Measuring Standard Error of Inflation in Pakistan: A Stochastic Approach

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"The answer to the question what is the Mean of a given set of magnitudes cannot in general be found, unless there is given also the object for the sake of which a mean value is required. There are as many kinds of average as many purposes; and we may almost say in the matter of prices as many purposes as many writers." Edgeworth (1888)

Abstract

Stochastic approach to index number (and its change) has recently attracted renewed attention of researchers as it provides the standard error of index number (and its change). One of the most important uses of index number is in the case of measurement of the general price level in an economy (and then inflation of course). In this study we estimate standard errors of month on month and year on year inflation in Pakistan under stochastic approach, following Clement and Izan (1987). We contribute in this study by providing mechanism and estimating the standard error of period average of YoY inflation and apply this to Pakistan data.

Key Words: Inflation, Pakistan

1. INTRODUCTION:

Stochastic approach to index number (and its growth) has recently attracted renewed attention of researchers as it provides the standard error of index number (and its growth). One of the most important uses of index number is in the case of measurement of the general price level in an economy (and then inflation of course). Stochastic approach to index numbers has been applied to measure inflation in studies like Clement and Izan (1987), Selvanathan (1989), Crompton (2000), Selvanathan (2003), Selvanathan and Selvanathan (2004), and Clement and Selvanathan (2007).

Historically, there are two main approaches to measure the index number: the functional approach and the stochastic approach. In the functional approach, prices and quantities of various goods and services are look upon as connected by certain typical observable relationship (Frisch,

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1936). The stochastic approach is less well known although it has a long history dating back to Jevons (1865) and Edgeworth (1888). In the stochastic approach prices and quantities are considered s two sets of independent variables. It assumes that (ideally) individual prices ought to change in the same proportion from one point of time to other. This assumption is based upon quantity theory of money - as the quantity of money increases all prices should increase proportionally. Any deviation of individual prices from such proportionality is seen as 'errors of observation' (and may be the result of non-monetary factors' affect on prices). Thus rate of inflation can be calculated by averaging over the proportionate changes in the prices of all individual goods and services. Keynes (1930) criticized the assumption that all prices must change equiproportionately so that that relative prices remain same by considering this as being 'root and cause erroneous'. In functional approach the deviations from proportionality are taken as expressions for those economic relations that serve to give economic meaning to index numbers (Frisch, 1936).

The recent interest of researcher in the stochastic approach to index number theory is due to Balk (1980), Clements and Izan (1981, 1987), Bryan and Cecchetti (1993) and Selvanathan and Rao (1994). Clements and Izan (1987) recognized the Keynes (1930) criticism on the assumption of identical systematic changes in prices and viewed the underlying rate of inflation as an unknown parameter to be estimated from the individual prices changes by linking index number theory to regression analysis.

The approach stochastic leads to familiar index number formulae such as Divisia, Laspeyres type index numbers. As uncertainty plays a vital role in this approach the foundations differ markedly from those of the functional approach, linking index number theory to regression analysis we not only get an estimate of the rate of inflation but also it sampling variance. With the relaxation of the assumption that prices of goods and services change equiproportionately, the individual prices in the basket of price index move disproportionately (which usually happens) and thus the overall rate of inflation may become less well defined (Selvanathan and Selvanathan 2006). In such situations the ability of stochastic approach becomes important by allowing us to construct confidence interval around the true rate of inflation with the help of standard error (of inflation). Confidence interval built around the true rate of inflation can be used in some practical purpose such as wage negotiations, wage indexation, inflation targeting (in interval), etc.

One of the criticisms on this new stochastic approach of Clements and Izan (1987) was on the restriction of homoscedasticity on the variance of the error term in the OLS regression (Diewert, 1995). Crompton (2000) also pointed out this deficiency and extended the new stochastic approach to derive robust standard errors for the rate of inflation by relaxing the earlier restriction on the variance of the error term by considering an unknown form of heteroscadasticity. Selvanathan (2003) presented some comments and corrections on Crompton's work. Selvanathan and Selvanathan (2004) showed how recent developments in stochastic approach to index number can be used to model the commodities prices in the OECD countries. Selvanathan and Selvanathan (2006) calculated annual rate of inflation for Australia, UK and US using stochastic approach[2]. These studies provided mechanism for calculating standard error for inflation. In this study we estimate standard errors of month on month (MoM) and year on year (YoY) inflation in Pakistan under stochastic approach, following Clement and Izan (1987). To the best of our knowledge no one has done work on the estimation of standard error of period average of inflation. Since State Bank of Pakistan (the central bank) targets 12-

[2] Clements, Izan and Selvanathan (2006) presented a review on the stochastic approach to index number theory.

motnh average of YoY inflation, we contribute by providing mechanism and estimating the standard error of YoY inflation; and 12-motnh average of YoY inflation in Pakistan.

The criticism on the assumption while estimating the standard error of inflation that when prices change they change equation proportionally has been responded by Clements and Izan (1987) by extending the stochastic approach by considering the underlying rate of inflation separate from the changes in relative prices. By applying Clements and Izan (1987) approach we have estimated the systematic change in relative prices for each of the 374 commodities in the CPI basket of Pakistan. However, in this paper we have presented the average systematic change in relative prices of different groups in CPI basket of Pakistan.

In the following section we provide details of existing mechanism of stochastic approach and its applications to index number theory in the context of price index. We then extend this stochastic approach to estimate YoY inflation, period (period) average inflation and their standard errors. In section 3 we present the results of the application of this approach on Pakistan for estimating MoM inflation, YoY inflation, and period (period) average inflation along with their standard errors. In section 4 we present the estimated average systematic change in relative prices of different groups in CPI basket of Pakistan following Clements and Izan (1987) extended approach. Concluding remarks follow in the last Section.

2. UNFOLDING THE STOCHASTIC APPROACH TO INDEX NUMBERS:

Different ways to apply the stochastic approach to index numbers give various forms of index numbers like Divisia, Laspeyres etc. Since Federal Bureaus of Statistics (Pakistan's statistical agency) uses Laspeyres index formula for measuring inflation in period t over the base period, we would like to confine following analysis to deriving Laspeyres index.

