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Abstract

This paper shows that a hybrid of the sticky-price and sticky-information models of price adjustment is able to deliver a hump-shaped inflation response to monetary shocks without counterfactually implying, as in Mankiw and Reis (2002) or Altig et al. (2005), that individual firms' prices change each quarter (whether they respond or not to the shock). Under the assumption that firms' price-setting decisions are strategically neutral, the inflation response to a transitory shock to the money-supply growth rate is hump-shaped for the hybrid model, whereas it is monotonic for both the sticky-price and sticky-information models. If the shock is permanent, then this response is hump-shaped for the sticky-information and the hybrid models, whereas it is flat for the sticky-price model.

Keywords: hump-shaped impulse response, inflation persistence, Phillips curve, strategic complementarity. JEL Classification Number: E31.

1 Introduction

Most macroeconomic models in the economic literature are unable to reconcile the two following stylized facts, at least not without abandoning rational expectations:

- **Frequency of individual price changes**

  The following quotation by Klenow and Willis (2006b) provides a good summary of the literature on the frequency of price changes: "The recent micro empirical literature [...] finds that nominal prices typically change at least once per year. Bils and Klenow (2004) and Klenow and Kryvtsov (2005) report that U.S. consumer prices change every six months or so, on average. Dyne et al. (2005), surveying a spate of recent studies, conclude that Euro Area prices typically change around once per year. Similarly, Taylor (1999) summarized the earlier evidence as saying prices change once a year on average."

- **Inflation dynamics**

  After a monetary shock, it takes more than one year for prices to completely adjust. The impact of a monetary shock on inflation is not only persistent, it is also hump-shaped. Mankiw (2001) argues that there is a broad consensus that shocks to monetary policy have a delayed and gradual effect on inflation. He refers to the traditional emphasis on the "long and variable lags" of monetary policy and the refrain of central bankers that they need to be forward-looking and respond to inflationary pressures even before inflation arises. He also indicates that it shows up in most empirical work. He refers to specific episodes (Paul Volcker started his historic disinflationary policy in the United States in October 1979, but the big declines in inflation came in 1981 and 1982) and to results from standard vector autoregressions.[1] The large VAR literature on the subject also confirms this finding. For example, Christiano and al. (2005) find that "inflation responds in a hump-shaped fashion, peaking after about two years."

[1] He mentions however that there is some debate about when the maximum impact occurs: Bernanke and Mark Gertler (1995) confirm the conventional wisdom that it occurs after a long lag, finding that monetary shocks have no effect on the price level at all during the twelve months after the shock, whereas Rotemberg and Woodford (1997) find shorter lags, with monetary shocks having a large impact after two quarters.

It may seem difficult to reconcile both stylized facts within the same model: when prices are kept constant less than one year (as the microeconomic stylized fact requires), it may seem difficult to account for the macroeconomic stylized fact that the impact of a monetary shock on inflation persists for more than one year. Taylor (1980) shows, however, that there is endogenous persistence: even if firms change their prices every year, price adjustment will not be complete after one year if price setting is staggered and if there is strategic complementarity in price setting (that is, firms tend to avoid large changes of their prices relative to those of competitors). Chari et al. (2000) respond that staggered price-setting cannot solve the persistence problem. The feature key to their findings is that their dynamic stochastic general equilibrium (DSGE) model yields strategic substitutability rather than strategic complementarity. Woodford (2003) argues that the parameterization of Chari et al. (2000) implies a "considerable degree of strategic substitutability," whereas they would find substantial strategic complementarity if they had taken into account the existence of firm-specific production factors.

Even though assuming a high degree of strategic complementarity may yield a sufficient degree of inflation persistence, Mankiw (2001) notes that a standard sticky-price model is unable to reproduce the hump-shaped response of inflation following a monetary shock. This is why Mankiw and Reis (2002), henceforth MR, have proposed an alternative to the standard sticky-price model: a sticky-information model.[2] The main new feature of their model is that nominal rigidity is not due to the cost of changing price tags and menus, but to the cost of acquiring information in order to re-optimize prices.[3] While the standard sticky-price model features a Calvo staggered price-setting process, motivated by menu costs, in which all firms face the same constant probability of having the opportunity to change prices, MR assume that in each period all firms face the same constant probability of being able to re-optimize current and future prices (henceforth, MR staggered information-updating process). Between two re-optimizations, a firm follows its price plan rather than keeping its price constant (since there is no menu cost, there is no reason to keep prices constant).[4]

[2] Another advantage of the sticky-information model is that in this model, unlike in the standard sticky-price model, anticipated disinflationary policies have no expansionary effects.

[3] As discussed in Ball et al. (2005), imperfect information is a short-cut to the harder task of modeling imperfect information-processing.

[4] In the sticky-information model, firms set their prices at a constant markup over marginal cost. Thus, they do not need to know aggregate variables but only their own

Altig et al. (2005) respond to Chari et al. (2000) by taking into account the existence of firm-specific production factors. This generates enough strategic complementarity to yield realistic dynamics of inflation even though firms re-optimize prices on average only every 1.5 quarters. They respond to the argument of Mankiw (2001) by generating the correct inflation impulse response thanks to a deviation from the standard Calvo staggering price-setting process: they assume that firms that cannot re-optimize index their prices to past inflation rather than keeping them constant.[5] Thus, they implicitly assume that the underlying nominal rigidity is not a menu cost but rather imperfect information on nominal variables or imperfect information-processing.

The models of Mankiw and Reis (2002) and Altig et al. (2005) both have their shortcomings. In particular, these two models yield a hump-shaped

marginal costs. This problem might be solved by assuming that firms do not know their own marginal costs. This is an odd assumption that my hybrid model will inherit through its sticky-information component.

[5]See Minford and Peel (2004), Mash (2005) and Collard and Dellas (2006) for a discussion of the role of this assumption.

inflation response at the cost of assuming that prices change every period[6] (they consider a period to be one quarter, as is usually done in this literature), which does not match the microeconomic stylized fact mentioned above. Since this shortcoming stems from the assumption that there are no menu costs, it is natural to try to fix this problem by introducing some menu costs into the model.[7] Mash (2005) combines the standard sticky-price model with a model of the same family as that of Altig et al. (the model of Christiano et al., 2005, which features indexation but not firm-specific capital). Calibrating the weight of both models to match the microeconomic evidence, he finds that the ability of his hybrid model to match the macroeconomic evidence on inflation persistence is severely compromised. But even if it had matched macroeconomic evidence, the deviation from rational expectations would still be problematic. Mash (2005) argues that if firms could choose their degree of indexation optimally, they would choose a value that corre-

[6] Although there is stickiness, prices change every period except in the special, and in the long run unrealistic, case of zero inflation.

[7] See also Collard and Dellas (2006) "In our view, this [assuming Calvo process without indexation] is the more realistic scenario as the evidence on price setting suggests that firms set their prices infrequently and discretely, and in between price jumps, prices remain constant".

sponds to their belief about the actual persistence of inflation, which would lower persistence over time, converging to a stable long-run value of zero for both actual and perceived persistence.

Collard and Dellas (2006) build a rational expectations model compatible with the two stylized facts mentioned above. They assume a standard Calvo price-staggering process completed with imperfect information. They assume that agents learn about the true aggregate state of the economy gradually, using a Kalman filter based on a set of signals on aggregate variables. They find that short-lived misperceptions of the state of the economy limit initial responses while propagating the shocks over time through the real rigidities.

In this paper I propose another alternative. I build a hybrid model incorporating both Calvo staggered price-setting and MR staggered information-updating, thus combining both underlying sources of nominal rigidity, i.e. menu costs and information costs.[8] Such a hybrid model can deliver the

[8] There are other papers that combine frictions based on menu costs and information costs. As in Collard and Dellas (2006), one of these frictions is, however, usually neither MR's sticky information nor sticky prices. Rotemberg and Woodford (1997) assume a one-period decision lag in a standard sticky-price model. Woodford (2003) extends this setting to an arbitrary number of lags. Kiley (1996) proposes a hybrid of a sticky-price

above-mentioned microeconomic stylized fact (and also yields heterogeneity of inflation expectations[9]). It avoids having the shortcoming of the sticky-information model, since in the hybrid model prices do not change every period.[10] It also delivers the same average duration between a shock and a firm's first response to it as in the sticky-price model, without having to

model (with endogenized probability of price adjustments) and an imperfect-information model, which, however, is different from the sticky-information model. What is usually called the hybrid model in the literature is a model such as that of Gali and Gertler (1999), in which some agents have backward-looking inflation expectations and the rest have rational expectations. Ball (2000) assumes that in forecasting inflation, agents use only an optimal univariate forecasting rule.

[9]Mankiw and al. (2004) argue that the sticky-information model is capable of explaining many features of the observed evolution of both the central tendency and the dispersion of inflation expectations over the past fifty years.

[10]If the money-supply growth rate is zero, then, even in the sticky-information model firms do not change their price every quarter. In this case, the hybrid model would not match the frequency of individual price changes better than the sticky-information model. The case of zero money-supply growth rate is, however, not empirically relevant, since the average money-supply growth rate is usually different from zero. As is often done in this literature (see for example Woodford, 2003), I will use equations linearized around a zero-inflation steady state to discuss cases in which the average long-run inflation is near zero but not necessarily equal to zero.

assume that prices are, on average, kept constant as long as in that model. The reason is that in the hybrid model, to respond to a shock, firms must not only have an opportunity to change their prices but also need to be informed about the shock. For simplicity and for comparability, I stay as close as possible to MR's framework, although this has some drawbacks, such as my model inheriting the partial-equilibrium feature of MR's model.

One challenge for new Keynesian models is to explain data while not assuming too much friction. Thus, adding two kinds of frictions may seem to be counterproductive. However, considering two types of frictions does not necessarily imply a larger overall amount of friction, but may only change the structure of the frictions involved. Moreover, I argue that the sticky information friction is not enough. Something else is necessary (except when inflation is zero) to replicate the empirical fact that prices do not change every period. Adding the Calvo process to MR's model makes it possible to replicate that fact, without necessarily compromising the ability of the model to match the macroeconomic evidence on the inflation response to monetary shocks (depending on the type of shock, it may even improve it).

