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Abstract

An extensive literature shows the importance of investment policy for sustainability of resource-based economies by examining the role of investment in current utility change (CUC) for a competitive optimizing economy. This paper extends some of these results by analysing the dependence of CUC on genuine investment, expressed in marginal resource productivity, under dynamic inefficiency. This inefficiency arises when a social planner, due to imperfection in knowledge or in institutions, does not take into account deviations of real economy from a theoretical model. These deviations or distortions, connected with the resource extraction, can influence utility, production, the balance equation, and the dynamics of the reserve, modifying the standard Hotelling rule. The analysis of this natural discrepancy between theory and real life implies that: first, institutional and resource policies in inefficient economies may be more important for CUC than investment policy; and secondly, under uncertainties in production possibilities and in damages from economic activities, sustainability requires a more cautious resource policy than is advised by a theory.

Key words:
nonrenewable resource; dynamic inefficiency; genuine investment; resource policy; sustainable development
JEL : O13; O47; Q32; Q38

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1. Introduction

Sustainability of real economies is always evaluated under uncertainties in future production possibilities and in various distortions such as institutional imperfections, economic wastefulness, damages from the processes of production and consumption to utility and to production itself. These uncertainties lead to the errors in the decisions of a social planner and cause deviations from an efficient and optimal path of economic development.

The literature on sustainability evaluation of resource-based economies offers an indicator of sustainable development, called genuine saving or genuine investment (GI), which is equal to increase in man-made capital minus resource depletion. This indicator was developed in the studies of the change in the current (present) value of consumption or utility at a specific moment of time in a dynamically efficient optimizing economy. Straightforward applications of these results to real-world situations can form the impression that, for sustainability, it is enough to invest in a proper way into man-made and human capital regardless of the pattern of extraction.

The current paper extends some of these studies by assuming that a real economy deviates from theoretical paths, but a planner, due to imperfection in knowledge or in institutions, uses the policies developed for an undistorted model. The paper provides the examples of distortions, which show that this natural discrepancy between a model and real life results in dynamically inefficient paths and, in some cases, unsustainability of the economy.

The idea of the indicator GI was offered in Hartwick (1977) as the "invest resource rent" rule for the problem formulated in Solow (1974) for the Dasgupta-Heal-Solow-Stiglitz (DHSS)[1] model of a perfectly competitive resource-based economy satisfying the standard Hotelling rule (HR) as a necessary condition of dynamic efficiency. For this model, zero GI with resource depletion measured in market prices leads to constant per capita consumption over time. Dixit et al. (1980) extended the Hartwick rule by showing for a more general production function that GI that is constant over time in present prices is a necessary and sufficient condition for a constant path of utility.[2] Dasgupta and Heal (1979, pp. 303-306), Hamilton and Hartwick

[1] This model with the Cobb-Douglas production function was developed in the works of Dasgupta and Heal (1974), Solow (1974), and Stiglitz (1974).

[2] Constant investment in present prices means that investment in current prices is growing with the rate of discount.

(2005), and Hamilton et al. (2006) analyzed the link between GI in current prices and current change in per capita consumption. Hamilton and Withagen (2007) derived the result of Dixit et al. (1980) (including also the result of Hamilton and Hartwick (2005)) in a more general setting (with internalized externalities), showing that instantaneous utility increases if and only if GI decreases in present prices. All these results were obtained for a competitive dynamically efficient economy.

Besides theoretical studies, variants of the indicator GI were used for practical evaluation of sustainability. For example, Pearce and Atkinson (1993) offered a simple indicator of weak sustainability[3] based on the assertion that "an economy is sustainable if it saves more than the *combined* depreciation on the two forms of capital" (man-made and natural). A variant of this indicator, modified for open economies, has been developed in Proops et al. (1999). These indicators were used in both papers to classify a number of countries into sustainable and unsustainable. Hamilton and Clemens (1999) developed a theory of genuine saving by adding the investment in human capital to traditional net savings and subtracting the value of resource depletion and environmental damage. The value of genuine saving was offered as an indicator of sustainability, and this indicator was used for comparing sustainability of a wide range of developing countries. Hence, as Hamilton and Hartwick (2005, p. 615) noted, "the magnitude of 'net investment' or 'genuine savings' has become a central focus in the measurement of the sustainability of an economy."

Proposition 1 (Section 2) of this paper extends Proposition 1 of Hamilton and Hartwick (2005), providing the link between GI and current utility change (CUC) in a dynamically inefficient economy.[4] The result shows that: 1) CUC may be determined only by the influence of inefficiency when this influence is not close to zero; 2) inefficiency asymmetrically affects the ability of investment to influence the sign of CUC, and this asymmetry is mutually inverted for large resource-poor and small resource-rich economies; 3) resource-based economies can be classified by the importance of investment or resource policies or both for CUC.

The examples of distortions (Section 3) include: 1) a resource-augmenting

[3]Weak sustainability of growth (development) is defined by Pezzey (1992) as nondecreasing per capita consumption (utility).

[4]A particular case of this result, when economy is dynamically efficient, can be obtained from the results of Dixit et al. (1980) and Hamilton and Withagen (2007).

technical change (Takayama 1980) that distorts the optimal dynamics of the resource stock and leads to a sustainable but dynamically inefficient economy when a planner does not take it into account; 2) irreversible damages to utility and production resulting from the resource use (e.g., Stollery 1998) that lead to inefficient and unsustainable outcomes when the planner ignores these effects; and 3) insecure property rights (Arrow et al. 2003) that also cause inefficient use of the resource and unsustainability of the economy, unless corrected by institutional transformations and resource policies.

