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Abstract

In this paper we show that financing constraints affect the optimal level of capital stock even when the financing constraint is ineffective. This happens when the firm rationally anticipates that access to external financing resources may be rationed in the future. We will show that with these expectations, the optimal investment policy is to invest less in any given period, thereby lowering the desired optimal capital stock in the long run

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Key words: Investment; constraints; uncertainty.

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1 Introduction

During the last twenty years a large literature on determinants of real investment have emphasized the role of financial factors in capital accumulation by firms. A considerable body of theoretical research has stated that credit rationing and financing constraints may be the result of optimizing behavior of lenders, rather than a consequence of exogenous forces (Hodgman (1960), Keeton (1979), Stiglitz and Weiss 1981). Further, a number of seminal papers have provided empirical evidence that real investment depends dramatically on financial factors such as internal finance and external debt (Fazzari et al. 1987, 1988). The main conclusion of this literature is that investment spending can be constrained by credit rationing in the short run. Such rationing causes the supply curve for funds to bend backwards when demand for loans exceeds a maximum (endogenous) amount. But, while in these models capital market imperfections shape the credit supply curve, the demand curve of capital remains usually unaffected, so that it is treated and analyzed as if firms were acting in perfect capital markets (Hubbard, 1998).

A second issue related to the effects of financing constraints is stressed by Jaffee and Stiglitz (1990). They observe that one of the main limitations of credit rationing models is the use of comparative statics to analyze the relationship between investment and finance. This approach, they argue, makes it difficult to focus on inter-period issues affecting the investment process. More specifically, Jaffee and Stiglitz observe that anticipated future credit rationing can have effects on current aggregate demand, "even when there is no credit rationing at present. Thus the impact of the credit rationing can not be assessed just by looking at those periods in which there is direct evidence for its presence" (p.874).

However, the more recent debate seems to have acknowledged this suggestion. Indeed, an interesting strand of research has focused on this issue studying the relationship between future financing constraints and current investment decisions. Among these the more remarkable works are D'autumne and Michel (1985), Milne and Robertson (1996), Kiyotaki and Moore (1997), Saltari and Travaglini (2001, 2003, 2006).

More precisely, Kiyotaki and Moore attempts to build on Jaffee and Stiglitz's original intuition, showing that, in the presence of credit rationing, transitory technological shocks can cause broad fluctuations in investment. In particular, the negative correlation between unexpected shocks and the price of capital goods can generate a negative cumulative effect on real investment. Indeed, in their model, shocks, by decreasing the prices of collateralized assets, can make the financing of new investment projects less and less attractive to an external lender. On the other hand, Saltari and Travaglini have shown that financial constraints need not be currently binding in order to affect current investment decisions. More precisely, they provide a framework in which firms are neither always constrained nor always unconstrained. They are concerned with those cases where a firm is free from financing constraints at the current time, but expects to face an upper bound at some later date. They show that the effects of future financing constraints are included in the market value of the firm, and thus are captured by marginal q.

In what follows we try to put together the basic intuitions of these previous works. Compared to the previous literature, the adding value of this paper lies in our attempt to provide a new perspective of the investment decisions, scrutinizing the long run impact of (potential) financing constraints on the optimal level of the capital stock chosen by the (constrained) firm. We get two main results. Firstly, we show that in a dynamic stochastic context, financing constraints affect firms' behavior, and, hence, its optimal capital stock, even when demand for loans is currently below the credit rationing threshold. This occurs because the firm rationally anticipates the possibility of future rationing. This forward-looking behavior modifies the current choice of the firm: it knows that a ceiling on its access to financial resources imposes an upper limit on capital increasing. Under this expectation we show that the net present value of the firm, is reduced even when the constraint is not currently effective. Secondly, we will show that in the presence of financing constraint the firm does not limit itself to reducing its investment when the upper limit is reached. But, what it actually does is to lower its desired optimal capital stock.

These results are obtained using a simple partial dynamic stochastic equilibrium model which compares a rationed firm's investment decisions with those taken by a non-rationed firm. No attempt is made to cover all possible applications of a general equilibrium model.

The paper is organized as follows. In section 2 we discuss a firm's investment policy under the alternative assumptions of a rationed and a non-rationed credit market. Section 3 compares these solutions. Section 4 draws a number of conclusions concerning the effects of credit rationing on aggregate investment.

