Collusion in board of directors

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Abstract

The aim of this paper is to study what is the best structure of a Board of Directors when collusive aspects between the Board and the CEO are taken into account. We analyze how shareholders should select the members of the Board in a framework with asymmetric information and uncertainty about the optimal projects for the firm. In particular, we examine the optimal degree of independence of the Board from a shareholders perspective. This allows us to state when it is beneficial for shareholders to have an insider-oriented board or an outsider oriented board with a majority of independent directors when collusion is a major threat.

Keywords: Collusion, Corporate Governance, Asymmetric Information, Uncertainty.

JEL Classification: D81, D82, G34.

1 Introduction

Collusion between Boards of Directors' members and the CEO may be a major problem for the governance of firms. Indeed, most of the recent corporate scandals in the US or in Europe have emphasized the importance of corporate governance in the management of firms. For instance, a significant proportion of Board members of the Vinci Group in Europe or Worldcom and Home Depot in the US proved to be ever loyal to their CEO. An example of such a behavior is highlighted

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in The Boston Globe (January 6, 2007): "Despite his failure to increase the value of Home Depot's stock, chief executive officer Robert Nardelli left the company this week with a $210 million farewell package, the result of an agreement he negotiated with the board of directors in 2000. Across America, a culture of collusion between board members and prospective CEOs inflates executive pay and needs to be checked by greater shareholder involvement." Those "collusive" directors (some of them referred to as the "Bernie's Boys" for Worldcom) vote in favor of the CEO's propositions and allow her to get among others things generous bonuses, severance packages and golden retirement pensions. In many of those cases of "bad governance," one of the main issues was an explicit or implicit collusion between Directors of the Board and the CEO.

The aim of this paper is to study what is the best structure of a Board of Directors when collusive aspects between the Board and the CEO are taken into account. The Sabarnes-Oxley Act, the NYSE and the NASDAQ regulations in the US request that independent directors play a more important role in boards of directors. In order to study the efficiency of such requirements, we examine what should be the optimal degree of independence of a Board of directors from a shareholders perspective. This allows us to state when it is beneficial for shareholders to have an insider-oriented board or an outsider oriented board with a majority of independent directors when collusion is a major threat.

In our setting, the CEO has to choose between two projects that differ by their level of risk. The CEO's ability to undertake projects (High or Low) is unknown by the shareholders. The level of risk of the projects and the CEO's ability are her private information. Selecting a too risky project while it is not optimal for the firm yields a private benefit to the CEO. This private benefit may represent her utility from deriving various advantages such as perks, or building empires. In order to limit the CEO's discretion, shareholders have the opportunity to select the members of the Board and thus to choose its degree of independence. The Board has both a supervising and a consulting job. In our model, the role of the Board is thus to bring information about the type of the project that has been advised by the CEO but also to monitor the CEO.

We also allow for the possibility of collusion between the board and the CEO. Collusion takes place through a bribe offered by the CEO to some Directors in order to induce them not to reveal to shareholders that she has made a bad decision for the firm. Such a bribe may be a monetary or a non monetary transfer (e.g. future salary increases, perks, insurance to stay in the Board,...). Consequently, the collection of information from the CEO by shareholders may be more difficult and more costly because collusion reduces the strengthness of monitoring by directors.

The composition of the board, and in particular his level of independence which is measured by the proportion of independent directors in the board, influences the CEO's behavior. Indeed, lower is this degree of independence, more the Board's information about the type of the project is precise, but also more the Board is prone to engage in collusion with the CEO, both due to his relationships with the CEO and his executive role in the firm for instance.

This framework allows us to derive the optimal compensation contract of the CEO which consists of a fixed and a variable part. More precisely, our results are the following. First, we consider as a benchmark the case of no board of directors (or equivalently the case of no CEO's monitoring by the directors). In this setting, we show that the variable part of the CEO's wage is higher for a high ability CEO than for a low ability CEO. Then, we allow shareholders to recruit a Board of Directors in order to monitor the CEO, assuming that collusion cannot emerge. An intesting result is that the Board behaves as a perfectly honest Board. The contract takes the same form as the one with no board i.e. no informational rent for a low ability CEO and a positive informational rent for a high ability CEO. Those informational rents correspond to the surplus a CEO can extract from the shareholders thanks to her informational advantage. However, informational rents are lower in this case than when there is no monitoring from the Board. This implies that it it less costly for shareholders to obtain information from the CEO when the Board monitors him. This enables us to characterize a threshold wage such that if the Board's wage is lower than this threshold, recruiting a Board of Directors in order to monitor the CEO is always beneficial for the shareholders.

Allowing for the possibility of collusion between the board and the CEO, we show that the optimal contract is collusion proof: it is optimal for the shareholders to offer a contract preventing collusion to emerge. The optimal contract is designed such that shareholders have to concede to the CEO the same informational rents as in the presence of a perfectly honest board. However, they also have to ensure that the coalition Board-CEO does not collude which is costly in terms of informational rents. We also prove that there exists a degree of independence of the Board above which it is not profitable for the coalition Board-CEO to engage in collusion. In this case, shareholders do not have to care about preventing collusion when designing the optimal contract. The Board will therefore behave as a perfectly honest Board.

To our knowledge, our paper is the first theoretical model to consider the possibility of explicit collusion between the Board of Directors and the CEO. However, collusion has received a large attention in the Mechanism Design literature. The seminal paper of Tirole (1986) studies a three-tier organization with a principal, a supervisor and an agent in a moral hazard framework. In Tirole (1986), the agent and the supervisor can collude. Tirole (1986) derives the optimal collusion-proof contract. In our model and in another context with adverse selection, we also tackle this problem and derive the optimal collusion-proof contract.[1] Faure-Grimaud, Laffont and Martimort (2003) also study, in an adverse selection model, the optimal design of organization and the value of delegation when the supervisor and the agent can collude against the principal.

[1] Our paper differs from Tirole (1986) as we introduce the possibility that the CEO (the agent) chooses between different projects of investments and that the possibility of collusion is affected by the composition of the board. Moreover, we study what would be the optimal supervisor (board of directors) in the context of corporate governance.

Finally, we also derive the optimal degree of independence of the Board. Contrary to the usual idea that an optimal Board should be independent, we find that shareholders may prefer to select a Board of Directors with a low degree of independence. Indeed, when designing the optimal structure of the Board, shareholders face a trade-off between the information that they may extract from the Board and the costs from both extracting it and avoiding collusion. We then characterize conditions under which the optimal structure is a Board with a low degree of independence. Those conditions are the following: the risk of both projects have to be close and the degree of independence necessary to have a perfectly honest Board should be high enough. However, when project 2 is too risky compared to project 1 or when the degree of independence necessary to have a perfectly honest Board is low enough, we find that the optimal structure is a Board with a high degree of independence. In this case, the shareholders should not care about collusion because collusion is not profitable for such Boards. Indeed, when the level of relative risks of the two projects is high, it is important to monitor the CEO and prevent him to choose too a risky and non profitable project for the shareholders. In this case, shareholders should choose a board with a high degree of independence. The problem is less acute when the risks of the two projects are close.

There is a large literature in corporate governance about the composition of Boards of Directors (Boone, Field, Karpoff and Raheja, 2007, Dahya and McConnell, 2007, Harris and Raviv, 2006, Linck, Netter and Yang, 2008, Raheja, 2005), the relationship between the Board of Directors and the CEO (Chhaochharia and Grinstein, 2009, or Hermalin and Weisbach, 2003) as well as the monitoring role (Cornelli, Kominek and Ljungqvist, 2010, or Harris and Raviv, 2006) and the advisory role of the boards of directors (Adams and Fereira, 2007). Nevertheless, the problem of potential collusion between the CEO and the board has received little attention.[2]

The closest paper is Adams and Ferreira (2007). In their model, there is a continuum of projects but the projects do not differ with their level of risks. The CEO is reluctant from transmitting information to the Board of Directors because of the Board's monitoring role. The composition of the board of directors influences the behavior of the CEO as the more independent is the board of directors the more the CEO is monitored and the less the CEO is inclined to share information with the board. We share their result stating that it is not always optimal to choose an independent board for shareholders. However, the force driving our result is different from theirs. In Adams and Ferreira, when the Board's independence level is low, there is a low probability for the CEO to lose control. This makes revelation of information less costly for him and implies that choosing such a Board may be optimal for the shareholders. In our article, shareholders choose a non independant board even though they know that CEO's monitoring will be weakened because of its greater ability

[2] For reviews of the Corporate Governance literature, see Bebchuk and Weisbach (2010), Adams, Hermalin and Weisbach (2010), Hermalin and Weisbach (2003) or Tirole (2001).

to collect information which allows them to make a better investment decision. Moreover, in their paper, they do not explicitly model collusion between the CEO and the board members.

Raheja (2006) studies the question of the optimal composition and the ideal size of Boards of Directors. In the model, the optimal board structure is determined by the trade-off between insiders' incentives to reveal their private information and the outsiders' costs to verify projects. We also derive the optimal composition of the board taking into account the collusive behavior of CEO and directors.

Another interesting question raised by this literature is the potential replacement of the CEO by the board of directors. Hermalin and Weisbach (1998) analyze the role of independent directors in boards and show that a bad CEO is more likely to be replaced when the board is independent. Independent directors are therefore a mean for controlling the performance of the firm and a threat for bad CEOs. Hermalin (2007) studies the decision of hiring an internal vs an external CEO. Less is known about the external CEO. The model he develops determines whether it is optimal to keep an existing CEO or to replace him at a certain cost. While we do not address the question of the replacement of the CEO, monitoring of the CEO by the directors may entail a high fine for him which may be interpreted as his dismissal.

The article is organized as follows. Section 2 describes the model. Section 3 analyzes the benchmark case of no board of directors while section 4 introduces monitoring of the CEO by the board. Section 5 studies the impact of collusion on our results. The optimal structure of the board is discussed in section 6. Finally, section 7 offers conclusions.

2 The Model

2.1 The CEO and the Projects of the Company

A firm can undertake a project which yields an uncertain payoff. The firm is run for the shareholders by a CEO, i.e. the CEO's task is to select the project that will be undertaken by the firm.

The CEO's ability to succeed in the projects may be either low, $\beta = \underline{\beta}$, with probability (γ) or high, $\beta = \overline{\beta}$ with probability $(1-\gamma)$. As $\underline{\beta}$ corresponds to a low CEO's ability and $\overline{\beta}$ to a high ability, we have $\overline{\beta} \geq \underline{\beta}$.

We assume that the firm can undertake two kinds of projects that differ with their level of risk. The implementation of those projects initially require a fixed investment I by the firm's shareholders. The characteristics of those projects are the following:

- Project 1 either succeeds, that is, yields verifiable income $R > 0$ or fails, that is, yields no income. The probability of success is denoted by (q_1). Moreover, this project may have a low probability of success, that is, $q_1 = \underline{p}\beta_i$ with probability (ν) or may have a high probability of success $q_1 = \overline{p}\beta_i$ with probability $(1-\nu)$ where $\beta_i \in \{\overline{\beta}; \underline{\beta}\}$ is the CEO's ability to succeed in the projects.

- In the same way, Project 2 either succeeds, that is, yields verifiable income $R > 0$ or fails, that is, yields no income. The probability of success is denoted by (q_2). Moreover, this project may have a low probability of success, that is, $q_2 = (\underline{p} - \varepsilon)\beta_i$ with probability (ν) or may have a high probability of success $q_2 = (\overline{p} + \varepsilon)\beta_i$ with probability $(1-\nu)$ where $\beta_i \in \{\overline{\beta}; \underline{\beta}\}$ is the CEO's ability to succeed in the projects.

The success and the failure of both projects are assumed to be perfectly correlated i.e. (ν) represents the probability that the economic context is bad for the type of projects considered by the firm while (ε) represents the relative volatility of project two compared to project one.

As the Net Present Value of the riskiest project has to be at least higher than the NPV of the other project, we should have:

$$\left(\nu(\underline{p} - \varepsilon)\beta_i + (1-\nu)\left(\overline{p} + \varepsilon\right)\beta_i\right) R - I \geq \left(\nu\underline{p}\beta_i + (1-\nu)\,\overline{p}\beta_i\right) R - I.$$

This is equivalent to:

$$\nu \leq \frac{1}{2}.$$

The CEO perfectly knows both her ability's type and the probability of success of the projects while shareholders only know their prior probability distributions.

The CEO may therefore send signals to shareholders about her type and the project she advises to select:

$$\begin{cases} \sigma_{1,1} = (\beta = \underline{\beta}, \text{Project 1}) & \Rightarrow q_2 = \underline{p} - \varepsilon \text{ and } q_1 = \underline{p} \\ \sigma_{1,2} = (\beta = \underline{\beta}, \text{Project 2}) & \Rightarrow q_2 = \overline{p} + \varepsilon \text{ and } q_1 = \overline{p} \\ \sigma_{2,1} = (\beta = \overline{\beta}, \text{Project 1}) & \Rightarrow q_2 = \underline{p} - \varepsilon \text{ and } q_1 = \underline{p} \\ \sigma_{2,2} = (\beta = \overline{\beta}, \text{Project 2}) & \Rightarrow q_2 = \overline{p} + \varepsilon \text{ and } q_1 = \overline{p} \end{cases}$$

The CEO's compensation (paid by the firm's shareholders) is composed by a fixed part $\alpha_{i,j}$ and a variable part $\mu_{i,j}\pi$ that depends on the profits from the project (π) where $i \in \{1, 2\}$ corresponds to the CEO's signal about her ability (called hereafter the CEO's type) and $j \in \{1, 2\}$ corresponds to the CEO's signal about the probability of success of the project (called hereafter the best project's type).

When Project 2 is selected while it has a low probability of success $q_2 = (\underline{p} - \varepsilon)$, the CEO receives a private benefit B which represents his private compensation for choosing a project that poorly performs. In this state of nature, the CEO should rationally choose Project 1 but this private benefit may induce him to misbehave.

The CEO's reservation wage is w.

2.2 The Board of Directors

Shareholders also have the opportunity to hire a Board. The Board has both a supervising and a consulting job, i.e. the Board may have information about the type of the project and can communicate it to shareholders but may also monitor the information communicated by the CEO.