The results of this study illustrate that, for sustainability, it is preferable to underestimate future production possibilities and overestimate damages since this policy of extra caution can reduce irreversible losses. Of course, this policy may lead to dynamic inefficiency caused by an overly conservative resource policy, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

The paper also notes that an indicator that reflects the changes in current utility can be, by construction, not sensitive to the changes in the ability of an economy to maintain non-declining utility during a long period of time. As Arrow et al. (2003) showed for imperfect economies, the accounting price[5] of a natural resource can be considerably higher than the market price, implying that the investment of the market resource rent and even the entire marked-valued output into man-made capital can be not enough to compensate for damages caused by the resource extraction. In other words, GI in accounting prices can be negative despite any effort in saving, implying that for some inefficient economies, institutional and resource policies are prerequisites of sustainability.

Section 4 discusses possible problems with using Proposition 1 for sustainability evaluation, and Section 5 concludes.

2. Investment and growth under distortions

In order to define a distorted economy, it is instructive to introduce first a "perfect" or undistorted optimizing economy. Following Hamilton and Hartwick (2005, p. 618), assume that the economy is closed, time t is continuous, consumption is aggregated into a single good C, labor is fixed, so that output $Q(t) = F(K, R)$ depends on man-made capital $K(t)$ and the resource

[5] The accounting price of the resource stock shows the change in the economy's long-term welfare when the resource stock is changed by one unit.

flow $R(t) = -\dot{S}(t)$, where $S(t)$ is the current resource stock ($\dot{S} := dS/dt$). The technology is stationary (F does not depend explicitly on t).

A number of studies, which results were used for practical evaluation of sustainability, assume that the economy satisfies the following:

• $F(K, R)$ is a regular production function that (a) denotes the maximum output for the given K and R, and (b) satisfies the Inada conditions, in particular $F_R > 0$ (*resource productiveness*), where $F_R := \partial F/\partial R$;

• output Q equals $F(K, R)$ (*static efficiency*);[6]

• the balance equation holds: $C + \dot{K} = F(K, R) - \delta K$, where $\dot{K}$ is investment and δK with $\delta = const$ is capital decay (*non-wastefulness*);

• the standard HR $\dot{F}_R = rF_R$[7] holds as a necessary condition of *dynamic efficiency*;

• the economy (a planner) maximizes a (social) welfare function (*optimality*).

In the real world, however, the resource use can be

– non-productive ($F_R = 0$) or counter-productive ($F_R < 0$);[8]

– productive, but static-inefficient ($Q < F(K, R)$);

– productive, static-efficient, but wasteful: ($C + \dot{K} < F(K, R) - \delta K$);

– productive, static-efficient, non-wasteful, but dynamically inefficient;

– productive, non-wasteful, efficient, but non-optimal.

This paper assumes that there is a vector $\mathbf{D}(t) = (D_1(t), D_2(t), D_3(t), D_4(t))$, called *distortion,* where D_i are the distortions in

$$\begin{aligned} &\text{production:} && F = F(K, R, D_1), && (1)\\ &\text{social utility:} && U = U(C, D_2), && (2)\\ &\text{the balance equation:} && \dot{K} = F(K, R, D_1) - C - \delta K - D_3, && (3)\\ &\text{the dynamics of the stock:} && \dot{S} = -R + D_4. && (4) \end{aligned}$$

The distortions can include imperfections, externalities, and any effects (including favorable for sustainability) that cause violation of the standard HR.

Assume, for simplicity, that $\mathbf{D}$ depends only on the extracted amount S_0-

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[6] Conventionally, efficiency is defined via the Pareto-optimality. Some studies, e.g. Hurwicz (1960), called this notion non-wastefulness.

[7] Here, $r(t) := F_K(t) - \delta$ is the market interest rate.

[8] The resource use is counter-productive when the decline in the resource stock results in the decline of output, e.g., as a result of a wildfire or oil spill.

$S(t)$.[9] For example, D_1 and D_2 can result from irreversible damages caused by economic activities (stock externalities, e.g., due to climate change); D_3 can stand for the growing cost of extraction (best-quality stock extracted first) or for static inefficiency and (or) for wastefulness of the economy ($D_3 > 0$); D_4 can be the productivity of the stock-augmenting investment, which is, first, growing with the extraction due to learning-by-doing and eventually declining due to the scarcity of the resource. Let D_5 be a deviation of the ratio $\dot{F}_R/F_R$ from a dynamically efficient path. Then the following result holds.

Lemma 1. *In economy (1)-(4),*

$$\dot{F}_R = [v(t) + \tau(t)]\, F_R, \tag{5}$$

where $v(t) := F_K - \delta,$[10] $\tau(t)$[11]$= \tau\left[\mathbf{D}(t)\right] := D_5 + \partial D_4/\partial(S_0 - S)$

$$+\frac{1}{F_R}\left[\frac{U_{D_2}\partial D_2/\partial(S_0 - S)}{U_C} + F_{D_1}\frac{\partial D_1}{\partial(S_0 - S)} - \frac{\partial D_3}{\partial(S_0 - S)}\right], \tag{6}$$

and $D_5 = 0$ *if the economy is dynamically efficient.*

Proof is in Appendix 1.

Deviation D_5 may depend on $\mathbf{D}$, for example, on D_4 when a planner does not take into account resource-augmenting investments ($D_5 = -\partial D_4/\partial(S_0 - S)$, Section 3.1), or on D_1 and D_2 when the planner ignores the damages caused by the resource extraction (Section 3.2). In this framework, dynamic efficiency is a relative notion. The planner's optimal path may be dynamically inefficient with respect to the first-best solution, e.g., because the planner underestimates future production possibilities and considers the first-best

[9] $\mathbf{D}$ can also depend on the rate of extraction, e.g., when damage includes the opportunity cost (Gaudet et al., 2006), or when damage is partly reversible. Then formula (6) below is more complicated, which, however, does not alter the conclusions of the paper. $\mathbf{D}$ can also depend on the amount of non-extracted resource, e.g., when the stock has an amenity value (D'Autume, Schubert, 2008). Then, if this value can be expressed in terms of utility, the problem can be reformulated by introducing the damage resulted from the resource extraction. In practice, this approach can be more precise, since the uncertainty in the extracted amount is essentially less than in the remaining stock. A review of studies with the modified HR can be found, e.g., in Gaudet (2007).