2 Financial constraints and the value of the firm

In this section we analyze how financing constraints affect the firm's behavior. In examining this question, it is useful to compare the behavior of a unconstrained firm, acting in a perfect capital market, with that of a constrained firm which takes its decisions in the presence of credit constraint.

2.1 Optimal capital stock without financing constraints

Let us suppose that the firm is risk neutral. Investment decisions are taken in a perfect capital market, where r is the equilibrium interest rate. Each firm operates with a large number of projects; it can buy new units of capital, and resell the old ones.

To simplify, we assume that the firm uses a single capital-input K with decreasing marginal productivity. The set of production possibilities changes continuously under a multiplicative shock θ, whose dynamics are given by

$$d\theta_t = \sigma\theta_t dz \tag{1}$$

where the shock θ is a geometric Brownian motion without *drift*, and dz is a Wiener process with $E(dz) = 0$ and $E(dz)^2 = dt$. The production function is

$$\theta_t f(K_t) = \theta_t K_t^{\alpha}, \text{ with } 0 < \alpha < 1 \tag{2}$$

The unconstrained firm chooses its optimal capital so as to maximize the present discounted value $V(\theta_t, K_t)$ of the expected cash flows:

$$V(\theta_t, K_t) = \max_{\{I_s\}} E_t \left\{ \int_t^{\infty} e^{-r(s-t)} \left[\theta_s f(K_s) - I_s\right] ds \right\} \tag{3}$$

where $dK = I_s ds$ is the investment flow, and the depreciation rate is equal to zero. The functional (3) can be rewritten using the Bellman equation:

$$rV_t dt = \max_{\{I\}} \left\{ \left[\theta_t f(K_t) - I_t\right] dt + E_t(dV) \right\} \tag{4}$$

Under the assumption of a perfect credit market the solution of (4) is[1]

$$V^{NR}(\theta_t, K_t) = \frac{\theta f(K_t^*)}{r} \tag{5}$$

This means that the value of the unrationed firm (NR) is equal to the present discounted value of expected cash flow corresponding to the desired capital stock K_t^*. Finally, combining the expression (5) with the first order condition for profit maximization ($V_K^{NR} = 1$) we obtain $\frac{\theta f'(K_t^*)}{r} = 1$, that is:

$$\theta^{NR}(K_t) = \frac{r}{f'(K_t)} = \frac{r}{\alpha}(K_t)^{1-\alpha} \tag{6}$$

[1] See the Appendix for the derivation of this result.

This last condition is particularly revealing because it describes the relationship between the value of the shock θ and the desired capital stock corresponding to θ. Graphically, this relation is the upward-sloping curve NR (see figure 1). Under the assumption of diminishing returns a firm which wishes to raise its capital stock and which therefore needs to increase its investment also requires a higher θ. In other words, given the starting level K_t, if the shock is higher than the one on the NR curve, the marginal value of the firm is $V_K^{NR} > 1$; the optimal policy is therefore to increase the capital stock so as to satisfy the condition $V_K^{NR} = 1$. Hence, if we start from a point on the NR curve, it is obvious that the higher the shock the higher is the increment in the desired capital stock. For the same reason below the NR curve where $V_K^{NR} < 1$, the firm finds it optimal to disinvest.

Using this argument we reach a first conclusion: if a firm acts without financial constraints the sole determinants of its investment decisions are the changing value of the shock θ and the corresponding value of its marginal productivity $f'(K_t)$. In any period fluctuations in θ can change the optimal capital stock but the size of past or future expected shocks has no effect on current investment decisions.

2.2 Optimal capital stock with financing constraints

The previous result shows that in a perfect credit market investment decisions in any given period are independent with respect to the size of the shock θ in other periods. Does this property change in the presence of credit rationing?

To analyze this problem, assume that the firm cannot finance new investment projects using internal finance or by issuing new shares; it can, however, request new funds from external lenders. Assume that the maximum amount of credit the firm can obtain is equal to the proportion $0 < m < 1$ of the starting endowment K_t, owned by the firm. It follows that the maximum capital stock that a firm can obtain in the next period is equivalent to $K_t(1+m)$.[2] In this model, the coefficient m is a parameter which is independent from the capital stock. Though this last assumption might appear too restrictive with respect to some models of credit rationing, it is rich enough to discuss the consequences of imperfect capital markets on the investment demand.[3]

[2] There are several definition of credit rationing. In this context we refer to *type 1 credit rationing:*: the credit contract defines the maximum amount of loan available at the going interest rate.