The structure of the Board is endogenous, in the sense that shareholders design it. Shareholders can choose the degree of independence of the Board. Lower is this degree of independence, more the Board's information about the type of the project is precise, but also more the Board is prone to engage in collusion with the CEO, both due to his relationships with the CEO and his executive role in the firm for instance.

We model the degree of independence of the board by a variable $\tau \in [\tau_{\min}, +\infty]$, with $\tau_{\min} \geq 1$, that acts as a discount factor for the collusion's rents. When his degree of independence, τ, increases, the amount of information hold by a Board decreases while his willingness to engage in collusion decreases. τ can also be interpreted as the degree of toughness and enforceability of the laws against collusion. Tougher are those laws, more difficult it is for the coalition Board-CEO to engage in collusion.

Let $\xi(\tau) = \frac{1}{\tau}$ be the probability that a Board with a degree of independence τ has gathered the true information about the economic context for the type of projects considered by the firm. When τ increases, Board members are more independent and less prone to collusion. However, as they have less information about the firm, their probability of knowing the truth is lower. We also assume that the CEO incurs a fine F when the Board reveals to the shareholders that she has announced that the project has a high probability of success while it is a project with a low probability of success, i.e. the case in which she gets the bonus B.

We are particularly interested in determining the value of the degree of independence τ such that the Board is Independent i.e. is completely honest and never accepts to engage in collusion with the CEO (this however means that he has a less precise information about the type of the project).

When collusion takes place, we assume that the CEO shares the collusive profits with the Board.

As it is usually the case in practice, the Board's wage is the total amount of the directors' fees which is constant and equals to w_0.

2.3 Multidimensional Screening Model

This model is a multidimensional screening model. Solving this kind of model is usually very complex (see Rochet and Chone, 1998). However, the structure of the model allows us to reduce this problem's complexity. Indeed, as the CEO's program can be specified as a function of only one parameter, $\theta_{i,j}$, we can rewrite the model as a usual four types unidimensional screening model. In this case, $\theta_{i,j}$ is defined in the following way:

$$\begin{cases} \theta_{1,1} = \underline{p\beta} \\ \theta_{1,2} = (\overline{p}+\varepsilon)\,\underline{\beta} \\ \theta_{2,1} = \underline{p}\overline{\beta} \\ \theta_{2,2} = (\overline{p}+\varepsilon)\,\overline{\beta} \end{cases}$$

Moreover, we assume that $\left(\underline{p}-\varepsilon\right)\overline{\beta} \geq (\overline{p}+\varepsilon)\,\underline{\beta}$, i.e. a high ability CEO undertaking a project with a low probability of success is more likely to succeed than a low ability CEO undertaking a project with a high probability of success. This assumption highlights the positive role of the CEO in her management of projects.

Denote the firm's profits $\pi^j\left(\theta_{i,j}\right) = \theta_{i,j}R - I$. The shareholders maximize their expected profits:

$$W = \nu\gamma\left[\left(1-\mu_{1,1}\right)\pi^1\left(\theta_{1,1}\right) - \alpha_{1,1}\right] + (1-\nu)\gamma\left[\left(1-\mu_{1,2}\right)\pi^2\left(\theta_{1,2}\right) - \alpha_{1,2}\right]$$
$$+\nu(1-\gamma)\left[\left(1-\mu_{2,1}\right)\pi^1\left(\theta_{2,1}\right) - \alpha_{2,1}\right] + (1-\nu)(1-\gamma)\left[\left(1-\mu_{2,2}\right)\pi^2\left(\theta_{2,2}\right) - \alpha_{2,2}\right]$$

3 No Board

When they do not hire a Board of Directors, shareholders maximize their expected profits under the usual Participation and Incentive constraints. PC_{ij} is the Participation constraint of a CEO with ability $i \in \{1,2\}$ when the project is of type $j \in \{1,2\}$. The Participation constraints ensure that the CEO will earn at least her reservation wage w. $IC_{ij\to kl}$ is the Incentive constraint of a CEO who reveals that her ability is $k \in \{1,2\}$ and the project is of type $l \in \{1,2\}$ while her true ability is i and the true type of the best project is j. The Incentives constraints ensure that the CEO earns a higher wage revealing the truth than lying to the shareholders. Through this process, shareholders induce the CEO to reveal her true type. Those constraints are stated here:

$$\alpha_{i,j} + \mu_{i,j}\pi^j\left(\theta_{i,j}\right) \geq w \tag{PC_{ij}}$$

$$\alpha_{HH} + \mu_{HH}\pi\left(\theta_{HH}\right) \geq \alpha_{HL} + \mu_{HL}\left[\overline{p}\overline{\beta}R - I\right] \tag{$IC_{HH\to HL}$}$$

$$\alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\,\overline{\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\,\overline{\beta}R - I\right] \tag{$IC_{HH\to LH}$}$$

$$\alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\,\overline{\beta}R - I\right] \geq \alpha_{LL} + \mu_{LL}\left[\overline{p}\overline{\beta}R - I\right] \tag{$IC_{HH\to LL}$}$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq \alpha_{HH} + \mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\overline{\beta}R - I\right] + B \tag{$IC_{HL\to HH}$}$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH}\left[\left(\underline{p} - \varepsilon\right)\overline{\beta}R - I\right] + B \tag{$IC_{HL\to LH}$}$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq \alpha_{LL} + \mu_{LL}\left[\underline{p}\overline{\beta}R - I\right] \tag{$IC_{HL\to LL}$}$$

$$\alpha_{LH} + \mu_{LH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] \geq \alpha_{HH} + \mu_{HH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] \tag{$IC_{LH\to HH}$}$$

$$\alpha_{LH} + \mu_{LH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\overline{p}\underline{\beta}R - I\right] \tag{$IC_{LH\to HL}$}$$

$$\alpha_{LH} + \mu_{LH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] \geq \alpha_{LL} + \mu_{LL}\left[\overline{p}\underline{\beta}R - I\right] \tag{$IC_{LH\to LL}$}$$

$$\alpha_{LL} + \mu_{LL}\left[\underline{p\beta}R - I\right] \geq \alpha_{HH} + \mu_{HH}\left[\left(\underline{p} - \varepsilon\right)\underline{\beta}R - I\right] + B \tag{$IC_{LL\to HH}$}$$

$$\alpha_{LL} + \mu_{LL}\left[\underline{p\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\underline{p\beta}R - I\right] \tag{$IC_{LL\to HL}$}$$

$$\alpha_{LL} + \mu_{LL}\left[\underline{p\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH}\left[\left(\underline{p} - \varepsilon\right)\underline{\beta}R - I\right] + B \tag{$IC_{LL\to LH}$}$$

Moreover, the Spence Mirrlees condition has to be satisfied, that is:

$$\mu_{HH} \geq \mu_{HL} \geq \mu_{LH} \geq \mu_{LL}$$

By assumption, we know that the following condition is satisfied:

$$\left(\underline{p} - \varepsilon\right)\overline{\beta} - \left(\overline{p} + \varepsilon\right)\underline{\beta} \geq 0 \tag{1}$$

As usual in this kind of problem, the binding constraints are[3]:

$$\alpha_{LL} + \mu_{LL}\left[\underline{p\beta}R - I\right] = w \tag{PC_{LL}}$$

$$\begin{aligned} \alpha_{LH} + \mu_{LH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] &= \alpha_{LL} + \mu_{LL}\left[\overline{p}\underline{\beta}R - I\right] \\ &= \alpha_{LL} + \mu_{LL}\left[\underline{p\beta}R - I\right] + \mu_{LL}R\underline{\beta}\Delta p \\ &= w + \mu_{LL}R\underline{\beta}\Delta p \end{aligned} \tag{$IC_{LH\to LL}$}$$

[3]We check that all constraints are satisfied in the Appendix.

$$
\begin{aligned}
\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] &= \alpha_{LH} + \mu_{LH}\left[\left(\underline{p} - \varepsilon\right)\overline{\beta}R - I\right] + B && (IC_{HL\to LH}) \\
&= \alpha_{LH} + \mu_{LH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] - \mu_{LH}R\left[\left(\overline{p} + \varepsilon\right)\underline{\beta} - \left(\underline{p} - \varepsilon\right)\overline{\beta}\right] + B \\
&= w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta} + \overline{\beta}\right)\right] + B
\end{aligned}
$$

$$
\begin{aligned}
\alpha_{HH} + \mu_{HH}\left[\left(\overline{p} + \varepsilon\right)\overline{\beta}R - I\right] &= \alpha_{HL} + \mu_{HL}\left[\overline{p}\overline{\beta}R - I\right] && (IC_{HH\to HL}) \\
&= \alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] + \mu_{HL}R\overline{\beta}\Delta p \\
&= w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta} + \overline{\beta}\right)\right] + \mu_{HL}R\overline{\beta}\Delta p + B
\end{aligned}
$$

Then we can characterize the optimal contract when there is no Board in the firm's organization. This is stated in the following Proposition:

Proposition 1 *When they do not hire a Board of Directors, shareholders must concede the following informational rents to a CEO*

$$
\begin{aligned}
U_{LL} &= w \\
U_{LH} &= w \\
U_{HL} &= w + \frac{B\left(\underline{p} - \varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]} \\
U_{HH} &= \begin{cases} w + \frac{B\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \text{ if } \varepsilon \leq \varepsilon_{nb} = \frac{\underline{\beta}\Delta p}{\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}} \\ w + \frac{B\Delta\beta(\overline{p}+\varepsilon)}{\underline{\beta}[\Delta p+2\varepsilon]} \text{ if } \varepsilon \geq \varepsilon_{nb} \end{cases}
\end{aligned}
$$

Moreover, the shareholders' expected profits are

$$
W_{NB} = \begin{cases} E(\pi) - w - \frac{(1-\gamma)B\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\frac{-\nu\overline{\beta}\Delta p+\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\overline{p}\underline{\beta}}\right] \text{ if } \varepsilon \leq \varepsilon_{nb} \\ E(\pi) - w - (1-\gamma)\left(\overline{p} + \varepsilon - \nu\Delta p - 2\nu\varepsilon\right)\frac{B\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)} \text{ if } \varepsilon \geq \varepsilon_{nb} \end{cases}
$$

A low ability CEO does not receive any rent whatever the type of project she advises to select. However, when her signal pushes shareholders to select the project with the highest volatility (Project 2), she receives a variable wage while she only gets a fixed wage when shareholders are induced to select Project 1.

A high ability CEO receives an informational rent which is higher when her signal induces shareholders to select the project with the highest volatility (Project 2) than when shareholders are induced to select Project 1. Moreover, the variable part of her wage is higher when project 2 is finally selected than when it is Project 1. But, in all cases, the variable part of a high ability CEO is higher that the one of a low ability CEO.

4 No Collusion

In this section, we assume that collusion is not possible between the Board of Directors and the CEO[4].When shareholders hire a Board, the CEO may incur a loss F when the Board has found that she has announced that the Project has a high probability of success while it is a low probability of success project, i.e. the case in which she has the bonus B. The Participation and Incentive constraints are now:

$$\alpha_{ij} + \mu_{ij}\pi_{ij} \geq w \tag{PC_{ij}}$$

$$\alpha_{HH} + \mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\overline{p}\overline{\beta}R - I\right] \tag{$IC_{HH\rightarrow HL}$}$$

$$\alpha_{HH} + \mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R - I\right] \tag{$IC_{HH\rightarrow LH}$}$$

$$\alpha_{HH} + \mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R - I\right] \geq \alpha_{LL} + \mu_{LL}\left[\overline{p}\overline{\beta}R - I\right] \tag{$IC_{HH\rightarrow LL}$}$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq \left(1-\xi(\tau)\right)\left\{\alpha_{HH} + \mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\overline{\beta}R - I\right] + B\right\} + \xi(\tau)\left(w-F\right) \tag{$IC_{HL\rightarrow HH}$}$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq \left(1-\xi(\tau)\right)\left\{\alpha_{LH} + \mu_{LH}\left[\left(\underline{p}-\varepsilon\right)\overline{\beta}R - I\right] + B\right\} + \xi(\tau)\left(w-F\right) \tag{$IC_{HL\rightarrow LH}$}$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq \alpha_{LL} + \mu_{LL}\left[\underline{p}\overline{\beta}R - I\right] \tag{$IC_{HL\rightarrow LL}$}$$

$$\alpha_{LH} + \mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R - I\right] \geq \alpha_{HH} + \mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R - I\right] \tag{$IC_{LH\rightarrow HH}$}$$

$$\alpha_{LH} + \mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\overline{p}\underline{\beta}R - I\right] \tag{$IC_{LH\rightarrow HL}$}$$

[4]We examine the case of collusion in the next section.

$$\alpha_{LH}+\mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R-I\right]\geq\alpha_{LL}+\mu_{LL}\left[\overline{p}\underline{\beta}R-I\right]\qquad(IC_{LH\rightarrow LL})$$

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq\left(1-\xi(\tau)\right)\left\{\alpha_{HH}+\mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\underline{\beta}R-I\right]+B\right\}+\xi(\tau)\left(w-F\right)\qquad(IC_{LL\rightarrow HH})$$

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq\alpha_{HL}+\mu_{HL}\left[\underline{p\beta}R-I\right]\qquad(IC_{LL\rightarrow HL})$$

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq\left(1-\xi(\tau)\right)\left\{\alpha_{LH}+\mu_{LH}\left[\left(\underline{p}-\varepsilon\right)\underline{\beta}R-I\right]+B\right\}+\xi(\tau)\left(w-F\right)\qquad(IC_{LL\rightarrow LH})$$

We also assume that the CEO faces a limited liability constraint, i.e., even if the Board found that the CEO has sent the wrong signal, she cannot get less than her reservation wage plus a fixed amount, K representing for instance the minimal compensation written in the CEO's labor contract. This gives:

$$\begin{aligned}\left(1-\xi(\tau)\right)\left\{w+B\right\}+\xi(\tau)\left(w-F\right)&\geq w+K\qquad(LL)\\ \Leftrightarrow B&\geq\frac{\xi(\tau)}{\left(1-\xi(\tau)\right)}F+K\end{aligned}$$

The binding constraints are:

