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Abstract

This paper introduces the dynamical framework which combines product and process innovations. The model contributes to the theoretical literature on innovations in two ways. First, it permits for the simultaneous dynamics of both types of innovations which is rarely considered in the literature. Second, the products being generated by the innovations are heterogeneous in their investment characteristics. This allows for the formation of the dynamic interdependency between both types of innovations. As a result the steady state levels of process innovations for each product are different and influence the dynamics of product innovations in turn.
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1 Introduction

The main goal of this paper is to introduce the new approach to modelling product and process innovations at the level of a single (possibly large) firm,

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which is a monopolist and produces multiple versions of the basic product in a given industry. Since late 1980-s it is widely acknowledged, that process and product innovations are not independent from each other. A lot of different empirical findings for different industries and economies support this conjecture [10], [13], [4], [14], [15].

The main purpose of theoretical modelling is then to capture in the simplest way possible the shape and form of this interdependency. This may be done through means of static models of equilibrium allocations of resources (intellectual, human, financial) between different types of innovative activity of a firm or through means of a dynamical model. The last one is more complicated but allows for the study of not only equilibrium allocations, but of the long-term dynamics of innovative processes and the short-run dynamics at the initial stage.

In this paper the dynamical perspective is adopted. The firm is free to choose between types of innovations at any given stage in its development and the evolution of both types of innovative activity may be analysed. This will give us more profound understanding of the key factors which affect innovations at the early and mature stages of the industry as well as to consider the differences in these factors across different types of innovations.

In the majority of models of product and process innovations the products which are being introduced are similar to each other, [6]. That's why this type of innovations are called homogeneous ones. The other strand of research literature proposes that all the products which are considered as the results of innovations are different from each other in some basic characteristics [17], [19]. This last approach is relatively new to the literature and is considered as more fruitful for modelling large multi product monopolists which perform the major part of product innovations, as for example in the metal producing industry [13] or pharmaceuticals industry. Hence the current paper follows the second approach. In the model proposed here products differ from each other by the difficulty of the optimization of their production processes which is reflected in their investment characteristics. These refer to the process innovations being associated with every new product. At the same time the monopolist is free to introduce new products to the market at a continuous basis. The rate of introduction defines the time when process innovations start. It is argued, that this kind of the dynamic dependence between different types of innovations reproduces the known stylized facts on innovations yet allowing for rather simple structural model.

The rest of the paper is organized as follows: first we consider the current state of research in the area of theoretical modelling of product and process innovations; then the basic model together with the underlying assumptions is introduced; the next section presents main results of the model which are discussed at the end of the paper.

2 State of Research

Before presenting the model the quick overview of different types of models built to capture the dependence between product and process innovations is required.

First consider, what are the methods used to install the link between two types of innovations in the literature. Frequently this purpose is achieved through the construction of the 2-stage static game, where on the first stage the decision upon the introduction of new good is being made and on the second - how much investment to put into the development of quality of this newly introduced product (conditional upon the successful introduction of it on the first stage). One example of such papers is [1] which is mainly devoted not to the interaction between both types of innovations themselves, but to the interrelation between organizational structure of the firm and its innovative decisions. In this literature quality innovations are the same as process innovations in other papers.

It is shown, that the complementarity between process and product innovations is the direct consequence of the complementarity between firm's manufacturing capabilities and its research capabilities. Current work correlates with this kind of literature in the idea of simultaneous decision making upon innovations of both types. However, it differs form this kind of models in accounting for dynamic characteristics of these new products. Moreover, heterogeneity of investment characteristics of these products and their qualities plays an essential role in the suggested model.

One other paper which corresponds to some extent to the suggested analysis is of Boone, [2]. There the process of innovations is also formulated as the 2-stage game, but the author elaborates on the incentives to innovate and their relation to the particular characteristics of the profit function. Both these examples are static in nature and they do not handle multi-product situations.

Later on it has been noted, that real innovative companies are often multi-product monopolies. Papers by Lambertini [17], [16] study the equilibrium characteristics of investments into innovations of such a monopoly. He allows for multi-product investments, and the number of existing products may also increase in the result of product innovations. However the whole model is static because it handles only the equilibrium points of innovative policy of a monopolist. Author does not study any dynamical characteristics of product and process innovations but only the equilibrium distribution of investments.

In the second paper Lambertini claims that the equilibrium level of quality investments is higher for the monopolist then the social optimum. However, more recent paper by Lin [19] suggests that this heavily depends on the level of economies of scope for the monopolist. In general to be able to answer this question one has to account for the dynamical perspective of multiple products development and the evolution of the product space. The recent paper by Lambertini [18] employs the dynamic approach to analyse the multi-product innovations with of a monopolist. The suggested paper differs from this last by explicit consideration of multiple products with heterogeneous investment characteristics and not only the varying products range, but also qualities of all the products which already exist.