[3] We do not attempt to derive this constraint endogenously. However, Hart and Moore (1994), and Kyotaki and Moore (1997) give an argument to show the nature of this contract: creditors know in advance the liquidation value of assets in place utilized as collateral. So they take care never to allow the loan to exceed the value of the collateralized

To evaluate how credit constraints alter the value of the firm, let us consider how investment policy changes in the presence of a credit ceiling. Contrary to the previous framework where an individual firm's flow of investment depends exclusively on the value of the shock, in the present context there is an upper limit to investment at $I_t = mK_t$. A firm observes θ and knows that, whatever its value, new investment cannot exceed the upper limit fixed by the loan constraint. As a result the current value of the firm is reduced by the potential profits which it will never gain because of the credit ceiling. In other words, the credit barrier reduces the firm's profit potential and its current value. The main consequence of this expectation is a change in the curve describing the dynamic, equilibrium relationship between shocks and desired changes in capital stock; at the current time the firm is already anticipating future credit rationing.

To calculate the value of the rationed firm, assume that $K_t(1+m)$ is the maximum capital stock the firm can hold when the shock θ reaches a particular trigger value, that is the constraint is binding. In this setting the general solution of the problem

$$rV_t dt = \max_{\{I_t\}} E_t \left\{ \left[\theta_t f\left(K_t\right) - I_t \right] dt + E_t\left(dV\right) \right\} \tag{7}$$

under the credit constraint $I_s \leq K_s m$ for any $s \geq t$, with $0 < m < 1$, is

$$V^{RZ}\left(K_t, \theta_t\right) = A\left(K_t\right) \theta_t^{n_1} + B\left(K_t\right) \theta_t^{n_2} + \frac{\theta_t f\left[K_t\right]}{r} \tag{8}$$

where RZ refers to the rationed firm and $n_1 > 1$ e $n_2 < 0$ are the roots of the characteristic equation.[4] $A(K_t)$ and $B(K_t)$ are "constants" of integration whose value depends on current assets. The last term on the right is the fundamental value. As usual it identifies the current value of future profits generated by the capital in place. If, as in the previous section, the flow of investment can change freely, the optimal solution for the firm is K_t^*, and the constants $A(K_t)$ and $B(K_t)$ must be equal to zero in order to satisfy the condition (5).

But if investment reaches the upper barrier at $K_t m$, we have to consider the value of the firm for small values of θ. Starting from such a value, it is unlikely that the shock θ will climb the barrier at any time in the immediate future; it is also unlikely, therefore, that the firm will invest at any time in the reasonable future. In this case the value of the firm is given by its current

capital goods K_t.

[4]The value of the two roots is respectively $n_1 = \frac{1}{2} + \sqrt{\frac{1}{4} + 2\frac{r}{\sigma^2}} > 1$ and $n_2 = \frac{1}{2} - \sqrt{\frac{1}{4} + 2\frac{r}{\sigma^2}} < 0$.

assets that is $V^{RZ}(K_t, \theta_t) = \frac{\theta f[K_t]}{r}$. But as θ tends to zero the expression $\theta_t^{n_2}$ tends to rise because $n_2 < 0$. This implies that for small values of θ the solution $B(K_t)\,\theta_t^{n_2}$ increases, implying an increase in the value of the firm and in investment, whereas the economic problem implies zero value and zero investment. To avoid this difficult we set $B(K_t) = 0$, in the value equation (8) rewriting the equation as:

$$V_t^{RZ} = A(K_t)\,\theta_t^n + \frac{\theta_t f(K_t)}{r} \tag{9}$$

where to simplify we write n for n_1. The remaining "constant" is determined by the behavior of the firm at the barrier $I_t = K_t m$. Since maximization requires $V_K^{RZ} = 1$, the optimal constrained policy of the firm must satisfy the condition

$$V_K^{RZ} = A^{'}(K_t(1+m))\,\theta_t^n + \frac{\theta_t f^{'}(K_t(1+m))}{r} = 1 \tag{10}$$

