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Applying the Structural Equation Model Rule-Based Fuzzy System with Genetic Algorithm for Trading in Currency Market

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Abstract

The present study uses the structural equation model (SEM) to analyze the correlations between various economic indices pertaining to latent variables, such as the New Taiwan Dollar (NTD) value, the United States Dollar (USD) value, and USD index. In addition, a risk factor of volatility of currency returns is considered to develop a risk-controllable fuzzy inference system. The rational and linguistic knowledge-based fuzzy rules are established based on the SEM model and then optimized using the genetic algorithm. The empirical results reveal that the fuzzy logic trading system using the SEM indeed outperforms the buy-and-hold strategy. Moreover, when considering the risk factor of currency volatility, the performance appears significantly better. Remarkably, the trading strategy is apparently affected when the USD value or the volatility of currency returns shifts into either a higher or lower state.

Keywords: Knowledge-based Systems, Fuzzy Sets, Structural Equation Model (SEM), Genetic Algorithm (GA), Currency Volatility

1. Introduction

With the onset of financial liberalization, internationalization, and new financial technology and innovation among countries, the global capitals flow more rapidly and massively in currency markets. In this context, currency exchange rates also become more volatile, unpredictable, and uncontrollable. In fact, the changes in the exchange rate reflect the relative activities of the economy between countries despite brief currency speculations. This condition implies that, to manage properly the risk and uncertainty of exchange rates, the interactive factors of the economy that actually result in the changes in the exchange rate should be examined. Once the trend or volatility of the exchange rate is well managed and supervised, international traders, financial institutes, and currency investors would find it advantageous to create effective and correct hedges, as well as to formulate investment strategies.

However, the exchange rates are determined by the demand and supply in the currency market, and sometimes, the central banks will intervene for the market's own benefit (Neely, 2005). In a liberalized market, a number of factors will reduce the

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demand for certain currencies, resulting in the depreciation of the value of such currency. Therefore, what factors will give rise to changes in demand and supply, i.e., market volatility? What factors should be managed to prevent the risk for exchange rate changes?

Economists have continuously proposed a number of exchange rate determination models and theories from various perspectives to determine the factors affecting exchange rates. Moffet and Karlsen (1994) found that the most important factors are inflation, interest rate, international balance of payments, and government fiscal policy, among others. However, a number of studies report that money supply and demand are the most important factors determining equilibrium exchange rates (Eichengreen et al., 2006), further arguing that monetary policy is the most influential tool in determining exchange rates (Bilson, 1981; Frenkle, 1981). A portfolio balance model (Branson et al., 1977; Branson and Henderson, 1985) assumes that domestic and foreign bonds are not interchangeable and that the portfolios held by investors affect the determination of exchange rates. Studies on foreign exchange risk began appearing in the 1970s. Among the best known is the regression analysis by Alder and Dumas (1984). Over the last two decades, several studies have employed Alder and Dumas as the basis for their determination of foreign exchange risk models (e.g. Williamson, 2001; Bodnar et al., 2002; Koedijk et al., 2002; Bodnar and Wong, 2003; Doidge et al., 2006).

However, referring to the factors or approaches found in only one or two specific references lacks comprehensiveness. On the other hand, judging the effect of these factors on the change in exchange rates is extremely deterministic. Thus, the current paper combines the theories on balance of payments, purchasing power parity, and flexible price monetary approaches to construct a knowledge-based system. Subsequently, the studied factors comprise money supply (M2), consumer pricing index (CPI), gross domestic production (GDP), rediscount rate, and stock price index as the five major observable variables to construct the structural equation model (SEM). Although the SEM includes theoretical constructs, it also handles measurement errors using the maximum likelihood estimation (MLE) to estimate the parameters (Anderson and Gerbing, 1988).

In the present paper, three latent (i.e. unobservable) variables are used, which are the value of the United States Dollar (USD), the value of the New Taiwan Dollar (NTD), and the USD index in SEM. The relationships between the three latent factors are very useful in building the knowledge-based system (Jöreskog, 1973). The volatilities of USD to TWD are also considered important in determining the exchange rate changes. Therefore, four factors are used as the input variables, with the change in exchange rates being the output variable for the fuzzy model. The knowledge-based system created through SEM can fit well with the fuzzy logical rules because such an approach reduces the number of observable variables and reveals the relationships between several latent variables. More specifically, a SEM comprises a measurement part, which represents the relationships between the latent variables and their observable variables, and a structural part, which represents the causal relations between the variables (Jöreskog and Sörborn, 1996).

To better train and test the non-linear fuzzy model, the fuzzy genetic algorithm (FGA) was employed to fit the fuzzy model. The fuzzy logic enables the processing of vague information through membership functions in contrast to Boolean characteristic mappings (Zadeh, 1965). Such an approach helps in the identification of the optimal parameters involved in fuzzy memberships, as well as the fuzzy rules (Karr, 1991, 1993; Chan et al., 1997). The fuzzy inference system presents a state-of-the-art framework that includes expert (explicable) knowledge in modeling nonlinear stochastic processes and complex systems (Zimmermann, 1996). The fuzzy and knowledge-based model provides the power behind the expert management that supervises the changes in exchange rates not only through the more concentrated and comprehensible factors, but also using the logical rules constructed by SEM and operated by Mamdani-type inference (Mamdani, 1976). Therefore, the application of a fuzzy expert system using the SEM is valuable and innovative for risk management in currency markets.

2. Methodology

2.1 Fuzzy logic

(1) The origin of fuzzy logic

The mathematical problems encountered in our world are classified into certain, random, and fuzzy phenomena. Although a random phenomenon is best addressed by the probability theory and statistics, the fuzzy phenomenon contrary to the certain, i.e., crisp phenomenon observed by the binary logic is perceived using the fuzzy logic that was proposed by Zadeh (1965, 1996). The fuzzy logic, which works in a similar manner as human reasoning, has the control and inference capabilities to implement a reasoning process based on the fuzzy set theory. Using the membership functions, the fuzzy sets are defined and characterized. For example, using some kind of linear/non-linear membership functions, a real-valued variable X is mapped to fuzzy numbers with a value between 0 and 1 and is denoted by $\mu_A(X)$. This value describes to what degree X belongs to fuzzy set A, similar to linguist terms such as "young" or "old" and "good" or "bad." To date, the fuzzy arithmetic, inference, and classification, among others, are developed and applied further to address the paradigm of system control or modeling.