One could think that a hybrid of the sticky-price and the sticky-information models would yield an average of the macroeconomic performance of the two pure models. If one believes as MR do that the sticky-information model yields better results than the sticky-price model, this would lead to the conclusion that the hybrid model may not fare as well as MR's model in the macroeconomic dimension. I show, however, that the hybrid model is not an average of the two pure models. In the case of strategic neutrality in price-setting,[11] the impulse response of inflation to an unexpected shock to the level of money supply is the same in the sticky-information model as in the sticky-price model.[12] Moreover, their common inflation response is strictly decreasing rather than hump-shaped. However, I will show for this case that the hybrid model is able to generate a hump-shaped inflation response, while both pure models deliver the same strictly decreasing inflation response.

The intuition is the following. In the sticky-price model, firms set their prices (when they have an opportunity to do so) equal to a weighted average

[11] That is, a firm's desired price does not depend on the prices set by its competitors. This is an assumption located between those of Woodford (2003) and Chari et al. (2000).

[12] MR, as well as Keen (2005), notice that the ability of a sticky-information model to produce a long delay in the peak inflation response depends critically on the degree of strategic complementarity.

of future desired prices (a desired price is the price a firm would choose if it faced no nominal rigidity). In the sticky-information model, when firms get new information, they set a price plan in which the price at each date is equal to the expected desired price. In case of strategic neutrality, however, the desired price does not depend on the aggregate price level but only on the money supply.[13] Now consider an unexpected and once-and-for-all change in the money-supply level. In this case, all firms set their prices equal to the new long-term equilibrium level as soon as they have the opportunity to change their prices knowing that the shock has taken place (from now on, I refer to this price adjustment as the "first informed price-adjustment"). They will not need to reset them later on. All firms change their prices by the same amount, determined by the difference between the new money-supply level and the old one. Thus inflation at a given date is proportional to the number of firms that have the opportunity to change their prices and have received the information that the shock has occurred. In both the sticky-price model and the sticky-information model, this number decreases since it is a constant fraction of a decreasing set of firms: in the sticky-price model

[13] MR note that in the case of strategic neutrality the desired price moves only with the money supply: firms adjust their prices immediately upon learning of the change in policy; as a result, inflation responds quickly (much as it does in the sticky-price model).

this is the set of the firms that have not yet had the opportunity to change their prices since the shock, whereas in the sticky-information model it is the set of firms that are not yet informed. In the hybrid case, firms setting their first informed prices today are either firms already informed in the last period and receiving the opportunity to change prices today, or firms that were not yet informed in the last period but are receiving information today as well as the opportunity to change prices. The key point is that the number of firms already informed in the last period and receiving the opportunity to change prices today is a hump-shaped function of time. Immediately after the shock, this set is small because only very few firms are informed. After a sufficiently long time, almost all firms are informed, but they have also almost all had the opportunity to change their prices, so the set is small again. In between, this set reaches a maximum.

This example shows that the hybrid model is not simply a weighted average of the two pure models, but can be superior to both. I am not arguing, however, that the sticky-information model cannot deliver a hump-shaped inflation response. Assuming strategic complementarity, MR have found that the sticky-information model delivers a hump-shaped response.[14] I find that this is true even with strategic neutrality if a permanent shock occurs to the money-supply growth rate rather than to the money-supply level. In this case, the inflation responses are qualitatively similar in the hybrid and in the sticky-information models, and both are clearly different from the sticky-price model response. Thus, assuming strategic neutrality in price setting, the hybrid and sticky-information models yield, qualitatively, the same inflation impulse response to a permanent shock to the money-supply growth rate. When this shock is more transitory in nature, then the inflation impulse response of the sticky-information model tends to lose its hump, whereas it stays hump-shaped in the hybrid model: if the shock to the growth rate is

[14] The following intuition explains why in the case of strategic complementarity the responses differ in the sticky-price and the sticky-information models. In the sticky-price model, the inflation response is maximal when the shock occurs, since the incentive to change prices is greatest at this time (later, the economy will be closer to its new equilibrium). In the sticky-information model firms do not need to overshoot their price changes in order to avoid being stuck in the future with prices that are out of line with those of competitors since they set a price plan rather than a price level (they can plan to increase their prices later when more firms are informed of the shock). The inflation response increases until a peak is reached, after which it converges toward zero as more and more firms are informed and most of the adjustment has already taken place.

completely transitory, then it is equivalent to a shock to the level, and the hybrid model delivers a hump-shaped inflation response whereas the two pure models deliver the same strictly decreasing response.

In this paper I assume strategic neutrality in price setting for three reasons. First, there is currently some debate about what degree of strategic complementarity is realistic. As mentioned above, the strategic-neutrality assumption is compatible with the literature. Second, shutting down the strategic-complementarity channel makes it possible to show that a hump-shaped inflation response can be generated even in the absence of strategic complementarity. The strategic-neutrality assumption also makes it easier to show in which sense the hybrid model is different from an average of the two pure models. The third reason is that strategic neutrality is the only case (except in the extreme cases in which the hybrid model reduces to one of the two pure models) in which the hybrid model yields an exact closed-form solution. As a first pass, it therefore seems reasonable to assume strategic neutrality since it is compatible with the literature, yields interesting results and is easier to compute. This however has a cost. Ball and Romer (1990) have shown that nominal frictions alone are not enough to cause business fluctuations generating large welfare losses. This suggests that assuming strategic neutrality would imply large menu costs or small welfare loss. These issues are difficult to discuss in my model since, as in MR, neither menu and information costs nor the utility function are explicit. Another issue is that strategic complementarity is likely to be necessary to get endogenous persistence. Thus, concerning the macro stylized facts, I will focus on the hump-shaped path of the inflation impulse response. I leave for further research the task of explaining endogenous persistence in a hybrid model with α smaller than 1.

On the way to computing inflation impulse responses, I derive the Phillips curve for the hybrid model without, at this stage, assuming strategic neutrality. One novel feature of this Phillips curve is that it involves a new kind of expectation operator. Since all firms are not perfectly informed, it is not surprising that the expectation $\overline{E}_t\left(\pi_{t+1}\right)$ of next period inflation which enters the Phillips curve is not the expectation $E_t\left(\pi_{t+1}\right)$ based on the best knowledge available at time t. However, $\overline{E}_t\left(\pi_{t+1}\right)$ is not the average of aggregate inflation expectations but the average of the firms' expectations about their own prices increases.

Dupor et al. (2006) and Klenow and Willis (2006) discuss hybrid models similar to the one in this paper (they propose, however, general equilibrium models). Dupor et al. (2006) find that both sticky prices and sticky information play an important role for inflation dynamics. Their work is more empirically oriented than mine, whereas I stay closer to MR's framework and find closed-form solutions for the impulse response of inflation. Klenow and Willis (2006) focus on making the link between their hybrid model and new microeconomic evidence. They find "modest support" for the sticky-information model and the hybrid model. Paustian and Pytlarczyky (2006) also develop a model merging sticky prices and sticky information, but in their model no firm faces both frictions: some firms face sticky prices, while others face sticky information. This setup allows them to assess the importance of sticky prices versus sticky information in a nested model that reduces to either specification in the extreme cases. They find that the data favors the sticky-price model over the sticky-information model. Besides these three papers, there are several empirical papers comparing the sticky-price and the sticky-information models without actually building a theoretical hybrid model. Their aim is usually to choose the best among the two or more models.[15]

The plan of this paper is as follows. Section 2 discusses the assumptions about when firms receive information or an opportunity to change their prices. Probability distributions resulting from merging the Calvo staggered price-setting process and the MR staggered information-updating process are computed (for example, the distribution of the time of the first informed price-adjustment after a shock). This time-dependent process must be imbedded in an economic environment in order to yield the magnitude of price changes (rather than their timing only). Section 3 presents the basic equations of this environment, staying as close as possible to MR's framework. Section 4 derives the Phillips curve. Section 5 focuses on the case of strategic neutrality in price setting and presents the inflation impulse response in the cases of three unanticipated shocks: a transitory shock to the money-supply growth rate (or equivalently a permanent shock to the money-supply level), a permanent shock to the money-supply growth rate, and the intermediate case of a persistent but not permanent shock to the money-supply growth rate. Section 6 presents concluding remarks.

[15] For example: Keen (2005), Korenok (2005), Korenok and Svanson (2006), Laforte (2005) and Trabandt (2006).

2 The hybrid price-setting and information-updating process

This section merges Calvo staggered price-setting and MR staggered information-updating and computes two relevant distributions. The way I merge these two processes is very simple, perhaps the most obvious way to model a firm facing both kinds of nominal rigidities (menu costs and imperfect information).

The Calvo sticky-price process assumes that each firm is always perfectly informed but each period it faces a constant probability λ of being exogenously given an opportunity to change its price (prices are kept constant between two such opportunities). One possible implicit story behind this assumption is that firms are hit by random idiosyncratic shocks, and because of high menu costs, they change their prices only when such a shock happens (assuming that idiosyncratic shocks are more important to firms than monetary shocks).

MR's staggered information-updating assumes that each firm can change its price every period at no cost, but each period it faces a constant probability γ of being given updated information exogenously (between two such opportunities, prices follow the old plan based on outdated information[16]). One possible implicit story behind this assumption is that firms receive information randomly[17] or that they have to make a report on the economic situation at random points in time (for reasons not connected to price setting) and may then use this information for the next time they set prices.

My hybrid process is based upon the assumption that each period a firm faces a probability λ of being given an opportunity to change its price and a probability γ of being given updated information. These two events are assumed to be independent (this is the case if opportunities to set prices are determined by random idiosyncratic shocks, news arrives randomly, and these two random processes are independent).[18] As in the sticky-price model, a firm keeps its price constant between two opportunities to change prices.

[16] MR assume that, between two re-optimizations, a firm does not know or does not take account of such information as how much it has sold.

[17] For example Carroll (2003) assumes that in any given period each individual faces a constant probability of reading the latest forecast in an article (individuals who do not encounter an article about inflation simply continue to believe the last forecast they read).