This expression alone is not sufficient to determine both $A'(K_t)$ and the trigger value of θ. To solve equation (9), we have to ensure that under an effective credit constraint, infinitesimal changes of θ do not induce the firm to modify its decisions, reducing investment. At the barrier we should thus have $V_{K\theta} = 0$, that is

$$V_{K\theta} = A'(K_t(1+m))\,n\theta_t^{(n-1)} + \frac{f^{'}(K_t(1+m))}{r} = 0$$

Putting together these two boundary conditions and remembering that $f(K) = K^\alpha$, we obtain

$$A'(K_t(1+m)) = -\frac{(n-1)^{n-1}}{(nr)^n}\alpha^n (K_t(1+m))^{n(\alpha-1)} \tag{11a}$$

$$\theta(K_t(1+m)) = \frac{n}{n-1}\frac{r}{\alpha}(K_t(1+m))^{1-\alpha} \tag{11b}$$

The expression (11a) is particularly eloquent. Since $A'(K_t(1+m))$ is negative, the solution $A^{'}(K_t(1+m))\,\theta^n$ associated with the positive root can be interpreted as the (implicit) marginal cost the firm suffers for not investing over the credit threshold $K_t m$. In other words, $A^{'}(K_t(1+m))\,\theta^n$ represents the marginal profit lost by the firm because of credit rationing. Intuition suggests that if $K_t(1+m)$ is the maximum capital stock available starting from K_t, the firm is being forced to give up the marginal profits which would derive from the potential expansion of capacity beyond $K_t m$. So, by using the solution for $A'(K_t(1+m))$ over the interval $[K_t(1+m), \infty)$, we obtain the market value of any additional investments that the firm cannot realize because of the credit constraint

$$A(K_t\,(1+m)) = \int_{K_t(1+m)}^{\infty} -\left[A'\,(k)\right] dk = \frac{(n-1)^{n-1}}{(nr)^{n}}\alpha^{n}\frac{(K_t\,(1+m))^{n(\alpha-1)+1}}{n\,(\alpha-1)+1}$$

The constant $A(K_t\,(1+m))$ is *negative*.[5] Substituting in (9) the economic interpretation is evident: since the firm anticipates the possibility of future constraints, *potential* credit rationing *reduces* the present value of expected profit and, as a consequence, the value of the firm.

3 Constraints and investment

The condition (11b) describes the relationship between changes in θ and fluctuations in desired capital stock under credit rationing. Under this condition, and unlike the situation described by equation (5), we have a multiplicative factor $\frac{n}{n-1}$. Given that $n > 1$, this factor is greater than one. The main consequence is that for the constrained firm the trigger value for the shock must exceed the critical threshold for the unconstrained firm to hold the same capital stock. In short, for identical shocks the constrained firm seeks a smaller capital stock. In figure 1 this proposition is illustrated by the RZ boundary curve. The boundary curve describes the trigger value of the shock θ corresponding to the maximum investment $K_t m$, or, equivalently, to the optimal capital $K_t\,(1+m)$. The equation of the RZ curve is given by the condition (11b)

$$\theta^{RZ}\,(K_t(1+m)) = \frac{n}{n-1}\frac{r}{\alpha}\,(K_t\,(1+m))^{1-\alpha}$$

This relationship implies that the RZ curve must be traced to the left of the NR curve.

This equation has some meaningful properties. The position of the RZ curve depends on the value of the positive root n, which is a function of r and σ^2. When the interest rate r rises the value of n rises as well, and correspondingly the critical value θ^{RZ} is reduced. This means that the credit

[5] Calculating the integral, we get

$$-\frac{(n-1)^{n-1}}{(nr)^{n}}\alpha^{n}\frac{k^{n(\alpha-1)+1}}{n\,(\alpha-1)+1}\Bigg|_{K_t(1+m)}^{\infty}$$

For convergence of the integral, α must be sufficiently less than one so that $n\,(\alpha-1)+1 < 0$.

r	σ	α	K^{RZ}	K^{NR}
0.04	0.15	0.5	76	100
0.06	0.15	0.5	80	100
0.04	0.30	0.5	59	100

Table 1: *Comparing capital stocks for some values of r, σ and α.*

constraint loosens. Intuitively, the negative correlation between θ^{RZ} and r is determined by the fact that an higher interest rate reduces the desired capital stock, and consequently decreases the probability to be rationed in the future. On the other hand, when the variance σ^2 rises the value of the root n reduces, and, hence, the trigger value θ^{RZ} rises. This is because the profit function is a convex function of the shock θ. But, since the firm is credit constrained the actual investment can never exceed $K_t m$, and the firm perceives as an exacerbation of the credit constraint the increase of σ^2.