(2) Fuzzy logic and inference

Unlike the binary classical logic that assigns value 1 for true and value 0 for false, the fuzzy logic is a mapping of membership function μ, which maps the universe of true values X onto the interval [0,1], written as $\mu : x \in X \rightarrow [0,1]$. The frequently used operations of fuzzy sets are the intersection and union. Suppose that Fuzzy sets A and B defined in the universe of discourse X have the membership functions μ_A and μ_B, respectively. The intersection and union of fuzzy sets A and B can then be written as:

$\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_A(x)) = \mu_A(x) \wedge \mu_A(x)$, that is one of the T-Norms

$\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_A(x)) = \mu_A(x) \vee \mu_A(x)$, that is one of the S-Norms

Moreover, fuzzy set relations, such as fuzzy Cartesian product and composition, also exist. For example, an n-ary fuzzy relation is a mapping of $R{:}X_1 \times X_2 \times \ldots \times X_n \rightarrow [0,1]$, which assigns membership grades to all n-tuples $(x_1, x_2, \ldots, x_n)$ from the Cartesian product $X_1 \times X_2 \times \ldots \times X_n$. Composition relations exist, such as max-min and max-product operations. Moreover, a fuzzy proposition P can be assigned to fuzzy set A, and its true value is given by $\mu(P) = \mu_A(x)$, with $0 \le \mu_A(x) \le 1$. Specifically, the degree of the truth for the fuzzy proposition P is indicated by the membership incline of x in the fuzzy set A. Logical fuzzy propositions, such as negation, disjunction, conjunction, and implication, may exist in fuzzy inference. The reasoning scheme is simplified as bypassing the relational calculus.

The expert knowledge has to be formulized using a set of linguist if-then fuzzy rules for the fuzzy inference. Thus, implementing all fuzzy operations as aforementioned is very complicated. Thus, the framework of min-max rule-based fuzzy inference is adopted (Mamdani, 1976) for its flexibility (Mamdani and Assilian, 1975) and practicality (Jager, 1995). Figure 1 depicts the min-max fuzzy inference, with two input variables X_1 and X_2 and two fuzzy rules, where A_i, B_i (for i=1,2), and C are fuzzy sets with their respective bell-shaped membership function. In this case, four major steps should be performed in this fuzzy inference, which are described as follows:

(i) In fuzzifying input variable X_1 and X_2, the function of antecedence is to implement the premise part of "if." The result of "and" or "or" operation β_i in the antecedence represents the degree of fulfillment for the rule i and can be written as:

$$\beta_i = \mu_{A_i}(x_1) \wedge \mu_{B_i}(x_2); \beta_i = \mu_{A_i}(x_1) \vee \mu_{B_i}(x_2) \text{ for } i=1,2$$

(ii) The "then" operation is the consequence represented by a fuzzy set C_i in output variable Z. The fuzzy implication reshapes the consequence with β_i given in the antecedence. The minimum implication is written as:

$$\mu_{C_i^{'}}(z) = \beta_i \wedge \mu_{C_i}(z) \text{ for } i=1,2$$

(iii) The outputs of $\mu_{C_i^{'}}(z)$ for i=1,2 are then aggregated to form a fuzzy set $C^{'}$ in output variable Z that is written as:

$$\mu_{C^{'}}(z) = \mu_{C_1^{'}} \vee \mu_{C_2^{'}} = [\beta_1 \wedge \mu_{C_1}(z)] \vee [\beta_2 \wedge \mu_{C_2}(z)]$$

(iv) Finally, the output of aggregation is defuzzified to obtain a crisp output z. Different methods of defuzzification may be used, such as mean of maximum, middle of max, center of area (COA), and Center of Gravity. The current paper uses method of COA to identify the centroid of the fuzzy set. For continuous and discrete cases, COA is written as:

$$z_{COA} = \frac{\int \mu_{C^{'}}(z) z dz}{\int \mu_{C^{'}}(z) z dz}; z_{COA} = \frac{\sum_{i=1}^{n} \mu_{C^{'}}(z_i) z_i}{\sum_{i=1}^{n} \mu_{C^{'}}} \quad (1)$$

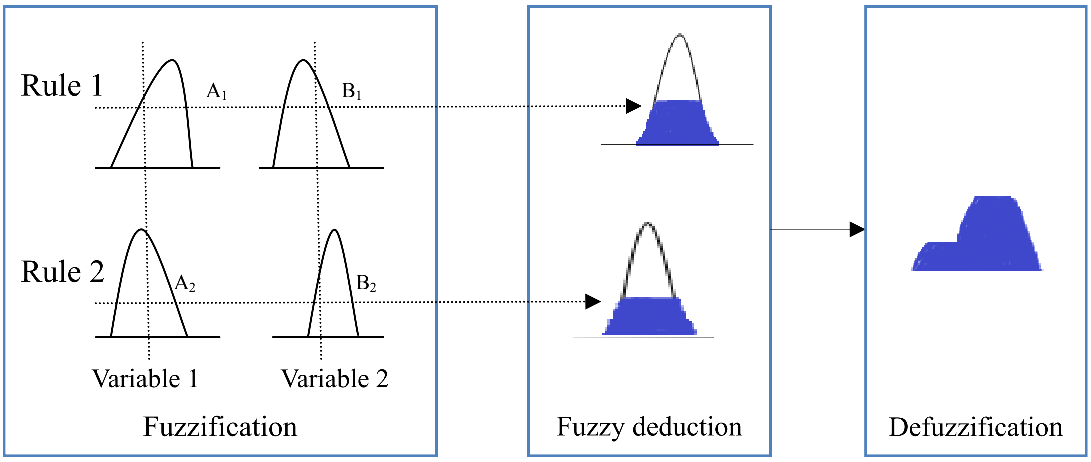


Figure 1 Fuzzy Inference

2.2 Genetic algorithm (GA)

A number of parameters embedded in the fuzzy inference should be determined to estimate the output variable effectively. For instance, the parameters or shape of the membership function, which may be Gaussian, trapezoidal, triangular, s-shaped, or z-shaped function, is a determinant for output estimation. To preclude the excessive non-linear property, the Gaussian membership function is applied in the present work. Suppose θ is the decision vector to be solved. The optimal problem is to minimize the objective function of root mean squared error (RMSE) in the fuzzy inference system, which is written as:

$$\underset{\theta}{Min}\,\mathrm{RMSE} = \sqrt{\sum_{i=1}^{n}\frac{(z_i - \hat{z}_i)^2}{n}} \tag{2}$$

where $\hat{z}_i$ is the estimation of output z_i for sample I, and n is the sample size.

The rule-based fuzzy inference is non-linear with max-min composition, implying that the functions in the fuzzy inference are somehow kinked and cannot be differentiable. Hence, the fuzzy inference system cannot be optimized using the classical method of direction search. In this complex context, GA based on the mechanics of natural selection and natural genetics is a better robust algorithm for implementing the fuzzy optimization. Three major types of operations exist in GA, namely, selection, reproduction, and mutation, according to the Darwinian evolutionary theory. For a detailed interpretation and demonstration, a number of previous studies may be used as reference (Goldberg, 1989; Koza, 1992).

However, the fuzzy inference system has a number of limitations. Aside from its nonlinear property and inefficiency in computing GA, one major criticism is that the fuzzy rules are not well developed and cannot be applied to the knowledge-based system. Thus, to embody the expert knowledge into the fuzzy rules appropriately, the structural equation modeling, as seen in the next section, is proposed to determine the concise and integrated structural relationships between input and output variables and to build the more convincing fuzzy rules as well.

2. 3 Structural equation modeling

The SEM technique enables the examination of complicated phenomena using hypothetical construct (latent) variables, measurement error, and correlative (interdependent) causation (Bollen and Long, 1993). The structural relationship between variables constructed by the linear relationship is referred to as the SEM (Tabachnick and Fidell, 1996). The SEM is currently one of the most useful tools for path analysis in marketing and consumer research (Gefen et al., 2000). SEM is also frequently used in traveler behavior studies and activity analyses (Kuppam and Pendyala, 2001).