Methodologically the current model is closer to the recent literature on vintage capital models although it concentrates on another type of questions. It is this strand of literature where the distributed parameter optimal control models are extensively used to describe the investment policy of an agent which has capital with different dates of appearance at hand. Then his policy should depend on the distribution of the mass of his capital in the past and hence the dynamic problem the agent has to solve is of distributed parameter optimal control type. Examples of such models are [3], [8] and others. This strand of literature uses vintage capital idea to describe policy of investments on industrial level, like in [8], [9] and also to contribute to the growth theories with embodied technological progress of the neoclassical type.

One of the few dynamic approaches to modelling heterogeneous innovations is the work of Hopenhayn&Mitchell [12], which handles the innovative process in a rather general way. However their work is mainly concentrated on the patent policy and handles innovative process in a sense of previous theories, namely of Shapiro, [11].

The suggested approach combines ideas of Hopenhayn&Mitchell and of Lambertini and Lin in a way that innovations are assumed to differ in their characteristics from each other as in [12] and at the same time the appearance of the new products on the market as in [18], [19] is allowed in the dynamic context.

The existing literature on theory of product and process innovations does not contain models similar to the suggested one. The model described further on ion the paper has two important features, novel to the area of product and process innovations:

- It allows for simultaneous dynamical optimization of product and process innovations;
- It allows for different investment characteristics of all the new products.

These two features allows to establish the dynamic and coherent link between both types of innovative activity. The formal exposition of the model follows.

3 Model

In this section the formal model is introduced together with the underlying economic intuition. The goal and structure of the model is also explained in this section.

3.1 Assumptions

Assume there is a single firm (a monopolist) in a given industry. The industry is mature and no growth of the demand is expected for existing products variety. Hence this monopolist is maximizing its profit by developing new products, which are then introduced to the market.

Assumption 1 *The only source of new profit for the monopolist is the development of new products which leads to the increase in the existing range of products over time, $n(t) > 0$.*

Assume the process of development of products is continuous in time and yield new products (which are new versions of some basic for the industry product) with some rate. Let us call this rate the rate of variety expansion.

Assumption 2 *The product innovations, are continuous in time and new products appear at a continuous basis, $\dot{n}(t) \geq 0$.*

Assume that the range of these new products is limited from above. The product innovations are limited to upgrades of some basic product which defines the industry (e.g. cell phones industry produces different versions of cell phones but not computers). We do not model fundamental inventions, which introduce totally new products to the economy by this model and hence it is natural to require that there is limited capacity of the industry for the variety of products which are somewhat similar to each other.

Assumption 3 *Product innovations are limited by the maximal possible range of products, $n(t) \leq N$*

Assume these newly introduced products initially require very much resources for their production and hence the monopolist allocates part of its R&D capacity on process innovations related to these new products. Every new product is than intensively studied with respect to opportunities for its costs minimization. As there are numerous new products (continuum of) there are numerous streams of such cost-minimizing processes associated with every product.

Assumption 4 *Every product has its own dimension of process innovations or 'quality' which depends on time,* $\forall i \in n(t) \exists q_i(t)$.

Assume at each point in time, the monopolist has to choose optimally the level of investments being made into the development of new products (product innovations) and into the development of production of already existing products (process innovations). Both these investment streams cannot be negative.

Assumption 5 *Product innovations and process innovations require different types of investments, which vary over time, while process innovations for every product are also different* $u(t) \geq 0, g_i(t) \geq 0$

Assume also that the monopolist is the long-run player and does not restrict its planning to some certain length of time. Hence, the innovations of both types occur continuously up to infinite time.

Assumption 6 *There is no terminal time for both processes of innovations,* $0 < t \leq \infty$

The last point to mention is that we assume that all innovations are certain. This is rather strong limitation, but allows to concentrate on the key issues of this paper: heterogeneity and form of interdependence between different types of innovations.

Assumption 7 *All innovations do not have any uncertainty associated with them.*

3.2 Dynamics

Here the form of the dynamic law, which governs the innovations of both types is formulated.

Observe that under the assumptions stated above, one has the process of new products introduction, $n(t)$ which describes the range of products which are already available for production. Hence product innovations are defined by the rate of increase of this range over time, $\dot{n}(t)$. This last cannot be negative, as the product which is already introduced to the market cannot be forgotten. The increase in the range of products which are already developed is proportional to the investments being made by the monopolist in this direction, $u(t)$. We assume no other internal factors, which may affect this product innovations rate. Hence, the dynamics of product innovations is described by the following differential equation:

$$\dot{n(t)} = \alpha \times u(t). \tag{1}$$

Here α is the efficiency of investments into the product innovations. It is constant and exogenously defined by the state of technology in the economy as a whole. According to this equation, the range of products which are already introduced cannot decrease over time. Eventually all the possible products are developed, as the range is restricted by N. This process is continuous and hence there is a continuous spectrum of products available on the market.