From (6) and from (11b) we can directly calculate desired capital stocks on the two alternative curves

$$K^{NR} = \left(\frac{r}{\alpha} \frac{1}{\theta^{NR}} \right)^{\frac{1}{\alpha - 1}}$$

and

$$K^{RZ} \equiv K_t \left(1 + m\right) = \left(\frac{r}{\alpha} \frac{n}{n-1} \frac{1}{\theta^{RZ}} \right)^{\frac{1}{\alpha - 1}}$$

For the same level of shock $\theta^{RZ} = \theta^{NR}$, combining the last two equations gives

$$K^{RZ} = K^{NR} \left(\frac{n}{n-1} \right)^{\frac{1}{\alpha - 1}}$$

This condition explains the previous proposition. Given that $\left(\frac{n}{n-1}\right)^{\frac{1}{\alpha-1}} < 1$, for the same shock, the RZ firm seeks a smaller capital stock than the NR firm.

Note that the root n affects dramatically both the desired capital stock and the solution $A\left(K_t(1+m)\right)\theta_t^n$. The following table shows the level of desired capital stock for the two alternative cases of constrained and unconstrained firm.

For plausible values of the parameters, i.e. $r = 4\%$, $\sigma = 15\%$ and $\alpha = 0.5$, if 100 is the capital stock of the NR firm, the desired capital of the RZ firm is much smaller and equal to $K^{NR}\left(\frac{n}{n-1}\right)^{\frac{1}{\alpha-1}} = 100 \cdot (76\%) = 76$.

Further, this example points out an aspect strongly debated in the investment theory: the relationship between uncertainty and demand for capital.

The previous data illustrate that for the NR firm the relationship between σ and K^{RZ} is negative, that is an increase in uncertainty reduces the desired capital stock. Obviously, in our setting this effect is negative. This is because there is no positive effect due to factors technological substitutability, since the firm acts with only a single input (Dixit and Pindyck, 1994).

3.1 More on constraints and optimal capital stock

So far we have considered how credit rationing acts on the RZ firm's desired capital stock. To push our analysis a bit further, let us see now what happens to investment policy. To discuss this point, a third curve, labelled CM, is drawn in figure 1. This line describes, for any starting level of K_t, the trigger value of θ^{RZ} that induces the firm to make the new maximum investment $I_t = K_t m$. Its equation is obtained from the condition (11b)

$$\theta^{CM}(K_t) = \frac{n}{n-1}\frac{r}{\alpha}\left(K_t\left(1+m\right)\right)^{1-\alpha}$$

The CM line is not a barrier control, but it is useful to describe the evolution of the investment. The horizontal distance between CM and RZ measures the maximum investment $K_t m$. In turn, any point included in the area among the two curves identifies the value of the shock which causes an investment smaller than the credit constraint.

Taking into consideration these three curves, it is now possible to describe the RZ firm's investment policy.

When $V_K^{RZ} > 1$, the marginal value of the RZ firm is higher than the user cost of the new investment. Thus the firm invests, but the value of the investment is no higher than the level fixed by the credit constraint.

For instance, if the initial capital is K_t and the shock is θ_0, the NR firm finds it optimal to invest until K^* (see figure 1). The credit constraint prevents the RZ firm from obtaining this capital stock: maximum investment must be no higher than $K_t m$; as a result the difference $K^* - K_t(1+m)$ gives a measure of credit rationing. In other words, when the shock reaches the trigger value θ^{CM} along the curve CM, the supply of lending becomes inelastic, ensuring that the investment will be no higher than $K_t m$. Given that the investment is free to range only over the area between the CM and the RZ curves and that it cannot climb to the upper ceiling, we see that the CM curve is a reflecting barrier for the stochastic process θ. At the trigger value θ^{CM} the firm invests to obtain $K_t(1+m)$, moving rightward along the RZ curve. This behavior is exemplified in the figure 1, where the shock $\theta^{CM} = \theta_0$ causes an investment equal to $K_t m$. It is clear that for $\theta > \theta_0$