2.1 *Derivation of Laspeyres index number*[3]

Following conventional notations let p represents price and q represent the quantity. We subscript these notations by it where $i(i = 1, 2, \dots, n)$ represents commodity and $t(t = 0, 1, 2, \dots\dots, T)$ the time. Under stochastic approach any observed price change is a reading on the 'underlying' rate of inflation and a random component (ϵ_{it}). If γ_t the price index relating expenditures in period t to expenditures in base period then following stochastic approach we can write

$$p_{it}q_{i0} = \gamma_t \, p_{i0}q_{i0} + \epsilon_{it} \qquad t(t = 0, 1, 2, \dots\dots, T) \tag{1}$$

And thus we have related the index number theory to regression analysis as now we can estimate the rate of inflation in period t by estimating the unknown parameter γ_t in (1). Rearranging (1) we get

$$\Rightarrow \frac{p_{it}}{p_{i0}} = \gamma_t + \frac{\epsilon_{it}}{p_{i0}q_{i0}} \tag{2}$$

Here $E\left(\frac{p_{it}}{p_{i0}}\right) = \gamma_t$, under the assumptions

$$E(\epsilon_{it}) = 0 \;, \; Cov(\epsilon_{it}, \epsilon_{jt}) = \sigma_t^2 p_{i0}q_{i0}\delta_{ij} \qquad (\delta_{ij} \text{ is the Kronecker delta}) \tag{3}$$

To remove heteroscadasticity in the error term we transform equation (1) into new form which gives homoscedastic variances in the error term across all the n commodities in any particular time period t. For this purpose we divided equation (1) by $\sqrt{p_{i0}\, q_{i0}}$ and obtain

$$y_{it} = \gamma_t x_{i0} + \eta_{it} \tag{4}$$

Where $y_{it} = \frac{p_{it}\, q_{i0}}{\sqrt{p_{i0}\, q_{i0}}}$; $\qquad x_{i0} = \sqrt{p_{i0}\, q_{i0}}$ and $\eta_{it} = \frac{\epsilon_{it}}{\sqrt{p_{i0}\, q_{i0}}}$

Now assumptions in (3) after above transformation are

[3] This sub-section (2.1) is mostly based upon Selvanathan and Selvanathan (2006) except for equation (1). Above equation (1) is slightly different from what is used in Selvanathan and Selvanathan (2006).

$$E(\eta_{it}) = 0 \text{ and } Cov(\eta_{it}, \eta_{jt}) = \sigma_t^2 \quad \delta_{ij}$$

Now we can now apply, say, least squares to (4) to have an estimator for inflation as below:

$$\hat{\gamma}_t = \frac{\sum_{i=1}^{n} x_{i0} y_{it}}{\sum_{i=1}^{n} x_{i0}^2}$$

$$= \frac{\sum_{i=1}^{n} (\sqrt{p_{i0}\, q_{i0}})(\frac{p\, q_{i0}}{\sqrt{p_{i0}\, q_{i0}}})}{\sum_{i=1}^{n} p_{i0}\, q_{i0}} = \frac{\sum_{i=1}^{n} p_{it}\, q_{i0}}{\sum_{i=1}^{n} p_{i0}\, q_{i0}} = \frac{\sum_{i=1}^{n} (\frac{p_{it}\, q_{i0}}{p_{i0}}) p_{i0}}{\sum_{i=1}^{n} p_{i0}\, q_{i0}} = \sum_{i=1}^{n} \frac{p_{it}}{p_{i0}} \times \frac{p_{i0}\, q_{i0}}{\sum_{i=1}^{n} p_{i0}\, q_{i0}}$$

We know $\frac{p_{i0}\, q_{i0}}{\sum_{i=1}^{n} p_{i0}\, q_{i0}}$ is the budget share of commodity i in base period and if we write it as

$w_{i0} = \frac{p_{i0}\, q_{i0}}{\sum_{i=1}^{n} p_{i0}\, q_{i0}}$ where $\sum_{i=1}^{n} w_{i0} = 1$, we have

$$\hat{\gamma}_t = \sum_{i=1}^{n} w_{i0} \frac{p_{it}}{p_{i0}} \tag{5}$$

Which is weighted average of the n price ratios (with base-period budget shares being weights) and is well-known Laspeyres price index. With the help of this price index we can have inflation (month on month and/or year on year) by using simple formulae as below:

$$Inflation\ (MoM) = (\frac{\hat{\gamma}_t}{\hat{\gamma}_{t-1}} - 1) \times 100 \tag{6}$$

$$Inflation\ (YoY) = (\frac{\hat{\gamma}_t}{\hat{\gamma}_{t-12}} - 1) \times 100 \tag{7}$$

Variance of the estimator in (5) is given by

$$Var(\hat{\gamma}_t) = \frac{\sigma_t^2}{\sum_{i=1}^{n} x_{i0}^2} \tag{8}$$

The parameter σ_t^2 can be estimated as

$$\hat{\sigma}_t^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_{it} - \hat{\gamma}_t x_{i0})^2 \tag{9}$$

By substitution estimated parameter of σ_t^2 from (9) together with the values of x_{i0} and y_{it} in (8) and rearranging we get

$$Var(\hat{\gamma}_t) = \frac{1}{n-1} \sum_{i=1}^{n} w_{i0} ({p_{it}}/{p_{i0}} - \hat{\gamma}_t)^2 \tag{10}$$

Thus, as the degree of relative prices variability increases the variance of the estimated index increases. This agrees with the intuitive notion that when the individual prices move very disproportionately, the overall price index is less well-defined (Dennis and Thomas (1976)).

Now question is in (4) and (10) we can have the estimated price index and its estimated variance respectively. From (4) we can find the estimated rate of inflation but here we not the variance of the estimated rate of inflation. For this purpose we have to proceed for inflation in the beginning rather than for index as we did in above subsection.

2.2 Application of Stochastic Approach to estimate headline inflation and its S.E.

Following notations used above if γ_t the price index relating expenditures in period t to expenditures in base period then following stochastic approach we can write

$$p_{it}q_{i0} = \gamma_t \, p_{i0}q_{i0} + \epsilon_{it} \qquad t(t = 0, 1, 2, \ldots\ldots, T) \tag{11}$$

Base period can be somewhere in distant past (say five year back) and at any point in time we define headline (or year on year – YoY) inflation as percentage change in price index over corresponding month last year then

$$\pi_t^H = \frac{\gamma_t - \gamma_{t-12}}{\gamma_{t-12}}$$

From (11) we can estimate of γ_t only. For estimate of γ_{t-12} we write (11) as

$$p_{it-12}q_{i0} = \gamma_{t-12} \, p_{i0}q_{i0} + \epsilon_{it-12} \qquad t(t = 0, 1, 2, \ldots\ldots, T) \tag{12}$$

Here again $E\left(\frac{p_{it-12}}{p_{i0}}\right) = \gamma_{t-12}$, under the similar assumptions as in (3)

$E(\epsilon_{it-12}) = 0$, $Cov(\epsilon_{it-12}, \epsilon_{jt-12}) = \sigma_{t-12}{}^2 p_{i0}q_{i0}\delta_{ij}$, ($\delta_{ij}$ is the Kronecker delta)

By subtracting (12) from (11) we have

$$p_{it}q_{i0} - p_{it-12}q_{i0} = (\gamma_t - \gamma_{t-12})p_{i0}q_{i0} + \epsilon_{it} - \epsilon_{it-12} \tag{13}$$