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]=w\qquad(PC_{LL})$$

$$\begin{aligned}\alpha_{LH}+\mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R-I\right]&=\alpha_{LL}+\mu_{LL}\left[\overline{p}\underline{\beta}R-I\right]\qquad(IC_{LH\rightarrow LL})\\&=\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]+\mu_{LL}R\underline{\beta}\Delta p\\&=w+\mu_{LL}R\underline{\beta}\Delta p\end{aligned}$$

$$\begin{aligned}\alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right]&=\left(1-\xi(\tau)\right)\left\{\alpha_{LH}+\mu_{LH}\left[\left(\underline{p}-\varepsilon\right)\overline{\beta}R-I\right]+B\right\}+\xi(\tau)\left(w-F\right)\qquad(IC_{HL\rightarrow LH})\\&=\left(1-\xi(\tau)\right)\left\{\begin{array}{c}\alpha_{LH}+\mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R-I\right]\\-\mu_{LH}R\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}-\left(\underline{p}-\varepsilon\right)\overline{\beta}\right]+B\end{array}\right\}+\xi(\tau)\left(w-F\right)\\&=\left(1-\xi(\tau)\right)\left\{w+\mu_{LL}R\underline{\beta}\Delta p+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B\right\}+\xi(\tau)\left(w-F\right)\end{aligned}$$

$$\begin{aligned}\alpha_{HH}+\mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R-I\right]&=\alpha_{HL}+\mu_{HL}\left[\overline{p}\overline{\beta}R-I\right]\qquad(IC_{HH\rightarrow HL})\\&=\alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right]+\mu_{HL}R\overline{\beta}\Delta p\\&=\left(1-\xi(\tau)\right)\left\{w+\mu_{LL}R\underline{\beta}\Delta p+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B\right\}\\&\quad+\xi(\tau)\left(w-F\right)+\mu_{HL}R\overline{\beta}\Delta p\end{aligned}$$

The optimal contract when there is a Board of Directors and when collusion is not achievable is characterized in the following Proposition:

Proposition 2 *When they hire a Board of Directors and when collusion is not possible, shareholders must concede the following informational rents to a CEO*

$$\begin{aligned}
U_{LL} &= w \\
U_{LH} &= w \\
U_{HL} &= w + (1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta\left(\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]}\right) \\
U_{HH} &= \begin{cases} w+\frac{\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2[(1-\xi(\tau))B-\xi(\tau)F]}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} & \text{if } \varepsilon \leq \varepsilon_{ib} = \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} \\ w+\frac{(\overline{p}+\varepsilon)\Delta\beta\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} & \text{if } \varepsilon \geq \varepsilon_{ib} \end{cases}
\end{aligned}$$

Moreover, the shareholders' expected profits are

$$W_{IB} = \begin{cases} E(\pi)-w-w_0-(1-\gamma)(1-\xi(\tau))(\underline{p}-\varepsilon)\Delta\beta\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right] & \text{if } \varepsilon \leq \varepsilon_{ib} \\ E(\pi)-w-w_0-(1-\gamma)\Delta\beta\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\nu(1-\xi(\tau))(\underline{p}-\varepsilon)+(1-\nu)(\overline{p}+\varepsilon)\right] & \text{if } \varepsilon \geq \varepsilon_{ib} \end{cases}$$

In this case, the optimal contract has the same form than without Board, i.e. no rent for a low ability CEO and a positive rent for a high ability CEO which is higher when Project 2 is selected following her advice. However, we can note that the informational rents extracted by a CEO when there is a Board of Directors having no possibility to collude are lower than when there is no Board whatever the CEO's type.

We can therefore immediately conclude that if the Board's wage is low enough, hiring an a Board is always beneficial for the shareholders when collusion is not possible, i.e. $W_{IB} \geq W_{NB}$ for all $w_0 \leq \widetilde{w_0}$.

Corollary 3 *There exists a Board's wage $\widetilde{w_0}$ such that for all $w_0 \leq \widetilde{w_0}$, hiring an a Board is always beneficial for the shareholders when collusion is not possible*

5 Collusive Board

We now examine a framework in which the CEO and the Board of Directors may collude when this is profitable for them.

In the following inequalities, w_L is the income of a board that announces that the project has a low probability of success, w_H is the income of a board that announces that the project has a high probability of success, $w_\emptyset$ is the income of a board that announces that it has no information regarding the project probability of success, w_0 is the income of a board when collusion cannot emerge as in the previous section.

The following constraints ensure that the CEO-Board coalition get more when telling the truth than colluding.

$$\gamma\left[U_{LL} - w + w_L\right] + (1-\gamma)\left[U_{HL} - w + w_L\right] \geq \gamma\left[\frac{U_{LH} - w}{\tau} + w_\emptyset\right] + (1-\gamma)\left[\frac{U_{HH} - w}{\tau} + w_\emptyset\right]$$
$$\Leftrightarrow w_L \geq \gamma\left[\frac{U_{LH} - w}{\tau} - (U_{LL} - w)\right] + (1-\gamma)\left[\frac{U_{HH} - w}{\tau} - (U_{HL} - w)\right] + w_\emptyset$$

$$\gamma\left[U_{LH} - w + w_H\right] + (1-\gamma)\left[U_{HH} - w + w_H\right] \geq \gamma\left[\frac{U_{LL} - w}{\tau} + w_\emptyset\right] + (1-\gamma)\left[\frac{U_{HL} - w}{\tau} + w_\emptyset\right]$$
$$\Leftrightarrow w_H \geq \gamma\left[\frac{U_{LL} - w}{\tau} - (U_{LH} - w)\right] + (1-\gamma)\left[\frac{U_{HL} - w}{\tau} - (U_{HH} - w)\right] + w_\emptyset$$

Since we have $U_{LL} \leq U_{LH} \leq U_{HL} \leq U_{HH}$ and $\tau \geq \tau_{\min} \geq 1$, necessarily

$$\gamma\left[\frac{U_{LH} - w}{\tau} - (U_{LL} - w)\right] + (1-\gamma)\left[\frac{U_{HH} - w}{\tau} - (U_{HL} - w)\right] \leq 0$$

.

We then have 4 constraints to satisfy:

$$\begin{aligned}
w_L &\geq \gamma\left[\frac{U_{LH} - w}{\tau} - (U_{LL} - w)\right] + (1-\gamma)\left[\frac{U_{HH} - w}{\tau} - (U_{HL} - w)\right] + w_\emptyset \ (1) \\
w_H &\geq \gamma\left[\frac{U_{LH} - w}{\tau} - (U_{LL} - w)\right] + (1-\gamma)\left[\frac{U_{HH} - w}{\tau} - (U_{HL} - w)\right] + w_\emptyset \ (2) \\
w_L &\geq w_0 \ (3) \\
w_H &\geq w_0 \ (4)
\end{aligned}$$

In the following, we will examine when it is in the shareholders' interest to avoid collusion between the Board and the CEO. Indeed, avoiding collusion is costly because shareholders have to pay higher wages to the Board in order to induce him to reveal the gathered information. If those informational rents are too high, it may therefore be optimal for the Board to let collusion happen.

5.1 Collusion-Proof contract

We first analyze a situation in which shareholders want to ensure that collusion in the Board is avoided. The only case they have to take into account is when the Board tells that there is a low probability of success (the Board is more likely to lie when the project is of a low probability of success; there is no point in lying when it is of a high probability of success). We therefore always have $w_L \geq w_H$. Shareholders can try to use w_L to pay the Board into revealing the truth: if they set w_L high enough, collusion might be avoided. The shareholders' expected profits have the following form:

$$\begin{aligned} W_{CP} &= E(\pi) - \gamma\nu U_{LL} - \gamma(1-\nu)U_{LH} - (1-\gamma)\nu U_{HL} - (1-\gamma)(1-\nu)U_{HH} \\ &\quad -\nu\xi(\tau)w_L - (1-\nu)\xi(\tau)w_H - (1-\xi(\tau))w_0 \end{aligned}$$

In that case, the constraint on w_L is binding. Since they want to maximize their income, shareholders set $w_H = w_\emptyset = w_0$ (because w_0 is the lowest wage of the board).

$$\begin{aligned} w_L &= \gamma\left[\frac{U_{LH}-w}{\tau} - (U_{LL}-w)\right] + (1-\gamma)\left[\frac{U_{HH}-w}{\tau} - (U_{HL}-w)\right] + w_\emptyset \\ &= \gamma\left[\frac{U_{LH}-U_{LL}}{\tau} + \frac{1-\tau}{\tau}(U_{LL}-w)\right] + (1-\gamma)\left[\frac{U_{HH}-U_{HL}}{\tau} + \frac{1-\tau}{\tau}(U_{HL}-w)\right] + w_\emptyset \\ w_H &= w_\emptyset = w_0 \end{aligned}$$

We can remark that there exists τ_0 such that $w_L \geq w_0 \Longleftrightarrow \tau \leq \tau_0$. This means that for $\tau \geq \tau_0$, engaging in collusion is not beneficial for the coalition Board-CEO and the optimal contract is the same as with an Independent Board. Actually, when $\tau \geq \tau_0$, the Board will not collude whatsoever happens. Shareholders don't need to induce the Board to say the truth because he will do it anyway. So, we have in this case

$$w_L = w_H = w_0$$

We are now characterizing τ_0

$$\begin{aligned} w_L \geq w_0 &\Longleftrightarrow \tau\left[\gamma(U_{LL}-w) + (1-\gamma)(U_{HL}-w)\right] \leq \gamma(U_{LH}-w) + (1-\gamma)(U_{HH}-w) \\ &\Longleftrightarrow \tau \leq \frac{U_{HH}-w}{U_{HL}-w} = \begin{cases} \frac{1}{1-\xi(\tau_0)}\frac{\overline{p}+\varepsilon}{\underline{p}-\varepsilon} \text{ if } \varepsilon \geq \varepsilon_{ib} \\ \frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \text{ if } \varepsilon \leq \varepsilon_{ib} \end{cases} \end{aligned}$$

And then, as $\xi(\tau) = \frac{1}{\tau}$:

$$\tau_0 = \begin{cases} 1 + \frac{\overline{p}+\varepsilon}{\underline{p}-\varepsilon} \text{ if } \varepsilon \geq \varepsilon_{ib} \\ \frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \text{ if } \varepsilon \leq \varepsilon_{ib} \end{cases}$$

However, on the interval $[\tau_{\min}, \tau_0]$, since shareholders have paid enough to avoid collusion, the CEO's rents are those of an Independent Board. For those degree of independence, since shareholders have paid enough to avoid collusion, the CEO's rents are those of an Independent Board:

$$
\begin{aligned}
U_{LL} &= w \\
U_{LH} &= w \\
U_{HL} &= w + (1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta \left(\frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} \right) \\
U_{HH} &= \begin{cases} w + \frac{\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2[(1-\xi(\tau))B-\xi(\tau)F]}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \text{ if } \varepsilon \leq \varepsilon_{ib} \\ w + \frac{(\overline{p}+\varepsilon)\Delta\beta\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} \text{ if } \varepsilon \geq \varepsilon_{ib} \end{cases}
\end{aligned}
$$

This is stated in the following Proposition.

Proposition 4 *Assume that collusion between the Board of Directors and the CEO is possible.*

- *In the optimal collusion proof contract, when they hire a Board of Directors, shareholders must concede the same rents to a CEO as in the presence of an Independent Board. In this case, the shareholders' expected profits are*

$$
W_{CP} = \begin{cases} \begin{array}{l} E(\pi) - w - w_0 \\ -(1-\gamma)(1-\xi(\tau))(\underline{p}-\varepsilon)\Delta\beta\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\begin{array}{c}\nu(1-\xi(\tau)) \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\end{array}\right] \end{array} & \textit{if } \varepsilon \leq \varepsilon_{ib} \\ E(\pi) - w - w_0 - (1-\gamma)\Delta\beta\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\begin{array}{c}\nu(1-\xi(\tau))^2(\underline{p}-\varepsilon) \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})(\overline{p}+\varepsilon)\end{array}\right] & \textit{if } \varepsilon \geq \varepsilon_{ib} \end{cases}
$$

- *Moreover, there exists τ_0 such that for Boards of Directors with a degree of independence $\tau \geq \tau_0$, it is not beneficial to engage in collusion.*

The second part of this Proposition means that from some degree of independence for the Board, it is so difficult for the coalition Board-CEO to engage in collusion that they prefer not to collude without any sharholders' intervention. For such Boards, the shareholders should therefore not care about collusion.

5.2 Collusion Free contract

We now characterize the optimal collusion free contract. In this case, shareholders would have to pay too much to avoid collusion. They therefore decide to let it happen because avoiding collusion will be too costly for them in terms of informational rents paid to the Board. The shareholders' expected profits have the following form:

$$\begin{aligned}W_{CF} &= E(\pi) - \gamma\nu U_{LL} - \gamma(1-\nu)U_{LH} - (1-\gamma)\nu U_{HL} - (1-\gamma)(1-\nu)U_{HH} \\ &\quad -\nu\xi(\tau)w_L - (1-\nu)\xi(\tau)w_H - (1-\xi(\tau))w_\emptyset\end{aligned}$$

This is optimal to set $w_L = w_0$. Inequalities (1) and (2) do not need to be satisfied. Subsequently, we have:

$$w_L = w_H = w_\emptyset = w_0$$

Since the Board is collusive, shareholders should not trust what it says for their own good. Therefore, the CEO's rents are those of a No Board case.

$$\begin{aligned}U_{LL} &= w \\ U_{LH} &= w \\ U_{HL} &= w + \frac{B\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]} \\ U_{HH} &= \begin{cases} w + \frac{B\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \text{ if } \varepsilon \leq \varepsilon_{nb} \\ w + \frac{B\Delta\beta(\overline{p}+\varepsilon)}{\underline{\beta}[\Delta p+2\varepsilon]} \text{ if } \varepsilon \geq \varepsilon_{nb} \end{cases}\end{aligned}$$

Proposition 5 *Assume that collusion between the Board of Directors and the CEO is possible. In the optimal collusion free contract, when they hire a Board of Directors, shareholders must concede the same rents to a CEO as without any Board.*

In this case, the shareholders' expected profits are

$$W_{CF} = \begin{cases} E(\pi) - w_0 - w - (1-\gamma)\left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right]\frac{B\Delta\beta(\underline{p}-\varepsilon)}{\underline{\beta}(\Delta p+2\varepsilon)} \; \textit{if } \varepsilon \leq \varepsilon_{nb} \\ E(\pi) - w_0 - w - (1-\gamma)\left[(\overline{p}+\varepsilon) - \nu(\Delta p + 2\varepsilon)\right]\frac{B\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)} \; \textit{if } \varepsilon \geq \varepsilon_{nb} \end{cases}$$

5.3 Optimal Contract with collusion

We will now use the specified form for the probability that a Board with a degree of independence τ has gathered the true information about the economic context for the type of projects considered by the firm, i.e. $\xi(\tau) = \frac{1}{\tau}$.