[18] Here I do not consider the case in which price setting and information updating are state dependent. In this case, the two processes may not be independent. For example, if firms choose to always update their information when they have an opportunity to change their prices, then the hybrid model would in fact be the same as the sticky-price model.

Firms remember past information when they are given the opportunity to reset prices, and so choose a price based on their last-updated information.

This hybrid model encompasses the two pure models. It yields the sticky-price model if $\gamma = 1$, and yields the sticky-information model if $\lambda = 1$. The possibility that λ be different from 1 is the only difference between my hybrid model and MR's sticky-information model since, as discussed in section 3, I otherwise preserve their economic environment (except for focusing on the strategic-neutrality case in section 5). Since I have assumed that the two pure processes are independent, any interaction between these processes will come only from the fact that a firm can respond to a shock in the hybrid model only if it is both aware of the shock and has an opportunity to change its prices after the shock has occurred.

2.1 Probability that a current price was set j periods ago based on information last updated $j+k$ periods ago

Let $\Omega_{j,k}$ be the probability that the price of a firm at time t was set at $t-j$ (and stayed constant since then) based on information last updated at $t-j-k$. The firm faced a probability λ of being able to change its price at $t-j$, a probability $(1-\lambda)^j$ of not being able to change its prices during the j periods until t. Thus, the probability that at t a firm had its last opportunity to change its price at $t-j$ is $\lambda(1-\lambda)^j$. Similarly, the probability that the information available at $t-j$ was last updated at $t-j-k$ is equal to $\gamma(1-\gamma)^k$: the probability γ of updating information at $t-j-k$ times the probability $(1-\gamma)^k$ of not being able to update information during the k periods until $t-j$. Since both processes are independent, the probability that the current price of a firm was set j periods ago based on information last updated $j+k$ periods ago is:

$$\Omega_{j,k} = \lambda\gamma(1-\lambda)^j(1-\gamma)^k \quad . \tag{2.1}$$

Because of the law of large numbers, $\Omega_{j,k}$ is also the proportion of firms in this situation.

For a time unit of a quarter, MR choose $\lambda = 0.25$ for the sticky-price model, and $\gamma = 0.25$ for the sticky-information model, because in each case a firm on average makes an adjustment once a year. How should λ and γ be chosen in the hybrid model? One possibility is $\lambda = 0.25$ and $\gamma = 0.25$. This, however, implies that the duration between a shock and the first informed price-adjustment is more than one year since overall nominal rigidities have been increased. Let's compute under which condition on λ and γ the average adjustment interval is equal to one year.

2.2 Lag between a shock and the first informed price-adjustment

Let's compute the probability Ξ_t that a firm sets its first informed price-adjustment t periods after the shock (the nature of this shock is not important here, since the price response is not computed in this section). In some particular settings (to be discussed below), this probability will be of particular importance since the inflation response will be proportional to it. Let's assume that an unanticipated monetary shock occurs at the beginning of the period $t = 0$, with some firms possibly already informed of this shock at $t = 0$ before setting their prices (or equivalently, the shock occurs at the end of period $t = -1$ when firms have already set their prices for period $t = -1$): at time $t = 0$ (and possibly later on as well) money supply differs from what all the firms previously expected. A firm that set its first informed price adjustment at t may have been first informed of the shock at $t - j$ (the probability that this happens is $\gamma(1-\gamma)^{t-j}$), and then had to wait j periods to receive an opportunity to change its price (the probability that this happens is $\lambda(1-\lambda)^{j}$). Thus, the probability that a firm sets its first informed price adjustment at t while having been first informed of the shock at $t-j$ is $\lambda\gamma(1-\lambda)^{j}(1-\gamma)^{t-j} = \Omega_{j,t-j}$. Since j could be anywhere between 0 and t, the probability Ξ_t that a given firm sets its first informed price adjustment at t is the sum over $0 \leq j \leq t$ of $\Omega_{j,t-j}$:

$$\Xi_t = \sum_{j=0}^{t} \Omega_{j,t-j} = \sum_{j=0}^{t} \lambda\gamma(1-\lambda)^{j}(1-\gamma)^{t-j} = \begin{cases} \lambda\gamma\frac{(1-\lambda)^{t+1}-(1-\gamma)^{t+1}}{\gamma-\lambda} & \text{if } \lambda \neq \gamma \\ \lambda^2(t+1)(1-\lambda)^{t} & \text{if } \lambda = \gamma \end{cases}. \tag{2.2}$$

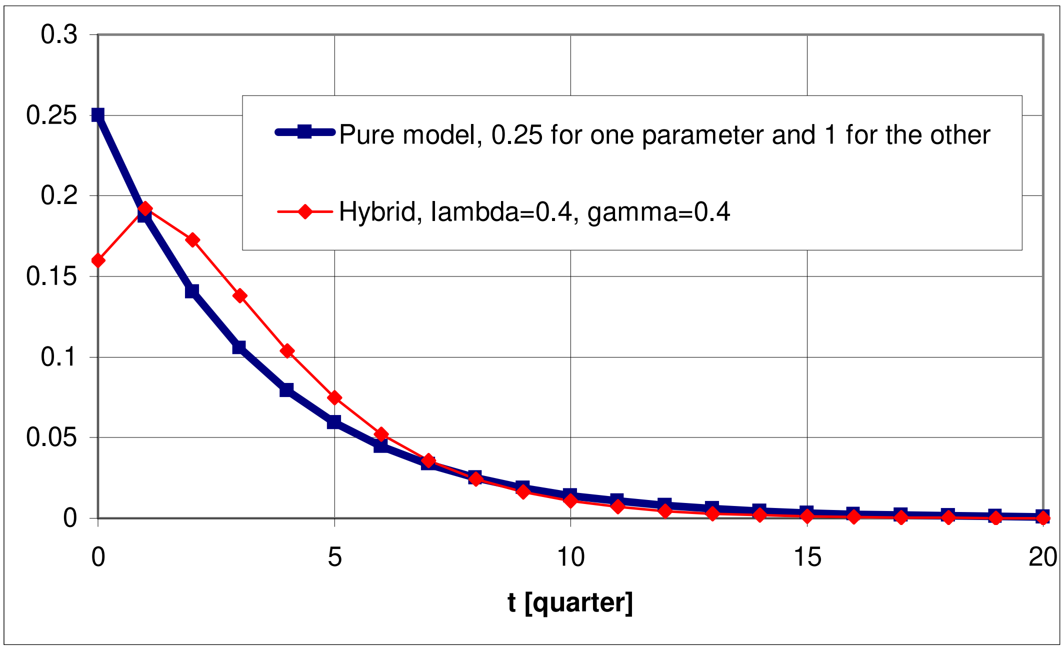
Except for the pure cases $\lambda = 1$ or $\gamma = 1$, the probability Ξ_t is a hump-shaped function of t. The maximum occurs at:

$$t_{\max} = \begin{cases} \frac{\ln\left(\frac{\ln(1-\lambda)}{\ln(1-\gamma)}\right)}{\ln\left(\frac{1-\gamma}{1-\lambda}\right)} - 1 \text{ if } \lambda \neq \gamma \text{ and } \lambda \neq 1 \neq \gamma \\ -\frac{1}{\ln(1-\lambda)} - 1 \quad \text{if } \lambda = \gamma \\ 0 \quad \text{if } \lambda = 1 \text{ or } \gamma = 1 \end{cases}. \tag{2.3}$$

The expectation of the distribution Ξ_t is $\frac{1}{\lambda} + \frac{1}{\gamma} - 2$. Taking into account that it is possible to have an informed price-adjustment at $t = 0$, a first informed adjustment at t implies a duration before the first informed adjustment of $t+1$. Thus, the average duration is $\frac{1}{\lambda}+\frac{1}{\gamma}-1$. In the sticky-price model, $\gamma = 1$ and this average duration is $\frac{1}{\lambda}$ (and $\lambda = 0.25$ yields an average duration of four quarters). Similarly, this duration is $\frac{1}{\gamma}$ in the sticky-information model (and $\gamma = 0.25$ yields an average duration of four quarters). In the hybrid case, an average duration of four quarters implies the following condition: $\frac{1}{\lambda}+\frac{1}{\gamma}=5$. Assuming that $\lambda=\gamma$, this yields $\lambda = 0.4 = \gamma$.

For an average duration of four quarters (in the hybrid case $\lambda=\gamma$ is also assumed), Figure 1 shows the distributions of time t of the first informed price-adjustment after a shock occurring at the beginning of period 0.

Figure 1: Distribution of time at which the first informed price-adjustment occurs (average duration: 1 year)



Two curves are shown in this figure. The pure sticky-price model and the pure sticky-information model yield the same strictly decreasing curve. The hybrid model with $\lambda = 0.4 = \gamma$ yields a maximum one quarter after the shock.[19]

The distribution for the hybrid curve is hump-shaped (at least for these

[19]Notice that even for the hybrid model, the curve may not be hump-shaped (in a discrete-time representation) if this hybrid is sufficiently close to a pure case.

parameter values), and is thus qualitatively different from the two pure models, which have the same decreasing curve. The intuition is the following. In both pure models the set of firms that have not yet had an informed price-adjustment decreases over time. Since the firms that set their first informed price-adjustment at a given date is a fixed fraction of this set, their number also decreases. In the hybrid model, the firms that set their first informed price-adjustment at a given date were either informed of the shock beforehand, or not even informed. The set of uninformed firms decreases over time. But the set of informed firms that have not yet had the opportunity to change their prices increases, in the hybrid model, at the beginning (at the very beginning it is empty since no firm is informed) and decreases only after having reached a maximum (in the long term, it decreases toward emptiness since the proportion of firms that have not yet had an informed price-adjustment converges toward zero).

3 The economic environment

The last section has only discussed the probability of some events assuming a constant probability λ of receiving an opportunity to change prices and an independent probability γ of updating information. To study the dynamics of inflation, however, it is necessary to know not only when firms change their prices, but also by how much. The price a firm wants to set depends on the economic environment in which the price-setting and information-updating process is imbedded. I follow the simple framework of MR, in which firms set their prices equal to a weighted average of current and future desired prices.