Dividing (13) by $E(\frac{p_{it-12}}{p_{i0}})$ and substituting $E\left(\frac{P_{it-12}}{P_{i0}}\right) = \gamma_{t-12}$ on right hand side, we get

$$\left[\frac{p_{it}q_{i0}-p_{it-12}q_{i0}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)}\right] = (\frac{\gamma_t-\gamma_{t-12}}{\gamma_{t-12}})p_{i0}q_{i0} + \frac{\epsilon_{it}-\epsilon_{it-12}}{\gamma_{t-12}} = \pi_t^H p_{i0}q_{i0} + e_{it} \tag{14}$$

Where $e_{it} = \frac{\epsilon_{it}-\epsilon_{it-12}}{\gamma_{t-12}}$. Again assuming that

$$E(e_{it}) = 0 \text{ and } Cov(e_{it}, e_{jt}) = \rho_t^2 p_{i0}q_{i0}\delta_{ij} \tag{15}$$

and proceeding as in subsection 2.1 we divide (14) by $\sqrt{p_{i0}q_{i0}}$ and get

$$\left[\frac{\frac{p_{it}}{p_{i0}}-\frac{p_{it-12}}{p_{i0}}}{E\left(\frac{p_{it-12}}{p_{i0}}\right)}\right]\sqrt{p_{i0}q_{i0}} = \pi_t^H\sqrt{p_{i0}q_{i0}} + \frac{e_{it}}{\sqrt{p_{i0}q_{i0}}} \tag{16}$$

Combining (3) and (5) we can write $\gamma_{t-12} = E\left(\frac{p_{it-12}}{p_{i0}}\right) = \sum_{i=1}^n w_{i0}\frac{p_{it-12}}{p_{i0}}$, Thus (16) becomes

$$\left[\frac{\frac{p_{it}}{p_{i0}}-\frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0}\frac{p_{it-12}}{p_{i0}}}\right]\sqrt{p_{i0}q_{i0}} = \pi_t^H\sqrt{p_{i0}q_{i0}} + \frac{e_{it}}{\sqrt{p_{i0}q_{i0}}}$$

If we take $Y_{it} = \left(\frac{\frac{p_{it}}{p_{i0}}-\frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0}\frac{p_{it-12}}{p_{i0}}}\right)\sqrt{p_{i0}q_{i0}}$, $X_{i0} = \sqrt{p_{i0}q_{i0}}$ and $\varphi_{it} = \frac{e_{it}}{\sqrt{p_{i0}q_{i0}}}$. Under assumptions

that $E(\varphi_{it})=0$ and $Cov(\varphi_{it,}\varphi_{jt}) = \varrho_t^2\delta_{ij}$, for equation

$$Y_{it} = \pi_t^H X_{io} + \varphi_{it} \tag{17}$$

Least square estimator of π_t^H is

$$\hat{\pi}_t^H = \frac{\sum_{i=1}^n X_{i0}Y_{it}}{\sum_{i=1}^n X_{i0}^2} = \frac{\sum_{i=1}^n \sqrt{p_{i0}q_{i0}}\left[\frac{\frac{p_{it}}{p_{i0}}-\frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0}\frac{p_{it-12}}{p_{i0}}}\right]\sqrt{p_{i0}q_{i0}}}{\sum_{i=1}^n p_{i0}q_{i0}} = \sum_{i=1}^n w_{i0}\left[\frac{\frac{p_{it}}{p_{i0}}-\frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^n w_{i0}\frac{p_{it-12}}{p_{i0}}}\right] = \frac{\hat{\gamma}_t-\hat{\gamma}_{t-12}}{\hat{\gamma}_{t-12}} \tag{18}$$

We knew this result from (7). Only benefit of all above process is that now we can have an estimate of the standard error of headline inflation which is

$$Var\,(\hat{\pi}_t^H) = \frac{\varrho_t^2}{\sum_{i=1}^n X_{i0}^2} \tag{19}$$

The parameter ϱ_t^2 can be estimated as

$$\hat{\varrho}_t^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_{it} - \hat{\pi}_t^H X_{i0})^2$$

By substitution of the estimated parameter ϱ_t^2 in (19) and rearranging we get

$$Var\ (\hat{\pi}_t^H) = \frac{1}{n-1}\sum_{i=1}^{n} w_{i0}\left[\frac{\frac{p_{it}}{p_{i0}} - \frac{p_{it-12}}{p_{i0}}}{\sum_{i=1}^{n} w_{i0}\frac{p_{it-12}}{p_{i0}}} - \hat{\pi}_t^H\right]^2 \quad (20)$$

Equation (20) shows that the variance of $\hat{\pi}_t^H$ increases with the degree of relative inflation variability[4]. Now we move towards estimating the period average inflation and its standard error.

2.3 *Application of Stochastic Approach to estimate period average inflation and its S.E.*

We know that period average, say 12 month period average) inflation can be calculated either by averaging the last 12 YoY inflation numbers or by taking YoY inflation of the 12-month (moving) averaged index number. We would like to use result above in subsection 2.1 for estimating the 12-month average inflation and those in subsection 2.2 for the standard error of period average inflation.

We have price index series as p_{it}. If 12-month averaged price index series is denoted by p_{it}^A then following the results in subsection 2.1, estimate of year on year inflation of p_{it}^A series will be

$$\hat{\gamma}_t^A = \sum_{i=1}^{n} w_{i0}\frac{p_{it}^A}{p_{i0}} \quad (21)$$

And thus

$$\hat{\pi}_t^A = \left(\frac{\hat{\gamma}_t^A}{\hat{\gamma}_{t-12}^A} - 1\right) * 100 \quad (22)$$

[4] Above procedure can be used to estimate the month on month inflation and its standard error.

Now for estimating the variance of the average inflation we use the result in above subsection 2.2 where we have derived the standard error of YoY inflation. If we replace the index with the average index in (20) we will get the standard error of average YoY inflation, that is

$$Var\ (\hat{\pi}_t^A) = \frac{1}{n-1} \sum_{i=1}^{n} w_{i0} \left[\frac{\frac{p_{it}^A}{p_{i0}} - \frac{p_{it-12}^A}{p_{i0}}}{\sum_{i=1}^{n} w_{i0} \frac{p_{it-12}^A}{p_{i0}}} - \hat{\pi}_t^A \right]^2 \qquad (23)$$

3. MEASURING STANDARD ERRORS OF INFLATION IN PAKISTAN:

In this section we present an application of the results derived in the previous section by using monthly data of prices of 374 commodities covering the period July 2001- June 2010 for Pakistan[5]. We present the estimated MoM inflation, YoY inflation, along with their standard errors for the case of Pakistan. As discussed above there are different ways to apply the stochastic approach to index numbers and each culminates in different form of index numbers like Divisia, Laspeyres etc. Just to compare our estimated results of inflation with those from Federal Bureaus of Statistics we have used such application of stochastic approach which produces Laspeyres index formula for measuring inflation in period t over the base period. Since State Bank of Pakistan targets (12-month) average inflation, particular attention has been paid to estimate period (period) average inflation and its standard error which is first empirical application of its type.