Each such product has the associated stream of process innovations. Denote by i the position of the product within the products range $n(t)$. Then to distinguish between process innovations for different products we will denote them by q_i. These innovations also grow only due to the investments being made into them. Since only one monopolist is modelled, there are no technology spillovers or acquisitions of competitors' innovations. At the same time, we assume that the improvements of every product are outdated as time flows. This process is that stronger the more refined the production technology already is. The given process innovation exhibits the decline of technology over time if no investments are being made into the production process innovation of this given product i. These considerations lead to the following form of dynamics of process innovations:

$$\dot{q_i(t)} = \gamma_i \times g_i(t) - \beta \times q_i(t). \tag{2}$$

Here γ_i denotes the efficiency of investments into the optimization of production of product i and β denotes the rate of decay of technology in the absence of investments. This one is assumed to be constant across all products, while the efficiency of investments is different. As a result the level of technology for every product might be different. The form of this difference depends on the form of the γ_i parameter dependence on i.

Observe that the equation (2) implies difference of dynamics of process innovations for all the products. Depending on the γ_i, the development of technology for every next product may be easier, harder or equally difficult then for preceding products. As in this paper we assumed that the product innovations is the process of appearance of new versions of the same basic product variety, it is natural to assume that next products are more complicated then preceding ones. To this end we specify the dependence of the efficiency of investments into production technology development as

$$\gamma_i = \gamma \times \sqrt{N - i}. \tag{3}$$

This specification makes process innovations less efficient for every next product but with the decreasing speed of this increase in difficulty. For the last product in the available range, $i = N$ this means zero level of technology, as the efficiency will become zero.

Now consider that product innovations actually introduce new products. These new products must have zero level of production technology at the moment of their appearance. This level might be increased through investments only. This requires a constraint on the initial level of production technology for any new product. At the same time, the position of this new product is dependent on the product innovations, as it may be seen from the formal form of constraint:

$$q_i(t)|_{i \leq n(t)} = 0. \tag{4}$$

This constraint introduces the notion of frontier or boundary product into the model. The boundary product is the current position of the product innovation process $n(t)$ in the available products range N. The quality of this boundary product is always zero. Observe, that every product among those which are to be developed becomes the boundary product exactly once, at the moment of its introduction. This moment, $t_i(0)$, triggers the process innovation associated with the product i and is in turn defined by the dynamics of product innovations, $n(t)$. Thus the last constraint is very important to the model: it establishes the dynamic dependence of process innovations from the product innovations. At the same time, the requirement of optimal allocation of resources between different types of investments established the dependence in the inverse direction: product innovations and rate of products variety expansion as the function of the process innovations. This form of dependence is the one being observed by empirical findings on the single firm's level [15]. It is captured in the objective function below.

3.3 Objective

The natural objective of the monopolist is the maximization of its profits, $\pi(t) \rightarrow max$ for any given time period. This paper concentrates on just one part of activities of such a monopolist, namely on the process of its innovative activities. To put this in line with profit maximization behaviour we assumed that markets for all existing products are mature, yield some constant profit with stable prices and output. Production policy of the monopolist is assumed to follow standard rules of monopolistic behaviour under profit maximization: given (constant) demand, the monopolist is setting the price and production as to maximize its profit. In mature markets the process innovations reached their maximum and thus no further improvements to the production process may be made. Hence, the production costs are also constant in time. These considerations lead to the conclusion that in mature markets the monopolist's production and pricing (and hence profits) are constant.

Proposition 1 *For those products which are already in mature stage, the production, price and profit of the monopolist are constant.*

Because of this one may abstract from this part of monopolist's activities in the optimization problem. Now consider those products which are being introduced in the given time frame and which production is subject to process innovations. For these products the profit of the monopolist is proportional to the costs decrease which is the result of process innovations, denoted by $q_i(t)$. If we abstract from the pricing policy and assume constant demand for each of such products, this would result in the profit function per unit of production of a linear form: $\pi_i(t) = \delta * q_i(t)$. Then normalizing δ coefficient to one, we may have q_i as the only profit parameter for any product within the product range N.

Since the problem under consideration is the dynamic one, for each product the profit maximization has to be defined over all the infinite time horizon. Hence, the monopolist has to maximize his instantaneous profit, which is equal to $q_i(t)$ over $0 < t \leq \infty$. The only parameter under the control of the monopolist is the investments into the process innovations which result in the increase of $q_i(t)$, reduction of costs and profit increase. Then the problem of profit maximization for every product i may be defined as:

$$V_i = \int_0^\infty e^{-rt} \times (q_i(t) - \frac{1}{2} \times g_i(t)^2)dt \rightarrow max_{g_i}. \tag{5}$$

under the condition of dynamics of process innovations governed by (2).
The investments into the process innovations for each product i, $g_i(t)$ are defined by the monopolist and control the rate of process innovations for every product separately from others; r denotes the discount rate, which is defined from time preferences of the monopolist and is exogenous to the problem.