[10] $v(t)$ is the market interest rate only with no distortion.

[11] $\tau(t)$ is the additive HR modifier or the influence of $\mathbf{D}$. This influence in equation (5) can be expressed in a multiplicative form: $\dot{F}_R = v\eta\left[\mathbf{D}\right]F_R$, where $\eta\left[\mathbf{D}\right] := 1 + \tau\left[\mathbf{D}\right]/v$. With no distortion, $\tau = 0$.

paths as infeasible (Section 3.1). The planner's optimal path may also appear inefficient when the planner does not account for damages while estimating social progress. In the latter case, the planner may even consider the first-best path as inefficient due to the difference between the units of measure for utility in the planner's and in the first-best solutions (Section 3.2).

In some cases, however, D_5 may not depend on $\mathbf{D}$, and instead, both D_5 and $\mathbf{D}$ may be determined by the same phenomena, e.g. imperfect institution (Section 3.3). Lemma 1 is not relevant in these cases.

Genuine investment defined in Hamilton and Hartwick (2005) is

$$G(t) := \dot{K}(t) + \dot{S}(t)F_R(t). \tag{7}$$

This measure includes not only current investment into man-made capital but the value of the currently extracted resource estimated in marginal resource productivity, which, with no distortion, coincides with the market price. Hence, G indicates a combination of investment and resource policies.

Since utility can be distorted by D_2, the dependence of utility on consumption can be nonmonotonic, and, therefore, consumption cannot substitute utility as a measure of well-being (see, e.g., Section 3.2). Hence, the proposition below establishes the link between G and $\dot{U}$ (instead of $\dot{C}$), which includes the link between G and $\dot{C}$ as a special case.

Proposition 1. *Current utility change is*

$$\dot{U} = (v - \dot{G}/G)GU_C + \Psi, \tag{8}$$

where $\Psi := \dot{S}F_R U_C D_5$ *is the influence of dynamic inefficiency.*

Proof is in Appendix 2.

When the economy is dynamically efficient ($D_5 = \Psi = 0$), Proposition 1 coincides with the result of Hamilton and Withagen (2007), expressed in present prices, and, with no distortion, with the result of Hamilton and Hartwick (2005). Equation (8) shows, that investment (7) can indeed determine $\dot{U}$ if Ψ is relatively small. However, $\dot{U}$ can be also completely determined by Ψ when the term $\left(v - \dot{G}/G\right) GU_C$ is close to zero.

Of course, sharp changes in G can determine an *instant* sign of $\dot{U}$ despite the large values of Ψ. Formula (8) shows that if there is a $t = \bar{t}$, such that $\Psi(\bar{t})$ has a large positive (negative) value, $\dot{U}(\bar{t})$ can be negative (positive) if $G(\bar{t})$ is negative (positive) and $\dot{G}(\bar{t})/G(\bar{t})$ has a large positive (negative) value. However, these cases are not relevant to sustainability due to the boundedness of investments, whereas distortions in general are less restricted. The boundedness of investment implies that the larger is Ψ, the shorter is the period of time when these cases are possible. Therefore, neglecting the short-run oscillations, it can be assumed, for determinateness, that $0 < v(t) < \infty$ and $\left|\dot{G}/G\right| < v$ along the long-run trends[12] for all $t \geqslant 0$ and the current investment $\dot{K}$ is bounded by the current output Q. Then a *feasible investment* can be defined as follows.

Definition 1. *Investment $\dot{K}(t) = w(t)Q(t)$ is feasible if $w(t) \in (0,1)$ and $|\dot{G}/G| < v$ for any $t \geqslant 0$.*

Definition 1 results in the following Corollary.

Corollary 1. *Equation (8) implies that*

$$\begin{aligned} \text{(I) } \dot{U} &\gtreqless 0 \;\; \textit{iff} \\ \Psi &\gtreqless -\left(v - \dot{G}/G\right) GU_C, \;\; \textit{or } G \gtreqless -\dot{S}F_R D_5 / \left(v - \dot{G}/G\right), \;\; \textit{or} \end{aligned} \tag{9}$$

$$D_5 \begin{cases} \lesseqgtr -\left(v - \dot{G}/G\right)\left(\dot{K}/\left(\dot{S}F_R\right)+1\right), \;\; \textit{when } \dot{S} < 0; \\ \gtreqless -\left(v - \dot{G}/G\right)\left(\dot{K}/\left(\dot{S}F_R\right)+1\right), \;\; \textit{when } \dot{S} > 0; \end{cases} \tag{10}$$

(II) *a feasible investment policy can change the sign of $\dot{U}$ iff*

$$\left.\begin{array}{r} -\left(v - \frac{\dot{G}}{G}\right) < \\ -\left(v - \frac{\dot{G}}{G}\right)\left(\frac{Q}{\dot{S}F_R}+1\right) < \end{array}\right\} D_5 \begin{cases} < -\left(v - \frac{\dot{G}}{G}\right)\left(\frac{Q}{\dot{S}F_R}+1\right), \;\; \textit{when } \dot{S} < 0; \\ < -\left(v - \frac{\dot{G}}{G}\right), \;\; \textit{when } \dot{S} > 0. \end{cases} \tag{11}$$

The following examples show that the impact of dynamic inefficiency on the efficacy of investment depends on the level of output and the share of the resource rent in output. Assume that $D_4 = 0$ $(\dot{S} < 0), v = 0.06$ and $\dot{G}/G = 0.03$ at $\bar{t} \geqslant 0$.

[12] The analysis can be easily complemented by the case with $\left|\dot{G}/G\right| \geqslant v$.