Table 1 in the Appendix we present the results for these three different forms of inflation for Pakistan economy.

[5] Prices for consumer price index (CPI) are collected by Federal Bureau of Statistics (Government of Pakistan) on month basis for 374 commodities.

Figures 1 (a) to 1 (c) present a scatter plot of the inflation versus the corresponding standard error for the three kinds of inflation, MoM, YoY and Moving average inflation; the solid line is the liner trend line.

Figure 1 (a): MoM Inflation in Pakistan and its S.E.

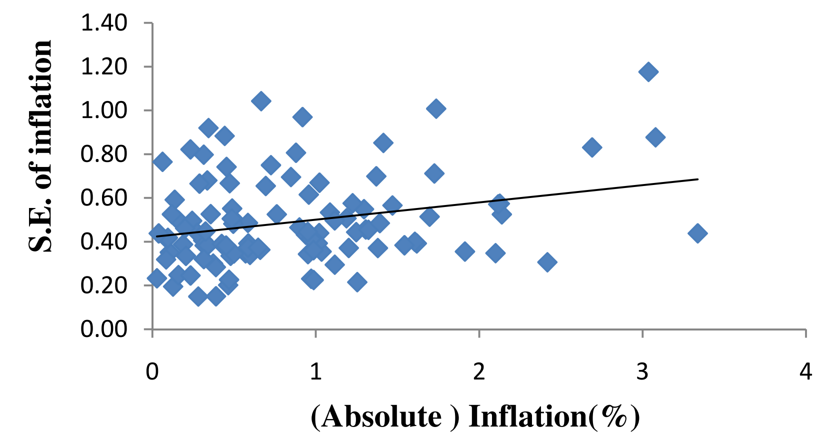
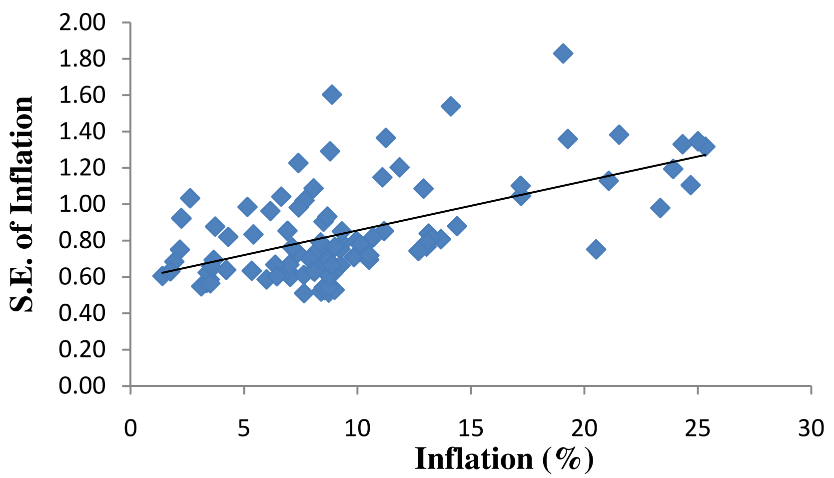
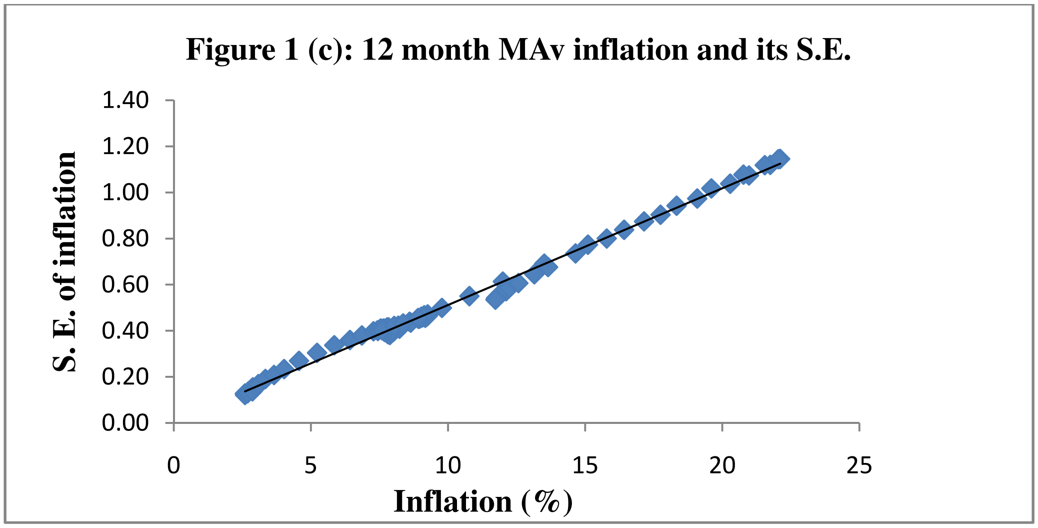


Figure 1 (b): YoY infaltion in Paksitan and its S.E.





From figure, since slope of each solid line is greater than zero, the standard error mostly increases along with increasing inflation. We can conclude that when the rate of inflation is higher it becomes more difficult to measure it precisely. Hence this agrees with the intuitive notion that when the individual prices move very disproportionately, the overall rate of inflation is less well defined.