In order to find the optimal contract in presence of collusion, W_{CB}, we have to compare W_{CP} and W_{CF} and to find which one is the highest depending on τ. Indeed, the shareholders will choose to design the contract (Collusion Proof or Collusion Free) in order to maximize their objective. As $\varepsilon_{ib} \leq \varepsilon_{nb}$ we only have three cases:

1. $\varepsilon \leq \varepsilon_{ib}$
2. $\varepsilon_{ib} \leq \varepsilon \leq \varepsilon_{nb}$
3. $\varepsilon_{nb} \leq \varepsilon$

The following Proposition characterizes the optimal contract when collusion is achievable.

Proposition 6 *For all $\tau \in [\tau_{\min}, \tau_0]$, the optimal contract is the collusion proof contract for all ε.*

This allows us to state that the shareholders's welfare, W_{CB} that depends on τ is, for all $\tau \in [\tau_{\min}, \tau_0]$:

$$W_{CB}(\tau) = max(W_{CP}; W_{CF}) = W_{CP}(\tau)$$

This is an important result as it means that when collusion is achievable and is profitable for the coalition Board/CEO, it is beneficial for the shareholders to offer a contract preventing collusion to emerge. However, this is costly in terms of informational rents.

This result and those of the previous sections allow us to characterize what is the optimal structure of the Board of Directors from the shareholders' perspective.

6 Optimal Structure of the Board

We are now able to find what is the optimal Board's degree of independence τ^* maximizing the piecewise continuous shareholders's welfare $W_{CB}(\tau)$.

We have to take care about corner solutions as $\tau \in [1; \tau_0]$.

In order to simplify the computations, we rewrite the intervals of discontinuity of $W_{CB}(\tau)$ in order to build them with respect to τ. This gives

$$\varepsilon \geq \varepsilon_{ib} = \frac{\overline{p}\underline{\beta}\Delta p - \frac{1}{\tau}\overline{p}\underline{p}\Delta\beta}{\frac{(\tau-1)}{\tau}\Delta\beta\overline{p} + \underline{p}\overline{\beta} - \overline{p}\underline{\beta}} \Leftrightarrow \tau \leq \frac{\frac{\Delta\beta\overline{p}}{[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}]}}{\frac{\Delta\beta\overline{p}}{[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}]} - \frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}} = \widehat{\tau}$$

Hence, when $\widehat{\tau} \geq \tau_0$ or $\left[\frac{\Delta\beta\overline{p}}{[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}]} - \frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right] \leq 0, \Longleftrightarrow \varepsilon \geq \frac{\overline{p}\underline{\beta}\Delta p}{\Delta\beta\overline{p}+[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}]} = \widehat{\varepsilon},$

$$\varepsilon \geq \varepsilon_{ib} \text{ for all } \tau$$

and when $\widehat{\tau} \leq \tau_0$ and $\left[\frac{\Delta\beta\overline{p}}{[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}]} - \frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right] \geq 0, \Longleftrightarrow \varepsilon \leq \frac{\overline{p}\underline{\beta}\Delta p}{\Delta\beta\overline{p}+[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}]} = \widehat{\varepsilon},$

$$\begin{aligned} \varepsilon &\geq \varepsilon_{ib} \text{ for } \tau \leq \widehat{\tau}, \text{ and} \\ \varepsilon &\leq \varepsilon_{ib} \text{ for } \tau \geq \widehat{\tau} \end{aligned}$$

The shareholders thus have the following objective function[5]:

When $\varepsilon \leq \widehat{\varepsilon}$ and $\widehat{\tau} \leq \tau_0$

$$W_{CB}(\tau) = \begin{cases} \begin{array}{c} E(\pi) - w - w_0 \\ -(1-\gamma)\Delta\beta \frac{\left[B - \frac{1}{\tau - 1}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\begin{array}{c} \nu(\frac{\tau-1}{\tau})^2(\underline{p} - \varepsilon) \\ +(1 - \nu + \frac{\nu}{\tau^2})(\overline{p} + \varepsilon) \end{array} \right] \end{array} & \text{if } \tau \leq \widehat{\tau} \\ \begin{array}{c} E(\pi) - w - w_0 \\ -(1-\gamma)(\frac{\tau-1}{\tau})(\underline{p} - \varepsilon)\Delta\beta \frac{\left[B - \frac{1}{\tau - 1}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\begin{array}{c} \nu(\frac{\tau-1}{\tau}) \\ +(1 - \nu + \frac{\nu}{\tau^2})\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta} - \underline{\beta}\overline{p}} \end{array} \right] \end{array} & \text{if } \widehat{\tau} \leq \tau \leq \tau_0 \\ \begin{array}{c} E(\pi) - w - w_0 \\ -(1-\gamma)(\frac{\tau-1}{\tau})(\underline{p} - \varepsilon)\Delta\beta \frac{\left[B - \frac{1}{\tau - 1}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta} - \underline{\beta}\overline{p}} \right] \end{array} & \text{if } \tau \geq \tau_0 \end{cases}$$

When $\varepsilon \geq \widehat{\varepsilon}$, or $\widehat{\tau} \geq \tau_0$

$$W_{CB}(\tau) = \begin{cases} \begin{array}{c} E(\pi) - w - w_0 \\ -(1-\gamma)\Delta\beta \frac{\left[B - \frac{1}{\tau - 1}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\begin{array}{c} \nu(\frac{\tau-1}{\tau})^2(\underline{p} - \varepsilon) \\ +(1 - \nu + \frac{\nu}{\tau^2})(\overline{p} + \varepsilon) \end{array} \right] \end{array} & \text{if } \tau \leq \tau_0 \\ \begin{array}{c} E(\pi) - w - w_0 \\ -(1-\gamma)\Delta\beta \frac{\left[B - \frac{1}{\tau - 1}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\nu(\frac{\tau-1}{\tau})(\underline{p} - \varepsilon) + (1-\nu)(\overline{p} + \varepsilon) \right] \end{array} & \text{if } \tau \geq \tau_0 \end{cases}$$

Recall that shareholders set the penalty F as high as possible, i.e. such that $\left[B - \frac{1}{\tau-1}F\right] = K$. The following Proposition summarizes our results:

Proposition 7 *When $\varepsilon \leq \widehat{\varepsilon}$, and $\tau_0 \geq \widehat{\tau}$, it is optimal for the shareholders to select a Board of Directors with a low degree of independence, i.e. $\tau^* = \widehat{\tau}$ and to offer contracts avoiding collusion between the Board and the CEO.*

In all other cases, it is optimal for the shareholders to select a Board of Directors with a high degree of independence, i.e. $\tau^* = \tau_0$. In this case, the shareholders should not care about collusion because collusion is not profitable for such Boards.

Contrary to the usual idea that the optimal Board should be independent, we find that shareholders may prefer to select a Board of Directors with a low degree of independence. However, the result is not due, as in Adams and Ferreira (2007), to the fact that the CEO is more prone to reveal information to a "friendly" Board.

Here, there is a trade-off between the information that shareholders may extract from the Board and the costs from both extracting it and avoiding collusion. Remind that the degree of

[5] As $\widehat{\tau} \leq \tau_0$ $\forall \varepsilon$

independence of the Board, τ, can also be interpreted as the degree of toughness and enforceability of the laws against collusion. Higher is τ, more difficult it is for the coalition Board-CEO to engage in collusion, but less they have information about the projects.

In other words, the optimal structure is a Board with a low degree of independence when:

- the risk of both projects are close, i.e. the projects among which the firm has to choose have similar level of risks, and
- the degree of independence necessary to have a perfectly honest Board is too high, i.e. the loss of information about the projects that would be associated to the choice of a perfectly honest Board would be too important.

The intuition for this result is the following. Shareholders should not care about hiring an independent Board when it is too costly to do so and when potential collusion between the CEO and the Board has not a big impact on the firm's decision which is the case when the projects are similar in terms of risk and the degree of independence necessary to have a perfectly honest Board is too high. Indeed, collusion allows the CEO to undertake projects with a level of risk that is higher than what would be optimal for shareholders. This means that closer are the risk of projects, lower are the costs of collusion. As hiring an more independent board leads to extract less information and as it would be too costly to choose a perfectly honest board (because τ_0 is high), choosing a board with a low degree of independence is therefore optimal.

However, in all other cases, i.e. when project 2 is much more risky than project 1 or when he degree of independence necessary to have a perfectly honest Board is low enough, it is optimal for shareholders to choose a Board with a high degree of independence. The optimal structure is therefore a perfectly hosnest Board and the shareholders should not care about collusion because collusion is not profitable for such Boards.

7 Conclusion

In this paper we shed light on the effect of collusion between a board of directors and a CEO. To the best of our knowledge this is the first paper to study formally collusion in this context.

In our paper, we have shown that when there is no board of directors (or equivalently the case of no CEO's monitoring by the directors) the variable part of the wage is higher for a high ability CEO than for a low ability CEO. When we assume that shareholders can recruit a Board of Directors in order to monitor the CEO but that collusion cannot emerge, the Board behaves as a perfectly honest Board. Allowing for the possibility of collusion between the board and the CEO, we show that the optimal contract is collusion proof: it is optimal for the shareholders to offer a contract preventing collusion to emerge. We also prove that there exists a degree of independence of the Board above which it is not profitable for the coalition Board-CEO to engage in collusion.

Finally, we also derive the optimal degree of independence of the Board. Contrary to the usual idea that an optimal Board should be independent, we find that shareholders may prefer to select a Board of Directors with a low degree of independence.

8 Appendix

Proof of Proposition 1. When they do not hire a Board of Directors, shareholders maximize their expected profits under the usual Participation and Incentive constraints. PC_{ij} is the Participation constraint of a CEO with ability $i \in \{H, L\}$ when the project is of type $j \in \{H, L\}$. The Participation constraints ensure that the CEO will earn at least her reservation wage w. $IC_{ij \to kl}$ is the Incentive constraint of a CEO who reveals that her ability is $k \in \{H, L\}$ and the project is of type $l \in \{H, L\}$ while her true ability is i and the true type of the project is j. The Incentives constraints ensure that the CEO earns a higher wage revealing the truth than lying to the shareholders. Through this process, shareholders induce the CEO to reveal his real type. As usual in this kind of problem, the binding constraints are :

$$\alpha_{LL} + \mu_{LL} \left[\underline{p}\underline{\beta}R - I\right] = w \tag{PC_{LL}}$$

$$\alpha_{LH} + \mu_{LH} \left[(\overline{p} + \varepsilon)\,\underline{\beta}R - I\right] = w + \mu_{LL}R\underline{\beta}\Delta p$$

$$\alpha_{HL} + \mu_{HL} \left[\underline{p}\overline{\beta}R - I\right] = w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta} + \overline{\beta}\right)\right] + B$$

$$\alpha_{HH} + \mu_{HH} \left[(\overline{p} + \varepsilon)\,\overline{\beta}R - I\right] = w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta} + \overline{\beta}\right)\right] + \mu_{HL}R\overline{\beta}\Delta p + B$$

In order to minimize the CEO's informational rents, shareholders set μ_{LL}, μ_{LH} and μ_{HL} as low as possible while satisfying the other Incentive constraints. We now check what are the conditions due to the other Incentive constraints (and will check later that Participation constraints are satisfied). There is no constraint on μ_{LL}, we can therefore set:

$$\mu_{LL} = 0$$

$$\alpha_{LL} + \mu_{LL} \left[\underline{p}\underline{\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH} \left[\left(\underline{p} - \varepsilon\right)\underline{\beta}R - I\right] + B = \tag{$IC_{LL \to LH}$}$$

$$\alpha_{LH} + \mu_{LH} \left[(\overline{p} + \varepsilon)\,\underline{\beta}R - I\right] - \mu_{LH}R\underline{\beta}\left[(\overline{p} + \varepsilon) - \left(\underline{p} - \varepsilon\right)\right] + B$$

$$\Leftrightarrow \mu_{LH} \geq \frac{B}{R\underline{\beta}\left[\Delta p + 2\varepsilon\right]}$$

and then

$$\mu_{LH} = \frac{B}{R\underline{\beta}\left[\Delta p + 2\varepsilon\right]}$$

$$\begin{aligned} \alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] &\geq \alpha_{LL} + \mu_{LL}\left[\overline{p}\overline{\beta}R - I\right] = w + \mu_{LL}R\left[\overline{p}\overline{\beta} - \underline{p\beta}\right] \qquad (IC_{HH\to LL}) \\ \Leftrightarrow w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + \mu_{HL}R\overline{\beta}\Delta p + B &\geq w + \mu_{LL}R\left[\overline{p}\overline{\beta} - \underline{p\beta}\right] \\ \Leftrightarrow \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + \mu_{HL}R\overline{\beta}\Delta p + B &\geq 0 \end{aligned}$$

which is satisfied, as $\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] \geq 0$.

$$\begin{aligned} \alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] &\geq \alpha_{LL} + \mu_{LL}\left[\underline{p}\overline{\beta}R - I\right] = w + \mu_{LL}R\underline{p}\Delta\beta \qquad (IC_{HL\to LL}) \\ \Leftrightarrow w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B &\geq w + \mu_{LL}R\underline{p}\Delta\beta \\ \Leftrightarrow \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B &\geq 0 \end{aligned}$$

As $\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] \geq 0$, $(IC_{HL\to LL})$ is not binding.