(a) Large resource-poor economy. Let $Q(\bar{t}) = 101$ and $\dot{S}(\bar{t})F_R(\bar{t}) = -1$. Then (Corollary 1) an investment policy can change the sign of $\dot{U}(\bar{t})$ iff

$$-0.03 < D_5 < 3.$$

(b) Small resource-rich economy. In the case with $Q(\bar{t}) = 11$ and $\dot{S}(\bar{t})F_R(\bar{t}) = -10$, an investment policy can affect the sign of $\dot{U}(\bar{t})$ iff

$$-0.03 < D_5 < 0.003.$$

It is intuitive that a large economy has more opportunities in investment than a small one, and so the range for D_5, in which investment is able to affect the sign of utility change, is larger in case (a) than in case (b). Another difference between these two cases is that investment in a large resource-poor economy can change the sign of $\dot{U}$ mostly when D_5 affects $\dot{U}$ negatively (positive D_5 reduces $\dot{U}$ when $\dot{S} < 0$). In this example, the range of $D_5 > 0$, in which investment is able to compensate for the damage from the inefficiency, is 100 times larger than the range of $D_5 < 0$, which positive effect can be annihilated by negative G. This asymmetry is inverted in a small resource-rich economy.

Assume that $\dot{S} < 0$.[13] Then boundedness of investments implies that, from the planner's point of view, the current state of an economy belongs to the one of the four following types determined by different roles of resource (institutional) and investment policies in the current change of utility depending on the level of distortion D_5.

(A) $D_5 \geqslant -\left(v - \dot{G}/G\right)\left[Q/\left(\dot{S}F_R\right) + 1\right]$: utility declines regardless of investment; non-negative values of $\dot{U}$ can be obtained only by reduction of the inefficiency if it is still possible.[14]

(B) $0 < D_5 < -\left(v - \dot{G}/G\right)\left[Q/\left(\dot{S}F_R\right) + 1\right]$: utility growth can be achieved by investment policy alone; the optimal saving rate may be higher than under $\Psi = 0$ in order to compensate not only for the shrinking natural capital but for the negative effect of inefficiency (e.g., van der Ploeg 2011).

[13] The inequalities below are inverted when $\dot{S} > 0$. An example that allows for the growing reserve stock is provided in Section 3.1. The case with $\dot{S} > 0$ may lead to some interesting implications that require a separate study.

[14] Possibility of reduction of inefficiency depends on the state of the economy with respect to tipping points. This problem is not considered in the present paper.

Without a policy reducing D_5, the level of utility can be lower than under $\Psi = 0$.

(C) $-\left(v - \dot{G}/G\right) < D_5 < 0$: utility growth can be achieved by investment policy alone; the optimal saving rate can be lower than under $\Psi = 0$ due to the positive effect from D_5; decline in utility is still possible when $G < -\dot{S}F_R D_5/\left(v - \dot{G}/G\right) < 0$.

(D) $D_5 < -\left(v - \dot{G}/G\right)$: utility grows regardless of investments; investment policy is important as a determinant of the level of utility along the growing path.

Types C and D may correspond to the economy where the planner underestimates future production possibilities (Section 3.1).

Condition (9) shows that, for $\Psi < 0$, the minimum investment G that provides non-declining utility can be essentially higher than zero. The next section shows that the value of G guaranteeing $\dot{U} \geqslant 0$ may not exist.

3. Dynamic inefficiency and sustainability: Examples

In the examples below, D_5^* denotes a deviation of the planner's optimal ratio $\dot{F}_R/F_R$ from a first-best path and $\widetilde{D}_5$ — a deviation of the first-best path from the planner's optimal path ($D_5^* = -\widetilde{D}_5$).

3.1. Resource-augmenting technical change

Assume that in a real economy, D_4 is the only distortion, namely, $\dot{S} = -R + S\phi(L_R/L)$,[15] where L_R/L is the share of the resource-augmenting research sector and $\phi(\cdot) \geqslant 0$ is the rate of growth of the resource stock due to research (Takayama, 1980). According to (6), this problem yields condition (5) with $\tau = -\phi$.

Assume also that a social planner does not use the information about ϕ and works with the undistorted model. Then, from the planner's point of view, the dynamics of the real economy without government intervention is non-optimal with $\widetilde{D}_5 = -\phi$,[16] which corresponds to the type C or D (depending on the behavior of ϕ) with growing consumption when $G >$

[15] Note that when $\phi > R/S$, the distortion D_4 results in $\dot{S} > 0$.

[16] Utility is not distorted here ($U_{D_2} = 0$); hence, formula (8) becomes $\dot{C} = \left(v - \dot{G}/G\right) G + \dot{S}F_R D_5$, since $\dot{U} = U_C\dot{C} + U_{D_2}\dot{D}_2$.

$\dot{S}F_R\phi/\left(v-\dot{G}/G\right)$ (the investment G may be negative here when $\dot{S}<0$). In fact, the reduced "optimal" planner's paths of resource extraction and consumption are dynamically inefficient with $D_5^*=-\widetilde{D}_5=-\partial D_4/\partial(S_0-S)=\phi$, and if D_4 and a correspondent increment in consumption are taken into account, then $D_5^*\equiv 0$ and consumption grows only when $G>0$. Hence, in this example, the discrepancy between theory and real life results in a sustainable but inefficient path. Dynamic inefficiency can be reduced only by adjustment of the resource policy when the planner updates the information about reserve estimates. This adjustment can result in sustainable and asymptotically efficient economy.

3.2. Irreversible damages to utility and production

Let a social utility and production are negatively affected in a real economy by the damage $D=D_1=D_2$ caused by a stock externality[17] ($D_{(S_0-S)}>0, U_D<0, F_D<0$). If a planner disregards the damage, then, according to Lemma 1, the planner's paths are dynamically inefficient with $D_5^*=-(F_D+U_D/U_C)D_{(S_0-S)}/F_R>0$.[18] The planner's problem reduces in this case to the one of Solow (1974) - Hartwick (1977), where, under the maximin criterion, the path of extraction starts from a higher level than in the efficient case,[19] and the economy follows a constant-consumption path (due to $G=0$) with a higher level than the initial level of the efficient path. Since for the planner, $U_D=F_D=0$, formula (8) becomes $\dot{C}=\left(v-\dot{G}/G\right)G$.