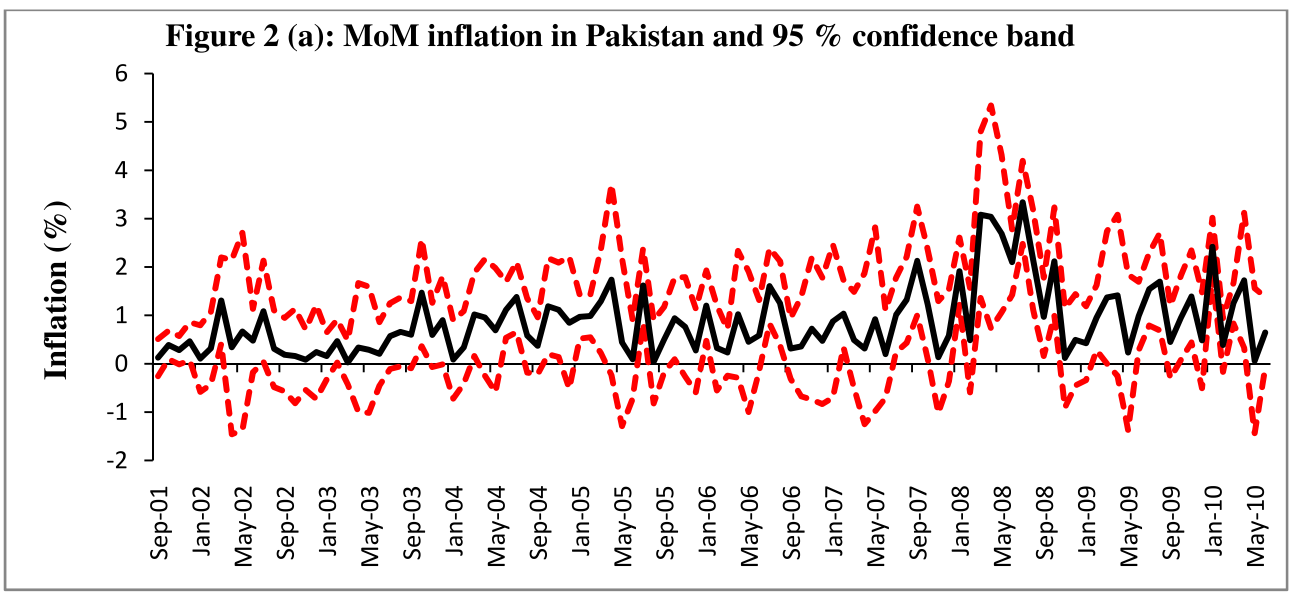


Figure 2 (b): YoY infaltion in Paksitan and 95% confidence band

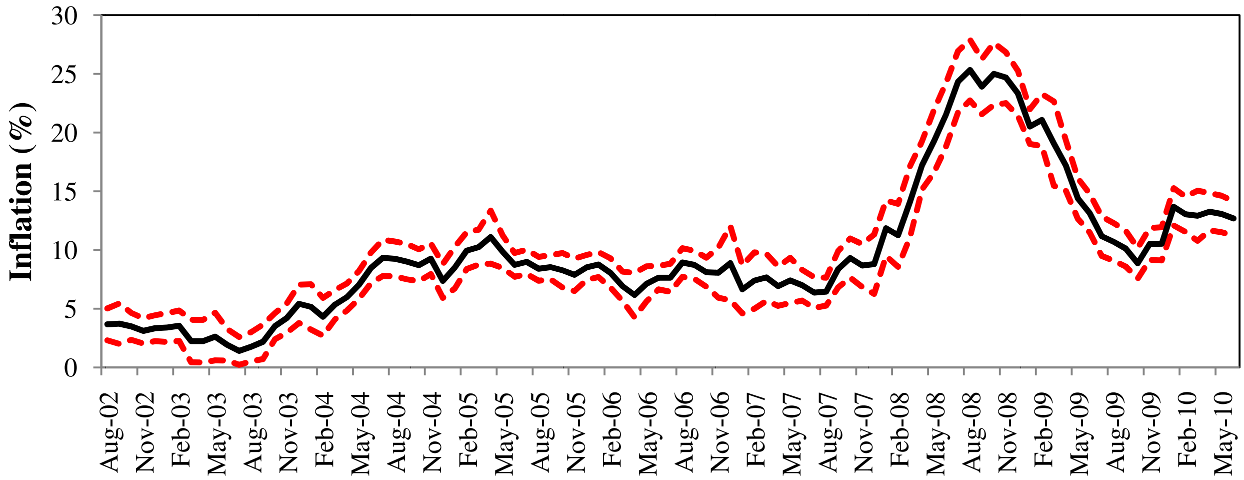


Figure 2 (c): 12 month MA infaltion in Paksitan and 95% confidence band

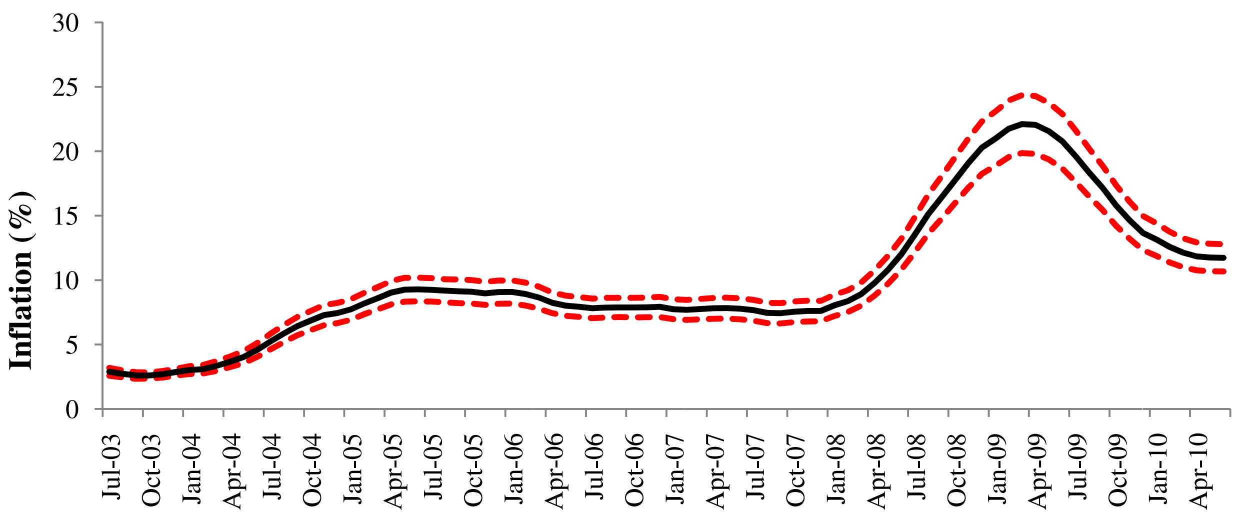


Figure 2 (a) to figure 2(c) present graph of all the three types of inflation along with respective 95% confidence band. From figure 2 (a) it is clear that the time when there is a jump in inflation like in April 2005 and May 2008, there is an increase in the width of the confidence bands. Similarly we can note that in other figures where the inflation is high, the width of confidence band is also increased.

4. MEASURING SYSTEMATIC CHANGE IN RELATIVE PRICES:

As we discussed in section 2, different ways to apply the stochastic approach to index numbers give various forms of index numbers like Divisia, Laspeyres etc. Keeping in mind the way FBS computes inflation, we followed such stochastic approach to end with Laspeyres formula. However, as criticized by Keynes (1930), following this approach we assumed that when prices change they change equiproportionately and thus relative prices remain same. Clements and Izan (1987) responded the Keynes criticism by considering common trend change in all prices as underlying rate of inflation separate from the systematic change in relative prices. Clements and Izan (1987) if we take p_{it} as the price of commodity i $(i = 1, 2, \dots, n)$ at time $t(t = 0, 1, 2, \dots\dots, T)$ then price log change $Dp_{it} = \log p_{it} - \log p_{it-1}$ can be considered as

$$Dp_{it} = \alpha_t + \beta_i + \xi_{it} \qquad i = 1, 2, \dots, n; \text{ and } t = 0, 1, 2, \dots\dots, T \tag{24}$$