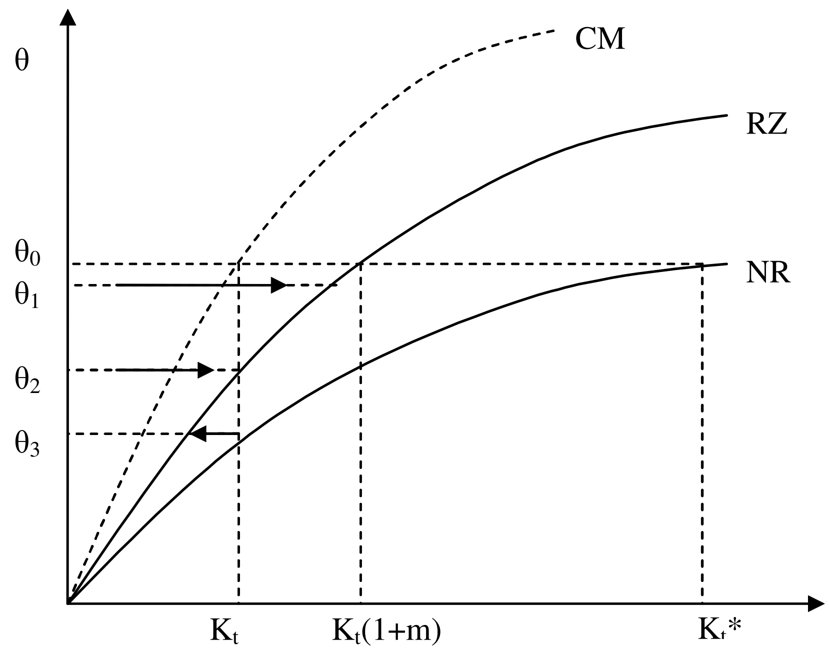

Figure 1: Investment under financial constraints

investment cannot increase further. There is, in other words a proportional rise in credit rationing.

A meaningful result is obtained when the value of the shock lies between the CM and RZ curves . With reference to figure 1, this occurs when $\theta_0 < \theta_1 < \theta_2$. Above the RZ curve $V_K^{RZ} > 1$. Here investment is positive but less than $K_t m$: the firm invests but for an amount just sufficient to return to the RZ curve . In this case, although the constraint is slack, we cannot derive the optimal capital stock from the NR curve : the *forward-looking* firm anticipates the possibility of future rationing even if the constraint is not effective at the current time. In technical terms the RZ firm finds it optimal to satisfy the marginal condition $V_K^{RZ} = 1$ even when the credit constraint is not effective. If investment was higher than necessary to satisfy the condition (11b), we would obtain (see figure 1) a point below the RZ curve where, given the credit constraint, $V_K^{RZ} < 1$. At this point, however, the firm will optimize its performance by reducing its capital stock. This result conforms to the intuition stated above: namely that the constraint on credit lead to a reduction in overall demand for capital. The size of the correction due to the credit ceiling is given by $A\left(K_t(1+m)\right)\theta_t^n$.

According to this result, note that if a monetary policy can improve the credit market reducing the coefficient m, the multiplier $\frac{n}{n-1}$ does not change. As a consequence, the RZ curve remains to the left of the NR curve.

All this leads to a remarkable result when the shock θ is smaller that the shock required to satisfy the RZ curve. This is the case for θ_3. Here the firm optimizes its performance by reducing its current assets so as to return to the curve at the point where $V_K^{RZ} = 1$. The main implication of this investment decision is to enforce the credit constraint. Since the volume of loans offered by the lender is proportional to capital stock, the smaller the current stock the smaller will be the available credit. In this way the maximizing behavior of the RZ firm can initiate a vicious circle. During a phase of economic recession the firm reduces its capital endowment. This hurts the firm in the next period because it owns less collateralizable assets. In the presence of credit rationing this reduces the demand for capital. From this point of view, credit rationing is caused not only by optimal decisions on the supply side (Hodgman, (1960)), but also by forward-looking expectations on the demand side.