As in MR, I assume:

$$p_t^* = p_t + \alpha y_t \ , \tag{3.1}$$

$$y_t = m_t - p_t \ . \tag{3.2}$$

Equation (3.1) says that a firm's desired price p_t^* depends on the overall price level p_t and the output gap y_t (where all variables are expressed in logs and potential output is normalized to zero). In periods of booms, marginal costs rise and each firm would like to raise its relative price. This equation could be derived from the firm's profit-maximization problem (although the real marginal cost of the firm would appear rather than the output gap) and α could be expressed in terms of deep parameters. Combining equations

(3.2) and (3.1) yields $p_t^* = (1 - \alpha) p_t + \alpha m_t$. Therefore $\alpha = 1$ corresponds to the strategic-neutrality case.

Equation (3.2) expresses aggregate demand as a function of p_t and an exogenous variable m_t, which can be interpreted as the log of money supply or, more broadly, as incorporating the many other variables that shift aggregate demand.[20] More generally, aggregate demand would also depend on the nominal interest rate. Here, however, I follow the simple approach of MR and exclude this possibility.

Let $x_{t,k}$ be the price actually set at time t by a firm receiving the opportunity to set its price at time t and holding information updated for the last time at time $t - k$. This firm sets $x_{t,k}$ equal to an average of its expected desired prices for time t and later, weighted by the probability that the price set at time t will not have changed:

$$x_{t,k} = \lambda \sum_{j=0}^{\infty} \left[(1 - \lambda)^j E_{t-k} \left(p_{t+j}^* \right) \right], \tag{3.3}$$

where E_{t-k} is the expectation of firms with information last updated at time $t - k$ (or equivalently the expectation of the best-informed firms at $t - k$).

[20] This equation is used to derive the impulse response function, but not the Phillips curve.

In the sticky-price case, a firm is always informed, thus $k = 0$ and equation (3.3) becomes $x_{t,k=0} = \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_t \left(p^*_{t+j} \right) \right]$. In the sticky-information case, $\lambda = 1$, and equation (3.3) is rewritten as $x_{t,k} = E_{t-k} \left(p^*_t \right)$.[21]

The aggregate price level is the average of prices set by the various cohorts of firms, weighted by the proportion of firms in each cohort:

$$p_t = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \Omega_{j,k} x_{t-j,k} \quad \text{where} \quad \Omega_{j,k} = \lambda\gamma (1-\lambda)^j (1-\gamma)^k \ . \qquad (3.4)$$

In the sticky-price case ($\gamma = 1$) equation (3.4) becomes $p_t = \lambda \sum_{j=0}^{\infty} (1-\lambda)^j x_{t-j,k=0}$.

In the sticky-information case ($\lambda = 1$) equation (3.4) becomes $p_t = \gamma \sum_{k=0}^{\infty} (1-\gamma)^k x_{t,k}$.

4 The Phillips curve

The Phillips curve yields a relationship between prices and the output gap. This is an intermediate stage before computing the inflation impulse re-

[21] $(1-\lambda)^j$ is, in fact, undetermined when $\lambda = 1$ and $j = 0$. Actually, this is calculated assuming λ infinitesimally close to 1 but not equal to 1. The resulting formula is indeed the same as the one used by Mankiw and Reis (2002) for the sticky-information case. A similar comment applies in other places in this paper.

sponse, since plugging equation (3.2) into the Phillips curve yields a relationship between prices and money supply. This section presents the Phillips curve without yet assuming strategic neutrality.

Appendix I shows that, after some tedious algebra, equations (3.1), (3.3) and (3.4) yield the following Phillips curve:

$$\pi_t - \overline{E}_t\left(\pi_{t+1}\right) = \frac{\lambda^2}{1-\lambda}\left[\alpha y_t + \varepsilon_t\left(p_t^*\right)\right], \tag{4.1}$$

where ε_t is an operator that takes the sum of expectation errors made at t (i.e. the average of expectation errors made by various cohorts weighted by the number of firms in each cohort). In the hybrid model, the sum of expectation errors made at t on p_t^*, is equal to:

$$\varepsilon_t\left(p_t^*\right) = \gamma\sum_{k=1}^{\infty}\left(1-\gamma\right)^k\left[E_{t-k}\left(p_t^*\right) - p_t^*\right]. \tag{4.2}$$

The term $\overline{E}_t\left(\pi_{t+1}\right)$ in equation (4.1) is defined as follows:

$$\overline{E}_t\left(\pi_{t+1}\right) = \lambda\left[\gamma\sum_{k=0}^{\infty}\left(1-\gamma\right)^k x_{t+1,k+1} - p_t\right]. \tag{4.3}$$

The expectation operator $\overline{E}_t$ obviously differs from the expectation E_t established on the basis of the best information available at t (or equivalently, made by the best-informed agents at t), since there is no reason why only the expectations of the best-informed firms should matter while the inflation expectations of firms setting their price at time based on old information would be completely neglected. What is perhaps more surprising is that the relevant expectation operator is not simply an average of the various inflation expectations.[22] Equation (4.3) gives the inflation expected to prevail at time $t+1$ when the aggregate price level at $t+1$ is expected to be $\gamma \sum_{k=0}^{\infty} (1-\gamma)^k x_{t+1,k+1}$ while the aggregate price level at time t is known to be p_t. One interpretation is the following: make a survey asking all firms[23] by how much they expect to increase their own prices from t to $t+1$ (don't ask them about their expectations for the increase of the aggregate price level); then $\overline{E}_t(\pi_{t+1})$ is the sum of these expected price increases. The proof is the following. Each firm will answer that it faces a probability $1-\lambda$ of keeping its price constant, and a probability λ of being able to reset its price. Thus, its expected increase of its own price is λ times the difference between the price it expects to set if it is able to reset it and its current price (all firms

[22] The average inflation expectation is $\lambda \left[\gamma \sum_{k=0}^{\infty} (1-\gamma)^k E_{t-k}(p_{t+1}) - p_t \right]$. See section 5 for a specific example in which the operator $\overline{E}_t(\pi_{t+1})$ is shown to be different from the average inflation expectations.

[23] Ask all firms once they know if they can reset their price at time t or not (firms that cannot reset their prices at time t are also to be included in the survey).

are aware of their current prices). Summing all these answers yields equation (4.3): λ times the difference between the sum $\gamma\sum_{k=0}^{\infty}(1-\gamma)^k x_{t+1,k+1}$ of all answers about the prices firms would expect to set at $t+1$ (if they can) and the sum p_t of their current prices.

It is easy to verify that if $\gamma = 1$, equation (4.1) boils down to the sticky-price Phillips curve $\pi_t - E_t(\pi_{t+1}) = \frac{\lambda^2}{1-\lambda}\alpha y_t$ (given, for example, in MR). In fact, when $\gamma = 1$, then $\varepsilon_t(p_t^*) = 0$ (i.e. there are no expectational errors at t on p_t^* since all firms are informed) and $\overline{E}_t(\pi_{t+1}) = E_t(\pi_{t+1})$ (since all firms have the same information set). In the other pure case, if $\lambda = 1$, then $\overline{E}_t(\pi_{t+1})$ disappears (firms do not need to take account of future inflation when setting their current prices, since they can change their prices in every period), and the Phillips curve is given by $\alpha y_t + \varepsilon_t(p_t^*) = 0$, which can be shown to be equivalent to the Phillips curve computed by MR for the sticky-information model.

The hybrid Phillips curve could be compared with the Phillips curves of other models. Three models would be particularly interesting in this respect. Woodford (2003) assumes that information-updating does not occur with a constant probability but is simply delayed by a fixed number of periods (thus extending a model he wrote with Rotemberg, in which the delay is always one period). Altig et al. (2005) assume that between two re-optimizations firms follow simple (non-optimal) indexation rules. Gali and Gertler (1999) assume that a fraction of firms set prices according to a rule of thumb (they index their prices according to last-period inflation) while the other firms have rational expectations.

Some differences between their Phillips curves and mine are due to a difference in frameworks. But even after adapting their models to MR's framework (this involves setting the preference for the present to zero and assuming that the real marginal cost is proportional to output) important differences remain. In the Rotemberg-Woodford model, when a firm sets its price for a given date, it always perfectly anticipates the aggregate price-level that will prevail at that date because all other firms will be setting their prices for that date on the basis of the same common information set. This is not the case in my hybrid model. An important difference between my hybrid model, on one hand, and the Gali-Gertler model or the model of Altig et al. (2005), on the other hand, is that their Phillips curves do not involve past expectations whereas my hybrid model does (it inherits this feature from the sticky-information model).

5 The inflation impulse response in the strategic-neutrality case

Strategic neutrality in price setting means that a firm's desired price does not depend on the prices set by competitors. After examining the relevance of this assumption, I discuss the inflation impulse response to three types of unanticipated shocks: i) a transitory shock to the money-supply growth rate or, equivalently, a permanent shock to the money-supply level (which is experiment 1 in MR), ii) a permanent shock to the money-supply growth rate (experiment 2 in MR), and iii) the intermediate case of a persistent but not permanent shock to the money-supply growth rate.

5.1 Strategic neutrality

How can the desired price of a firm be independent of the prices set by competitors? A first answer would be that in the standard monopolistic-competition model à la Dixit-Stiglitz, firms set their prices at a constant markup over the marginal cost. Thus the prices of competitors do not directly influence the desired price. This answer, however, does not take into account the possibility that the prices of competitors may indirectly influence the desired price, through their impact on marginal costs.

During the staggered price-setting process, a firm that adjusts its prices is motivated by several incentives. First, it might want to adjust less than what it would if all other firms had the opportunity to adjust, because it faces competition from those firms that have not adjusted yet. This strategic complementarity in price setting may hold even if firms simply choose a markup that is a constant proportion of the marginal cost without taking prices set by competitors directly into account. For example, assume there is a reduction in money supply. Then a firm that adjusts its prices downward, while some other firms have not yet done so, will face greater demand than otherwise. If marginal productivity decreases with output (or if there are some firm-specific factors), then marginal costs increase with output (assuming that the prices of production factors bought on the economy-wide market stay constant), and, even if the firm faced no nominal rigidities, it would want to set its price higher than it would if other firms didn't face nominal rigidities in order not to be overburdened by a overly high demand.