In reality, however, the change in well-being is[20]

$$\dot{U}=\left(v-\dot{G}/G\right)GU_C+(F_DU_C+U_D)\dot{D}, \tag{12}$$

which is negative under the "undistorted" rules of investment and extraction ($G=0$), since $(F_DU_C+U_D)\dot{D}<0$. Denote $\Gamma=\min\dot{G}/G$ for all feasible G.

[17] E.g., D can result from irreversible climate change (Stollery, 1998).

[18] As usual, $U_C>0$.

[19] When damage affects only production, Stollery (1998, p. 735) showed that the optimal extraction starts from a lower initial level and declines slower than in the case with no damage. The same result for the damage in utility was obtained in Bazhanov (2011, formula (33), Fig. 4).

[20] Formula (12) can be obtained from formula (8) using the expression for D_5^* and the fact that $\dot{D}=-\dot{S}D_{(S_0-S)}$.

Then, according to Corollary 1, investment G that provides $\dot{U} > 0$ does not exist when the damage is large, namely, when

$$(F_D U_C + U_D)\dot{D} < -(v - \Gamma)\left(Q + \dot{S}F_R\right) < 0.$$

Hence, the "undistorted" policies result in inefficiency and unsustainability of this economy.

The growth of utility can be achieved in this case only when the planner recognizes the damage and reconsiders the measure of progress in the society. This done, the planner, situated in the Solow-Hartwick case, can obtain a sustainable and optimal paths by changing the resource policy alone, namely, by reducing extraction, while the investment rule remains the same.[21]

3.3. Insecure property rights

Following Arrow et al. (2003, p. 664), assume that the owner i $(i = 1 \ldots N; N \geqslant 2)$ extracts a liquid resource from the pool with the stock S_i. All N owners are identical, non-cooperative, and the pools are separated by porous barriers. The resource diffuses from larger pools to smaller ones with the same rate $\lambda > 0$.[22] Then the depletion equations are

$$\dot{S}_i = \lambda \sum_{j \neq i} (S_j - S_i) - R_i, i = 1 \ldots N,$$

where $R_i = R_i(t)$ is the rate of extraction of the owner i at the moment t. The necessary conditions for PV-maximization of the each owner's utility yield equation (5) with $\tau \equiv D_5 = (N-1)\lambda > 0$ (socially efficient paths require $N = 1$). This inefficiency results in the distorted path of extraction

$$R_D = \textstyle\sum_{i=1}^N R_i = \left[(\rho + D_5)/\eta\right] S_0 e^{-(\rho + D_5)t/\eta}$$

with the higher initial rate $R_D(0)$ and faster decline $\dot{R}_D$ than for the efficient path

$$R = [\rho/\eta]\, S_0 e^{-\rho t/\eta}.$$

In these formulas, $\rho > 0$ is the social discount rate, and $\eta > 1$ — the elasticity of marginal utility. Hence, the distorted equation for the whole

[21] Stollery (1998) showed that the Hartwick rule ($G = 0$) is still optimal in this economy.
[22] No barriers corresponds to $\lambda \to \infty$.

reserve is $\dot{S} = -R + D_4$, where $D_4 = R - R_D$. Lemma 1 is not relevant in this example, since D_4 does not depend directly on the extracted resource $S_0 - S$, and deviation D_5 does not depend on D_4; both these distortions result from imperfect institutions, expressed in $N \geqslant 2$ and $\lambda > 0$.[23]

Let $v = 0.06$, and $\dot{G}/G = -0.04$. Since $\dot{S} < 0$ and $D_5 \geqslant 0$ for any $t \geqslant 0$, a feasible investment policy can change the sign of $\dot{U}$ iff (Corollary 1)

$$(N-1)\lambda < -0.1\left[Q/\left(\dot{S}F_R\right) + 1\right] > 0.$$

When this condition is not satisfied, only institutional changes and resource policies can prevent decline in utility. An investment results in not declining utility here iff

$$G \geqslant -10(N-1)\lambda\dot{S}F_R > 0 \ \text{ or } w \geqslant -\left[10(N-1)\lambda + 1\right]\left(\dot{S}F_R/Q\right),$$

which is very restrictive for $N > 1$.

It is illustrative to consider two cases.

(a) Large resource-poor economy ($Q = 101$; $\dot{S}F_R = -1$). In this case, $\dot{K}$ can change the sign of $\dot{U}$ iff $(N-1)\lambda < 10$, which means, e.g., that, for $\lambda = 1$, utility declines for any investment (type A) if $N \geqslant 11$. Let $N = 5$. Then the saving rate, compensating for the shrinking resource and inefficiency, should be no less than $w_{\min} = \frac{41}{101}$ (or $G/Q \geqslant \frac{40}{101}$), whereas with no distortion ($N = 1$), utility grows for any $w > \frac{1}{101}$ (or $G > 0$).