Product innovations are taking place simultaneously with process innovations. They result in the continuous increase in the variety of products which appear in the market. For each such product the potential profit over all its life-cycle is defined by (5). However, the product innovations process also has an influence on the value generation for the monopolist. This influence is describe by the introduction of new products. The likeliness of the introduction of new product is defined by the stream of potential profits from its subsequent production. Hence, the decision of increasing or decreasing investments into the product innovations is governed by this potential profit stream, as the newly invented product has zero level of production technology and as such cannot be produced with costs lower than its price. This is where the constraint (4) comes in. As a result, one may define the profit generated by the introduction of new product as the evaluation of potential profit stream from the subsequent development of the production of this product. Hence the total profit generated by the product innovations is the integral over all such potential profit streams:

$$V(n) = \int_0^{n(t)} (\alpha \times u(t) \times V_i - \frac{1}{2} \times u(t)^2) di. \tag{6}$$

where V_i is the potential profit stream from the development of process innovations for product i. As the (6) is integrated over all $i \in N$, V_i is evaluated for all products including the boundary one, $i = n(t)$.

In both (5) and (6) it is assumed that costs of investments are quadratic. As it can be seen above, to derive the total profit of the monopolist first one has to solve the problem of profit maximization for every product i. This will define the optimal rate of process innovations, q_i, which in turn define the total profit from product i. Then the problem of determining the profit generated by all the products together, as defined by (6) may be considered.

For this purpose it is natural to employ the Hamilton-Jacobi-Bellman approach, where the profit generated by the development of each product and by the product innovations themselves are defined as process and product innovations value functions. We briefly discuss the method of solution in the next subsection.

3.4 Solution

First observe, that the problem of profit maximization from the process innovations may be solved independently of the problem for product innovations due to the infinite time horizon in the model [7]. In this case stream of investments into the production of every product i, starting from the time of its introduction, $t_i(0)$, is fully defined by the rate of the monopolist's investments into this process.

Proposition 2 *Process innovations q_i are independent from product innovations $n(t)$ except for the time of introduction of the product i into the market, $t_i(0)$.*

Eventually the process innovations lead to the maximal level of production technology which may be continuously maintained and remain unchanged afterwards. Hence one may construct the value function of the problem of maximization of (5) and rewrite it in the form of HJB equation (together with (2)).

$$rV_i = max\{q_i(t) - \frac{1}{2}g_i(t)^2 + \frac{dV(i)}{dq_i}(\gamma\sqrt{N-i}g_i(t) - \beta_i q_i(t))\}. \tag{7}$$

This HJB equation brings together the dynamics of process innovations for the product i and the value generated by such innovations, which is the increase in the profit from this product. Observe, that the time derivative of the value function V_i is absent from the equation since the time horizon is infinite. Thus the HJB equation is time-autonomous.

For this type of HJB equations the linear specification of the value function is the only relevant one [7]. This means that the value generated by the process innovations depends linearly on the level of the production technology, q_i at each point in time. This leads to the constant rate of investments into this technology for every product. The results of this solution method are presented in the next section.

Now consider the product innovations. It has been noted before, that the value of this process to the monopolist consists solely in the introduction of new product which has to be developed afterwards to bring profit. Hence the value of this part of the innovations process is dependent on the value being generated by every potential product, (7). The monopolist decides upon the intensity of introduction of new products into the market, $\alpha u(t)$, at each point in time. This intensity is controlled through the stream of investments into the product innovations, $u(t)$. At the same time careful consideration of (6) shows that at each point in time the value of the introduction of the next product depends only on this product's production technology level, which may be eventually reached.

Proposition 3 *The current value of the product innovations process, $V_{n(t)}$, depends only on the production technology benefits of the next boundary product, $V_q = V_{i=n(t)}$ and not on the production technologies of the products which are already in the market, $i < n(t)$.*

Then the value generation of the product innovations process may be described by the HJB equation, which combines (6), (1) and the resulting optimal current value of the process innovation, $V_{i=n(t)}$:

$$rV_{n(t)} = max\{\alpha u(t) \times V_{i=n(t)} - \frac{1}{2}u(t)^2 + \alpha u(t) \times \frac{\partial V_{n(t)}}{\partial n(t)}\}. \tag{8}$$

After obtaining the optimal investment strategies for product innovations and the subsequent optimal product innovations dynamics one would have the full solution for the model. The results are described below.