However, the price of production factors bought on the economy-wide market need not stay constant. This can lead the firm to decrease its prices by more than needed to reach the frictionless equilibrium (strategic substitutability in price setting). Assuming that the real wage is pro-cyclical, it will be lower during the transitory recession generated by the reduction in money supply. This tends to decrease marginal cost and thus lead to lower prices. If the complementarity and the substitutability incentives cancel each other out, there is strategic neutrality in price setting: a firm would choose to set its prices independently of the aggregate price level.

Whether there is complementarity or substitutability in price-setting is a much debated issue in the literature. Using a dynamic stochastic general equilibrium (DSGE) model with sticky prices, Chari et al. (2000) find that strategic substitutability arises from realistic deep-parameter values of their model. On the other hand, Woodford (2003) argues that Chari and al. would find strategic complementarity in firms' price decisions if they had taken into account the existence of firm-specific production factors. Incorporating sticky information into different DSGE models, Keen (2005) finds strategic substitutability, whereas Trabandt (2006) finds strategic complementarity. My reading of the current state of this debate is that it is not settled yet, and that the middle ground of assuming strategic neutrality would be compatible with the literature.

5.2 Permanent shocks to the money-supply level

Let's consider the case of a shock δ to the level of money supply. Appendix II shows that in this case, for $t \geq 0$, the aggregate price level p_t in the hybrid model is given by:

$$\begin{aligned} p_t &= m_t + \delta \left[\frac{\lambda}{\gamma - \lambda} (1-\gamma)^{t+2} - \frac{\gamma}{\gamma - \lambda} (1-\lambda)^{t+2} \right] \quad \text{for } \lambda \neq \gamma \qquad (6.1) \\ p_t &= m_t - \delta (1-\lambda)^{t+1} \left[1 + (t+1)\lambda \right] \quad \text{for } \lambda = \gamma \ . \end{aligned}$$

Whereas, in the sticky-price and the sticky-information models, p_t is respectively given by:

$$p_{t,\gamma=1} = m_t - \delta (1-\lambda)^{t+1} \qquad \text{for } \gamma = 1$$

$$p_{t,\lambda=1} = m_t - \delta (1-\gamma)^{t+1} \qquad \text{for } \lambda = 1.$$

Thus the pure sticky-price and the pure sticky-information cases yield the same price dynamics if they are calibrated to get the same average duration between two re-optimizations, that is, if the parameter λ used in the sticky-price case is the same as the parameter γ used in the sticky-information case.

If $\lambda \neq \gamma$, the price level in the hybrid case p_t can be expressed as a linear combination of the two pure cases' price levels:

$p_t = \frac{\gamma(1-\lambda)p_{t,\gamma=1} - \lambda(1-\gamma)p_{t,\lambda=1}}{\gamma-\lambda}$ where the weights $\frac{\gamma(1-\lambda)}{\gamma-\lambda}$ and $-\frac{\lambda(1-\gamma)}{\gamma-\lambda}$ have opposite signs (thus, it is a linear combination, but not a weighted average although the sum is 1). Or equivalently: $p_t = p_{t,\gamma=1} + \lambda\frac{1-\gamma}{\lambda-\gamma}\left(p_{t,\lambda=1} - p_{t,\gamma=1}\right)$.

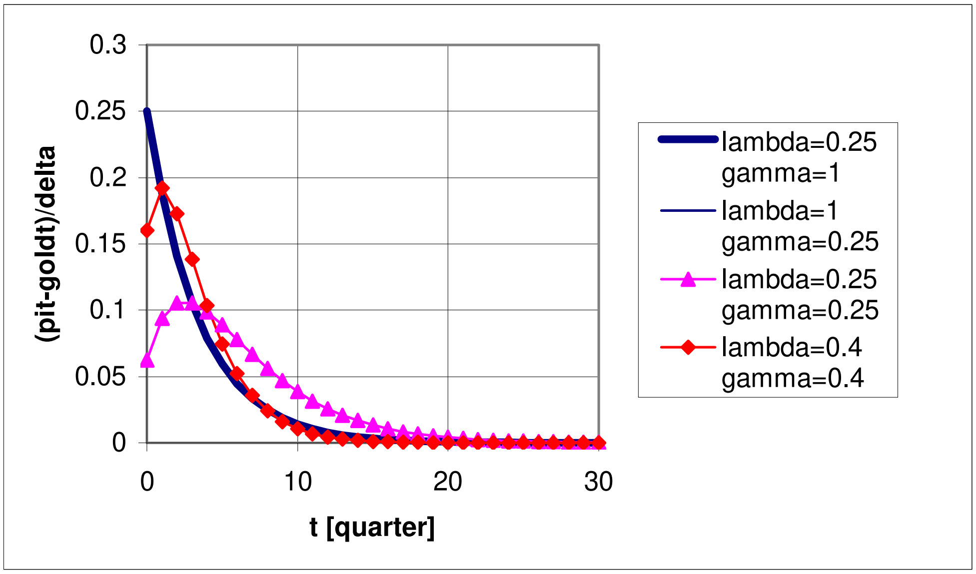
Equation (6.1) implies that the inflation response for $t \geq 0$ is given by:

$$\pi_t - g = \begin{cases} \delta\ \Xi_t & \text{for } \ t \geq 0 \\ 0 & \text{for } \ t < 0 \end{cases}, \tag{6.2}$$

where g is the money-supply growth rate for $t \neq 0$.

The inflation response after the shock is proportional to the probability Ξ_t given in equation (2.2) that a firm sets its first informed price-adjustment at t. Thus Figure 1 already shows the impulse response of inflation for different configurations of the parameters representing the degree of price and information stickiness. This figure is reproduced below (a curve giving the impulse response for $\lambda = 0.25 = \gamma$ is added). As discussed in section 2, setting λ and γ equal to 0.4 yields the same average duration before the first informed adjustment as in the sticky-price model ($\lambda = 0.25$ & $\gamma = 1$) or the sticky-information model ($\lambda = 1$ & $\gamma = 0.25$). On the other hand, the hybrid model calibrated at $\lambda = 0.25 = \gamma$ generates a greater degree of nominal rigidity.

Figure 2: Inflation impulse response to a permanent shock to the money-supply level



With these parameter values, the sticky-information model yields the same inflation response as the sticky-price model. Moreover, after the initial jump, it decreases monotonically, whereas the impulse response in the hybrid model is hump-shaped. When for the hybrid model the probability of being informed is the same as the probability of having the opportunity to reset prices, and probabilities are calibrated as in MR to yield an average duration of one year before an informed price-adjustment takes place, then the maximum occurs one quarter after the shock. If the overall nominal rigidities are larger ($\lambda = 0.25 = \gamma$), then the maximum response of inflation occurs later.

The intuition as to why inflation is proportional to the probability of setting a first informed price-adjustment is easiest to understand if the level of money supply is constant before the shock, is modified by the shock, and remains at its new level after the shock. In this scenario, the price set after the shock by an informed firm is at its long-term equilibrium. In this case firms change their prices only once: they adjust to the long-term equilibrium as soon as they can set their first informed price-adjustments. Since all firms had the same price before the shock and end up with the same new long-term equilibrium, the inflation response is proportional to the number of first informed price-adjustments.

The impulse response of inflation computed for the special case of a zero money-supply growth rate remains the same for any other constant growth rate. To see this, consider equation (II.2):

$$p_t = (1-\lambda)\, p_{t-1} + \lambda^2 \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \left[(1-\lambda)^j\, E_{t-k} \left((1-\alpha)\, p_{t+j} + \alpha m_{t+j} \right) \right] \right].$$

If the money-supply dynamics are of the form $\tilde{\tilde{m}}_t = \tilde{\tilde{m}}_{-1} + \tilde{\tilde{g}}\,(t+1)$, then $p_t = m_t$ in the absence of shocks. Therefore, equation (II.2) can be rewritten replacing p by $\tilde{p} = p - \tilde{\tilde{m}}$ and m by $\tilde{m} = m - \tilde{\tilde{m}}$. Choosing $\tilde{\tilde{m}}_{-1} = m_{-1}$ and

$\widetilde{\widetilde{g}} = g$, it follows that $\widetilde{\widetilde{m}}_t = m_{new,t}$ (where $m_{new,t}$ extends to all time the affine relationship between t and m that holds after the shock), and thus $\widetilde{m}_t = 0$ after the shock.

If we know how to solve (II.2) for the price dynamics in the case where money supply is constant after the shock, and want to know the price dynamics when money supply grows at a constant rate after the shock, all we need to do is subtract $\widetilde{m}_t = m_{new,t}$ from the p and m variables to be in the setting in which money supply is zero after the shock, solve for the dynamics of $\widetilde{p}$, and add back $m_{new,t}$. Adding back $m_{new,t}$ will change the aggregate price level but not the difference between inflation with the shock and inflation without the shock (the constant $\widetilde{\widetilde{m}}_{-1}$ disappears when inflation is computed, and the term $\widetilde{\widetilde{g}}$ disappears when the difference between inflation with and without the shock is computed). The point is that, and this is true in the general case since the argument is based on equation (II.2), the inflation impulse response is invariant to a transformation of the money-supply dynamics consisting in adding a money-supply component with a constant growth rate.