(b) Small resource-rich economy ($Q = 11$; $\dot{S}F_R = -10$). $\dot{K}$ can change the sign of $\dot{U}$ iff $(N-1)\lambda < 0.01$; e.g., for $\lambda \geqslant 0.01$ and $N \geqslant 2$, utility declines regardless of any feasible investment. Let $\lambda = 0.009$ and $N = 2$. Then not declining utility is possible when almost all output is being invested, namely, $w \geqslant \frac{10.9}{11}$ (or $G/Q \geqslant \frac{0.9}{11}$), although, for this resource-dependent economy,

[23] Formally, the link $D_5(D_4)$ is $D_5 = -\rho - (\eta/t)\, W\left[D_4 t/S_0 - (\rho t/\eta)\, e^{-(\rho t/\eta)}\right]$, where $W[\cdot]$ is the Lambert W function (see, e.g., Corless et al., 1996). Numerically, using computational software (e.g., Maple), this formula gives $D_5 = (N-1)\lambda$ for any t when $arg\{W[\cdot]\} > -1/e$. Also, formally, D_4 changes with $S_0 - S(t)$, since both are changing in time. However, it can be shown that D_4 cannot be represented as a function of $S_0 - S$ only. Namely, the assumption $D_4 = D_4(S_0 - S)$, given τ, the expression $D_4 = R - R_D$, and using Lemma 1, results in $D_4 = (N-1)\lambda(S_0 - S) + D_4(0)$, since $D_1 = D_2 = D_3 = 0$. Here, $D_4(0) = -(N-1)\lambda S_0/\eta$. However, since the reserve S_0 is fixed, D_4 must result only in intertemporal redistribution of the resource, namely, the condition $\int_0^\infty D_4 dt = 0$ must hold, which is not true for D_4 derived in this way.

even with no distortion ($N = 1$), the saving rate yielding at least constant utility must be very high, namely, $w_{\min} = -\dot{S}F_R/Q \approx 0.91$.

Hence, the use of the policies for the undistorted model in this example also results in inefficiency and unsustainability; moreover, a feasible investment compensating for the inefficiency and providing non-declining current utility may not exist.

4. Multiple resources and distortions

Proposition 1 can be generalized in a straightforward way for n resources. Then formula (7) becomes

$$G(t) := \dot{K}(t) + \sum_{i=1}^{n} \dot{S}_i(t) F_{R_i}(t), \tag{13}$$

and, applying the same approach for the proof, the combined influence of all the inefficiencies on utility can be defined as

$$\Psi := U_C \sum_{i=1}^{n} \dot{S}_i F_{R_i} D_{5i}. \tag{14}$$

Equations (8) and (14) show that the combined effect of all distortions on the current change of aggregate utility can be positive despite the unsustainable extraction of some resources. The problem originates from the assumption that the components of consumption are substitutes. This assumption implies a specific way of aggregation of all the factors that can influence consumption and utility. Then, for example, according to formula (14), a common pool situation ($D_5 > 0$) for one resource can be compensated by the resource-augmenting investment for another resource, when a planner does not take into account both these distortions. An indicator using this aggregation will show that the current utility should grow, despite the known problems in the future.

The influence of distortions in the resource extraction on $\dot{U}$ is similar to the one of investment policies due to the boundedness of the resource stock. Indeed, formula (8) with $\Psi \equiv 0$ shows that utility can grow in the short run due to declining investments even when $G < 0$ (when $\dot{G}/G > v$), although this case has nothing to do with sustainability, since the boundedness of the saving rate and the resource reserve will eventually result in $\dot{G}/G < v$ and in

$\dot{U} < 0$. Future growth in utility is possible, but only after a period of decline. The case with $G > 0$ is different because this "reserve of investment" can be used during the infinite period of time by maintaining this level of G or asymptotically diminishing it to zero, which will positively affect utility change during this period. However, a positive G by itself cannot guarantee sustainability, because utility will decline when $\dot{G}/G > v$.

In the same way, the instant sign of Ψ shows only current influence of distortions on $\dot{U}$, whereas sustainability depends on the trends. Hence, Proposition 1 and an indicator based on formula (8) can, of course, provide useful policy recommendations for underinvesting and overextracting economies; however, such an indicator, calibrated for a specific economy, cannot guarantee even theoretically the existence of an economic program with non-declining utility during a long period of time, and it cannot show if the ability of the economy to maintain non-declining utility is improving.[24]

5. Concluding remarks

This study assumed that a social planner of a resource-based economy constructed optimal paths using a model that is not sufficiently adequate to the problem, e.g., due to imperfection in knowledge or in institutions. In order to imitate this discrepancy between theory and real life, the study has extended the result of Hamilton and Hartwick (2005) regarding the role of genuine investment (GI) in current utility change (CUC) by assuming that 1) there are distortions that affect utility, production, balance equation, and the dynamics of the resource stock; and 2) the planner does not take into account some of these distortions. As a result, the planner's paths are only second-best optimal and dynamically inefficient. Proposition 1 shows the link between GI and CUC depending on the influence of this inefficiency.

Proposition 1 and Corollary 1 have shown that CUC can be determined by GI, measured in the marginal resource productivity, only when the influence of the disregarded distortions is close to zero. These results entail a classification of the status of a resource-based economy by the importance of investment or resource policies or both for CUC. The study has shown that the distortions asymmetrically affect the ability of investment to control CUC, and this asymmetry is mutually inverted for capital-rich-resource-poor and capital-poor-resource-rich economies.

[24] E.g., in an overconsuming economy with $\dot{U} > 0$, this ability is declining.

The examples of neglected distortions demonstrated that a dynamically inefficient economy may be sustainable, when the planner underestimates future production possibilities, or unsustainable, when the planner ignores institutional imperfections or damages from economic activities. The examples also have shown that a feasible level of GI that provides a positive CUC may not exist.

Since an economy is unsustainable when current utility declines, the results of this study imply that for sustainability of real economies, 1) institutional and resource policies may be more important than investment policies when the level of inefficiency is high; 2) it is preferable that a resource policy is more conservative than is prescribed by a theory. In the former conclusion, investment policy, of course, is still important as a determinant of the level of utility along a growing or declining path and as a determinant of growth when the level of inefficiency is low. In the latter one, an overly conservative resource policy may result in dynamic inefficiency, but with updates in knowledge, the policy can be corrected, and the economy can be asymptotically efficient.