4 Results

4.1 Process Innovations

First-order conditions of maximization with respect to the investments g_i yield investment strategy of the monopolist which differs between products (in accordance to the investment efficiency, γ_i) but is otherwise constant. This investment strategy is valid only starting from the time of actual introduction of the product i into the market:

$$g_i^{opt} = \frac{\gamma * \sqrt{(N-i)}}{(r+\beta)}; \quad \forall t \geq t_i(0). \tag{9}$$

This investment rule yields constant investments rate for every product and is proportional to the efficiency of investments, γ while is negatively influenced by the decay rate of technology, β. At the same time for every product i the rate of investments is lower than for preceding one, $i-1$ because of the increasing complexity of the production technology. This is described by the investment efficiency specification (3). The observations derived from the (9) are summarized in the following Proposition.

Proposition 4 *Optimal investments of the monopolist in the process innovations, g_i^{opt} differ for all new products i but are constant in time. They are proportional to the efficiency of investments γ and are in inverse relation to the decay rate of technology β.*

With this investment rule, one may define the optimal path of the production technology for each new product, q_i^{opt}, starting from the time of the product's introduction, $t_i(0)$. Up to this time the technology is at zero level. Hence, the technology dynamics is piecewise defined: it is zero till time $t_i(0)$ and is the solution to (2) with optimal investments from (9) afterwards:

$$q_i(t)^{opt} = \begin{cases} 0, t < t_i(0); \\ \frac{\gamma^2(N-i)}{\beta(r+\beta)^2} \times (1 - e^{-\beta t}), t \geq t_i(0). \end{cases} \tag{10}$$

The inspection of (10) leads to the following observation:

Proposition 5 *Production technology q_i never declines but increases with decreasing speed up to its maximal level, $\bar{q}_i$ which is different for different products.*

The decreasing speed of improvements in production technologies comes from the increasing burden of maintaining the existing level of technology, q_i, while investments are constant in time. This is rather stylized approach to process innovations, as in reality the investments are adjusted to the existing level of technology. However, the general pattern of process innovations in the model is in line with empirical findings [10], [13]. There is a limit for improving the production process of any product and as more refined technology is used it is more and more difficult to improve it further. Eventually the monopolist reaches the point where no new refinements would be profitable: new investments are totally spend to maintaining the existing level of technology. At this point the product enters the mature stage of development and no new profit increases may be derived from it. This is the reason for the monopolist to continue with introduction of new products as their technology are easier to improve that of those which are already mature.

Proposition 6 *Process innovations q_i have maximal level of development $\bar{q}_i$, which decreases in i and is different for all products.*

To illustrate the form of the dynamics of process innovations, consider the evolution of production technologies for several products:

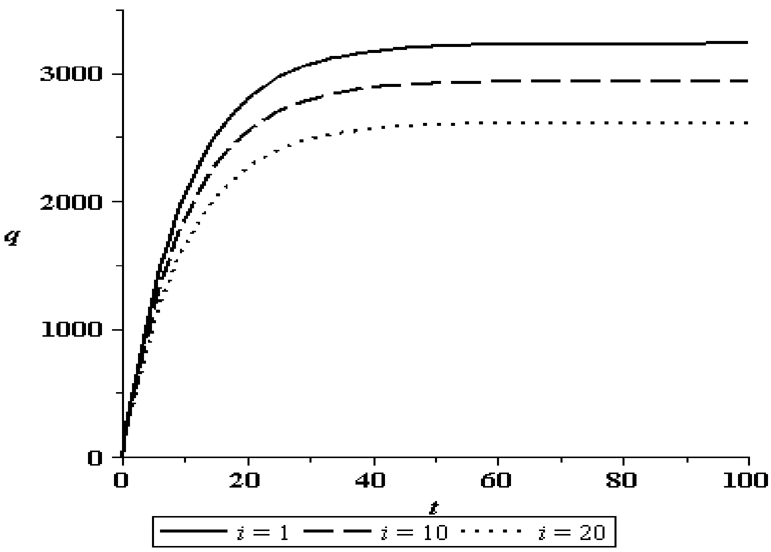

Figure 1: Difference in technologies for different products

Observe that all the technologies start to grow at different times since (4). On (Figure 1) we assumed that all the products are introduced at the same initial time $t = 0$ which is not the case. To define the time of introduction of every product, one has to solve for the product innovation process first.

The (Figure 1) illustrates the dynamics of development of process innovations. For every next product the growth of technology is slower then for the preceding one. This is the result of increasing complexity of process innovations across products and decreasing in i rates of investments (9). Note, that these different dynamics of production technologies for different products are the direct consequence of the *heterogeneity* of investment characteristics of different new products i. The special form of this heterogeneity, assumed in the model establishes the specific form of such difference, namely that each next product has lower long-run production technology level then all the preceding ones. However this may be easily changed by assuming the form of the investment efficiency function γ_i different from (3). These observations are summarized in the following Proposition:

Proposition 7 *The differences in process innovations across products are fully defined by the differences in investment efficiencies, γ_i, which stress the increased complexity of every new product relative to existing ones.*

4.2 Product Innovations

Now consider that together with process innovations q_i the monopolist is undertaking the continuous process of product innovations due to the fact of limited capacity for profit generation from improvements of technology for every product. In terms of the model the monopolist decides upon the rate of introduction of new products which is governed by the investment strategy $u(t)$.