$$\begin{aligned} \alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\underline{\beta}R - I\right] &\geq \alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\underline{\beta}R - I\right] \qquad (IC_{LH\to HH}) \\ &= \alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] - \mu_{HH}R(\overline{p}+\varepsilon)\Delta\beta \\ \Leftrightarrow w + \mu_{LL}R\underline{\beta}\Delta p &\geq w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] \\ &\quad + \mu_{HL}R\overline{\beta}\Delta p + B - \mu_{HH}R(\overline{p}+\varepsilon)\Delta\beta \\ \Leftrightarrow \mu_{HH} &\geq \frac{B}{R\underline{\beta}\left[\Delta p + 2\varepsilon\right]} = \mu_{LH}. \end{aligned}$$

This is satisfied from the Spence Mirlees condition.

$$\alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\underline{\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\overline{p}\underline{\beta}R - I\right] = \alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] - \mu_{HL}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right] \qquad (IC_{LH\to HL})$$

$$\begin{aligned} \Leftrightarrow w + \mu_{LL}R\underline{\beta}\Delta p &\geq w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] - \mu_{HL}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right] + B \\ \Leftrightarrow \mu_{HL} &\geq \frac{B\left[\underline{p} - \varepsilon\right]\Delta\beta}{R\underline{\beta}\left[\Delta p + 2\varepsilon\right]\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right]} = \mu_{HL}^{1} \end{aligned}$$

$$\alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] = w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R(\overline{p}+\varepsilon)\Delta\beta \qquad (IC_{HH\to LH})$$

$$\begin{aligned} \Leftrightarrow w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + \mu_{HL}R\overline{\beta}\Delta p + B &\geq w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R(\overline{p}+\varepsilon)\Delta\beta \\ \Leftrightarrow \mu_{HL} &\geq \frac{B\Delta\beta}{R\Delta p\overline{\beta}\underline{\beta}} = \mu_{HL}^{2} \end{aligned}$$

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq\alpha_{HL}+\mu_{HL}\left[\underline{p\beta}R-I\right]=\alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right]-\mu_{HL}R\underline{p}\Delta\beta \tag{$IC_{LL\to HL}$}$$

$$\Leftrightarrow w\geq w+\mu_{LL}R\underline{\beta}\Delta p+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]-\mu_{HL}R\underline{p}\Delta\beta+B$$

$$\mu_{HL}\geq\frac{B\left(\underline{p}-\varepsilon\right)}{R\underline{p\beta}\left[\Delta p+2\varepsilon\right]}$$

This is always verified as $\frac{B\left(\underline{p}-\varepsilon\right)}{R\underline{\beta}\left[\Delta p+2\varepsilon\right]\underline{p}}\leq\mu_{LH}$ and due to the Spence Mirlees condition.

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq\alpha_{LH}+\mu_{LH}\left[\left(\underline{p}-\varepsilon\right)\underline{\beta}R-I\right]+B= \tag{$IC_{LL\to LH}$}$$

$$\alpha_{LH}+\mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R-I\right]-\mu_{LH}R\underline{\beta}\left[\left(\overline{p}+\varepsilon\right)-\left(\underline{p}-\varepsilon\right)\right]+B$$

$$\Leftrightarrow w\geq w+\mu_{LL}R\underline{\beta}\Delta p-\mu_{LH}R\underline{\beta}\left[\Delta p+2\varepsilon\right]+B$$

$(IC_{LL\to LH})$ is thus not binding.

$$\alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right]\geq\alpha_{HH}+\mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\overline{\beta}R-I\right]+B \tag{$IC_{HL\to HH}$}$$

$$=w+\mu_{LL}R\underline{\beta}\Delta p+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+\mu_{HL}R\overline{\beta}\Delta p-\mu_{HH}R\overline{\beta}\left[\Delta p+2\varepsilon\right]+2B$$

$$\Leftrightarrow\mu_{HH}\geq\frac{B\left[\underline{p}-\varepsilon\right]\Delta\beta}{R\underline{\beta}\left[\Delta p+2\varepsilon\right]^2\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]}\Delta p+\frac{B}{R\overline{\beta}\left[\Delta p+2\varepsilon\right]}=\mu_{HH}^1$$

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq\alpha_{HH}+\mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\underline{\beta}R-I\right]+B= \tag{$IC_{LL\to HH}$}$$

$$\alpha_{HH}+\mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R-I\right]-\mu_{HH}R\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}-\left(\underline{p}-\varepsilon\right)\underline{\beta}\right]+B$$

$$\Leftrightarrow\mu_{HH}\geq\begin{array}{c}\mu_{LH}\frac{\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]}{\left[\left(\overline{p}\overline{\beta}-\underline{p\beta}\right)+\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]}\\+\mu_{HL}\frac{\overline{\beta}\Delta p}{\left[\left(\overline{p}\overline{\beta}-\underline{p\beta}\right)+\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]}\\+\frac{2B}{R\left[\left(\overline{p}\overline{\beta}-\underline{p\beta}\right)+\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]}\end{array}=\mu_{HH}^2$$

We therefore have:

$$\begin{aligned}\mu_{LL}&=0\\\mu_{LH}&=\frac{B}{R\underline{\beta}\left[\Delta p+2\varepsilon\right]}\\\mu_{HL}&=\max\left\{\mu_{LH};\mu_{HL}^1;\mu_{HL}^2\right\}\\\mu_{HH}&\geq\max\left\{\mu_{HL};\mu_{HH}^1;\mu_{HH}^2\right\}\end{aligned}$$

We now have to show that $\mu_{HL}=\begin{cases}\mu_{HL}^1\text{ if }\varepsilon\leq\frac{\underline{\beta}\Delta p}{\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}}=\varepsilon_{nb}\\\mu_{HL}^2\text{ if }\varepsilon\geq\frac{\underline{\beta}\Delta p}{\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}}=\varepsilon_{nb}\end{cases}$

We only have six cases:

1. $\mu_{LH} \leq \mu_{HL}^1 \leq \mu_{HL}^2 \Longleftrightarrow \mu_{HL} = \mu_{HL}^2$ if $\varepsilon\Delta\beta \leq \Delta p\underline{\beta} \leq \varepsilon\left[\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}\right]$. Indeed, we have :

$$\begin{aligned}
\mu_{LH} \leq \mu_{HL}^1 \leq \mu_{HL}^2 \quad &\Longleftrightarrow \quad \frac{1}{\Delta p + 2\varepsilon} \leq \frac{\left(\underline{p} - \varepsilon\right)\Delta\beta}{\left(\Delta p + 2\varepsilon\right)\left(\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right)} \leq \frac{\Delta\beta}{\Delta p\overline{\beta}} \\
&\Longleftrightarrow \quad \begin{cases} \underline{p}\overline{\beta} - \overline{p}\underline{\beta} \leq \left(\underline{p} - \varepsilon\right)\Delta\beta \\ \left(\underline{p} - \varepsilon\right)\Delta p\overline{\beta} \leq \left(\Delta p + 2\varepsilon\right)\left(\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right) \end{cases} \\
&\Longleftrightarrow \quad \begin{cases} \varepsilon\Delta\beta \leq \Delta p\underline{\beta} \\ \Delta p\underline{\beta} \leq \varepsilon\left(\Delta\beta + \overline{\beta}\frac{\underline{p}}{\overline{p}} - \underline{\beta}\right) \end{cases}
\end{aligned}$$

For the following cases (2, 3 and 4), we use the same inequalities to obtain.

2. $\mu_{HL}^1 \leq \mu_{LH} \leq \mu_{HL}^2 \Longleftrightarrow \mu_{HL} = \mu_{HL}^2$ if $\Delta p\underline{\beta} \leq \varepsilon\Delta\beta$

3. $\mu_{LH} \leq \mu_{HL}^2 \leq \mu_{HL}^1 \Longleftrightarrow \mu_{HL} = \mu_{HL}^1$ if $\varepsilon\left[\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}\right] \leq \Delta p\underline{\beta} \leq 2\varepsilon\Delta\beta$

4. $\mu_{HL}^2 \leq \mu_{LH} \leq \mu_{HL}^1 \Longleftrightarrow \mu_{HL} = \mu_{HL}^1$ if $\Delta p\underline{\beta} \geq 2\varepsilon\Delta\beta$

5. $\mu_{HL}^2 \leq \mu_{HL}^1 \leq \mu_{LH} \Longleftrightarrow$ impossible. Indeed, we would eventually obtain

$$\varepsilon\Delta\beta \geq \Delta p\underline{\beta} \geq \varepsilon\left(\Delta\beta + \overline{\beta}\frac{\underline{p}}{\overline{p}} - \underline{\beta}\right)$$

which is not possible because the last term is strictly superior to the first one.

6. $\mu_{HL}^1 \leq \mu_{HL}^2 \leq \mu_{LH} \Longleftrightarrow$ impossible

We therefore have the result of the lemma.

And then :

$$U_{LL} = \alpha_{LL} + \mu_{LL}\left[\underline{p}\underline{\beta}R - I\right] = w$$

$$U_{LH} = \alpha_{LH} + \mu_{LH}\left[\left(\overline{p} + \varepsilon\right)\underline{\beta}R - I\right] = w + \mu_{LL}R\underline{\beta}\Delta p = w$$

$$U_{HL} = \alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] = w + \frac{B\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta} + \overline{\beta}\right)\right]}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]} + B$$

$$\Longleftrightarrow U_{HL} = w + \frac{B\left(\underline{p} - \varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]}$$

Moreover, when $\varepsilon \leq \frac{\underline{\beta}\Delta p}{\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}} = \varepsilon_{nb}$

$$U_{HH} = \alpha_{HH} + \mu_{HH}\left[\left(\overline{p} + \varepsilon\right)\overline{\beta}R - I\right] = w + \frac{B\left(\underline{p} - \varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]} + \frac{B\left[\underline{p} - \varepsilon\right]\Delta\beta\overline{\beta}\Delta p}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right]}$$

$$\Longleftrightarrow U_{HH} = w + \frac{B\left(\underline{p} - \varepsilon\right)\overline{p}\left(\Delta\beta\right)^2}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right]}$$

Moreover, when $\varepsilon \geq \frac{\underline{\beta}\Delta p}{\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{nb}$

$$U_{HH} = \alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] = w + \frac{B\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]} + \frac{B\Delta\beta}{\underline{\beta}}$$
$$\Longleftrightarrow U_{HH} = w + \frac{B\Delta\beta\left(\overline{p}+\varepsilon\right)}{\underline{\beta}\left[\Delta p + 2\varepsilon\right]}$$

To sum up, here are the CEO' informational rents when there is No Board:

$$\begin{array}{rcl} U_{LL} & = & w \\ U_{LH} & = & w \\ U_{HL} & = & w + \dfrac{B\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p+2\varepsilon\right]} \\ U_{HH} & = & \left\{\begin{array}{c} w + \frac{B\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \text{ if } \varepsilon \leq \varepsilon_{nb} \\ w + \frac{B\Delta\beta(\overline{p}+\varepsilon)}{\underline{\beta}[\Delta p+2\varepsilon]} \text{ if } \varepsilon \geq \varepsilon_{nb} \end{array}\right. \end{array}$$

We can verify now that we have

$$U_{LL} \leq U_{LH} \leq U_{HL} \leq U_{HH}$$

When $\varepsilon \leq \varepsilon_{nb}$, we need to see if $\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\overline{p}\underline{\beta}} \geq 1$, which is true since $\underline{p}\overline{\beta}-\overline{p}\underline{\beta} = \overline{p}\Delta\beta - \overline{\beta}\Delta p$. Subsequently, we have $U_{HL} \leq U_{HH}$. When $\varepsilon \geq \varepsilon_{nb}$, since $\underline{p}-\varepsilon \leq \overline{p}+\varepsilon$, we also have $U_{HL} \leq U_{HH}$.
Rewriting the shareholders' expected profits depending on those informational rents, when there is no board, we have:

$$W_{NB} = E(\pi) - \gamma\nu U_{LL} - \gamma\left(1-\nu\right)U_{LH} - \left(1-\gamma\right)\nu U_{HL} - \left(1-\gamma\right)\left(1-\nu\right)U_{HH}$$

This gives, for $\varepsilon \leq \frac{\underline{\beta}\Delta p}{\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{nb}$

$$W_{NB} = E(\pi) - w - \frac{\left(1-\gamma\right)B\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{\beta}\left(\Delta p+2\varepsilon\right)}\left[\frac{-\nu\overline{\beta}\Delta p + \overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\overline{p}\underline{\beta}}\right]$$

And for $\varepsilon \geq \frac{\underline{\beta}\Delta p}{\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{nb}$

$$W_{NB} = E(\pi) - w - \left(1-\gamma\right)\left(\overline{p}+\varepsilon-\nu\Delta p - 2\nu\varepsilon\right)\frac{B\Delta\beta}{\underline{\beta}\left(\Delta p+2\varepsilon\right)}$$

■

Proof of Proposition 2. The binding constraints are:

$$\alpha_{LL} + \mu_{LL}\left[\underline{p}\underline{\beta}R - I\right] = w \qquad (PC_{LL})$$

$$\alpha_{LH}+\mu_{LH}\left[\left(\overline{p}+\varepsilon\right)\underline{\beta}R-I\right]=w+\mu_{LL}R\underline{\beta}\Delta p$$

$$\alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right]=(1-\xi(\tau))\left\{w+\mu_{LL}R\underline{\beta}\Delta p+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B\right\}+\xi(\tau)\left(w-F\right)$$

$$\alpha_{HH}+\mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R-I\right]=\begin{array}{c}(1-\xi(\tau))\left\{w+\mu_{LL}R\underline{\beta}\Delta p+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B\right\}\\ +\xi(\tau)\left(w-F\right)+\mu_{HL}R\overline{\beta}\Delta p\end{array}$$

Again, in order to minimize the informational rents, shareholders will set μ_{LL}, μ_{LH} and μ_{HL} as low as possible while satisfying the other incentive constraints. We now check what are the conditions due to the other Incentive constraints (and will check later that Participation constraints are satisfied).