Besides regular consequences caused by deviation of real life from theory, an indicator based on CUC may not reflect sustainability of an economy, because, by construction, it does not show the change in the ability of the economy to maintain non-declining utility during a long period of time. Arrow et al. (2003) showed that, in order to evaluate current sustainability change, the accounting prices should be used for measuring GI; an indicator based on these prices presumably should contain not only the values of current investment and the rate of extraction, but the amounts of capital, resource reserve, and the information about moderate (preferably underestimating) assumptions concerning the paths of production possibilities.

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7. Appendix 1. Proof of Lemma 1

Since optimal paths are always efficient, a necessary condition of dynamic efficiency for economy (1)-(4) can be obtained from optimality conditions, e.g., in the problem of PV-maximization[25] of $\int_0^\infty U(C, D_2)\pi(t)dt$ with a discount factor $\pi(t)$. The Hamiltonian of this problem is $H = U(C, D_2)\pi(t) + \mu_K(F - C - \delta K - D_3) + \mu_S(D_4 - R)$, and the Pontryagin-type necessary conditions are

$$
\begin{aligned}
H_C &= U_C\pi(t) - \mu_K = 0, && (15)\\
H_R &= \mu_K F_R - \mu_S = 0, && (16)\\
\dot{\mu}_K &= -\frac{\partial H}{\partial K} = -\mu_K\left(F_K - \delta\right), && (17)\\
\dot{\mu}_S &= -\frac{\partial H}{\partial S} = -\pi(t)U_{D_2}\frac{\partial D_2}{\partial\left(S_0 - S\right)}\frac{\partial\left(S_0 - S\right)}{\partial S} - \\
&\quad \mu_K\left(F_{D_1}\frac{\partial D_1}{\partial\left(S_0 - S\right)}\frac{\partial\left(S_0 - S\right)}{\partial S} - \frac{\partial D_3}{\partial\left(S_0 - S\right)}\frac{\partial\left(S_0 - S\right)}{\partial S}\right) - \\
&\quad \mu_S\frac{\partial D_4}{\partial\left(S_0 - S\right)}\frac{\partial\left(S_0 - S\right)}{\partial S}. && (18)
\end{aligned}
$$

Equation (18) with μ_K from (15) becomes

$$
\begin{aligned}
\dot{\mu}_S &= \pi(t)U_{D_2}D_{2_{(S_0-S)}} + \\
&\quad U_C\pi(t)\left(F_{D_1}D_{1_{(S_0-S)}} - D_{3_{(S_0-S)}}\right) + \mu_S D_{4_{(S_0-S)}}. \qquad (19)
\end{aligned}
$$

The time derivative of equation (16) is $\dot{\mu}_S = \dot{\mu}_K F_R + \mu_K \dot{F}_R$, which, combined with (19), results in

$$
\begin{aligned}
\dot{\mu}_K F_R + \mu_K \dot{F}_R &= \pi(t)\left[U_{D_2}D_{2_{(S_0-S)}} + U_C\left(F_{D_1}D_{1_{(S_0-S)}} - D_{3_{(S_0-S)}}\right)\right] + \\
&\quad \mu_S D_{4_{(S_0-S)}}.
\end{aligned}
$$

The last equation after dividing through by F_R and substitutions for $\dot{\mu}_K$ (from (17)) and μ_S (from (16)) becomes

$$
\begin{aligned}
-\mu_K\left(F_K - \delta\right) + \mu_K\frac{\dot{F}_R}{F_R} &= \frac{\pi(t)}{F_R}\left[U_{D_2}D_{2_{(S_0-S)}} + U_C\left(F_{D_1}D_{1_{(S_0-S)}} - D_{3_{(S_0-S)}}\right)\right] + \\
&\quad \mu_K D_{4_{(S_0-S)}},
\end{aligned}
$$

which, divided through by μ_K with substitution for μ_K from (15), yields $\dot{F}_R/F_R = F_K - \delta + \tau\left(\mathbf{D}\right)$, where $\tau\left(\mathbf{D}\right)$ is defined by formula (6)■

[25] The maximin, formulated as $\max_{r,c}\int_0^\infty \overline{U}\delta e^{-\delta t}dt \equiv \overline{U} = const(r, c)$ with the additional constraint $U(C, D_2) = \overline{U}$, yields the same result.

8. Appendix 2. Proof of Proposition 1

The proof follows the approach of Hamilton and Hartwick (2005, Proposition 1), which was first applied in Hartwick (1977). The differences are that the current proof uses: 1) utility as a measure of well-being (due to distortion D_2); 2) formula (5) to substitute for $\dot{F}_R$ instead of the standard HR. Namely, equations (3), (1), and (7) give

$$\begin{aligned}
\dot{C} &= \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 \\
&= \dot{K}F_K + \dot{R}F_R + \dot{D}_1 F_{D_1} - \delta\dot{K} - \ddot{K} - \dot{D}_3 + R\dot{F}_R - R\dot{F}_R \\
&= (F_K - \delta)\dot{K} - (F_K - \delta + \tau)F_R R - \left[\ddot{K} - d\left(RF_R\right)/dt\right] + \dot{D}_1 F_{D_1} - \dot{D}_3.
\end{aligned}$$

From (4), $R = -\dot{S} + D_4$. Then

$$\begin{aligned}
\dot{C} &= (F_K - \delta)\dot{K} - (F_K - \delta + \tau)F_R\left(-\dot{S} + D_4\right) \\
&\quad - \left[\ddot{K} - d\left\{\left(-\dot{S} + D_4\right)F_R\right\}/dt\right] + \dot{D}_1 F_{D_1} - \dot{D}_3 \\
&= (F_K - \delta)\dot{K} + (F_K - \delta)F_R\dot{S} - (F_K - \delta)F_R D_4 - \tau F_R R \\
&\quad - \left[\ddot{K} + d\left\{\dot{S}F_R\right\}/dt\right] + d\left\{D_4 F_R\right\}/dt + \dot{D}_1 F_{D_1} - \dot{D}_3
\end{aligned}$$

or

$$\dot{C} = vG - \dot{G} - vF_R D_4 + d\left\{D_4 F_R\right\}/dt + \dot{D}_1 F_{D_1} - \dot{D}_3 - \tau F_R R. \tag{20}$$