Let's compute $\overline{E}_t\left(\pi_{t+1}\right)$ for $t = 0$ in the simple case where the level of money supply is 0 before the shock, is modified to δ by the shock, and remains at δ after the shock. Firms unaware of the shock expect to keep their prices constant even if they are allowed to reset them. Similarly, firms aware of the shock, and having had the opportunity to make an informed price-adjustment, have fully adjusted already, and thus expect to keep their prices constant even if they could reset prices in the future (expectations are polled at a point in the period $t = 0$ when each firm knows if it can reset its price at $t = 0$ or not). The only firms expecting to change their own prices are those that are informed of the shock but haven't had an opportunity for an informed price-adjustment. These firms, which at $t = 0$ are a proportion $\gamma(1-\lambda)$ of all firms, expect to increase their own prices by an amount δ if they have an opportunity to do so. Thus, $\overline{E}_0(\pi_1) = \gamma(1-\lambda)\lambda\delta$: the probability $\gamma(1-\lambda)$ of being informed without having had an opportunity for an informed price-adjustment times the probability λ of having an opportunity to adjust prices next period times the price change δ. The average inflation expectations can also be computed. Firms unaware of the shock expect zero inflation. Firms aware of the shock (whether they had an opportunity to make an informed price adjustment or not), which at $t = 0$ are a proportion γ of all firms, know that inflation at time $t = 1$ will be $\delta\ \Xi_1$. Thus, the average next period inflation expected at $t = 0$ is

$\gamma\delta\Xi_1$. From equation (2.2), $\Xi_1 = 2\lambda\gamma\left(1-\frac{\lambda+\gamma}{2}\right)$. Hence, the ratio at $t=0$ of average inflation expectations to $\overline{E}_0(\pi_1)$ is $2\gamma\frac{1-\frac{\lambda+\gamma}{2}}{1-\lambda}$. This ratio is equal to 1 if $\gamma = 1$, i.e. in the pure sticky-price case. If $\gamma \neq 1$, this ratio is usually different from 1 (the only other exception is when $\gamma = 1-\lambda$). This example proves that the operator $\overline{E}$ can be different from the average expectations.

5.3 Permanent shocks to the money-supply growth rate

Let's continue to assume that firms are strategically neutral, but that at time $t=0$ there is a shock to the money-supply growth rate rather than to the level of money supply. Equation (II.13) in Appendix II implies that in this case the level of inflation for $t \geq 0$ is given by:

$$\begin{aligned}
\pi_t &= g + (g - g_{old}) \begin{bmatrix} -t\frac{\lambda\gamma(1-\gamma)^{t+1}}{\gamma-\lambda} \\ +\frac{\lambda\gamma(1-\gamma)(1-\lambda)\left[(1-\lambda)^t-(1-\gamma)^t\right]}{(\gamma-\lambda)^2} \\ -(1-\gamma)^{t+1} \end{bmatrix} \text{for } \lambda \neq \gamma \qquad (6.3) \\
\pi_t &= g + (g - g_{old})(1-\lambda)^t\left[\frac{\lambda^2}{2}t(t+1) - (1-\lambda)\right] \quad \text{for } \lambda = \gamma.
\end{aligned}$$

Notice that (for $t \geq 0$):

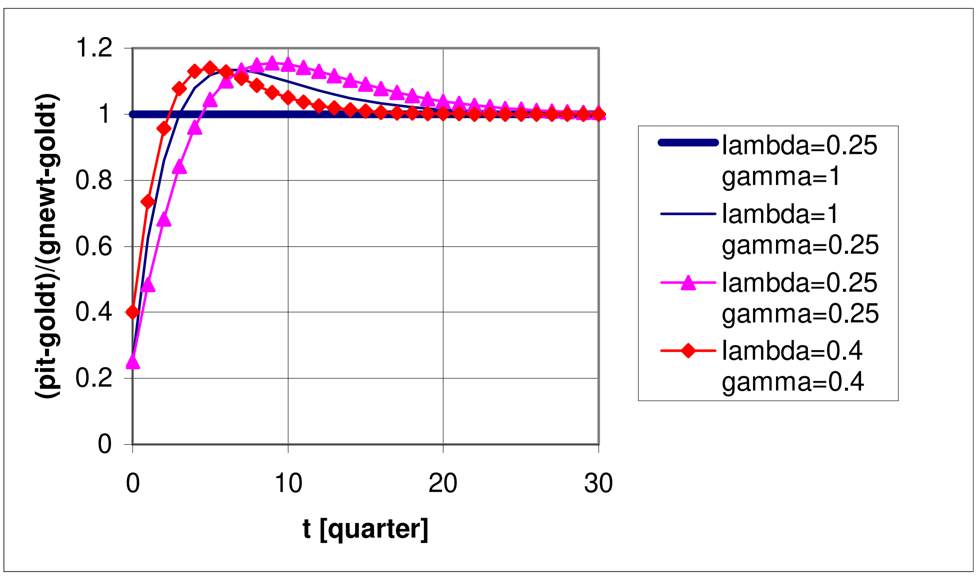
If $\gamma = 1$ then $\pi_t = g$.

If $\lambda = 1$ then $\pi_t = g + \left[-(1-\gamma)^{t+1} + \gamma(1-\gamma)^t t\right](g - g_{old})$.

If $\lambda = \gamma$ then $\pi_t = g + \left[-(1-\lambda)^{t+1} + \lambda^2(1-\lambda)^t t(t+1)/2\right](g - g_{old})$.

Figure 3 shows the impulse response to a money-supply growth shock, using the same numerical example as for Figure 2.

Figure 3: Inflation impulse response to a permanent shock to the money-supply growth rate



These curves show the impulse response at time t as a proportion of the long-run response. All curves converge toward 1 in the long run. For the sticky-price case ($\gamma = 1$) the inflation response is flat, whereas it is hump-shaped for the sticky-information case ($\lambda = 1$), reaching a maximum at

$t = \frac{1-\gamma}{\gamma} - \frac{1}{\ln(1-\gamma)}$. The impulse response is also hump-shaped for the hybrid case both when $\lambda = \gamma = 0.25$ and when $\lambda = \gamma = 0.4$. The two hybrid curves are hump-shaped, but the maximum occurs later for the first curve. There is a jump in inflation at time $t = 0$ for all four curves, but this jump is much larger for the sticky-price curve than for the other curves. Overall, the two hybrid curves have the same qualitative features as the sticky-information curve, both contrasting sharply with the sticky-price curve. In contrast with the case of a shock to the money-supply level, the sticky-information curve is similar to the hybrid case here. The advantage of the hybrid curve relates to micro evidence rather than macro evidence: in the sticky-information model every firm changes its price every period (in contrast with micro evidence), whereas this is not the case in the hybrid model.

5.4 Persistent but not permanent shocks to the money-supply growth rate

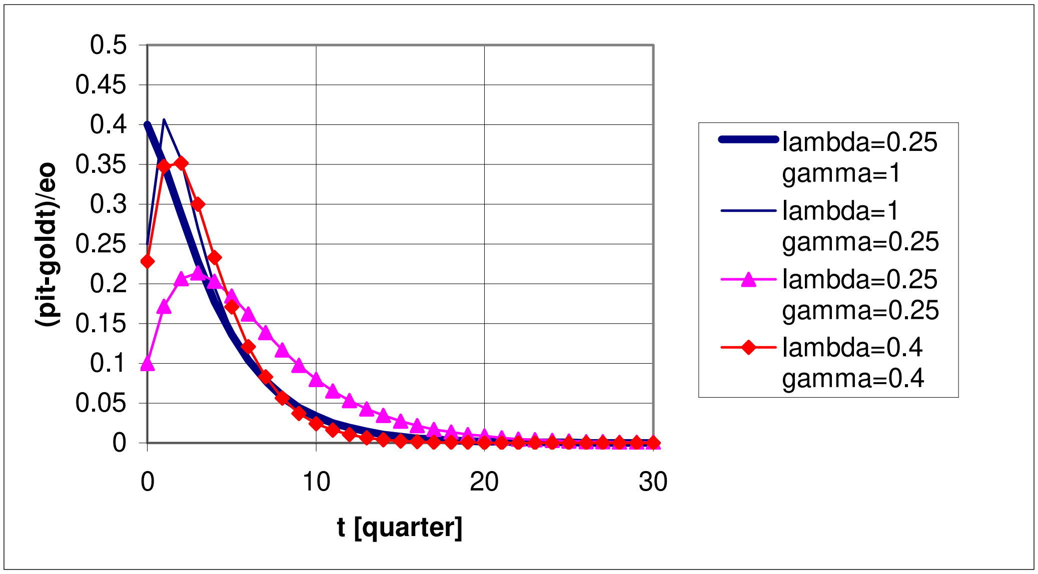
Let's consider another type of monetary shock. Suppose that

$$m_{old,t} = m_{-1} + (t+1)\, g_{old} \text{ for } t < 0 \ ,$$
$$m_t = m_{-1} + (t+1)\, g_{old} + \varepsilon_0 \sum_{i=0}^{t} \rho^i \text{ for } t \geq 0 \ ,$$

where ρ is the autocorrelation of the money-supply growth rate. If $\rho = 0$, then the shock is completely transitory, corresponding to the permanent shock to the level of money supply discussed in section 5.2. If $\rho = 1$, then the shock is permanent, corresponding to the permanent shock to the money-supply growth rate discussed in section 5.3. Intermediate values of ρ correspond to a persistent but not permanent shock to the money-supply growth rate: the money-supply growth rate changes the most at $t = 0$, then changes each period by a smaller amount and converges toward its initial value g_{old}. During this process, a change of the money-supply level of $\frac{\varepsilon_0}{1-\rho}$ takes place gradually. In considering this experiment, MR regard the value of $\rho = 0.5$ as being realistic for U.S. quarterly data.

In the strategic-neutrality case, it is possible to find a closed-form solution for the aggregate price dynamics (see equation II.14 in appendix II). Figure 4 shows the impulse response of inflation using the same numerical example as for Figures 2 and 3.

Figure 4: Inflation impulse response to a shock to the money-supply growth rate with autocorelation $\rho = 0.5$



This figure shows that the inflation impulse response is not hump-shaped in the sticky-price model whereas it is hump-shaped in the sticky-information model and in the hybrid model. The inflation impulse responses of the sticky-information and the hybrid models are still qualitatively similar. Section 5.2 has shown that decreasing the persistence of the shock ultimately favors the hybrid model. For the intermediate case of $\rho = 0.5$, a small advantage for the hybrid model can already be observed in the sense that the inflation jump at the shock is smaller and the hump appears later in the hybrid model (the exact date at which the hump appears depends on the calibration).

6 Conclusion

Most macroeconomic models cannot explain the two following stylized facts simultaneously: i) individual firms change prices every six months to a year and ii) inflation impulse responses to monetary shocks are hump-shaped. This paper presents a hybrid sticky-price and sticky-information model compatible with both of these facts.