$$\mu_{LL}=0$$

$$\begin{aligned}\alpha_{HH}+\mu_{HH}\left[\left(\overline{p}+\varepsilon\right)\overline{\beta}R-I\right]&\geq\alpha_{LL}+\mu_{LL}\left[\overline{p}\overline{\beta}R-I\right]\\&=w+\mu_{LL}R\left[\overline{p}\overline{\beta}-\underline{p\beta}\right]=w\end{aligned}\qquad(IC_{HH\to LL})$$

$$\alpha_{LL}+\mu_{LL}\left[\underline{p\beta}R-I\right]\geq(1-\xi(\tau))\left\{\alpha_{LH}+\mu_{LH}\left[\left(\underline{p}-\varepsilon\right)\underline{\beta}R-I\right]+B\right\}+\xi(\tau)\left(w-F\right)=\qquad(IC_{LL\to LH})$$

$$(1-\xi(\tau))\left\{\begin{array}{c}w+\mu_{LL}R\underline{\beta}\Delta p\\-\mu_{LH}R\underline{\beta}\left[\left(\overline{p}+\varepsilon\right)-\left(\underline{p}-\varepsilon\right)\right]+B\end{array}\right\}+\xi(\tau)\left(w-F\right)$$

$$\Leftrightarrow\mu_{LH}\geq\frac{B-\frac{\xi(\tau)}{(1-\xi(\tau))}F}{R\underline{\beta}\left[\left(\overline{p}+\varepsilon\right)-\left(\underline{p}-\varepsilon\right)\right]}$$

As $\frac{\xi(\tau)}{(1-\xi(\tau))}F-B\leq 0$, we have

$$\mu_{LH}=\frac{B-\frac{\xi(\tau)}{(1-\xi(\tau))}F}{R\underline{\beta}\left[\left(\overline{p}+\varepsilon\right)-\left(\underline{p}-\varepsilon\right)\right]}$$

$$\alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right]\geq\alpha_{LL}+\mu_{LL}\left[\underline{p}\overline{\beta}R-I\right]=w+\mu_{LL}R\underline{p}\Delta\beta\qquad(IC_{HL\to LL})$$

$$\Leftrightarrow(1-\xi(\tau))\left\{\begin{array}{c}\mu_{LL}R\underline{\beta}\Delta p\\+\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B\end{array}\right\}-\xi(\tau)F\geq\mu_{LL}R\underline{p}\Delta\beta$$

$$\Leftrightarrow\mu_{LH}\geq\frac{\frac{\xi(\tau)}{(1-\xi(\tau))}F-B}{R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]}$$

As $\frac{\xi(\tau)}{(1-\xi(\tau))}F - B \leq 0$, this is satisfied

$$\alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] \geq \alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] = w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R(\overline{p}+\varepsilon)\Delta\beta \tag{$IC_{HH\to LH}$}$$

$$\Leftrightarrow \left\{\begin{array}{c} (1-\xi(\tau))\left\{\begin{array}{c} w + \mu_{LL}R\underline{\beta}\Delta p \\ +\mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B \end{array}\right\} \\ +\xi(\tau)(w-F) + \mu_{HL}R\overline{\beta}\Delta p \end{array}\right\} \geq w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R(\overline{p}+\varepsilon)\Delta\beta$$

$$\mu_{HL} \geq \frac{(\overline{p}+\varepsilon)\Delta\beta - (1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta}{\overline{\beta}\Delta p}\left(\frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{R\underline{\beta}\left[(\overline{p}+\varepsilon) - \left(\underline{p}-\varepsilon\right)\right]}\right)$$

$$\Leftrightarrow \mu_{HL} \geq \mu_{LH}\frac{\left[(\overline{p}+\varepsilon) - (1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\right]\Delta\beta}{\overline{\beta}\Delta p} = \mu^1_{HL}$$

$$\alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\underline{\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\overline{p}\underline{\beta}R - I\right] \tag{$IC_{LH\to HL}$}$$

$$= \alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] - \mu_{HL}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right]$$

$$\Leftrightarrow w + \mu_{LL}R\underline{\beta}\Delta p \geq \left\{\begin{array}{c} (1-\xi(\tau))\left\{\begin{array}{c} w + \mu_{LL}R\underline{\beta}\Delta p \\ +\mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B \end{array}\right\} \\ +\xi(\tau)(w-F) - \mu_{HL}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right] \end{array}\right\}$$

$$\mu_{HL} \geq \frac{(1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta}{\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right]}\mu_{LH} = \mu^2_{HL}$$

We can verify that

$$\mu_{HL} = \left\{\begin{array}{l} \mu^1_{HL} \text{ if } \varepsilon \geq \frac{\underline{\beta}\Delta p - \xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}} = \varepsilon_{ib} \\ \mu^2_{HL} \text{ if } \varepsilon \leq \frac{\underline{\beta}\Delta p - \xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}} = \varepsilon_{ib} \end{array}\right.$$

Indeed, we have :

$$\begin{aligned} \mu_{HL} = \mu^1_{HL} \quad &\Longleftrightarrow \quad \mu^1_{HL} \geq \mu^2_{HL} \\ &\Longleftrightarrow \quad \frac{\left[\Delta p + 2\varepsilon + \xi(\tau)\left(\underline{p}-\varepsilon\right)\right]\Delta\beta}{\overline{\beta}\Delta p} \geq \frac{(1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{p}\overline{\beta} - \overline{p}\underline{\beta}} \\ &\Longleftrightarrow \quad \varepsilon\left[\begin{array}{c} (2-\xi(\tau))\left(\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right) \\ +(1-\xi(\tau))\overline{\beta}\Delta p \end{array}\right] \geq (1-\xi(\tau))\underline{p}\overline{\beta}\Delta p - \left(\Delta p + \xi(\tau)\underline{p}\right)\left(\underline{p}\overline{\beta} - \overline{p}\underline{\beta}\right) \\ &\Longleftrightarrow \quad \varepsilon \geq \frac{\underline{\beta}\Delta p - \xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}} \end{aligned}$$

Moreover, when $\varepsilon \geq \frac{\underline{\beta}\Delta p - \xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta + \frac{\underline{p}}{\overline{p}}\overline{\beta} - \underline{\beta}}$, one can easily check that :

$$\mu_{HL} = \mu^1_{HL} \geq \mu_{LH}$$

and when $\varepsilon \leq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}}$

$$\mu_{HL} = \mu_{HL}^2 \geq \mu_{LH}$$

$$\begin{aligned}
\alpha_{LL} + \mu_{LL}\left[\underline{p}\underline{\beta}R - I\right] \geq \alpha_{HL} + \mu_{HL}\left[\underline{p}\underline{\beta}R - I\right] &= \alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] - \mu_{HL}R\underline{p}\Delta\beta \\
\Leftrightarrow \mu_{HL}R\underline{p}\Delta\beta \geq (1-\xi(\tau))\left\{\mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B\right\} &- \xi(\tau)F \\
\Leftrightarrow \mu_{HL} \geq \frac{(1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{p}\Delta\beta}\left(\frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{R\underline{\beta}\left[(\overline{p}+\varepsilon) - \left(\underline{p}-\varepsilon\right)\right]}\right) \\
\Leftrightarrow \mu_{HL} \geq \frac{(1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{p}\Delta\beta}\mu_{LH}
\end{aligned}$$

Since $(1-\xi(\tau))\,\underline{p}\Delta\beta \leq \underline{p}\Delta\beta$ and since $\mu_{HH} \geq \mu_{LH}$, $IC_{LL\to HL}$ is also satisfied. Finally, we get

$$\begin{aligned}
\mu_{HL} &= \max\left\{\mu_{HL}^1; \mu_{HL}^2\right\} \\
&= \left\{\begin{array}{l} \mu_{HL}^1 \text{ if } \varepsilon \geq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{ib} \\ \mu_{HL}^2 \text{ if } \varepsilon \leq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{ib} \end{array}\right.
\end{aligned}$$

$$\alpha_{LL} + \mu_{LL}\left[\underline{p}\underline{\beta}R - I\right] \geq (1-\xi(\tau))\left\{\alpha_{HH} + \mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\underline{\beta}R - I\right] + B\right\} + \xi(\tau)(w-F) = \qquad (IC_{LL\to HH})$$

$$(1-\xi(\tau))\left\{\alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] - \mu_{HH}R\left[(\overline{p}+\varepsilon)\overline{\beta} - \left(\underline{p}-\varepsilon\right)\underline{\beta}\right] + B\right\} + \xi(\tau)(w-F)$$

$$\Leftrightarrow \mu_{HH} \geq \left\{\begin{array}{c} (1-\xi(\tau))\left\{\mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B\right\} \\ -\xi(\tau)F + \mu_{HL}R\overline{\beta}\Delta p \end{array}\right\}\left(\frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{R\left[(\overline{p}+\varepsilon)\overline{\beta} - \left(\underline{p}-\varepsilon\right)\underline{\beta}\right]}\right) = \mu_{HH}^1$$

$$\alpha_{HL} + \mu_{HL}\left[\underline{p}\overline{\beta}R - I\right] \geq (1-\xi(\tau))\left\{\alpha_{HH} + \mu_{HH}\left[\left(\underline{p}-\varepsilon\right)\overline{\beta}R - I\right] + B\right\} + \xi(\tau)(w-F) \qquad (IC_{HL\to HH})$$

$$= (1-\xi(\tau))\left\{\begin{array}{c} (1-\xi(\tau))\left\{w + \mu_{LL}R\underline{\beta}\Delta p + \mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + B\right\} \\ +\xi(\tau)(w-F) + \mu_{HL}R\overline{\beta}\Delta p - \mu_{HH}R\overline{\beta}\left[\Delta p + 2\varepsilon\right] + B \end{array}\right\} + \xi(\tau)(w-F)$$

$$\Leftrightarrow \mu_{HH} \geq \frac{(1-\xi(\tau))\,\mu_{HL}R\overline{\beta}\Delta p - \xi(\tau)\mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] + (1-\xi(\tau))\,B - \xi(\tau)F}{R\overline{\beta}\left[\Delta p + 2\varepsilon\right]} = \mu_{HH}^2$$

$$\begin{aligned}
\alpha_{LH} + \mu_{LH}\left[(\overline{p}+\varepsilon)\underline{\beta}R - I\right] &\geq \alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\underline{\beta}R - I\right] \qquad (IC_{LH\to HH}) \\
&= \alpha_{HH} + \mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R - I\right] - \mu_{HH}R(\overline{p}+\varepsilon)\Delta\beta
\end{aligned}$$

$$\Leftrightarrow \mu_{HH} \geq \frac{\begin{array}{c}(1-\xi(\tau))\,\mu_{LH}R\left[\underline{p}\overline{\beta} - \overline{p}\underline{\beta} - \varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right] \\ +\mu_{HL}R\overline{\beta}\Delta p\end{array} + (1-\xi(\tau))\,B - \xi(\tau)F}{R(\overline{p}+\varepsilon)\Delta\beta} = \mu_{HH}^3$$

We thus have:

$$
\begin{aligned}
\mu_{LL} &= 0 \\
\mu_{LH} &= \frac{B-\frac{\xi(\tau)}{(1-\xi(\tau))}F}{R\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} \\
\mu_{HL} &= \begin{cases} \mu_{HL}^{1} \text{ if } \varepsilon \geq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{ib} \\ \mu_{HL}^{2} \text{ if } \varepsilon \leq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{ib} \end{cases} \\
\mu_{HH} &\geq \max\left\{\mu_{HL};\mu_{HH}^{1};\mu_{HH}^{2};\mu_{HH}^{3}\right\}
\end{aligned}
$$

$$U_{LL} = \alpha_{LL}+\mu_{LL}\left[\underline{p}\underline{\beta}R-I\right] = w$$

$$U_{LH} = \alpha_{LH}+\mu_{LH}\left[(\overline{p}+\varepsilon)\underline{\beta}R-I\right] = w+\mu_{LL}R\underline{\beta}\Delta p = w$$

$$
\begin{aligned}
U_{HL} &= \alpha_{HL}+\mu_{HL}\left[\underline{p}\overline{\beta}R-I\right] = (1-\xi(\tau))\left\{\begin{array}{c} w+\mu_{LL}R\underline{\beta}\Delta p \\ +\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B \end{array}\right\}+\xi(\tau)(w-F) \\
&= w+(1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta\left(\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]}\right)
\end{aligned}
$$

$$
\begin{aligned}
U_{HH} &= \alpha_{HH}+\mu_{HH}\left[(\overline{p}+\varepsilon)\overline{\beta}R-I\right] = (1-\xi(\tau))\left\{\begin{array}{c} w+\mu_{LL}R\underline{\beta}\Delta p \\ +\mu_{LH}R\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}-\varepsilon\left(\underline{\beta}+\overline{\beta}\right)\right]+B \end{array}\right\} \\
&\quad +\xi(\tau)(w-F)+\mu_{HL}R\overline{\beta}\Delta p \\
&= \begin{cases} w+\frac{\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2[(1-\xi(\tau))B-\xi(\tau)F]}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \text{ if } \varepsilon \leq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{ib} \\ w+\frac{(\overline{p}+\varepsilon)\Delta\beta\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} \text{ if } \varepsilon \geq \frac{\underline{\beta}\Delta p-\xi(\tau)\underline{p}\Delta\beta}{(1-\xi(\tau))\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} = \varepsilon_{ib} \end{cases}
\end{aligned}
$$

To sum up, here are the CEO utilities when there is an Independent Board:

$$
\begin{aligned}
U_{LL} &= w \\
U_{LH} &= w \\
U_{HL} &= w+(1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta\left(\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]}\right) \\
U_{HH} &= \begin{cases} w+\frac{\left(\underline{p}-\varepsilon\right)\overline{p}(\Delta\beta)^2[(1-\xi(\tau))B-\xi(\tau)F]}{\underline{\beta}[\Delta p+2\varepsilon]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \text{ if } \varepsilon \leq \varepsilon_{ib} \\ w+\frac{(\overline{p}+\varepsilon)\Delta\beta\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} \text{ if } \varepsilon \geq \varepsilon_{ib} \end{cases}
\end{aligned}
$$

We can verify now that we have

$$U_{LL} \leq U_{LH} \leq U_{HL} \leq U_{HH}$$

When $\varepsilon \leq \varepsilon_{ib}$, we need to see if $\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\overline{p}\underline{\beta}} \geq 1$, which is true since $\underline{p}\overline{\beta}-\overline{p}\underline{\beta} = \overline{p}\Delta\beta - \overline{\beta}\Delta p$. Subsequently, we have $U_{HL} \leq U_{HH}$. When $\varepsilon \geq \varepsilon_{ib}$, since $(1-\xi(\tau))\left(\underline{p}-\varepsilon\right) \leq \overline{p}+\varepsilon$, we also have $U_{HL} \leq U_{HH}$. One can remark that types (HL) and (HH) informational rents are lower with an Independent Board than without Board.

$$\begin{aligned} U_{HLib} \leq U_{HLnb} &\iff (1-\xi(\tau))\left(\underline{p}-\varepsilon\right)\Delta\beta\left(\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]}\right) \leq \frac{B\left(\underline{p}-\varepsilon\right)\Delta\beta}{\underline{\beta}\left[\Delta p+2\varepsilon\right]} \\ &\iff (1-\xi(\tau))\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right] \leq B \text{ , which is true} \end{aligned}$$

Moreover, we can prove that $\varepsilon_{ib} \leq \varepsilon_{nb}$. Indeed,

$$\begin{gathered} \varepsilon_{ib}-\varepsilon_{nb} \leq 0 \\ \iff \frac{\underline{\beta}\Delta p-\xi\underline{p}\Delta\beta}{(1-\xi)\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} - \frac{\underline{\beta}\Delta p}{\Delta\beta+\frac{\underline{p}}{\overline{p}}\overline{\beta}-\underline{\beta}} \leq 0 \\ \iff 2\xi\Delta\beta\left[\overline{p}\underline{\beta}-\underline{p}\overline{\beta}\right] \leq 0 \end{gathered}$$

which is true since we have $\underline{\beta}\overline{p}-\underline{p}\overline{\beta} \leq 0$.

