$ is exploited according to Definition 5 if and only if profits are positive. Theorem 1 proves, however, that this equivalence no longer holds in general for commodity k exploitation.

Theorem 1: *There exist an economy* E *and an RS in which the* equivalence *between* positive profits *and the* existence of commodity k -exploited agent*s does not hold*.

Proof. 1. Following a similar argument as in Yoshihara and Veneziani (2011), it can be proved that there exists an economy $E = \left\langle H; (P, \mathbf{b}); (\boldsymbol{\omega}^\nu)_{\nu \in N} \right\rangle$ with an unequal distribution of the initial aggregate endowment of good k , such that an RS $\left((\mathbf{p}, w), (\boldsymbol{\alpha}^\nu)_{\nu \in N} \right)$ with $\mathbf{p}\hat{\boldsymbol{\alpha}} - w\alpha_0 = 0$ and $\underline{\alpha}_k > 0$ exists.

2. At this RS, every capitalist receives zero income. This implies that $\tau^{\mathbf{c}^\nu} = 0$ for every $\nu \in N$. Then, given that $\underline{\alpha}_k = \sum_{\nu \in N} \underline{\alpha}_k^\nu \leq \sum_{\nu \in N} \omega_k^\nu$ at the RS, there exists at least one agent $\nu \in N$ such that $\underline{\alpha}_k^\nu > \tau^{\mathbf{c}^\nu} \underline{\alpha}_k$. This implies the existence of commodity k -exploitation, even though $\mathbf{p}\hat{\boldsymbol{\alpha}} - w\alpha_0 = 0$. **Q.E.D.**

In this paper, it is assumed that capitalist income consists solely of profit revenues.

However, Theorem 1 can be extended to economies in which capitalists also supply one unit of labour to earn a wage as in Veneziani and Yoshihara (2011) and Yoshihara (2010),[7] which seems a more plausible behavioural assumption whenever agents aim to maximise revenues.

Theorem 1 implies that the notion of commodity k exploitation is not relevant in Marxian exploitation theory if the New Interpretation is adopted, since the GCET no longer holds. Although commodity k exploitation as defined in Definition 7 does represent an unequal exchange-type of social relation among producers with commodity k as the value numéraire, Theorem 1 implies that the notion of exploitative social relations does not convey any relevant information about capitalist economies unless labour is the value numéraire.

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[7]In this case, every agent in H earns the same wage income, and so $\tau^{\mathbf{c}^{\nu}} = \frac{1}{|H|}$ for all $\nu \in H$. Then, again there can exist $\nu \in N$ such that $\underline{\alpha}_k^{\nu} > \tau^{\mathbf{c}^{\nu}} \underline{\alpha}_k$, unless $\left(\omega_k^{\nu}\right)_{\nu \in H} = \left(\frac{1}{|H|}\omega_k, \ldots, \frac{1}{|H|}\omega_k\right)$ (with $H = N$).

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