The SEM group includes “covariance structure analysis,” “latent variable analysis,” “confirmatory factor analysis,” and the “analysis of linear structural relationships.” By combining multiple regression and factor analyses, the SEM is capable of simultaneously analyzing correlations of a group of mutually dependent variables (Hair, 1998). The SEM functions are essentially used in exploring the causality between multivariables or univariables that are not supposed to be directly measured. Although some presumptions have to be made over observed variables and measurement errors, the unobservable or latent variables are constructed. In the meantime, the structural model is established between endogenous and exogenous constructs. Thus, the theoretical structure of SEM comprises the “structural model” and “measurement model” (Hatcher, 1998). Specifically, the structural model is used to define the linear relationship between the endogenous and exogenous latent variables, whereas the measurement model defines the linear relationship between the latent variables and the observed variables. Thus, the linear relationship developed from the second model is typically used to extort the observation data before processing the analysis (Lin, 1984).

3. Data Methods and Description

3.1 Study design

The present study uses the SEM to build an overall Mamdani-type fuzzy influence system for analyzing reasonable relationships between various observed and latent variables of the economic index. The observed variables include the USD value, NTD value, and the USD index, plus the USD/NTD volatility of exchange rate returns that acts as the risk factor for control. Note that USD/NTD indicates that 1 US dollar is expressed in New Taiwan Dollars. The output and input variables are converted into linguistic variables or fuzzy sets using various suitable membership functions. The major fuzzy rules are then constructed by the SEM of path analysis. To optimize the fuzzy influence system, the GA is applied to fit the parameters embedded in the membership functions of each linguist variable. Finally, the fuzzy trading system is constructed to empirically verify the decision of trading.

3.2 Study samples

The study samples are derived from the macroeconomic database of Taiwan Economic Journal (TEJ) and the Financial Statistics Monthly edited by the Economic Research Department at the Central Bank of the Republic of China. The study period covers January 1, 1996 to December 2010, yielding a total of 180 pieces of data that are regarded as the values of observed variables. To test the robustness of the model, the data are divided into training period samples and verification period samples. The training period is from February 1996 to September 2007, whereas the verification period is from July 2008 to December 2010. Using the available variables, the present study converts the macroeconomic variables into monthly increase rates for the SEM rule-based fuzzy model. The measurement scales for different economic indexes vary widely. Thus, the present study also converts a variety of data into standardized scores before analyzing or comparing the statistics so as to achieve a good fit between the model and the data sample.

3.3 Conceptual model construction

According to the theory of SEM and the relevant literature, several variables can affect the change in exchange rates. The data must be meaningful and informative for the change in exchange rates. However, including all the variables in the model construction would be costly and would also make the achievement of the goal difficult. Thus, although the macroeconomic system widely covers the monetary, financial securities, and labor markets, the current research followed Walras' Law by removing the labor market and selecting five important variables from the remaining three markets: M2, CPI, GDP, rediscount rate, and stock price index. The present study also adopted various economic observed values from Taiwan and the US, using these values for the observed variables that are classified into the 1st and 2nd Groups. The observed values taken from Taiwan (hereafter referred to as the NTD value) are the endogenous variables (dependent variables), whereas those from the US (hereafter referred to as the USD value) are the exogenous variables (independent variables). To better understand the interaction of economic variables with the exchange rates of the major world currencies, the present work selects six major international currencies, namely the Euro (EUR), Japanese Yen (JPY), Great Britain Pound (GBP), Canadian Dollar (CAD), Swedish Kronor (SEK), and the Swiss Franc (CHF), and attempts to compare the volatility of these currencies against the US dollar. Based on the weight of the USD index, the present study computes the values of USD index that are observable and can be classified as Group 3. The USD index is considered as an endogenous or mediator variable.

By analyzing foreign exchange related literature, the current research compiles the data from expert knowledge and infers several hypotheses through the SEM. The hypotheses include a positive effect of USD value on NTD value and on the USD index, a positive effect of the USD index on the NTD value, and an indirect effect of USD value on NTD value through the USD index. The result can be found in Sec. 4.1.

The current research uses the SEM, as shown in Figure 2, to construct the conceptual model. The measurement and structural models are also built by estimating the model parameters. The details are shown as follows:

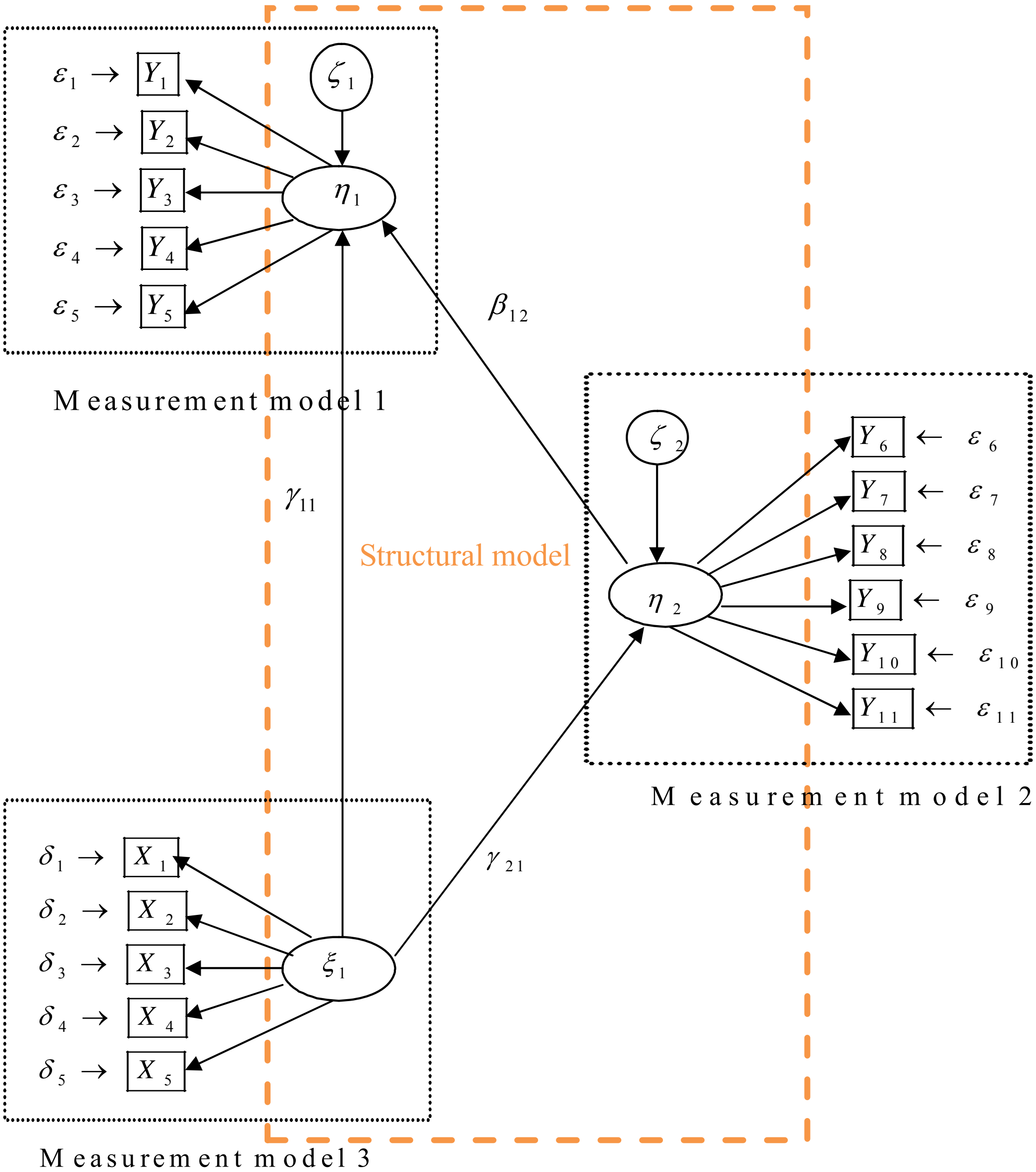


Figure 2 SEM conceptual model

(1) Measurable equation

(a) Measurement model 1

TW $M_2 = f$(NTD Value, measurement error) $\quad Y_1 = \lambda_{11}^y \eta_1 + \varepsilon_1$

TW CPI $= f$(NTD Value, measurement error) $\quad Y_2 = \lambda_{21}^y \eta_1 + \varepsilon_2$

TW GDP $= f$(NTD Value, measurement error) $\quad Y_3 = \lambda_{31}^y \eta_1 + \varepsilon_3$

TW Rediscount rate $= f$(NTD Value, measurement error) $\quad Y_4 = \lambda_{41}^y \eta_1 + \varepsilon_4$

TW Stock price index $= f$(NTD Value, measurement error) $\quad Y_5 = \lambda_{51}^y \eta_1 + \varepsilon_5$

(b) Measurement model 2

USD/EUR $= f$(US dollar index, measurement error) $\quad Y_6 = \lambda_{62}^y \eta_2 + \varepsilon_6$

USD/GBP $= f$(US dollar index, measurement error) $\quad Y_7 = \lambda_{72}^y \eta_2 + \varepsilon_7$

USD/JPY $= f$(US dollar index, measurement error) $\quad Y_8 = \lambda_{82}^y \eta_2 + \varepsilon_8$

USD/CAD $= f$(US dollar index, measurement error) $\quad Y_9 = \lambda_{92}^y \eta_2 + \varepsilon_9$

USD/CHF $= f$(US dollar index, measurement error) $\quad Y_{10} = \lambda_{102}^y \eta_2 + \varepsilon_{10}$

USD/SEK $= f$(US dollar index, measurement error) $\quad Y_{11} = \lambda_{112}^y \eta_2 + \varepsilon_{11}$

(c) Measurement model 3

US $M_2 = f$(USD Value, measurement error) $\quad X_1 = \lambda_{11}^x \xi_1 + \delta_1$

US CPI $= f$(USD Value, measurement error) $\quad X_2 = \lambda_{21}^x \xi_1 + \delta_2$

US GDP $= f$(USD Value, measurement error) $\quad X_3 = \lambda_{31}^x \xi_1 + \delta_3$

US Rediscount rate $= f$(USD Value, measurement error) $\quad X_4 = \lambda_{41}^x \xi_1 + \delta_4$

US Stock price index $= f$(USD Value, measurement error) $\quad X_5 = \lambda_{51}^x \xi_1 + \delta_5$

(2) Structural equation

NTD Value = γ_{11}(USD Value)+ β_{12}(US dollar index)+residual error

$\eta_1 = \gamma_{11}\xi_1 + \beta_{12}\eta_2 + \zeta_1$

US dollar index= γ_{21}(USD Value)+residual error

$\eta_2 = \gamma_{21}\xi_1 + \zeta_2$

3.4 SEM structure

The SEM provides an intact comprehensive system for data analysis and theoretical research. Using SEM, the causality in the structural model and the construct validity in the measurement model may be simultaneously analyzed and evaluated. The details of this approach are described below (Hu and Jen, 2008):

(1) Structural model

The structural model shows the causality between a host of latent variables. The cause-and-effect relationship in the model is generally derived from other theoretical assumptions. The "cause" assumed in the model is called the exogenous variable, whereas the "effect" is the endogenous variable. The following shows the structural model of SEM:

$$\eta = B\eta + \Gamma\xi + \zeta \tag{3}$$

where ξ is exogenous variable, η is endogenous variable, Γ stands for the coefficient matrices of the influence effect of exogenous variables on endogenous variables, B refers to the coefficient matrices of the influence effect of endogenous variables on endogenous variables, and ζ is the vector of the "residual error."

Three basic hypotheses are given in this model: (i) The variables are represented by deviation scores, where the average value is 0; (ii) no correlation exists between ξ and ζ ; and (iii) the diagonal line of B is 0, where $I - B$ is a non-singular matrix.

(2) Measurement model

The observed variables use measureable secondary data to observe the effect of latent variables. The measurement model is used to elaborate on the relationship between the latent and observed variables.

Generally, the measurement model comprises two equations that define the association between endogenous latent variables η and endogenous observed variables y and between exogenous latent variables ε and exogenous observed variables x. In fact, the measurement model can be considered a measurement and a kind of reliability description of the observed variables as follows:

$$y = \Lambda_y \eta + \varepsilon \tag{4}$$

where y refers to the endogenous observed variables, Λ_y stands for the coefficient matrices of the relationship between y and η, and ε is measurement error of y.

$$x = \Lambda_x \xi + \delta \tag{5}$$

where x refers to the exogenous observed variables; Λ_x stands for the coefficient matrices of the relationship between x and ξ ; δ is measurement error of x ; and Λ_y and Λ_x are the approximate regression coefficients for y and x, respectively.

Based on the aforementioned two straight lines of the measurement model, observed variables can be used to indirectly infer latent variables by assuming the following hypotheses: (i) no correlation exists between the measurement error and η, ξ, or ζ, but η, ξ, and ζ can be correlated; and (ii) similarly, no correlation exists between the residual error (ζ) and the measurement error (ε and δ).