In the case of a permanent shock to the growth rate of money supply, the inflation responses are hump-shaped both in the hybrid and in the sticky-information models, and both are clearly different from the monotonic response of the sticky-price model. Reducing the persistence of the shock ultimately favors the hybrid model. If the shock is completely transitory, the hybrid model delivers a hump-shaped inflation response to monetary policy, whereas the two pure cases yield the same strictly decreasing response. Intuitively, this result relates to the hump-shaped dynamics of the number of informed firms that have not yet had the opportunity to re-optimize their prices since the shock occurred. In the intermediate case of a shock to the money-supply growth rate with an autocorrelation coefficient equal to 0.5, the hybrid and the sticky-information models yield qualitatively similar inflation impulse responses.

On the way to computing inflation impulse responses, I derive the Phillips curve for the hybrid model (without, at this stage, assuming anything about strategic neutrality). One novel feature of this Phillips curve is that it involves a new kind of expectation operator.

This study could be extended by considering other kinds of monetary shocks or deviation from strategic neutrality. It is not clear that in these settings the hybrid model will be so distinctly superior to the pure models as it is in the examples discussed here. A priori, it could be expected that the hybrid model behaves more as an average of the pure models would in the case of anticipated shocks. Whether this is the case or not could be checked.

The results of this paper have been obtained under the assumption of strategic neutrality in the pricing decisions of firms. It would be important to discuss the case of strategic complementarity since this may increase persistence. Would strategic complementarity penalize the hybrid model relative to the sticky-information model? Strategic complementarity is likely to make the sticky-information inflation impulse-response more hump-shaped. It would be interesting to know how the inflation impulse response changes according to the degree of strategic complementarity in the hybrid model. Another issue is that strategic complementarity may allow the hybrid model to remedy a shortcoming of the sticky-information model that has not yet been discussed in this paper: in the sticky-information model, a monetary shock would have no impact on inflation once all firms are informed of the shock.[24] This would be obvious if information updating were assumed to occur at intervals of constant duration (then all firms would be informed after that duration) rather than with a constant probability (in which case there are always some firms that are not informed yet). But a hybrid of the standard sticky-price model and a sticky-information model in which information is updated at constant intervals would become a sticky-price model as soon as all firms are informed. Thus, the hybrid model may inherit the endogenous persistence that, assuming strategic complementarity, the sticky-price model features in most cases.[25]

[24] Collard and Dellas (2003) and Dupor and Tsuruga (2005) criticize the sticky-information model on this account.

[25] This phenomenon is best thought of as a "contract multiplier," as Taylor (1980) put it. In the special case where perfect adjustment is immediate (this happens in the case

Other interesting further research would include: i) discussing the case in which the price-adjustment opportunity and information updating are not independent or are state-dependent, and ii) extending the model to a general-equilibrium setting.

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of a permanent shock to the money-supply growth rate), the contract multiplier cannot generate additional persistence.

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Appendix I: Derivation of the hybrid Phillips curve

Two intermediate equations

Let's first derive two equations that will be useful later.

Equation (3.3) yields:

$$x_{t,k} = \lambda E_{t-k}\left(p_t^*\right) + (1-\lambda)\, x_{t+1,k+1} \tag{I.1}$$

because $x_{t,k} = \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_{t-k}\left(p_{t+j}^*\right)\right] = \lambda E_{t-k}\left(p_t^*\right) + \lambda \sum_{j=1}^{\infty} \left[(1-\lambda)^j E_{t-k}\left(p_{t+j}^*\right)\right]$

$= \lambda E_{t-k}\left(p_t^*\right) + (1-\lambda)\, \lambda \sum_{j=0}^{\infty} \left[(1-\lambda)^j E_{t-k}\left(p_{t+1+j}^*\right)\right]$

$= \lambda E_{t-k}\left(p_t^*\right) + (1-\lambda)\, x_{t+1,k+1}.$

Equation (3.4) yields:

$$p_t = \lambda\gamma \sum_{k=0}^{\infty} \left[(1-\gamma)^k x_{t,k}\right] + (1-\lambda)\, p_{t-1} \tag{I.2}$$

because

$$p_t = \lambda\gamma \sum_{j=0}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-j,k}\right]$$

$$= \lambda\gamma \left[\sum_{k=0}^{\infty} (1-\gamma)^k x_{t,k}\right] + \lambda\gamma \sum_{j=1}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-j,k}\right],$$

where

$$\lambda\gamma \sum_{j=1}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-j,k}\right]$$
$$= (1-\lambda)\lambda\gamma \sum_{j=0}^{\infty} \left[(1-\lambda)^j \sum_{k=0}^{\infty} (1-\gamma)^k x_{t-1-j,k}\right]$$
$$= (1-\lambda) p_{t-1}$$

Rewriting the hybrid Phillips curve

The hybrid Phillips curve $\pi_t - \overline{E}_t(\pi_{t+1}) = \frac{\lambda^2}{1-\lambda}[\alpha y_t + \varepsilon_t(p_t^*)]$,

where $\overline{E}_t(\pi_{t+1}) = \lambda \left[\gamma \sum_{k=0}^{\infty} (1-\gamma)^k x_{t+1,k+1} - p_t\right]$

and $\varepsilon_t(p_t^*) = \gamma \sum_{k=1}^{\infty} (1-\gamma)^k [E_{t-k}(p_t^*) - p_t^*]$,

can be rewritten as:

$$\pi_t - \lambda \left[\gamma \sum_{k=0}^{\infty} (1-\gamma)^k x_{t+1,k+1} - p_t\right] = \frac{\lambda^2}{1-\lambda}\left[\alpha y_t + \gamma \sum_{k=1}^{\infty} (1-\gamma)^k [E_{t-k}(p_t^*) - p_t^*]\right],$$

which, using $p_t^* = p_t + \alpha y_t$ and $\pi_t = p_t - p_{t-1}$, becomes after some algebraic transformations:

$$\frac{p_t - (1-\lambda) p_{t-1} - (1-\lambda)\gamma \sum_{k=0}^{\infty} \left[(1-\gamma)^k \lambda x_{t+1,k+1}\right]}{\lambda^2} = \gamma \sum_{k=0}^{\infty} (1-\gamma)^k E_{t-k}(p_t + \alpha y_t). \tag{I.3}$$

Deriving the hybrid Phillips Curve

Equation (I.3) is true since

$$\frac{p_t-(1-\lambda)p_{t-1}-(1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2}$$

$$=\frac{\lambda\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k x_{t,k}\right]-(1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2} \qquad \text{using equation (I.2)}$$

$$=\frac{\lambda\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\left[\lambda E_{t-k}(p_t^*)+(1-\lambda)x_{t+1,k+1}\right]\right]-(1-\lambda)\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k\lambda x_{t+1,k+1}\right]}{\lambda^2} \qquad \text{using equation (I.1)}$$

$$=\gamma\sum_{k=0}^{\infty}\left[(1-\gamma)^k E_{t-k}\left(p_t^*\right)\right] \qquad \text{algebra}$$

$$=\gamma\sum_{k=0}^{\infty}(1-\gamma)^k E_{t-k}\left(p_t+\alpha y_t\right) \qquad \text{using equation (3.1)}$$

Appendix II: The inflation impulse response

This appendix gives some equations for computing the impulse responses and explains why computation of the impulse response is easier in the pure cases (sticky prices or sticky information) and in the strategic neutrality case than in the general case.

II.1) The general case

Plugging equations (3.1), (3.2), (3.3), (4.2) and (4.3) into the hybrid Phillips curve (4.1) yields:

$$\begin{aligned} & (1-\lambda)\,\lambda\gamma\sum_{k=0}^{\infty}(1-\gamma)^k\,x_{t+1,k+1} \\ & -\left[1-\lambda^2\gamma\,(1-\alpha)\right]p_t+(1-\lambda)\,p_{t-1} \\ = \quad & -\lambda^2\gamma\left\{\alpha m_t+\sum_{k=1}^{\infty}(1-\gamma)^k\left[E_{t-k}\left((1-\alpha)\,p_t+\alpha m_t\right)\right]\right\}. \end{aligned} \tag{II.1}$$

Plugging into equation (II.1) $x_{t+1,k+1}$ expressed in terms of p and m according to equation (3.3) yields:[26]

$$p_t=(1-\lambda)\,p_{t-1}+\lambda^2\gamma\sum_{k=0}^{\infty}(1-\gamma)^k\left[\sum_{j=0}^{\infty}\left[(1-\lambda)^j\,E_{t-k}\left((1-\alpha)\,p_{t+j}+\alpha m_{t+j}\right)\right]\right]. \tag{II.2}$$

[26] Alternatively, plug equations (3.1), (3.2) and (3.3) into equation (I.2) of appendix I.

In the general case this equation involves an infinity of aggregate prices (all aggregate prices from $t-1$ and thereafter). Some algebraic transformations can, however, reduce this dimensionality to a third-order recursive equation with variable coefficients. This equation is solvable (not necessarily analytically, but at least numerically). The resolution of the general case is left for further research. I will focus below on three special cases in which computation is simple.

II.2) Three simple cases

Sticky information

In the sticky-information case, $\lambda = 1$, and the future aggregate prices on the right-hand side of equation (II.2) disappear (and so does the last-period aggregate price), which yields a simple equation in p_t:

$$p_t = \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[E_{t-k} \left((1-\alpha) p_t + \alpha m_t \right) \right]. \qquad \text{(II.3)}$$

This equation can be solved (assuming rational expectations) once the nature of the monetary shock is specified.

In the absence of shocks, $E_{t-k}(p_t) = p_t$ and $E_{t-k}(m_t) = m_t$, and equation (II.3) yields $p_t = m_t$ (whatever the dynamics of m).

If there is only a lone shock occurring at time $t = 0$, and if it is not anticipated, then (II.3) becomes:

$$\frac{p_t - m_t}{m_{old,t} - m_t} = \frac{(1-\gamma)^{t+1}}{\alpha + (1-\gamma)^{t+1}(1-\alpha)}, \tag{II.4}$$

where $m_{old,t}$ is the actual money-supply before the shock, or the money supply that would have prevailed after time $t = 0$ if no shock had occurred. m_t still denotes the actual money-supply ($m_t = m_{old,t}$ for $t < 0$, but is different from $m_{old,t}$ at $t = 0$, and may be different later on as well). Equation (II.4) gives a measure of the incompleteness of the price adjustment made at time t (as a proportion of the adjustment that would have been made at time t in the absence of nominal rigidities). For $t < 0$, equation (II.4) yields $p_t = m_t$. Inflation can be computed by extracting p_t from equation (II.4) and substracting a lagged version of this equation.