At each point in time the monopolist considers the potential profit from improving technology of the boundary product (the one he is going to introduce). This latter is described by the current value of the process innovations for each product, V_i. The exact form of this value is obtained from the form of evolution of technology for this product, (10). The monopolist cares only about boundary product, which has the index $i = n(t)$. For this product the production technology is at the zero level, $q_i = 0$ and the resulting value function (5) does not depend on the state of technology. This value is the solution to (7) using (9) and (10). As a result the value $V_{i=n(t)}$ being used in (8) is:

$$V_{i=n(t)}|_{q_i=0} = \frac{\gamma^2 \times (N - n(t))}{2r(r+\beta)^2} = V_{i=n(t)}. \tag{11}$$

This value function depends only on the level of product innovations at each point in time, but not on the process innovations dynamics. It depends on parameters of the efficiency of investments into the improvement of technology, γ and the decay rate of technology, β. The current value of a boundary product, (11) reflects the heterogeneous nature of the technologies for different products as it changes with changing $n(t)$ which is defined from product innovations dynamics.

Using the HJB equation for product innovations, (8) with (11) inserted into it, one may derive the first order conditions for optimal investment rule, $u(t)^{opt}$ and the resulting optimal investments strategy as functions of the value generated by the boundary product, $V_{i=n(t)}$. As this last is known, the optimal investment strategy for the product innovations is the function of the current range of products in the market, $n(t)$:

$$(12)\quad u(t)^{opt} = \frac{\alpha r \gamma^2}{r(r+\beta)(\sqrt{r^4+2r^3\beta+r^2\beta^2+2\alpha^2\gamma^2}+r(r+\beta))}(N-n(t)).$$

As the main drive of product innovations comes from the profit generation by the improvements of technology for the boundary product the investment strategy for product innovation depends on the efficiency of investments into process innovations, γ. It also depends on the decay rate of production technology, β as well as from the efficiency of investments into the product innovations themselves, α. The careful consideration of the form of (12) leads to the conclusion, that the rate of investments is limited by the available potential for products introduction, N and negatively depends on the existing variety of products, $n(t)$. The higher is the existing variety, the wider is the range of products which are already developed. With the increase of this range the rate of product innovations is slowing down and reaches zero at the maximal level of variety, N. There is no explicit representation of the investment or research capacity of the monopolist in the model except for this maximal achievable level of products' variety N. However the dynamics of optimal investments into process and product innovations follows the pattern of limited research capacity: process innovations into the development of each product stop after reaching some mature stage, $\bar{q}_i$ while the development of new products provides new possibilities for improvements. At the same time new products development is slowing down with the range of already introduced products.

Proposition 8 *Product innovations investments, $u(t)^{opt}$, are decreasing to zero while the process of variety expansion reaches its limit N, and all the possible versions of the basic product are already introduced, $n(t) = N$.*

Now with the help of optimal investment strategy (12) one may derive the evolution path of the product innovation process $n(t)$ through its dynamics, (1). This is an ordinary differential equation of the first order which has the explicit solution.

$$n^{opt}(t) = N - e^{-\frac{\alpha^2\gamma^2 rt}{\sqrt{(r+\beta)^2 r^2 (r^4+2r^3\beta+r^2\beta^2+2\alpha^2\gamma^2)}+(r+\beta)^2 r^2}}(N - n_0). \tag{13}$$

Here n_0 is the range of products already introduced to the market at the initial time.

This equation demonstrates the positive rate of product innovations at each point in time. However the decreasing rate of investments over time, $u(t)^{opt}$ yields decreasing intensity of new products' introduction as the process approaches the N.

Product innovations depend on the process innovations in a dynamical way. This may be seen directly from the form of (13), but also from the fact that $n(t)^{opt}$ is the function of the value generated by the boundary product technology, (11). The last observation comes from the form of the HJB equation for product innovations, (8). At every point in time the product innovations and their intensity $\alpha \times u(t)$ are governed by the expected current value of development of the technology associated with the current product, $q_{i=n(t)}$ which is different and decreasing across products as (10) demonstrates. This establishes the dynamic and time-varying dependence of products variety expansion (product innovations) from process innovations (production technology).

Proposition 9 *Product innovations $n(t)$ depend in a dynamic way from process innovations q_i and the form of this dependence is defined by the investment efficiency function, γ_i*

As the direct consequence of this observation evolution paths of product innovations being started with different initial range of already introduced products, n_0 are convergent. The higher is the range of products which are already introduced, the less opportunities the monopolist has to develop new products, as the total range which might be developed in the given industry with given basic product is limited by N. Hence, more resources are allocated to the process innovation development for already introduced products while new products are introduced at a slower pace. Recall that the form of process innovations investment efficiency, (3) implies that it is more profitable to develop simpler products from the initial range, than new ones. These latter are developed only as long as it is no longer possible to refine the technology for previous products. The product innovations are governed by the expected profit from the development of next boundary product. With higher initial range n_0 current value of the profit from the development of next product is lower then for the case of low initially developed range.