(3) Inference of covariance matrix

Given SEM's basic assumptions and under the hypothesis of normal data distribution, the covariance matrix of Vector $z' = (y', x')$ can be theoretically obtained. The process is described below:

(a) Definition of variables

Λ_y: pxm coefficient matrix describing the relationship between y and η

Λ_x: qxm coefficient matrix describing the relationship between x and ξ

B : m×m coefficient matrix describing the influence effect of η on itself

Γ: m×n coefficient matrix describing the influence effect of ξ on η

Φ: nxn covariance matrix of ξ

Ψ: m×m covariance matrix of ζ

Θ_ε: pxp covariance matrix of ε

Θ_δ: qxq covariance matrix of δ

(b) The structural equation model is shown as below:

$$\eta = \Gamma\xi + B\eta + \zeta;\ y = \Lambda_y\eta + \varepsilon;\ x = \Lambda_x\xi + \delta$$

The covariance matrix Σ_{xx} of exogenous observed variables is obtained as follows:

$$\begin{aligned}\Sigma_{xx} &= Cov(xx) = E(xx') \\ &= E[(\Lambda_x\xi + \delta)(\Lambda_x\xi + \delta)'] \\ &= \Lambda_x E(\xi\xi')\Lambda_x' + \Lambda_x E(\xi\delta') + \Lambda_x E(\delta\xi) + E(\delta\delta') \\ &= \Lambda_x\Phi\Lambda_x' + 0 + 0 + \Theta_\delta \\ &= \Lambda_x\Phi\Lambda_x' + \Theta_\delta\end{aligned} \tag{6}$$

The covariance matrix Σ_{yy} of endogenous observed variables is obtained as follows:

$$
\begin{aligned}
\Sigma_{yy} &= Cov(yy) = E(yy') \\
&= E[(\Lambda_y \eta + \varepsilon)(\Lambda_y \eta + \varepsilon)'] \\
&= \Lambda_y E(\eta\eta')\Lambda_y' + \Lambda_y E(\eta\varepsilon') + \Lambda_y E(\varepsilon\eta) + E(\varepsilon\varepsilon') \qquad (7) \\
&= \Lambda_y E(\eta\eta')\Lambda_y' + 0 + 0 + \Theta_\varepsilon \\
&= \Lambda_y \Sigma_{\eta\eta} \Lambda_y' + \Theta_\varepsilon
\end{aligned}
$$

The covariance matrix $\Sigma_{\eta\eta}$ of endogenous latent variables is obtained as follows:

$$
\begin{aligned}
\Sigma_{\eta\eta} &= E(\eta\eta') \\
&= E[(I-B)^{-1}\Gamma\xi + (I-B)^{-1}\zeta][(I-B)^{-1}\Gamma\xi + (I-B)^{-1}\zeta]' \\
&= (I-B)^{-1}\Gamma E(\xi\xi')\Gamma'(I-B)^{-1'} + (I-B)^{-1}E(\zeta\zeta')(I-B)^{-1'} \qquad (8) \\
&= (I-B)^{-1}\Gamma\Phi\Gamma'(I-B)^{-1'} + (I-B)^{-1}\Psi(I-B)^{-1'} \\
&= (I-B)^{-1}[\Gamma\Phi\Gamma' + \Psi](I-B)^{-1'}
\end{aligned}
$$

Equation (8) is then substituted into (7) to derive the following:

$$
\Sigma_{yy} = \Lambda_y[(I-B)^{-1}(\Gamma\Phi\Gamma' + \Psi)(I-B)^{-1'}]\Lambda_y' + \Theta_\varepsilon \qquad (9)
$$

The covariance matrix Σ_{xy} of exogenous observed variables and endogenous observed variables is obtained as follows:

$$
\begin{aligned}
\Sigma_{xy} &= Cov(xy) = E(xy') \\
&= E[(\Lambda_x \xi + \delta)(\Lambda_y \eta + \varepsilon)'] \\
&= \Lambda_x E(\xi\eta')\Lambda_y' + \Lambda_x E(\xi\varepsilon') + E(\delta\eta')\Lambda_y' + E(\delta\varepsilon') \qquad (10) \\
&= \Lambda_x E(\xi\eta')\Lambda_y' + 0 + 0 + 0 \\
&= \Lambda_x \Sigma_{\xi\eta} \Lambda_y'
\end{aligned}
$$

The covariance matrix $\Sigma_{\eta\xi}$ of endogenous latent variables and exogenous latent variables is obtained as follows:

$$
\begin{aligned}
\Sigma_{\eta\xi} &= E(\eta\xi) \\
&= E[(I-B)^{-1}\Gamma\xi\xi' + (I-B)^{-1}\zeta\xi'] \\
&= (I-B)^{-1}\Gamma E(\xi\xi') + (I-B)^{-1}E(\zeta\xi') \qquad (11) \\
&= (I-B)^{-1}\Gamma\Phi + 0 \\
&= (I-B)^{-1}\Gamma\Phi
\end{aligned}
$$

$\Sigma_{\xi\eta} = \Sigma_{\eta\xi} = \Phi\Gamma(I-B)^{-1'}$, and substituting this formula into Equation (10) yields the following:

$$
\Sigma_{xy} = \Lambda_x \Phi\Gamma(1-B)^{-1'}\Lambda_y' \qquad (12)
$$

Finally, the covariance matrix is written as $\Sigma = \begin{pmatrix} \Sigma_{yy} & \Sigma_{yx} \\ \Sigma_{xy} & \Sigma_{xx} \end{pmatrix}$. Combining Equations (6), (9), and (12) yields the following:

$$\Sigma = \begin{bmatrix} \Lambda_y[(I-B)^{-1}(\Gamma\Phi\Gamma' + \Psi)(I-B)^{-1'}]\Lambda'_y + \Theta_\varepsilon & \Lambda_y(1-B)^{-1}\Gamma\Phi\Lambda'_x \\ \Lambda_x\Phi\Gamma'(1-B)^{-1'}\Lambda'_y & \Lambda_x\Phi\Lambda'_x + \Theta_\delta \end{bmatrix} \quad (13)$$

3.5 Linguistic variables and linguistic terms

Table 1 describes the linguistic variables used in the present study, as well as their linguistic terms. The input linguistic variables are the "NTD value," the "USD value," the "USD index," and the "Volatility of currency (USD/NTD) returns," whereas the output linguistic variable is termed "Trading strategy." Each linguistic variable has three linguistic terms, i.e., membership functions.

3.6 The membership function

The membership function describes to what degree of the truth the input variable represents the membership of vaguely defined sets. The shape of the membership function can dramatically affect the result of the fuzzy system. The membership functions are generally determined in accordance with experts'' recommendations and operation habits. Linear membership functions, including the Gaussian membership function, are good candidates for use in the fuzzy system. The path analysis derived through the SEM is significantly more impartial and unprejudiced in operating the fuzzy sets rationally.

Table 1 Names and types of variables and their linguistic terms

Linguistic variables	Type	Linguistic terms
NTD value	Input variable	Low, Average, High
USD value	Input variable	Low, Average, High
USD index	Input variable	Low, Average, High
Volatility of currency returns	Input variable	Low, Average, High
Trading strategy	Output variable	Sell, Hold, Buy

3.7 Establishment of fuzzy rules

Fuzzy rules are the most important part of the fuzzy system. Establishing rules that comply with the status quo requires rule-of-thumb specifications. If the goodness-of-fit of SEM is inappropriate, the logic of the rules will also be improper. Hence, determining various rules of thumb is necessary to improve the fuzzy rules. More reliable decisions can thus be made to achieve better investment performance. Therefore, based on the rules of thumb inferred from the SEM and the additional rules involved in the volatility factor, the present study analyzes the fuzzy input variables, i.e., latent variables and creates 18 fuzzy rules, as shown in Table 2.