Where α_t is the common trend change in all prices (the underlying rate of inflation) and β_i is the change in relative prices of commodity i. Assuming the random component of change in prices, ξ_{it}, to be independent over commodities and time, and that the variances $[Var(\xi_{it})]$ are inversely proportional to corresponding arithmetic averages of budget shares; Clements and Izan (1987) showed that least squares estimates of α_t and β_i subject to budget constraint[6] as given below:

$$\hat{\alpha}_t = \sum_{i=1}^{n} \bar{w}_i \, Dp_{it} \tag{25}$$

$$\hat{\beta}_i = \frac{1}{T}\sum_{t=1}^{T}\left[\frac{1}{\theta_t^2} \Big/ \sum_{t=1}^{T}\frac{1}{\theta_t^2}\right](Dp_{it} - \hat{\alpha}_t) \tag{26}$$

With respective variances of these estimators as below:

$$Var(\hat{\alpha}_t) = \frac{\theta_t^2}{(n-1)} \tag{27}$$

[6] Budget share weighted average of the systematic component of relative price change is zero

$$Var(\hat{\beta}_i) = \frac{1}{(n-1)\sum_{t=1}^{T}\frac{1}{\theta_t^2}}\left(\frac{1}{\overline{w}_i} - 1\right) \tag{28}$$

Where θ_t^2 is the sum (over commodities) of squares of estimated random component of price changes, that is

$$\theta_t^2 = \sum_{i=1}^{n}\left(\hat{\xi}_{it}\right)^2$$

$$= \sum_{i=1}^{n}\overline{w}_i(Dp_{it} - \hat{\alpha}_t)^2 + \sum_{i=1}^{n}\overline{w}_i(\overline{Dp_i} - \overline{\hat{\alpha}})^2 - 2\sum_{i=1}^{n}\overline{w}_i(Dp_{it} - \hat{\alpha}_t)\,\overline{w}_i(\overline{Dp_i} - \overline{\hat{\alpha}}) \tag{29}$$

While it is obvious that $\overline{Dp_i} = \frac{1}{T}\sum_{t=1}^{T} Dp_{it}$ and $\overline{\hat{\alpha}} = \frac{1}{T}\sum_{t=1}^{T}\hat{\alpha}_t$

Our contribution in this section of the study is an application of the Clements and Izan (1987) extended stochastic approach to index number to Pakistan's monthly data of prices of 374 commodities covering the period July 2001- June 2010. As in above section 3, this approach also gives us the (trend) inflation rate and its standard errors which are presented in Appendix. In addition to inflation and its standard error, the beauty of Clements and Izan (1987) extended stochastic approach is that it also gives us the systematic change in relative prices of each commodity. It may be difficult to extract any meaning result from the detailed presentation of systematic change in relative prices of each of the 374 commodities in CPI basket for Pakistan[7]. However, it will be meaningful if we present the average systematic change in relative prices of various groups in the CPI basket as in Table A2 of the Appendix. From the table it is clear that coefficients of relative prices of all groups are significant. The estimated relative prices of Food Beverages & Tobacco group are increased by highest percentage point amongst all the three ways to measure the inflation from month on month to 12-month moving averages.

[7] Detailed results can be obtained from the author, if desired.

5. CONCLUSION:

Stochastic approach to index number (and its growth) has recently attracted renewed attention of researchers after contributions from Clement and Izan (1987), Selvanathan (1989), Selvanathan (2003), Selvanathan and Selvanathan (2004), and Clement and Selvanathan (2007). Stochastic approach remained sidelined until Clement and Izan (1987) responded to the criticism of Keynes (1930) on the assumption of equiproportionate change in all prices while measure inflation. Clements and Izan (1987) recognized the Keynes (1930) criticism on the assumption of identical systematic changes in prices and viewed the underlying rate of inflation as an unknown parameter to be estimated from the individual prices changes by linking index number theory to regression analysis. By linking index number theory to regression analysis we not only get an estimate of the rate of inflation but also it sampling variance. With the relaxation of the assumption that prices of goods and services change equiproportionately, the individual prices in the basket of price index move disproportionately (which usually happens) and thus the overall rate of inflation may become less well defined (Selvanathan and Selvanathan 2006). In such situations the ability of stochastic approach becomes important by allowing us to construct confidence interval around the true rate of inflation with the help of standard error (of inflation).

In this study we estimate standard errors of month on month (MoM) and year on year (YoY) inflation in Pakistan under stochastic approach, following Clement and Izan (1987). To the best of our knowledge no one has done work on the estimation of standard error of period average of inflation. Since State Bank of Pakistan (the central bank) targets 12-motnh average of YoY inflation, we contribute by providing mechanism and estimating the standard error of YoY inflation; and 12-motnh average of YoY inflation in Pakistan. From the analysis of change in relative prices we found that estimated relative prices of Food Beverages & Tobacco group are increased by highest percentage point amongst all the three ways to measure the inflation from month on month to 12-month moving averages.

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Appendix

Table A1: Official rate of inflation, stochastic approach estimates of inflation and extended stochastic approach estimates of inflation