Sticky prices

In the pure sticky-price model, equation (II.2) becomes (after plugging in $\gamma = 1$):

$$p_t = (1-\lambda)\,p_{t-1} + \lambda^2 \sum_{j=0}^{\infty} \left[(1-\lambda)^j\, E_t\left((1-\alpha)\,p_{t+j} + \alpha m_{t+j}\right)\right]. \tag{II.5}$$

The future aggregate price levels are still present, but they can be eliminated easily by writing equation (II.5) for $t+1$, taking the expectation at t, and substracting equation (II.5) for t divided by $1-\lambda$. This yields:

$$E_t p_{t+1} - \left(2 + \frac{\alpha\lambda^2}{1-\lambda}\right) p_t + p_{t-1} = -\frac{\alpha\lambda^2}{1-\lambda} m_t. \tag{II.6}$$

This is a second-order recursive equation with constant coefficients. There are several ways to solve this equation analytically. For example, the dimensionality can be further reduced to

$$p_t = \theta p_{t-1} + (1-\theta)^2 \sum_{i=0}^{\infty} \theta^i E_t m_{t+i}, \tag{II.7}$$

where θ is solution of $2 + \frac{\alpha\lambda^2}{1-\lambda} = \theta + \frac{1}{\theta}$ such that $\theta < 1$. The solution of (II.7) is:

$$p_t = (1-\theta)^2 \sum_{k=0}^{\infty} \sum_{i=0}^{\infty} \theta^{i+k} E_{t-k} m_{t-k+i}. \tag{II.8}$$

Assuming that there is only a lone unanticipated shock occurring at time $t = 0$, equation (II.8) could be expressed in terms of $m_{old,t}$ and m_t. In the sticky-price model it particularly makes sense to focus on cases in which the growth rate of the money supply (without taking the logs) is constant everywhere except when a shock occurs, because only in such cases would the output gap be zero in the absence of shocks (or if the shock lies infinitely far in the past): in the absence of shocks, $E_t(p_{t+1}) = p_{t+1}$ and equation (II.6) becomes $\Delta\pi_{t+1} = \frac{\alpha\lambda^2}{1-\lambda}(p_t - m_t)$, which yields $p_t = m_t$ only if m_t is an affine function of time. Let's thus consider that money-supply dynamics is given by $m_t = m_{old,t} = m_{-1} + (t+1) g_{old}$ until a shock occurs at $t = 0$ such that $m_t = m_{-1} + \delta + (t+1) g$ for $t \geq 0$. Then, the solution is:

$$\begin{aligned} p_t &= m_t - \theta^{t+1}\delta \text{ for } t \geq 0 \\ p_t &= m_{old,t} \text{ for } t < 0. \end{aligned} \tag{II.9}$$

Thus, adjustment is perfect and immediate if $\delta = 0$ (even if there is a change in the growth rate).

Strategic neutrality

In the case of strategic neutrality ($\alpha = 1$), equation (II.2) becomes:

$$p_t = (1-\lambda) p_{t-1} + \lambda^2 \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \left[(1-\lambda)^j E_{t-k}(m_{t+j}) \right] \right]. \tag{II.10}$$

All future aggregate prices have disappeared (as in the sticky-information model), but the last-period aggregate price is still present: this is a first-order recursive equation with constant coefficients that can be solved easily:

$$p_t = \lambda^2 \gamma \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{l=0}^{\infty} \sum_{j=0}^{\infty} \left[(1-\lambda)^{j+l} E_{t-l-k} \left(m_{t-l+j} \right) \right] \right]. \tag{II.11}$$

Let's assume that there is only one shock, that it is not anticipated and that it occurs at $t = 0$. As above, $m_{old,t}$ is the actual money-supply before the shock or the money supply that would have prevailed after time $t = 0$ if no shock had occurred. Then $E_{t-l-k}\left(m_{t-l+j}\right) = m_{old,t-l+j}$ except if $t - k \geq l$ and $t + j \geq l$ (that is, except if $t + j \geq l$) in which case it is equal to m_{t-l+j}. Then equation (II.11) becomes for $t \geq 0$:

$$p_t = \lambda^2 \gamma \left\{ \begin{array}{c} \sum_{k=0}^{t} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \sum_{l=0}^{t-k} \left[(1-\lambda)^{j+l} \left(m_{t-l+j} - m_{old,t-l+j} \right) \right] \right] \\ + \sum_{k=0}^{\infty} (1-\gamma)^k \left[\sum_{j=0}^{\infty} \sum_{l=0}^{\infty} \left[(1-\lambda)^{j+l} m_{old,t-l+j} \right] \right] \end{array} \right\}. \tag{II.12}$$

Under which conditions is the output gap zero in the absence of a shock? As shown above, this is the case for the sticky-price model only if monetary supply is an affine function of time. Since the strategic-neutrality case overlaps with the sticky-price case, being an affine function of time is a necessary condition for money-supply dynamics to yield a zero output gap in the absence of a shock for every parameter value of the strategic-neutrality case. That this necessary condition is also sufficient is most easily seen in the Phillips curve (4.1) itself.

- **Permanent shock to the money-supply level and growth rate**

  Let's focus, as in the sticky-price model, on money-supply dynamics given by $m_t = m_{old,t} = m_{-1} + (t+1)\, g_{old}$ until an unanticipated shock occurs at $t = 0$ such that $m_t = m_{new,t} = m_{-1} + \delta + (t+1)\, g$ for $t \geq 0$. Then, for $t \geq 0$, the solution is:[27]

$$p_t = \begin{array}{c} m_t + \delta \left[\frac{\lambda}{\gamma-\lambda} (1-\gamma)^{t+2} - \frac{\gamma}{\gamma-\lambda} (1-\lambda)^{t+2} \right] \\ + (g - g_{old}) \left[\begin{array}{c} (t+1)(1-\gamma)^{t+2} \frac{\lambda}{\gamma-\lambda} \\ +\gamma \frac{(1-\gamma)(1-\lambda)\left[(1-\gamma)^{t+1} - (1-\lambda)^{t+1}\right]}{(\gamma-\lambda)^2} \end{array} \right] \end{array} \text{ for } \lambda \neq \gamma \tag{II.13}$$

$$p_t = m_t - \left\{ \begin{array}{c} \delta (1-\lambda)^{t+1} \left[1 + (t+1)\lambda\right] \\ + (g - g_{old}) (t+1) (1-\lambda)^{t+1} \left(1 + \frac{\lambda}{2} t\right) \end{array} \right\} \text{ for } \lambda = \gamma.$$

  From this equation, inflation could be computed. In particular, if the monetary shock occurs only to the level of money supply (that is, $g =$

[27] If there is a $m_{new,t}$ instead of a m_t in (II.13), then this formula is also valid for $t = -1$ (with $m_{new,t=-1} = m_{-1} + \delta$).

g_{old}), then inflation is simply given (for $t \geq 0$) by $\pi_t - g = \delta\ \Xi_t$, where Ξ_t is the probability that a firm sets its first informed price-adjustment t periods after the shock.

- **Persistent but not permanent shocks to the money-supply growth rate**

  Let's consider the following shock:

  $m_{old,t} = m_{-1} + (t+1)\, g_{old}$ for $t < 0$

  $m_t = m_{-1} + (t+1)\, g_{old} + \varepsilon_0 \sum_{i=0}^{t} \rho^i$ for $t \geq 0$.

  This implies that $\Delta m_t - g_{old} = \rho\,(\Delta m_{t-1} - g_{old}) + \varepsilon_t$, where $\varepsilon_t = \varepsilon_0$ if $t = 0$ and zero otherwise.

  Thus, ρ is the autocorrelation of the money-supply growth rate. If $\rho = 0$, then the shock is completely transitory (or equivalently, it is a permanent shock to the level of money supply). If $\rho = 1$, then this is a permanent shock to the money-supply growth rate. Then, for $t \geq 0$, the solution is:

a) for $\lambda \neq \gamma$

$$p_t = m_t + \frac{1}{1-\rho}\varepsilon_0 \left\{ \begin{array}{c} \frac{\lambda}{\gamma-\lambda}\left(1-\gamma\right)^{t+2} - \frac{\gamma}{\gamma-\lambda}\left(1-\lambda\right)^{t+2} \\ +\rho^{t+1} \\ +\frac{\rho}{1-\frac{1-\rho}{\lambda}}\frac{1}{\rho+\frac{1-\rho}{\lambda}}\frac{1}{1-\lambda-\rho(1-\gamma)} \left[\begin{array}{c} \gamma\left(1-\lambda\right)^{t+2} \\ +\left(1-\lambda\right)\rho^{t+1}\left(1-\gamma\right)^{t+2} \\ -\left(1-\lambda\right)\rho^{t+1} \\ +\rho^{t+2}\left(1-\gamma\right)\left[\begin{array}{c} 1 \\ -\left(1-\gamma\right)^{t+1} \end{array} \right] \end{array} \right] \end{array} \right\}. \quad \text{(II.14)}$$

b) for $\lambda = \gamma$

$$p_t = m_t + \varepsilon_0 \frac{1}{1-\frac{1-\rho}{\lambda}}\frac{1}{1+(1-\rho)\frac{1-\lambda}{\lambda}} \left\{ \begin{array}{c} -\frac{1-\rho^{t+1}}{1-\rho}\rho\left(1-\lambda\right)^{t+1} \\ +\lambda\left(1-\lambda\right)^{t+1}\left(\frac{1-\rho^{t+1}}{1-\rho} - (t+1)\right)\frac{\rho}{1-\rho} \\ +\frac{1-\lambda}{\lambda^2}\left(1-\rho\right)\left[\left(1-\lambda\right)^{t+1} + \left(1-\lambda\right)^{t+1}\lambda\left(t+1\right) - \rho^{t+1}\right] \end{array} \right\}.$$

Here again, the inflation dynamics can be computed.