3.8 Fuzzy trading system

The present work primarily aims to fuzzify input variables and risks and then use the GA for fitness so that the relationships between input and output variables could be more precisely identified. The primary method is to properly convert the four input variables into the output variable by operating the high, low, or average fuzzy set of each variable. The numerical values of economic variables are thus transferred into the linguistic variables of USD and NTD values through the Gaussian membership functions. Therefore, the fuzzy rules derived from the SEM can be evaluated through the sample data, and the fuzzy inference system can then be fitted to determine the parameters embedded in fuzzy sets. A fuzzy trading system is constructed and ready for verification using the test data.

Table 2 Fuzzy Rules

Rule No.	Specification
Rule 01	IF NTD Value High and USD Value High then Strategy is Buy U/N
Rule 02	IF NTD Value low and USD Value low then Strategy is Sell U/N
Rule 03	IF NTD Value average and USD Value average then Strategy is Hold U/N
Rule 04	IF USD Value High and USDX High then Strategy is Buy U/N
Rule 05	IF USD Value low and USDX low then Strategy is Sell U/N
Rule 06	IF USD Value average and USDX average then Strategy is Hold U/N
Rule 07	IF NTD Value High then Strategy is Sell U/N
Rule 08	IF NTD Value low then Strategy is Buy U/N
Rule 09	IF USD Value High then Strategy is Buy U/N
Rule 10	IF USD Value low then Strategy is Sell U/N

Rule 11	IF USDX High then Strategy is Buy U/N
Rule 12	IF USDX low then Strategy is Sell U/N
Rule 13	IF USD Value High and VCR High then Strategy is Buy U/N
Rule 14	IF USD Value low and VCR low then Strategy is Sell U/N
Rule 15	IF USD Value average and VCR average then Strategy is Hold U/N
Rule 16	IF USDX High and VCR High then Strategy is Buy U/N
Rule 17	IF USDX low and VCR low then Strategy is Sell U/N
Rule 18	IF USDX average and VCR average then Strategy is Hold U/N

Notes: 1. "USDX" =USD index. "VCR" =Volatility of currency returns. "Strategy" =Trading strategy. "U/N" =USD/NTD.

2. Volatility of currency returns is the volatility estimated from GARCH model.

4. Analysis of Empirical Results

4.1 Structural model analysis

The SEM can be used for path analysis and the decomposition of influence. The effect of one latent variable on another is called a direct effect, whereas the effect that occurs from a third latent variable is referred to as an indirect effect. The direct effect plus the indirect effect is called the total effect. The decomposition chart of the influence effects is shown in Table 3, and the present study's overall empirical structural model is illustrated in Figure 3.

The model reports that the USD value has a significant effect on the USD and the NTD value, the USD index has a significant effect on the NTD value and through the USD index, the USD value has an indirect effect on the NTD value.

Table 3 The Decomposition of the Influence Effects

Latent independent variables	Latent dependent variables	Direct effect	Indirect effect	Total effect
USD value	USD index	0.11*	－	0.11*
	NTD value	0.91*	0.008	0.918*
USD index	NTD value	0.07*	－	0.07*

Notes: 1. [*] represents *p*-value <0.05; 2. [－] represents non-effect.

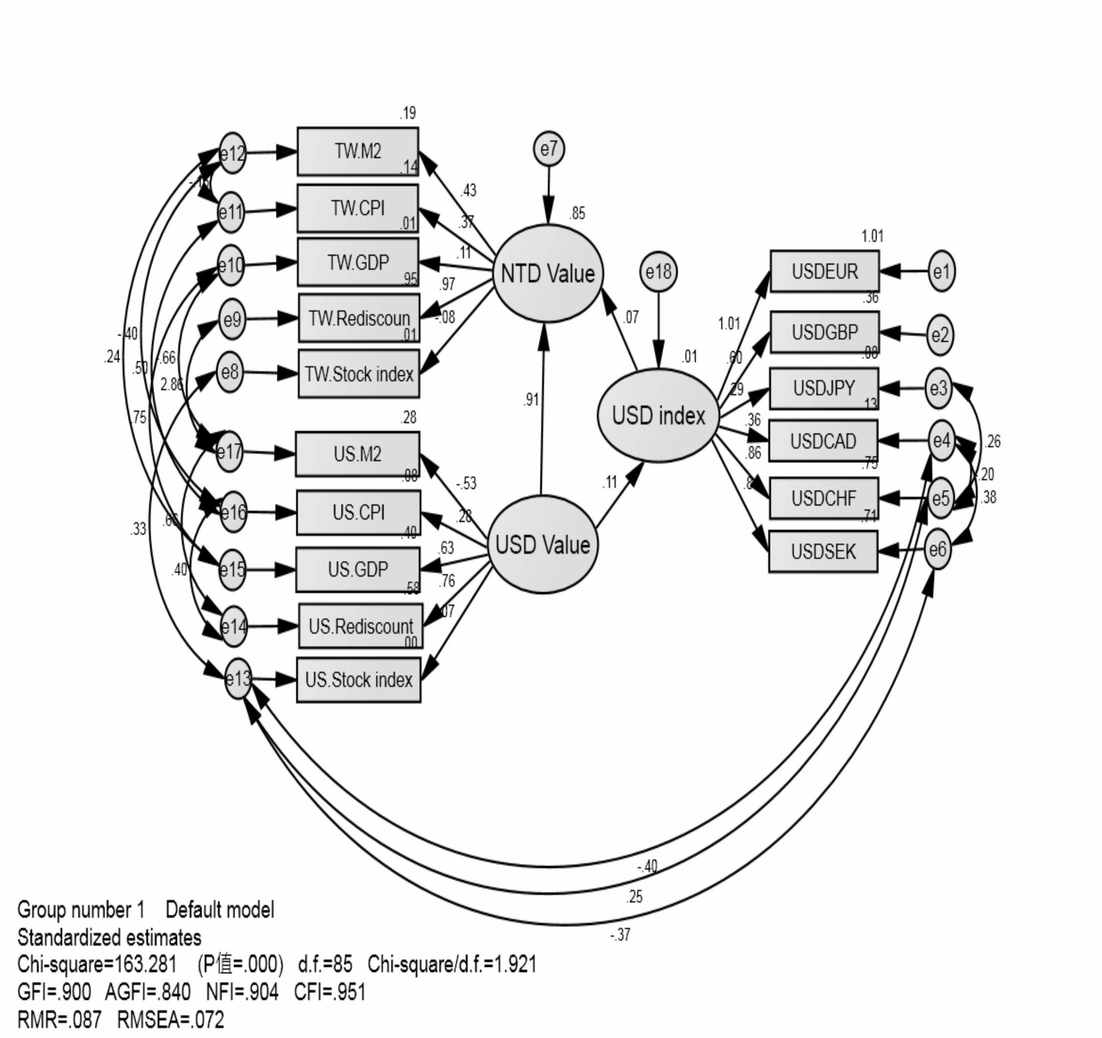


Figure 3 The Overall Empirical Structural Model

4.**2** Model fit analysis

According to the evaluation indicators revealed in Figure 3, the Chi-square value is 163.281, the degree of freedom is 85, and the *p*-value is 0.000, representing a highly significant level. The Chi-square value ratio is 1.921 (163.281÷85), which is within the value of criterion≤3, representing a proper overall fit between the theory model and the overall observed data.