Thus the product innovations' pace will be slower in the first case. So the convergence of product innovations evolution paths is the direct consequence of the proposition above.

As a result the influence of the initially researched capacity eventually wears down. This is illustrated by the (Figure 2).

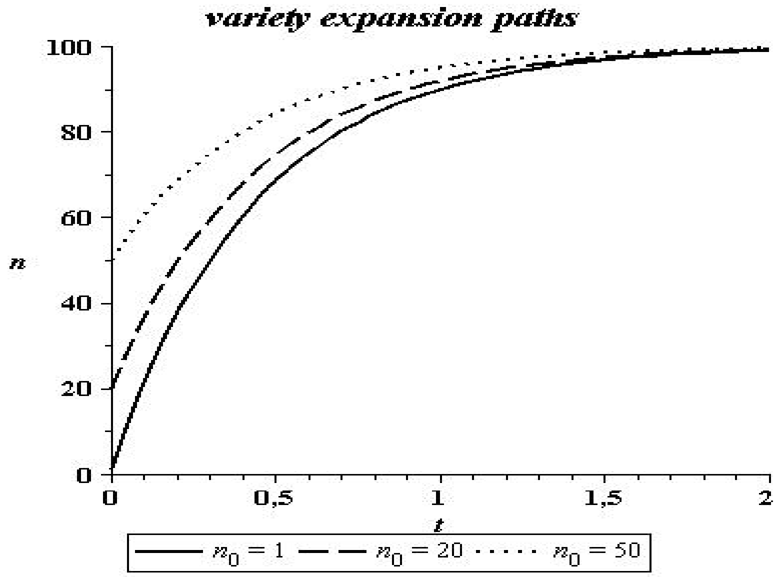

Figure 2: Convergence of different product innovations paths

4.3 Interdependence between both types of innovations

Now consider the general pattern of innovative activities of the monopolist. Putting together the results derived from (10) and (13) one may observe the overall process. The monopolist is allocating the resources between two types of innovations in the following way. At each point in time the rate of product innovations is defined from the current value of future profit from the development of the next boundary product, $V_{i=n(t)}$ and the efficiency of investments themselves, α. Process innovations q_i for each such a new product start only after the introduction of the product i, $t(0)_i$ which is defined by the product innovations process $n(t)$ as its inverse function.

$$t(0)_i = f^{-1}(n^{opt}(t))|_{n(t)=i};$$

$$t(0)_i = -\frac{\sqrt{r^2(r+\beta)^2(r^4+2r^3\beta+r^2\beta^2+2\alpha^2\gamma^2)}+r^2(r+\beta)^2}{\alpha^2\gamma^2 r}\times$$

$$\times \ln(\frac{N-i}{N-n_0}) > 0. \tag{14}$$

This moment is different for all the potential products. The rate of investments into the improvement of production technology for each such product depends on its efficiency of investment, γ_i and decay rate of technology, identical for all products, β. The efficiency of investments is defined as a function of the product's position relative to other products, i, in the products' range N. In this paper it is assumed to reflect the increased complexity of investments in every next product and is defined as (3). Observe that the function (14) specifies explicitly what is the time of the start of development of process innovations for every product i. From the function $t(0)_i$ one may conclude that the rate of introduction of new products slows down as the position of the new product approaches the limit range, N. Then at every such moment in time the investment opportunities for the monopolist are different. These opportunities are lesser for every new product introduced and developed, since the space of products N is the maximal range of different versions of the same basic product which may be introduced into the market. As a result every next product differs from all others by the level of its production technology which may be achieved, $\bar{q}_i$.