Month	Month on Month					Headline (Year on Year)					12-month moving average				
	Official rate of inflation	Stochastic approach estimates of		Extended stochastic approach estimates of		Official rate of inflation	Stochastic approach estimates of		Extended stochastic approach estimates of		Official rate of inflation	Stochastic approach estimate of		Extended stochastic approach estimates of	
		inflation	S.E.	Inflation	S.E.		Inflation	S.E.	Inflation	S.E.		Inflation	S.E.	Inflation	S.E.
Jul-03	0.57	1.01	0.35	0.89	0.24	1.41	1.72	0.60	1.80	0.54	2.89	3.03	0.16	1.54	0.48
Aug-03	0.66	0.76	0.36	0.62	0.15	1.76	2.03	0.63	2.04	0.52	2.73	2.68	0.14	1.52	0.48
Sep-03	0.60	0.53	0.35	0.32	0.18	2.18	2.34	0.75	2.21	0.54	2.60	2.50	0.13	1.50	0.50
Oct-03	1.47	1.57	0.57	1.20	0.33	3.51	4.94	0.56	3.47	0.50	2.60	2.37	0.12	1.64	0.50
Nov-03	0.60	0.69	0.34	0.74	0.14	4.22	5.08	0.64	4.29	0.57	2.70	2.52	0.13	1.89	0.49
Dec-03	0.90	1.17	0.47	1.12	0.17	5.41	6.92	0.83	5.43	0.56	2.87	2.68	0.14	2.22	0.46
Jan-04	-0.09	0.11	0.41	-0.07	0.27	5.15	6.67	0.98	5.26	0.58	3.02	3.06	0.16	2.54	0.42
Feb-04	-0.34	-0.48	0.38	-0.23	0.31	4.31	5.43	0.82	4.62	0.59	3.09	3.29	0.17	2.79	0.40
Mar-04	1.02	0.81	0.44	0.74	0.26	5.34	6.19	0.63	5.52	0.51	3.35	3.73	0.19	3.21	0.38
Apr-04	0.96	0.81	0.62	0.42	0.35	5.99	6.39	0.59	6.02	0.48	3.66	4.07	0.21	3.56	0.37
May-04	0.69	0.78	0.65	1.02	0.43	7.04	7.57	0.60	7.03	0.50	4.03	4.56	0.23	4.08	0.36
Jun-04	1.12	1.00	0.29	1.12	0.20	8.45	9.10	0.67	7.89	0.53	4.57	5.26	0.27	4.70	0.36
Jul-04	1.38	1.58	0.37	1.20	0.25	9.33	9.72	0.78	8.21	0.56	5.23	5.94	0.30	5.23	0.36
Aug-04	0.59	0.64	0.39	0.62	0.14	9.25	9.59	0.75	8.20	0.52	5.86	6.58	0.34	5.75	0.37
Sep-04	0.37	0.27	0.30	0.28	0.12	9.00	9.30	0.76	8.16	0.55	6.43	7.03	0.36	6.27	0.40
Oct-04	1.19	1.19	0.51	0.76	0.14	8.70	8.89	0.69	7.72	0.49	6.86	7.41	0.38	6.62	0.41
Nov-04	1.12	0.99	0.50	1.05	0.21	9.26	9.22	0.66	8.04	0.51	7.29	7.77	0.40	6.94	0.42
Dec-04	-0.85	-0.97	0.70	-0.05	0.38	7.37	6.91	0.73	6.87	0.54	7.45	7.79	0.40	7.03	0.40
Jan-05	0.97	1.04	0.23	0.95	0.19	8.51	7.89	0.90	7.88	0.63	7.72	7.90	0.40	7.21	0.38
Feb-05	0.99	1.03	0.22	0.83	0.17	9.95	9.53	0.80	8.95	0.66	8.19	8.23	0.42	7.54	0.39
Mar-05	1.29	1.56	0.55	0.84	0.27	10.25	10.35	0.76	9.05	0.60	8.60	8.56	0.44	7.81	0.40
Apr-05	1.74	2.10	1.01	0.94	0.41	11.10	11.76	1.15	9.56	0.61	9.03	9.00	0.46	8.11	0.40
May-05	-0.44	-0.84	0.88	0.16	0.31	9.84	9.96	0.71	8.70	0.51	9.26	9.18	0.47	8.22	0.39
Jun-05	0.10	-0.28	0.42	0.74	0.19	8.74	8.57	0.51	8.32	0.46	9.28	9.13	0.47	8.27	0.39
Jul-05	1.62	1.55	0.39	1.06	0.18	8.99	8.54	0.53	8.17	0.48	9.25	9.04	0.46	8.28	0.38
Aug-05	0.04	0.12	0.44	0.12	0.23	8.40	7.98	0.52	7.67	0.46	9.17	8.90	0.46	8.24	0.37
Sep-05	0.50	0.44	0.35	0.34	0.15	8.53	8.17	0.53	7.73	0.48	9.13	9.12	0.47	8.21	0.36

Oct-05	0.94	0.53	0.43	0.51	0.16	8.27	7.46	0.75	7.48	0.55	9.09	9.01	0.46	8.18	0.37
Nov-05	0.76	0.94	0.53	0.93	0.18	7.89	7.41	0.70	7.36	0.54	8.97	8.84	0.45	8.08	0.38
Dec-05	-0.27	-0.33	0.44	-0.10	0.23	8.51	8.10	0.54	7.30	0.45	9.06	8.93	0.46	8.11	0.38
Jan-06	1.20	1.28	0.37	0.96	0.31	8.76	8.37	0.54	7.32	0.40	9.08	8.96	0.46	8.06	0.39
Feb-06	0.33	0.42	0.45	0.69	0.32	8.05	7.71	0.63	7.17	0.48	8.92	8.81	0.45	7.93	0.40
Mar-06	0.23	0.25	0.25	0.16	0.17	6.91	6.32	0.64	6.49	0.51	8.64	8.50	0.44	7.73	0.41
Apr-06	1.02	1.09	0.67	0.68	0.29	6.16	5.27	0.96	6.24	0.58	8.23	7.98	0.41	7.45	0.42
May-06	0.45	0.32	0.74	0.34	0.22	7.12	6.51	0.76	6.42	0.58	8.01	7.71	0.40	7.28	0.43
Jun-06	0.59	0.49	0.37	0.94	0.30	7.65	7.33	0.51	6.62	0.43	7.92	7.61	0.39	7.14	0.43
Jul-06	1.61	1.64	0.40	1.09	0.24	7.63	7.42	0.61	6.65	0.52	7.81	7.52	0.39	7.01	0.42
Aug-06	1.25	1.38	0.44	0.97	0.15	8.93	8.77	0.62	7.50	0.48	7.86	7.59	0.39	7.00	0.40
Sep-06	0.32	0.18	0.32	0.38	0.11	8.73	8.49	0.61	7.55	0.51	7.88	7.45	0.38	6.97	0.39
Oct-06	0.36	0.44	0.53	0.20	0.22	8.11	8.39	0.63	7.24	0.46	7.87	7.49	0.38	6.96	0.38
Nov-06	0.73	1.10	0.75	0.68	0.28	8.07	8.55	1.09	6.98	0.52	7.89	7.61	0.39	6.97	0.37
Dec-06	0.47	0.88	0.67	0.50	0.33	8.88	9.88	1.60	7.59	0.73	7.92	7.76	0.40	7.03	0.38
Jan-07	-0.88	-1.10	0.81	-0.36	0.52	6.64	7.29	1.04	6.26	0.57	7.74	7.68	0.39	6.96	0.37
Feb-07	1.04	1.19	0.36	0.95	0.21	7.39	8.12	1.23	6.53	0.64	7.69	7.71	0.40	6.91	0.35
Mar-07	0.49	0.27	0.51	0.18	0.31	7.67	8.14	1.02	6.55	0.66	7.75	7.85	0.40	6.93	0.34
Apr-07	0.31	0.07	0.80	-0.05	0.31	6.92	7.05	0.85	5.82	0.58	7.81	8.01	0.41	6.91	0.33
May-07	0.92	0.97	0.97	0.85	0.33	7.41	7.74	0.98	6.32	0.72	7.83	8.10	0.42	6.89	0.35
Jun-07	0.20	-0.09	0.46	0.92	0.20	7.00	7.12	0.67	6.30	0.61	7.77	8.07	0.41	6.87	0.36
Jul-07	1.01	1.08	0.39	1.10	0.26	6.37	6.53	0.67	6.31	0.62	7.66	7.99	0.41	6.84	0.36
Aug-07	1.32	1.46	0.45	1.06	0.23	6.45	6.61	0.60	6.40	0.55	7.45	7.80	0.40	6.74	0.38
Sep-07	2.13	2.32	0.57	1.54	0.23	8.37	8.89	0.79	7.57	0.57	7.43	7.84	0.40	6.75	0.41
Oct-07	1.23	1.27	0.58	0.90	0.17	9.31	9.79	0.85	8.27	0.61	7.54	7.99	0.41	6.85	0.44
Nov-07	0.14	-0.15	0.59	0.40	0.26	8.67	8.44	0.93	8.00	0.67	7.60	7.97	0.41	6.90	0.45
Dec-07	0.58	0.74	0.49	0.85	0.23	8.79	8.29	1.29	8.35	0.77	7.60	7.84	0.40	6.92	0.47
Jan-08	1.91	1.94	0.35	1.58	0.25	11.86	11.62	1.20	10.29	0.82	8.04	8.21	0.42	7.27	0.51
Feb-08	0.49	0.18	0.55	0.13	0.41	11.25	10.50	1.36	9.46	0.88	8.36	8.42	0.43	7.50	0.54
Mar-08	3.08	3.22	0.88	2.35	0.34	14.12	13.75	1.54	11.64	0.86	8.91	8.89	0.46	7.95	0.56
Apr-08	3.04	3.17	1.18	2.60	0.43	17.21	17.28	1.05	14.29	0.73	9.78	9.74	0.50	8.67	0.57
May-08	2.69	2.79	0.83	2.73	0.41	19.27	19.40	1.36	16.17	0.87	10.78	10.74	0.55	9.55	0.58
Jun-08	2.10	1.91	0.35	2.17	0.24	21.53	21.79	1.38	17.43	0.92	12.00	11.97	0.61	10.53	0.59
Jul-08	3.34	3.33	0.44	3.03	0.39	24.33	24.49	1.33	19.36	0.79	13.51	13.48	0.69	11.67	0.60
Aug-08	2.14	2.33	0.53	1.84	0.29	25.33	25.57	1.32	20.13	0.76	15.10	15.09	0.77	12.86	0.61
Sep-08	0.97	0.86	0.41	0.79	0.20	23.91	23.78	1.19	19.38	0.71	16.42	16.34	0.84	13.86	0.62
Oct-08	2.12	2.15	0.57	1.72	0.26	25.00	24.85	1.35	20.19	0.79	17.75	17.62	0.90	14.84	0.64
Nov-08	-0.12	-0.29	0.53	0.62	0.40	24.68	24.68	1.10	20.41	0.66	19.09	18.99	0.97	15.89	0.63