Moreover, the other goodness-of-fit indices were all found to reach or are approximate close to the value of the criterion (GFI>0.9; AGFI>0.9; NFI>0.9; CFI>0.9; RMR<0.08; RMSEA<0.08). This finding illustrates that the overall model of the structural path diagram has a good fit to what is actually observed. In other words, the model based on the SEM factor analysis can appropriately interpret the actual data.

4.3 The buy/sell trading signals

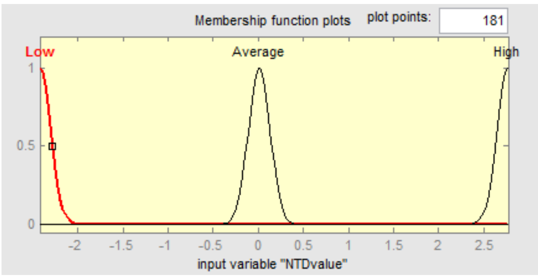
The trading of USD/NTD will give rise to different returns at different times. The buy-and-sell signals are actually determined by the output variable of the fuzzy inference system. If the output variable is greater than a specific value, it is viewed as the buy signal, and if it is smaller than a specific value, it is viewed as the sell signal. These buy/sell specific points in trading strategy are determined to minimize the error square between the output variable and the actual exchange rate of return. As a result of the fuzzy inference trained by the sample data, the buy signal appears when the output variable exceeds 3.3428, and the sell signal emerges when the output variable is less than -3.9531.

4.4 Membership functions of variables

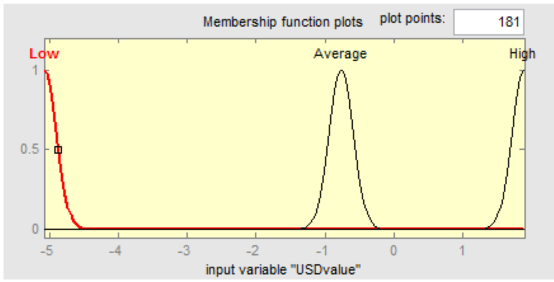
The membership functions of each variable are obtained with their respective optimal parameters, which are then fine-tuned by a GA based on the fuzzy rules and inference in previous section. As a result, Figure 4 exhibits all the empirical membership functions of the input and output variables.

Figure 4(a) shows the membership functions of the input variable NTD value. The optimal membership is obtained by genetically searching the parameters with respective function types (low, average, high) to minimize the objective function of RMSE, as shown in Equation (2). The results of the optimal membership parameters for the NTD value are -2.4226 and 2.7706, indicating that when the NTD value is below -2.4226, a "low value" level is achieved, and when it is above 2.7706, a "high value" level is reached. Figure 4(b) reveals the membership function of the input variable USD value. Similarly, the results of the optimal membership parameters for the USD value are obtained as -5.0779 and 1.8926. A USD value below -5.0779reflects a "low value" level, whereas a USD value above 1.8926reflects a "high value" level. Figure 4(c) shows that the optimal membership parameters for the input variable USDX are -1.4202 and 0.4685, implying that when the USDX is below -1.4202, a "low index" level is achieved, and when it is above 0.4685, a "high index" level is reached. Figure 4(d) shows that the optimal membership parameters for the input variable volatility of currency (USD/NTD) returns are 0.1449 and 0.5461. Hence, when the volatility of currency returns is below 0.1449, a "low volatility" level is achieved, and when it is above 0.5461, a "high volatility" level is reached. Figure 4(e) demonstrates that the optimal membership parameters for the trading strategy Sell and Buy are -3.9531 and 3.3428. A trading strategy value below -3.9531 creates a “Sell USD/NTD” signal, a value above 3.3428 creates a “Buy USD/NTD” signal, and a value between -3.9531 and 3.3428generates a “Hold” signal, wherein nothing is available for trading.

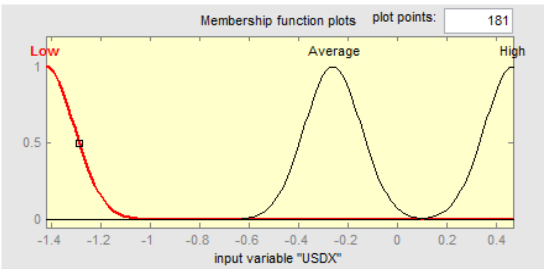
The fuzzy GA (FGA) is used to fit the fuzzy model and to optimize the parameters involved in fuzzy memberships and the fuzzy rules. Figure 5 exhibits the objective function (fitness) converging stably and smoothly to the best value (around 1800) when the number of generations reaches approximately 40. This finding indicates that the fuzzy inference system can effectively fit the training samples of exchange rates through the FGA.



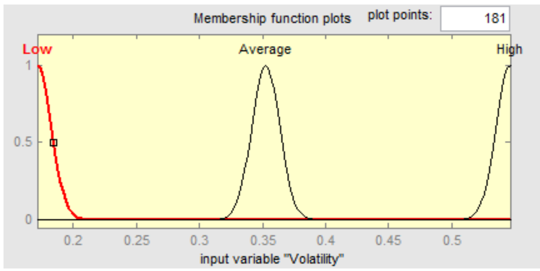
(a) Membership functions of NTD value



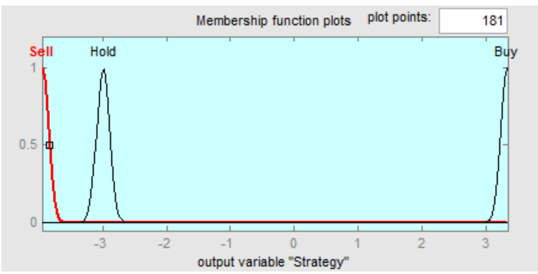
(b) Membership functions of USD value



(c) Membership functions of USDX



(d) Membership functions of VCR



(e) Membership functions of trading strategy

Figure 4 Optimal membership functions of the fuzzy input and output variables

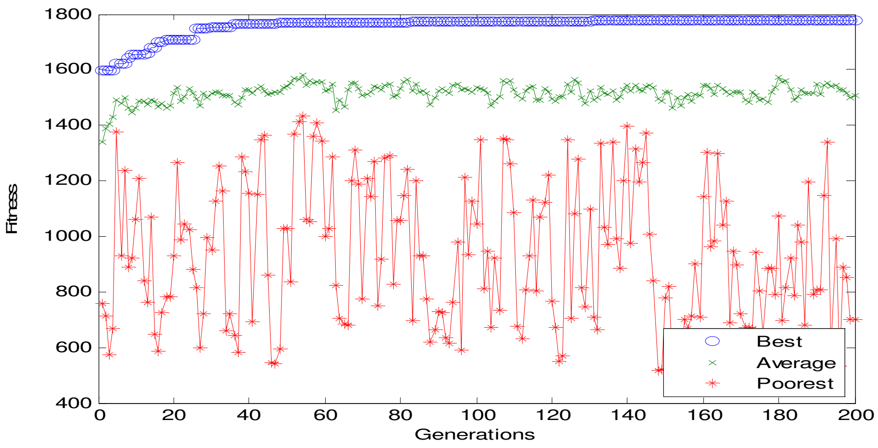
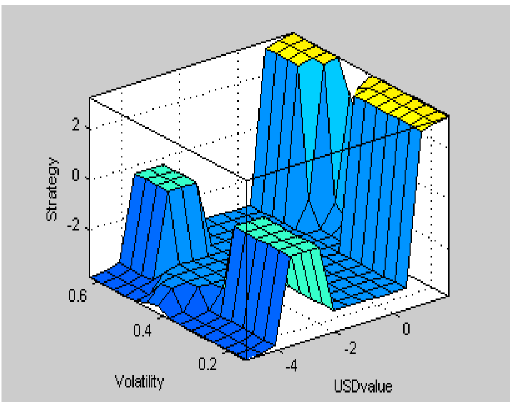