4 Conclusions

In this paper we have studied firm's behavior under financing constraints. In our model credit rationing affects investment decisions in all periods, even when the constraint is not directly effective. When investment decisions are constrained by credit rationing the limited financial resources act as an upper bound on investment. In this context, the constraint produces an overall decrease in the net present value of the firm capturing the discounted expected loss of investment and profits. As a result, the constrained firm is always worth less than its fundamental value. In additon, this result implies that the capital demand of a constrained firm is always smaller than the capital demand of the identical unconstrained firm.

The novel outcome of this model lies in the argument that financing constraints affects not only the supply curve of funds, but also the demand curve of capital. Furthermore, if temporary productivity shocks reduce the net worth of current capital stock in some periods, the firm can rationally desire to lessen its capital stock (used as collateral), thereby reinforcing the negative effects of the constraints. When this happens, the firm initiates a vicious circle.

Aizeman and Marion (1999), studying the correlation between real investment and financial constraints, have claimed that credit rationing introduces a non-linearity in the supply curve, hampering the expansion of investment in good times, without mitigating the drop in the bad times. In their paper, however, the demand for capital is unaffected by credit rationing. Our results cast doubt on that conclusion. As we have shown, the forward-looking firm anticipates the possibility of future rationing, even in periods when the constraint is not directly effective. As a result the probability of future rationing may exert negative effects on capital demand in any period, thereby altering the overall pattern of investment.

The partial equilibrium model employed in this paper is technically simple. Nonetheless, this framework could be extended to develop the dynamic perspective which characterizes the model. Hubbard (1998) has suggested the need for a comprehensive framework within which to study the incremental effects of financing constraints and irreversible capital on investment decisions. Obviously, such a challenge can only be met is we assume that the capital is at least partially reversible, so that capital goods can be used as collateralized assets in the loan contract.

Finally, many authors have stressed that the investment models *à la* Jorgenson, as well as models with irreversibility, represent observed investment dynamics in an unsatisfactory way. Such dynamics, it is argued, are more stable that those suggested by theoretical models (Bertola and Caballero (1994). The stylized facts seem to strengthen the argument that fluctuating interest rates do not provide a satisfactory explanation for inter-period investment profiles. These observations suggest that sluggishness in aggregate investment can be a consequence of quantitative constraints rather than prices and that poor aggregate investment can be the result of heterogeneous investment decisions penalized by credit constraints.

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5 Appendix

In this appendix we derive the solution to the maximization problem of the unconstrained firm.

Using Ito's lemma and the stochastic process (1), we obtain an expression for the expected capital gain

$$E_t\left(dV\right) = V_K dK + \frac{1}{2}\sigma^2\theta_t^2 V_{\theta\theta} dt$$

Substituting this expression into the Bellman equation (4)

$$rV_t dt = \max_{\{I\}}\left\{\left[\theta_t f\left(K_t\right) - I_t\right] dt + E_t\left(dV\right)\right\}$$

we can write

$$rV_t = \max_{\{I\}}\left\{\theta_t f\left(K_t\right) - I_t + V_K I_t + \frac{1}{2}\sigma^2\theta_t^2 V_{\theta\theta}\right\}$$

The first order condition is

$$V_K = 1 \tag{12}$$

Substituting for V_K in the previous equation we have the following differential equation

$$rV_t = \theta_t f\left(K_t\right) + \frac{1}{2}\sigma^2\theta_t^2 V_{\theta\theta} \tag{13}$$

whose characteristic equation is

$$r = \frac{1}{2}\sigma^2 n\left(n-1\right)$$

The general solution of (13) is

$$V\left(K_t, \theta_t\right) = A\left(K_t\right)\theta_t^{n_1} + B\left(K_t\right)\theta_t^{n_2} + \frac{\theta_t f\left(K_t\right)}{r}$$

Ruling out speculative bubbles, i.e. setting to zero the two constants $A\left(K_t\right)$ and $B\left(K_t\right)$, we get

$$V\left(\theta_t, K_t\right) = \frac{\theta_t f\left(K_t^*\right)}{r}$$

which is equation (5). This is what we call in the main text the fundamental value, that is the present discounted value of expected cash flow corresponding to the desired capital stock.