This implies that we only have three possible cases to consider for U_{HH}

1. When $\varepsilon \leq \varepsilon_{ib}$

$$\begin{aligned} U_{HHib}-U_{HHnb} &= \frac{\left(\underline{p}-\varepsilon\right)\overline{p}\,(\Delta\beta)^2\left[(1-\xi(\tau))\,B-\xi(\tau)F\right]}{\underline{\beta}\left[\Delta p+2\varepsilon\right]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} - \frac{B\left(\underline{p}-\varepsilon\right)\overline{p}\,(\Delta\beta)^2}{\underline{\beta}\left[\Delta p+2\varepsilon\right]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \\ sign(U_{HHib}-U_{HHnb}) &= sign(-\xi(\tau)\,(B+F)\left(\underline{p}-\varepsilon\right)(\Delta\beta)^2) \leq 0 \end{aligned}$$

2. When $\varepsilon_{ib} \leq \varepsilon \leq \varepsilon_{nb}$

$$\begin{aligned} U_{HHib}-U_{HHnb} &= \frac{(\overline{p}+\varepsilon)\,\Delta\beta\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} - \frac{B\left(\underline{p}-\varepsilon\right)\overline{p}\,(\Delta\beta)^2}{\underline{\beta}\left[\Delta p+2\varepsilon\right]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]} \\ sign(U_{HHib}-U_{HHnb}) &= sign\left[(\overline{p}+\varepsilon)\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]\left[\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right]-B\left(\underline{p}-\varepsilon\right)\overline{p}\Delta\beta\right] \end{aligned}$$

Since $B \geq \left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]$ we need to prove that $\left(\underline{p}-\varepsilon\right)\overline{p}\Delta\beta \geq (\overline{p}+\varepsilon)\left(\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right)$

$$\begin{aligned} \left(\underline{p}-\varepsilon\right)\overline{p}\Delta\beta-(\overline{p}+\varepsilon)\left(\underline{p}\overline{\beta}-\overline{p}\underline{\beta}\right) &= \underline{p}\overline{p}\Delta\beta-\overline{p}\underline{p}\overline{\beta}+\overline{p}\overline{p}\underline{\beta}-\varepsilon\left(\overline{p}\Delta\beta-\overline{\beta}\underline{p}+\overline{p}\underline{\beta}\right) \\ &= \overline{p}\left[\underline{\beta}\Delta p-\varepsilon\left(\Delta\beta-\frac{\underline{p}}{\overline{p}}\overline{\beta}+\underline{\beta}\right)\right] \end{aligned}$$

Since $\varepsilon \leq \varepsilon_{nb}$, we have $\underline{\beta}\Delta p-\varepsilon\left(\Delta\beta-\frac{\underline{p}}{\overline{p}}\overline{\beta}+\underline{\beta}\right) \geq 0$

3. When $\varepsilon_{ib} \leq \varepsilon_{nb} \leq \varepsilon$

$$U_{HHib}-U_{HHnb} = \frac{(\overline{p}+\varepsilon)\,\Delta\beta\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}\left[(\overline{p}+\varepsilon)-\left(\underline{p}-\varepsilon\right)\right]} - \frac{B\Delta\beta\,(\overline{p}+\varepsilon)}{\underline{\beta}\left[\Delta p+2\varepsilon\right]}$$

Since $B \geq \left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]$ we have $U_{HHib} \leq U_{HHnb}$.

We can now calculate the income of the shareholders. There are two cases to consider. When $\varepsilon \leq \varepsilon_{ib}$,

$$W_{IB} = E(\pi) - w - w_0 - (1-\gamma)(1-\xi(\tau))(\underline{p}-\varepsilon)\Delta\beta \frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta} - \underline{\beta}\overline{p}}\right]$$

When $\varepsilon \geq \varepsilon_{ib}$,

$$W_{IB} = E(\pi) - w - w_0 - (1-\gamma)\Delta\beta \frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\nu(1-\xi(\tau))(\underline{p}-\varepsilon) + (1-\nu)(\overline{p}+\varepsilon)\right]$$

We have to find for which values of τ, the contract is collusion proof.

1. $\varepsilon \leq \varepsilon_{ib}$

$$\begin{aligned} W_{CP} - W_{CF} &= -(1-\gamma)(1-\xi(\tau))(\underline{p}-\varepsilon)\Delta\beta \frac{\left[B - \frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p + 2\varepsilon)} \left[\begin{array}{c} \nu(1-\xi(\tau)) \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \end{array}\right] \\ &\quad +(1-\gamma)\left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta} - \underline{\beta}\overline{p}}\right] \frac{B\Delta\beta(\underline{p}-\varepsilon)}{\underline{\beta}(\Delta p + 2\varepsilon)} \geq 0 \text{ for all } \tau \in [\tau_{\min}, \tau_0] \end{aligned}$$

with $\tau_0 = \frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}$ for $\varepsilon \leq \varepsilon_{ib}$. Indeed,

$$W_{CP} - W_{CF} = \frac{(1-\gamma)(\underline{p}-\varepsilon)\Delta\beta}{\underline{\beta}(\Delta p + 2\varepsilon)} \left(\begin{array}{c} \left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right]B \\ -\left[(1-\xi(\tau))B - \xi(\tau)F\right]\left[\begin{array}{c} \nu(1-\xi(\tau)) \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \end{array}\right] \end{array}\right)$$

As, we have $\xi(\tau)^{ED}(\tau) = \frac{1}{\tau}$, this gives

$$W_{CP} - W_{CF} = \frac{(1-\gamma)(\underline{p}-\varepsilon)\Delta\beta}{\underline{\beta}(\Delta p + 2\varepsilon)} \left(\begin{array}{c} \left[\left(\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F) + \nu B\right]\tau^2 \\ -\nu\left[B + F + B\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right]\tau \\ +\nu(B+F)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \end{array}\right)$$

This polynomial in τ with a positive second degree term has 2 positive roots. If those roots are both lower than τ_0, then, $W_{CP} - W_{CF} \geq 0$ for all $\tau \in [\tau_{\min}, \tau_0]$. The lowest root is

$$\tau_1 = \frac{\nu\left[B + F + B\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right] - \sqrt{\begin{array}{c} \nu^2\left[B + F + B\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right]^2 \\ -4\nu(B+F)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\left[\left(\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F) + \nu B\right] \end{array}}}{2\left[\left(\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F) + \nu B\right]}$$

We have

$$\tau_1 \geq \tau_0$$

$$\iff \left(\begin{array}{c} 4\left(\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)^2 \left[\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F)+\nu B\right]^2 \\ -4\nu\left(B+F+B\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\left[\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F)+\nu B\right] \\ -4\nu(B+F)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\left[\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F)+\nu B\right] \end{array} \right) \geq 0$$

$$\iff 4\left(\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)^2 \left[\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F)+\nu B\right]\left[\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)(B+F)\right] \geq 0$$

which is true. $W_{CP}-W_{CF}$ is therefore positive for all $\tau \in [\tau_{\min}, \tau_0]$. The optimal contract is the collusion proof contract for $\varepsilon \leq \varepsilon_{ib}$.

2. $\varepsilon_{ib} \leq \varepsilon \leq \varepsilon_{nb}$

$$\begin{aligned} W_{CP}-W_{CF} &= -(1-\gamma)\Delta\beta\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\nu(1-\xi(\tau))^2(\underline{p}-\varepsilon)+(1-\nu+\frac{\xi(\tau)\nu}{\tau})(\overline{p}+\varepsilon)\right] \\ &\quad +(1-\gamma)\left[\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right]\frac{B\Delta\beta(\underline{p}-\varepsilon)}{\underline{\beta}(\Delta p+2\varepsilon)} \geq 0 \text{ for all } \tau \in [\tau_{\min}, \tau_0] \end{aligned}$$

with $\tau_0 = \frac{\overline{p}+\varepsilon}{\underline{p}-\varepsilon}+1$ if $\varepsilon \geq \varepsilon_{ib}$. Indeed,

$$W_{CP}-W_{CF} = \frac{(1-\gamma)\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)}\left(\begin{array}{c} \left[\begin{array}{c}\nu \\ +(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\end{array}\right]B(\underline{p}-\varepsilon) \\ +\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]\left[\begin{array}{c}\nu(1-\xi(\tau))^2(\underline{p}-\varepsilon) \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})(\overline{p}+\varepsilon)\end{array}\right] \end{array}\right)$$

As, we have $\xi^{ED}(\tau)=\frac{1}{\tau}$, this gives

$$W_{CP}-W_{CF} = \frac{(1-\gamma)\Delta\beta(\underline{p}-\varepsilon)}{\underline{\beta}(\Delta p+2\varepsilon)(\tau-1)}\left(\begin{array}{c} \left[(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right]B\tau^3 \\ \left[\begin{array}{c}-\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)B+2\nu B \\ +\left(\nu+(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)(B+F)\end{array}\right]\tau^2 \\ -\nu\left[B\left(1+\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)+2\nu(B+F)\right]\tau \\ +\nu(B+F)\left(1+\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right) \end{array}\right)$$

We are now able to show that this degree 3 polynomial, denote it $P(\tau)$, is negative for all $\tau \in$

$[\tau_{\min},\tau_0]$. Indeed

$$\frac{\partial P(\tau)}{\partial\tau}=\left(\begin{array}{c}3\tau^2B(1-\nu)\left[\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right]\\ +2\tau\left[-\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)B+2\nu B+\left(\nu+(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)(B+F)\right]\\ -\left[\nu B\left(1+\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)+2\nu\left(B+F\right)\right]\end{array}\right)$$

Moreover, as

$$\varepsilon\geq\varepsilon_{ib}\Longleftrightarrow\frac{\tau\left(\underline{\beta}\overline{p}(\tau_0-2)\right)-\varepsilon\overline{\beta}\tau_0}{\overline{p}\Delta\beta}\leq 1$$

we have

$$\begin{aligned}&\tau^2B(1-\nu)\left[\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right]\\ =\;&\tau B(1-\nu)\left(\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)\left(\frac{\tau\left(\underline{\beta}\overline{p}(\tau_0-2)\right)-\varepsilon\overline{\beta}\tau_0}{\overline{p}\Delta\beta}\right)\leq\tau B(1-\nu)\left(\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)\end{aligned}$$

and thus

$$\begin{aligned}\frac{\partial P(\tau)}{\partial\tau}\;&\leq\;\left(\begin{array}{c}3\tau B(1-\nu)\left(\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)\\ +2\tau\left[-\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)B+2\nu B+(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}(\tau_0-1)(B+F)\right]\\ -\left[\nu B\tau_0+2\nu\left(B+F\right)\right]\end{array}\right)\leq 0\\ &\Longleftrightarrow\;\tau\leq\frac{\nu B\tau_0+2\nu\left(B+F\right)}{\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)B+\nu B+\left(\nu+(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)(B+F)}\end{aligned}$$

Moreover,

$$\frac{\nu B\tau_0+2\nu\left(B+F\right)}{\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)B+\nu B+\left(\nu+(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)(B+F)}\leq\tau_0$$

Hence, $\frac{\partial P(\tau)}{\partial\tau}$ is negative for all $\tau\in[\tau_{\min},\tau_0]$. Finally, we will show that $(W_{CP}-W_{CF})(\tau_0)\geq 0$

$$\begin{gathered}\left(W_{CP}-W_{CF}\right)(\tau_0)\geq 0\Longleftrightarrow\\ \frac{(1-\gamma)\Delta\beta(\underline{p}-\varepsilon)}{\underline{\beta}(\Delta p+2\varepsilon)\tau_0}\left(\begin{array}{c}\left[(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right]B\tau_0^3\\ \left[\begin{array}{c}-\left(\nu+(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right)B+2\nu B\\ +\left(\nu+(1-\nu)\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)(B+F)\end{array}\right]\tau_0^2\\ -\left[\nu B\tau_0+2\nu\left(B+F\right)\right]\tau_0+\nu\left(B+F\right)\tau_0\end{array}\right)\geq 0\\ \Longleftrightarrow\left(\begin{array}{c}B(1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\tau_0\left(\tau_0-1\right)\left[\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right]\\ +(1-\nu)\tau_0\left(\tau_0-1\right)F+\nu\left(\tau_0-1\right)(B+F)\end{array}\right)\geq 0\end{gathered}$$

However, as

$$
\begin{aligned}
\varepsilon &\leq \varepsilon_{nb} \Longleftrightarrow \\
\frac{\overline{\beta}\Delta p}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}(\underline{p}-\varepsilon) &\geq \frac{\overline{p}\Delta\beta\underline{p}+\left[\underline{p}\overline{\beta}-\underline{\beta}\overline{p}\right]\underline{p}-\underline{p}\overline{\beta}\Delta p}{\left[\underline{p}\overline{\beta}-\underline{\beta}\overline{p}\right]}-(\underline{p}-\varepsilon)
\end{aligned}
$$

we have, together with $\overline{p}\Delta\beta \geq \overline{\beta}\Delta p$

$$
\begin{aligned}
\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)} &\geq \frac{\overline{\beta}\Delta p}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)} \\
&\geq \frac{\overline{p}\Delta\beta\underline{p}+\left[\underline{p}\overline{\beta}-\underline{\beta}\overline{p}\right]\underline{p}-\underline{p}\overline{\beta}\Delta p}{\left[\underline{p}\overline{\beta}-\underline{\beta}\overline{p}\right](\underline{p}-\varepsilon)}-1-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)} \geq 0
\end{aligned}
$$

As $\left[\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right] \geq 0$, $(W_{CP}-W_{CF})(\tau_0)$ is thus positive and as $\frac{\partial P(\tau)}{\partial\tau}$ is negative for all $\tau \in [\tau_{\min},\tau_0]$, $W_{CP}-W_{CF}$ is therefore positive for all $\tau \in [\tau_{\min},\tau_0]$. The optimal contract is the collusion proof contract for $\varepsilon_{ib} \leq \varepsilon \leq \varepsilon_{nb}$.