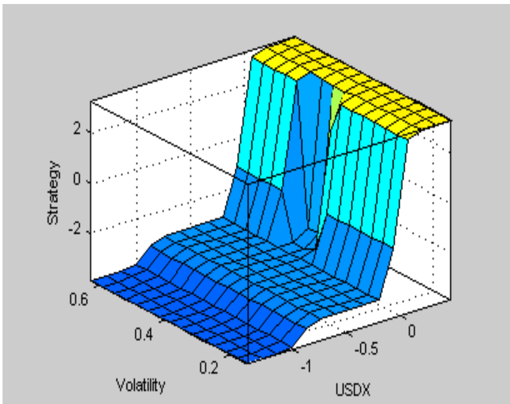
Figure 5 Convergence of GA

4.5 Changes in input and output variables

Following the training, several three-dimensional charts between the values of trading strategy versus those of input factors are built according to the optimal fuzzy rules and inference. The value of trading strategy is apparently noticeable against USD value and volatility of currency returns or against USDX value and volatility of currency returns. The results are shown in Figure 6.



(a) Variation in strategy vs. USD value and volatility



(b) Variation in strategy vs. USDX and volatility

Figure 6 Variation in trading strategy

Figure 6(a) reveals that the weight of the trading value (importance level) rises as the volatility or USD value becomes significantly lower or higher. Figure 6(b) shows that the USD index dominates the volatility in affecting the value of the trading strategy. However, a comparison of both figures shows that when the USD value or the USD index is higher, i.e., when the USD/NTD buy point emerges, the effect of volatility of currency returns also becomes greater. This finding indicates the importance of volatility risk control for making trading decisions in foreign exchange markets. Remarkably, when the USD value is lower, thus expressing the sell signal for USD/NTD trading, the effect of USD value becomes greater.

4.6. Investigation of investment performance

The current paper compares the investment performance of the "Fuzzy trading strategy with Volatility of currency returns," in which risk is considered, and the "Fuzzy trading strategy without Volatility of currency returns," in which risk is disregarded, as well as the general "Buy-Hold," i.e., three methods. Table 4 shows the buy/sell points and return of investment (ROI) rates generated according to the "fuzzy trading strategy," in which B represents the "buy signal," H represents a "hold," and S represents the "sell signal."

The rate of ROI gained from the first operation method is 45.7031%, and that from the second operation method is 32.9582%, in contrast to merely 0.0461% using the "Buy-Hold" method. As this comparison emphasizes, applying the "fuzzy trading strategy" to investments can substantially improve the rate of currency return. Furthermore, considering the volatility of currency returns can yield better profit than disregarding such volatility.

5. Conclusion

To construct reasonable fuzzy rules, the present study employed the SEM to determine the suitable path diagram between various economic indices of observed variables and their respective latent variables. A risk factor of control, i.e., volatility of currency returns, was also considered in fuzzy sets. The current research constructed several fuzzy rules based on SEM path analysis and fitted the parameters of the memberships of fuzzy sets to the returns of currency data using a GA. According to the empirical results, the present work has the following conclusions:

(i) Regardless of whether or not the volatility of currency returns is considered, investments made with the fuzzy trading strategy achieve a significantly better rate of return than those made using the Buy-Hold method. The returns were 45.7031% and 32.9582% using the two previous methods, whereas a value of only 0.0461% was obtained using the proposed method.

(ii) Investment using the fuzzy trading strategy, in which the volatility of currency returns is considered, yields better results than when such volatility is disregarded.

(iii)The trading strategy is apparently affected as the USD value or the volatility of currency returns shifts into either a higher or lower state.

In summary, the fuzzy trading system has incorporated the SEM path and factor analyses into fuzzy rules. Moreover, the inclusion of the volatility of currency returns in the fuzzy trading system certainly helped in acquiring larger excess returns, thus outperforming the Buy-Hold strategy.

Table 4 Comparison between “Fuzzy trading strategy with Volatility of currency returns” and “Fuzzy trading strategy without Volatility of currency returns” and “Buy-Hold”

Date	Fuzzy trading strategy with VCR		Fuzzy trading strategy without VCR		Buy-Hold
	Signal	ROI (%)	Signal	ROI (%)	ROI (%)
Jul-08	'B'	3.1130	'B'	3.1698	0.7745
Aug-08	'B'	3.2802	'B'	3.2831	2.9949
Sep-08	'B'	3.0962	'B'	3.1577	1.9168
Oct-08	'H'	2.2484	'H'	2.6278	2.6717
Nov-08	'B'	3.1105	'B'	3.1667	0.8900
Dec-08	'S'	-3.4853	'H'	-2.9652	-1.3151
Jan-09	'S'	-3.8913	'S'	-3.8577	2.8234
Feb-09	'H'	-1.5544	'H'	0.6239	3.3428
Mar-09	'S'	-3.2332	'H'	-1.2597	-3.0002
Apr-09	'H'	1.4745	'H'	-2.6509	-2.0373
May-09	'S'	-3.8544	'S'	-3.8567	-1.7699
Jun-09	'S'	-3.8872	'S'	-3.8512	0.5132
Jul-09	'B'	3.1129	'H'	-2.9447	0.0000
Aug-09	'B'	3.1133	'H'	-2.9451	0.3194
Sep-09	'B'	3.1087	'H'	0.0239	-2.2205
Oct-09	'B'	3.1127	'B'	3.1693	1.0350
Nov-09	'B'	3.1121	'B'	3.1686	-1.0816
Dec-09	'B'	3.1020	'B'	3.1581	-0.4828
Jan-10	'B'	3.1130	'B'	3.1697	-0.1250
Feb-10	'B'	3.1116	'B'	3.1680	0.2965
Mar-10	'B'	3.1132	'B'	3.1701	-0.8325
Apr-10	'B'	3.1129	'B'	3.1696	-1.2683
May-10	'S'	-3.8917	'S'	-3.3296	2.5362
Jun-10	'B'	3.1135	'B'	3.1705	0.1643
Jul-10	'B'	3.2814	'B'	3.2838	-0.6527
Aug-10	'B'	3.1201	'B'	3.1813	0.1060
Sep-10	'B'	3.1837	'B'	3.2213	-2.4342
Oct-10	'B'	3.1127	'B'	3.1694	-1.7646
Nov-10	'B'	3.1361	'B'	3.1894	0.2207
Dec-10	'B'	3.1179	'B'	3.1771	-1.5747
Total ROI (%)	45.7031		32.9582		0.0461

Note: “VCR” =Volatility of currency returns.

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