Using Lemma 1,

$$\begin{aligned}
\tau F_R R &= \frac{U_{D_2}\partial D_2/\partial(S_0 - S)}{U_C}R + F_{D_1}\frac{R\partial D_1}{\partial(S_0 - S)} - \frac{R\partial D_3}{\partial(S_0 - S)} \\
&\quad + \frac{F_R R\partial D_4}{\partial(S_0 - S)} + D_5 F_R R.
\end{aligned}$$

Since D_i, by assumption, depend only on the extracted stock $S_0 - S$, equations $R = -\dot{S} + D_4$ and $\dot{D}_i = -\dot{S}\partial D_i/\partial(S_0 - S) = R\partial D_i/\partial(S_0 - S) - D_4\partial D_i/\partial(S_0 - S)$ yield $R\partial D_i/\partial(S_0 - S) = \dot{D}_i + D_4\partial D_i/\partial(S_0 - S)$. Then

$$\begin{aligned}
\tau F_R R &= D_5 F_R R + \frac{U_{D_2}}{U_C}\dot{D}_2 + F_{D_1}\dot{D}_1 - \dot{D}_3 + \dot{D}_4 F_R \\
&\quad - D_4\left[\frac{U_{D_2}\partial D_2/\partial(S_0 - S)}{U_C} + \frac{F_{D_1}\partial D_1}{\partial(S_0 - S)} - \frac{\partial D_3}{\partial(S_0 - S)} + \frac{F_R\partial D_4}{\partial(S_0 - S)}\right]
\end{aligned}$$

or

$$\begin{aligned}
\tau F_R R &= \frac{U_{D_2}}{U_C}\left(\dot{D}_2 + \frac{D_4 \partial D_2}{\partial(S_0 - S)}\right) + F_{D_1}\left(\dot{D}_1 + \frac{D_4 \partial D_1}{\partial(S_0 - S)}\right) \\
&\quad - \left(\dot{D}_3 + \frac{D_4 \partial D_3}{\partial(S_0 - S)}\right) + F_R\left(\dot{D}_4 + \frac{D_4 \partial D_4}{\partial(S_0 - S)}\right) + D_5 F_R R.
\end{aligned}$$

Substitution of the last formula into (20) results in

$$\begin{aligned}
\dot{C} &= vG - \dot{G} - vF_R D_4 + D_4 \dot{F}_R - \frac{U_{D_2}}{U_C}\left(\dot{D}_2 + \frac{D_4 \partial D_2}{\partial(S_0 - S)}\right) \\
&\quad - F_{D_1}\frac{D_4 \partial D_1}{\partial(S_0 - S)} + \frac{D_4 \partial D_3}{\partial(S_0 - S)} - F_R \frac{D_4 \partial D_4}{\partial(S_0 - S)} - D_5 F_R R,
\end{aligned}$$

which, after substitution for $\dot{F}_R$ from (5), becomes

$$\begin{aligned}
\dot{C} &= vG - \dot{G} + \tau F_R D_4 - \frac{U_{D_2}}{U_C}\left(\dot{D}_2 + \frac{D_4 \partial D_2}{\partial(S_0 - S)}\right) \\
&\quad - F_{D_1}\frac{D_4 \partial D_1}{\partial(S_0 - S)} + \frac{D_4 \partial D_3}{\partial(S_0 - S)} - F_R \frac{D_4 \partial D_4}{\partial(S_0 - S)} - D_5 F_R R.
\end{aligned} \tag{21}$$

Note that

$$\begin{aligned}
\tau F_R D_4 &= D_5 F_R D_4 + \frac{U_{D_2}}{U_C}\frac{D_4 \partial D_2}{\partial(S_0 - S)} + F_{D_1}\frac{D_4 \partial D_1}{\partial(S_0 - S)} \\
&\quad + F_R \frac{D_4 \partial D_4}{\partial(S_0 - S)} - \frac{D_4 \partial D_3}{\partial(S_0 - S)}.
\end{aligned}$$

Substitution of this expression into (21), using (4), results in

$$\dot{C} = vG - \dot{G} - \frac{U_{D_2}}{U_C}\dot{D}_2 + \dot{S} F_R D_5,$$

which, after substitution into $\dot{U} = U_C \dot{C} + U_{D_2}\dot{D}_2$, yields formula (8) of the proposition.

When economy (1)-(4) is dynamically efficient ($D_5 = 0$), formula (8) can be obtained from the result of Dixit et al. (1980, Theorem 1) or from a generalization of this result in Hamilton and Withagen (2007). Namely, in terms of the present value prices of utility, capital, and the resource, defined in Lemma 1 as $\pi(t), \mu_K$, and μ_S, these results claim that

$$\pi(t)\dot{U} = -\frac{d}{dt}\left(\mu_K \dot{K} + \mu_S \dot{S}\right),$$

which, using formulas (15)-(17), can be rewritten as follows:

$$\begin{aligned}\pi(t)\dot{U} &= -\frac{d}{dt}\left[\mu_K\left(\dot{K}+\frac{\mu_S}{\mu_K}\dot{S}\right)\right] = -\frac{d}{dt}\left[\mu_K\left(\dot{K}+F_R\dot{S}\right)\right] \\ &= -\frac{d}{dt}\left[\mu_K G\right] = -\mu_K\left(\frac{\dot{\mu}_K}{\mu_K}G+\dot{G}\right) = \mu_K\left[\left(F_K-\delta\right)G-\dot{G}\right].\end{aligned}$$

Then, with the use of formula (15) and the notation $v := F_K - \delta$, it becomes

$$\dot{U} = \left(v - \dot{G}/G\right) GU_C,$$

which is formula (8) for $D_5 = 0$■

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