Dec-08	-0.50	-0.25	0.48	0.23	0.17	23.34	23.45	0.98	19.79	0.59	20.29	20.24	1.04	16.84	0.60
Jan-09	-0.42	-0.55	0.39	-0.13	0.31	20.52	20.43	0.75	18.08	0.55	20.97	20.94	1.07	17.44	0.56
Feb-09	0.95	1.22	0.34	1.15	0.23	21.07	21.69	1.13	19.10	0.70	21.75	21.84	1.12	18.19	0.55
Mar-09	1.37	1.94	0.70	0.95	0.25	19.07	20.17	1.83	17.69	0.87	22.11	22.34	1.15	18.59	0.54
Apr-09	1.41	1.10	0.85	0.98	0.29	17.19	17.76	1.10	16.07	0.75	22.04	22.32	1.14	18.68	0.52
May-09	0.23	0.04	0.82	0.40	0.40	14.39	14.61	0.88	13.74	0.69	21.55	21.81	1.12	18.39	0.48
Jun-09	0.99	0.70	0.36	1.32	0.27	13.14	13.25	0.84	12.89	0.73	20.77	21.01	1.08	17.90	0.45
Jul-09	1.54	1.61	0.38	1.30	0.22	11.17	11.37	0.85	11.17	0.72	19.60	19.84	1.02	17.14	0.44
Aug-09	1.70	1.66	0.51	1.30	0.27	10.69	10.64	0.82	10.63	0.71	18.33	18.38	0.94	16.29	0.45
Sep-09	0.45	0.38	0.38	0.39	0.19	10.12	10.11	0.78	10.23	0.68	17.15	17.05	0.87	15.49	0.48
Oct-09	0.95	0.89	0.45	0.66	0.15	8.87	8.74	0.66	9.17	0.63	15.79	15.61	0.80	14.56	0.51
Nov-09	1.39	1.27	0.48	1.35	0.22	10.51	10.45	0.69	9.90	0.55	14.65	14.35	0.74	13.69	0.51
Dec-09	-0.49	-0.36	0.51	0.04	0.23	10.52	10.32	0.72	9.72	0.56	13.65	13.19	0.68	12.87	0.51
Jan-10	2.42	2.47	0.31	2.28	0.29	13.68	13.68	0.81	12.13	0.58	13.15	12.59	0.65	12.42	0.52
Feb-10	0.39	0.37	0.28	0.58	0.22	13.04	12.73	0.77	11.56	0.58	12.57	11.84	0.61	11.87	0.52
Mar-10	1.25	1.29	0.22	0.99	0.16	12.91	12.01	1.09	11.61	0.66	12.12	11.15	0.57	11.42	0.51
Apr-10	1.73	2.07	0.71	1.85	0.42	13.26	13.09	0.81	12.48	0.65	11.84	10.76	0.55	11.16	0.51
May-10	0.06	-0.19	0.76	0.17	0.20	13.07	12.83	0.79	12.24	0.64	11.75	10.57	0.54	11.06	0.51
Jun-10	0.65	0.34	0.37	0.78	0.17	12.69	12.43	0.74	11.70	0.62	11.73	10.43	0.54	11.01	0.51

Table A2: Group-wise change in relative price (July 01-June 10)

		Month on Month Inflation			Headline (Year on Year)			12-month moving average		
Group	Weight in CPI basket	Actual change in relative price (%)	Estimated change in relative price (%)	S.E.	Actual change in relative price (%)	Estimated change in relative price (%)	S.E.	Actual change in relative price (%)	Estimated change in relative price (%)	S.E.
Food Beverages & Tobacco	0.403	0.14	0.18	0.001	1.62	1.72	0.01	1.47	1.72	0.01
Apparel, Textile & Footwear	0.061	-0.26	-0.21	0.004	-3.38	-3.15	0.09	-3.18	-3.22	0.07
House Rent	0.234	-0.06	-0.01	0.001	-0.51	-0.36	0.02	-0.55	0.04	0.02
Fuel & Lighting	0.073	0.03	-0.18	0.003	0.73	0.37	0.08	0.99	-0.44	0.06
Household Furniture & Equipment	0.033	-0.24	-0.18	0.008	-2.89	-2.37	0.18	-2.59	-2.16	0.14
Transport & Communication	0.073	-0.06	-0.14	0.003	-0.34	-1.06	0.08	-0.09	-1.37	0.06
Recreation & Entertainment	0.008	-0.45	-0.51	0.035	-6.20	-6.33	0.75	-6.17	-6.78	0.55
Education	0.035	-0.16	-0.20	0.008	-1.77	-1.48	0.75	-1.87	-1.58	0.13
Cleaning, Laundry & Personal Appearance	0.059	-0.17	-0.17	0.005	-2.08	-2.21	0.01	-2.04	-2.14	0.07
Medicare	0.021	-0.28	-0.27	0.140	-3.77	-2.86	0.29	-3.34	-3.42	0.22