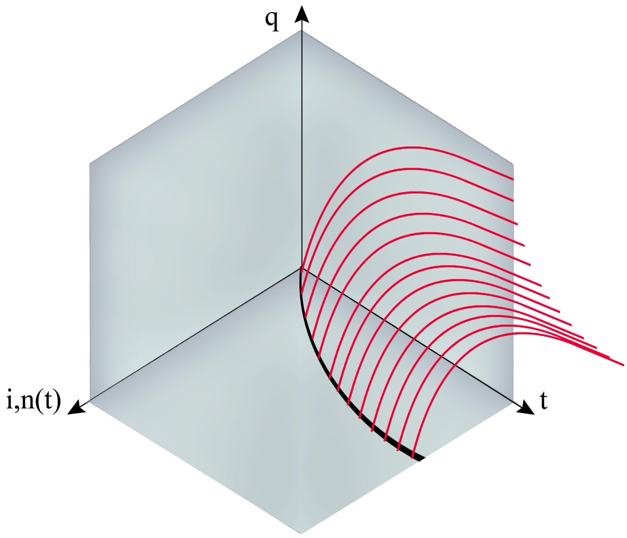

Figure 3: Reconstruction of product and process innovations

The (Figure 3) puts together all the information about the monopolist's behaviour: the solid black line represents the product innovations while red lines are process innovations for every of the introduced products i. It may be seen that every such a process eventually reaches its boundary. Starting from that point the production technology for this product cannot be further refined and the profit from the product i cannot further increase. The profit-seeking behaviour of the monopolist pushes him to the product innovations (i. e. introduction of further new versions of the basic product) and the improvement of technology for other yet underdeveloped products. In infinite time this process reaches its limits when the monopolist introduces all possible versions of the product, $i = N$. At this point the industry as a whole enters the mature stage of its technological cycle and new, fundamental inventions are necessary to boost its growth.

5 Discussion

In this paper the simple and rather stylized model of the monopolist firm engaged in the product and process innovation activities is introduced. Main distinguishing features of the model are:

- The model is fully dynamic and both types of innovations are directly controlled through investments by the monopolist;

- Process innovations directly influence the profit generation and thus product innovations intensity while the latter define only the starting point of process innovations for each new product;

- New products all differ from each other by their production technologies and the level of the process innovations into different products is different.

The first feature is rarely realized in the literature, as usually one or the other type of innovations is assumed to be fixed at some certain level while the other is directly controlled. We argue that the simultaneous control over both types of innovative activities is essential for the optimal management of the multi product monopoly which is a typical situation in many industries. If such a monopolist would only introduce new products but will not develop production technology from its starting level, then this monopolist will not be able to derive new profit from this new product since it has a zero level of process development from the start. Of course the profit is still derived from the product but it is not optimal not to improve the production technology with the goal to cut production costs as this yields an immediate increase in profits without changes in price or production policies.

At the same time process innovations may not be stimulated without the product innovations and their dynamics. If one would assume the range of existing products constant and not changing eventually the monopolist will improve the technology of all the products which are already produced without the development of new ones. Then all the improvements to production technologies of already introduced products, n_0 will start at the same moment of time and the monopolist will not have the criteria for the time management of these innovations. The product innovations process gradually introduces new products and thus the monopolist has time to allocate his efforts between different products. One may conclude that the product innovations process is a generator of new process innovations for every new product although it does not directly influence the subsequent process of technology improvements. This one-way dependence of product innovations from the process innovations is in line with the empirical literature [15].

The next important feature of the model is the explicit heterogeneity of the products which are being introduced into the market by the monopolist. This heterogeneity is covered by the form of the investment efficiency parameter, γ_i. With $\gamma_i = \gamma$ all the new products would be identical in their investment characteristics, while any other form of this function makes them different. In the current setting it is assumed that every next product has the decreased efficiency of investments then the preceding one. As it can be seen from the previous section, this leads to different levels of technology development for all the new products. Every next introduced product has the maximal level of process innovations lower then the preceding one. Due to the increasing weight of maintaining the existing level of technology it is less profitable to develop the production technology of the existing versions of the basic product only and refuse from the introduction and development of new ones even if they are more complicated. It is the increasing burden of maintaining the refined production technology which drives the introduction of new, more complicated and otherwise less profitable products into the market. Some portion of resources is allocated to investments into the introduction of new products, $u(t)$ at any given moment in time. As more and more products are introduced, more resources are needed for maintaining the existing level of technology for all these products and the process of new products introduction slows down. Observe that without the heterogeneity this will not be the case since every next product will essentially be the same as the basic one in terms of its production technology and no additional investments will be then needed for its development.

In such a situation there will be no limits for the introduction of new versions of the basic product while this is obviously not the case. For any given industry there is a limited research capacity for the refinements of the basic product being sold by the industry. In the other case no new industries will be never formed, while the cliometric analysis [5] clearly indicates that the process of the creation of new industries and the decay of the old ones is one of the important components of the technological growth in the modern economy [20].

At last observe that the dynamics of innovations, generated by the suggested stylized model is in line with empirical findings in the field [10], [13]. While the industry is young, there are a lot of opportunities for the introduction of new products and for the development of process innovations associated with them. As the industry matures, these opportunities shrink and it is harder to bring something new into the industry. Unlike the space of ideas which marks more fundamental level of difference between invention and innovation this work concentrates solely on two types of innovations which are always limited in their nature without the underlying process of creation of new knowledge in academia.

The suggested approach allows for a number of immediate and fruitful extensions for analysis of more applied questions. One may model the strategic interactions between several firms instead of a single monopolist in this framework. Such strategic interactions would possibly lead to the endogenous specialization of innovative activities of agents. Another extension is to allow for limited life cycles of new introduced products. This would allow for the analysis of patenting policy efficiency in the framework of heterogeneous innovative products.

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