3. $\varepsilon_{ib} \leq \varepsilon_{nb} \leq \varepsilon$

$$
\begin{aligned}
W_{CP}-W_{CF} &= -(1-\gamma)\Delta\beta\frac{\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]}{\underline{\beta}(\Delta p+2\varepsilon)}\left[\begin{array}{c}\nu(1-\xi(\tau))^2(\underline{p}-\varepsilon) \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})(\overline{p}+\varepsilon)\end{array}\right] \\
&\quad +(1-\gamma)\left[(\overline{p}+\varepsilon)-\nu(\Delta p+2\varepsilon)\right]\frac{B\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)} \geq 0 \text{ for all } \tau \in [\tau_{\min},\tau_0]
\end{aligned}
$$

with $\tau_0 = \frac{\overline{p}+\varepsilon}{\underline{p}-\varepsilon}+1$ if $\varepsilon \geq \varepsilon_{ib}$. Indeed,

$$
W_{CP}-W_{CF} = \frac{(1-\gamma)(\underline{p}-\varepsilon)\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)}\left(\begin{array}{c}\left[\nu+(1-\nu)(\tau_0-1)\right]B \\ -\left[B-\frac{\xi(\tau)}{(1-\xi(\tau))}F\right]\left[\begin{array}{c}\nu(1-\xi(\tau))^2 \\ +(1-\nu+\frac{\xi(\tau)\nu}{\tau})(\tau_0-1)\end{array}\right]\end{array}\right)
$$

As, we have $\xi^{ED}(\tau)=\frac{1}{\tau}$, this gives

$$
W_{CP}-W_{CF} = \frac{(1-\gamma)(\underline{p}-\varepsilon)\Delta\beta}{\underline{\beta}(\Delta p+2\varepsilon)(\tau-1)}\left(\begin{array}{c}\left[\nu(2B+F)+(1-\nu)(\tau_0-1)F\right]\tau^2 \\ -\nu\left[B\tau_0+2(B+F)\right]\tau \\ +\nu(B+F)\tau_0\end{array}\right)
$$

This polynomial in τ with a positive second degree term has 2 positive roots. If those roots are both lower than τ_0, then, $W_{CP}-W_{CF} \geq 0$ for all $\tau \in [\tau_{\min},\tau_0]$. The lowest root is

$$
\tau_2 = \frac{\nu\left[B\tau_0+2(B+F)\right]-\sqrt{\begin{array}{c}\nu^2\left[B\tau_0+2(B+F)\right]^2 \\ -4\nu(B+F)\tau_0\left[\nu(2B+F)+(1-\nu)(\tau_0-1)F\right]\end{array}}}{2\left[\nu(2B+F)+(1-\nu)(\tau_0-1)F\right]}
$$

We have

$$\tau_2 \geq \tau_0$$

$$\Longleftrightarrow 4\tau_0^2 \begin{bmatrix} \nu(2B+F) \\ +(1-\nu)(\tau_0-1)F \end{bmatrix}^2 - 4\nu\tau_0 [B\tau_0 + (B+F)] \begin{bmatrix} \nu(2B+F) \\ +(1-\nu)(\tau_0-1)F \end{bmatrix} \geq 0$$

$$\Longleftrightarrow 4\tau_0(\tau_0-1)[\nu(2B+F)+(1-\nu)(\tau_0-1)F][\nu(B+F)+(1-\nu)\tau_0 F] \geq 0$$

which is true. $W_{CP} - W_{CF}$ is therefore positive for all $\tau \in [\tau_{\min}, \tau_0]$. The optimal contract is the collusion proof contract for $\varepsilon_{ib} \leq \varepsilon_{nb} \leq \varepsilon$.

Proof of Proposition 6. When $\varepsilon \leq \widehat{\varepsilon}$ and $\widehat{\tau} \leq \tau_0$, we have

$$W_{CB}(\tau) = \begin{cases} -\left[B - \frac{1}{\tau-1}F\right] \begin{bmatrix} \nu(\frac{\tau-1}{\tau})^2(\underline{p}-\varepsilon) \\ +((1-\nu)+\frac{\nu}{\tau^2})(\overline{p}+\varepsilon) \end{bmatrix} & \text{if } \tau \leq \widehat{\tau} \\ -\left[B - \frac{1}{\tau-1}F\right] \begin{bmatrix} \nu(\frac{\tau-1}{\tau})^2 \\ +\left[(1-\nu)(\frac{\tau-1}{\tau}) + \nu\frac{(\tau-1)}{\tau^3}\right] \frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \end{bmatrix} & \text{if } \widehat{\tau} \leq \tau \leq \tau_0 \\ -\frac{1}{\tau}[(\tau-1)B-F]\left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right] & \text{if } \tau \geq \tau_0 \end{cases},$$

■

When $\varepsilon \geq \widehat{\varepsilon}$ or $\widehat{\tau} \geq \tau_0$, we have:

$$W_{CB}(\tau) = \begin{cases} -\frac{1}{(\tau-1)\tau^2}[(\tau-1)B-F] \begin{bmatrix} \nu(\tau-1)^2(\underline{p}-\varepsilon) \\ +((1-\nu)\tau^2+\nu)(\overline{p}+\varepsilon) \end{bmatrix} & \text{if } \tau \leq \tau_0 \\ -\frac{1}{\tau}[(\tau-1)B-F]\left[\nu(\tau-1)(\underline{p}-\varepsilon)+(1-\nu)(\overline{p}+\varepsilon)\right] & \text{if } \tau \geq \tau_0 \end{cases},$$

Assume first $\varepsilon \leq \widehat{\varepsilon}$ and $\widehat{\tau} \leq \tau_0$. As F has to be set as high as possible, we have $\left[B - \frac{1}{(\tau-1)}F\right] = K$, due to the CEO's limited liability constraint. We thus have, if $\tau \leq \widehat{\tau}$

$$\begin{aligned} \frac{\partial W_{CB}(\tau)}{\partial \tau} &= -\left(K\left[\nu(\underline{p}-\varepsilon)\left(\frac{2\tau^2(\tau-1)-2\tau(\tau-1)^2}{\tau^4}\right) - \frac{2\nu}{\tau^3}(\overline{p}+\varepsilon)\right]\right) \\ &= -\left(2K\nu\frac{[(\underline{p}-\varepsilon)(\tau-1)-(\overline{p}+\varepsilon)]}{\tau^3}\right) \end{aligned}$$

However,

$$(\underline{p}-\varepsilon)(\tau-1)-(\overline{p}+\varepsilon) \leq 0$$

as $\tau \leq \tau_0 = 1 + \frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}$. And then $\frac{\partial W_{CB}(\tau)}{\partial \tau} \geq 0$.

If $\widehat{\tau} \leq \tau \leq \tau_0$,

$$W_{CB}(\tau) = -K \begin{bmatrix} \nu(\frac{\tau-1}{\tau})^2 \\ +\left[(1-\nu)(\frac{\tau-1}{\tau}) + \nu\frac{(\tau-1)}{\tau^3}\right] \frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}} \end{bmatrix}$$

The first derivative of this objective function is in this case:

$$
\begin{aligned}
\frac{\partial W_{CB}(\tau)}{\partial \tau} &= -\left(K\left[\nu\left(\frac{2(\tau-1)}{\tau^3}\right)+\left[\frac{(1-\nu)}{\tau^2}+\nu\frac{3-2\tau}{\tau^4}\right]\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta}-\underline{\beta}\overline{p}}\right]\right) \\
&= -\frac{K}{\tau^4}\left[2\nu\tau(\tau-1)+(1-\nu)\tau^2\tau_0+\nu(3-2\tau)\tau_0\right] \\
&= -\frac{K}{\tau^4}\left[2\nu\tau^2-2\nu\tau+(1-\nu)\tau^2\tau_0+3\nu\tau_0-2\nu\tau\tau_0\right] \\
&= -K\left[\frac{2\nu}{\tau^2}-\frac{2\nu}{\tau^3}+\frac{(1-\nu)\tau_0}{\tau^2}+\frac{3\nu\tau_0}{\tau^4}-\frac{2\nu\tau_0}{\tau^3}\right] \\
&= \frac{K}{\tau^4}\left[-\tau^2(2\nu+(1-\nu)\tau_0)+2\nu\tau(1+\tau_0)-3\nu\tau_0\right]
\end{aligned}
$$

The sign of this expression is equivalent to the sign of a second degree concave polynomial in τ. This polynomial has two positive roots. We will show below that it is negative in $\widehat{\tau}$ and τ_0 and that its derivative in $\widehat{\tau}$ and τ_0 is also negative. This implies that it is negative for all τ in $[\widehat{\tau},\tau_0]$ and that consequently $W_{CB}(\tau)$ is non increasing on this interval. Indeed, we have

$$
\begin{aligned}
\frac{\partial W_{CB}(\tau)}{\partial \tau}|\tau_0 &= -\frac{K}{\tau_0^4}\left[(1-\nu)\tau_0^3+\nu\tau_0\right]\leq 0 \\
&= -K\left[\frac{(1-\nu)}{\tau_0}+\frac{\nu}{\tau_0^3}\right]\leq 0
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial W_{CB}(\tau)}{\partial \tau}|\widehat{\tau} &= -\frac{K}{\widehat{\tau}^2}\left[2\nu\left(1-\frac{1}{\frac{\tau_0}{\tau_0-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}}}\right)+(1-\nu)\tau_0+\frac{3\nu\tau_0}{\left(\frac{\tau_0}{\tau_0-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}}\right)^2}-\frac{2\nu\tau_0}{\left(\frac{\tau_0}{\tau_0-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}}\right)}\right] \\
&= -\frac{K}{\widehat{\tau}^2}\left[\begin{array}{c} 2\nu\left(\frac{\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}}{\tau_0}\right)+(1-\nu)\tau_0 \\ +\frac{3\nu\left(\tau_0-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)^2}{\tau_0}-2\nu\left(\tau_0-\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)\end{array}\right] \\
&= \frac{K}{\widehat{\tau}^2}\left[-\tau_0^2+4\nu\tau_0\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}-2\nu\left(\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)-3\nu\left(\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)^2\right]
\end{aligned}
$$

This is always negative as $\Delta=16\nu^2\left(\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)^2-4\nu\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\left(2+3\frac{(\overline{p}+\varepsilon)}{(\underline{p}-\varepsilon)}\right)\leq 0$ because $\nu\leq\frac{1}{2}$.

Moreover, it is easy to check that the derivative of the second degree concave polynomial in τ (having the same sign as $\frac{\partial W_{CB}(\tau)}{\partial\tau}$) is negative in τ_0 and in $\widehat{\tau}$ (because $\nu\leq\frac{1}{2}$). This implies that

$$
\frac{\partial W_{CB}(\tau)}{\partial \tau}\leq 0 \text{ for all } \tau\in[\widehat{\tau},\tau_0].
$$

If $\tau \geq \tau_0$,

$$\begin{aligned}\frac{\partial W_{CB}(\tau)}{\partial \tau} &= -\frac{(\tau-1)}{\tau}K\left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta} - \underline{\beta}\overline{p}}\right] \\ &= -\frac{1}{\tau^2}K\left[\nu + (1-\nu)\frac{\overline{p}\Delta\beta}{\underline{p}\overline{\beta} - \underline{\beta}\overline{p}}\right] \leq 0\end{aligned}$$

When $\varepsilon \geq \widehat{\varepsilon}$, or $\widehat{\tau} \geq \tau_0$ we have, if $\tau \leq \tau_0$

$$\frac{\partial W_{CB}(\tau)}{\partial \tau} = -\left(K\frac{2\nu}{\tau^3}\left[(\tau-1)\left(\underline{p}-\varepsilon\right) - \left(\overline{p}+\varepsilon\right)\right]\right) \geq 0$$

If $\tau \geq \tau_0$

$$W_{CB}(\tau) = -K\left[\nu\frac{(\tau-1)^2}{\tau}(\underline{p}-\varepsilon) + (1-\nu)\frac{\tau-1}{\tau}(\overline{p}+\varepsilon)\right]$$

$$\begin{aligned}\frac{\partial W_{CB}(\tau)}{\partial \tau} &= -K\left[\nu\frac{2\tau(\tau-1)-(\tau-1)^2}{\tau^2}(\underline{p}-\varepsilon) + (1-\nu)\frac{1}{\tau^2}(\overline{p}+\varepsilon)\right] \\ &= -K\left[\nu(\tau-1)\frac{\tau+1}{\tau^2}(\underline{p}-\varepsilon) + (1-\nu)\frac{1}{\tau^2}(\overline{p}+\varepsilon)\right] \leq 0\end{aligned}$$

■

This allows us to conclude that when $\varepsilon \leq \widehat{\varepsilon}$ and $\tau_0 \geq \widehat{\tau}$, it is optimal for the shareholders to select a Board of Directors with a low degree of independence, i.e. $\tau^* = \widehat{\tau}$. In all other cases, it is optimal for the shareholders to select a Board of Directors with a high degree of independence, i.e. $\tau^* = \tau_0$.

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