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Bazdresch, Santiago

University of Minnesota

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# Empirical Policy Functions as Benchmarks for Evaluation of Dynamic Capital Structure Models

Santiago Bazdresch

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## Abstract

This paper presents a set of benchmark moments for evaluation or estimation of quantitative capital structure models. The moments are directly related to the models being studied: the main features of each models' empirical policy functions. The paper describe a general method for estimating these benchmarks and shows that they capture a substantial part of the actual variation in firms actions in the data. Two versions of these benchmarks are presented: one dimensional ones and two dimensional ones. In both cases we express these as the total change in the control variable and the change relative to the change in the state variable. The empirical policy functions turn out to be smooth and mostly monotonous. Three key numbers that we suggest quantitative dynamic models have match closely are that within firms, for every 10% increase in debt relative to assets investment relative to assets declines 3.7%, debt issuance relative to market value decreases 1.1% and equity issuance relative to market value increases 0.5%.

*Keywords: dynamic models of capital structure, policy function, value function, model evaluation*

*JEL Classification Codes: C14, C52, C61; G31, G32*

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The author is at the Carlson School of Management, University of Minnesota. Contact: [santiago@umn.edu](mailto:santiago@umn.edu), 612-624-6081.

# 1 Introduction

While there is a large literature addressing the issue of capital structure dynamics, there is no agreement yet on what is a good standard model. One of the problems in finding such a model is that there is no agreed-upon evaluation method for the large list of models out there. Each one is estimated or calibrated to fit a particular set of moments or particular features of the data, but there is no single objective that they are all aiming for.<sup>1</sup> Most of the existing capital structure theory is described in static settings which are very difficult to contrast directly with the data, and therefore the issue of the right benchmarks to match has not been considered important before. However, one of the main advantages of writing fully dynamic capital structure models is that we can easily compare them to the data and therefore that we can impose more discipline on them by asking them to match certain features of it quantitatively.

This paper argues that for each model in the literature there is a set of natural moments that can serve as benchmarks for evaluating it quantitatively. Moreover, these benchmarks are to a large extent common across existing models. A large fraction of these models are state space models where the firm sets the value of financial and real control variables as a function of its current state. Often solving the model implies finding the policy functions of the firm or a numerical approximation of these policy functions. While these models are slightly different they can typically be written in a form where the exogenous state of nature is productivity and the endogenous state of nature is the level of debt. We argue that the moments that are natural benchmarks candidates are the empirical counterparts of the firms' optimal policy functions. In these models the policy function is often the amount of debt issuance that the firm undertakes. Slightly different models allow for more complicated state and policy variables, for example adding capital, cash and dividends, but there is little variation overall. Therefore a small set of benchmarks can be established that allows us to compare models

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<sup>1</sup>For example Hennessy and Whited (2005), Titman and Tsyplakov (2007), Gamba and Triantis (2008), Palazzo(2011), Hackbarth, Hennessy and Leland (2007)

to each other meaningfully.

The empirical counterparts of policy functions are a natural benchmark because the intuition of the model is embedded in them. There is very little intuition in finding that a model finds average leverage to be, say, 40%. There is also little intuition in finding the cross section or time series correlation among variables, the variance or the estimated speed of adjustment. Instead, these ‘empirical policy functions’ codify the main issues that Corporate Finance typically has cared about. For example the questions: In which state of nature do firms issue debt? How much higher is the investment rate of low debt firms? are two ‘policy function’ questions. Furthermore the essential test for dynamic models should be whether they get the ‘dynamics’ in the data correctly, abstracting away from any firm heterogeneity. Therefore we argue that the right estimates

Also, the nature of the standard models in this area is such that the timing assumptions have a important impact on the resulting variances and correlations across variables. Slight changes in the timing, the variance or the auto-correlation of the shocks in the models leads to dramatic changes in the estimated variances, correlations and regression estimates in model simulations. This also suggests that the key quantitative benchmarks should not be these variances, correlations and regression estimates, but should instead be the direct predictions of the model: how do firms react to different states of the world.

We show that the benchmarks we present are relevant in the sense that they capture a non-negligible fraction of the total variation in firms behavior. Therefore models that meet these benchmarks rationalize important features of the data.

This paper proceeds as follows: The first section describes the empirical method in the paper; the second section describes a generic model and presents the estimated empirical policy functions; the third section exemplifies the evaluation method by describing a particular model and showing where the model is successful and where it fails; section four concludes.

## 2 Generic Model and Example

The most popular dynamic capital structure models can be described in terms of the Bellman equation:

$$V(x) = \max_y (D(x, y) + E_{x'|x, y}[\beta V(x')])$$

where  $x' \sim g(x, y)$ .

Here  $V(x)$  represents the market value of the firm's equity when the state variable takes value  $x$ ,  $D(x, y)$  represents dividends paid to shareholders or equity issuance,  $\beta$  is the discount factor and  $E[\cdot]$  is the expectation over the value of  $x'$  given  $x$  and  $y$ . Also,  $g(x, y)$  represents the law of motion of the state variable. Typically the variable  $x$  contains the firm's productivity  $\pi$  and its leverage  $L$ . The control variable  $y$  typically consists of leverage changes  $\Delta L$ .

The firm is assumed to observe  $x$  and then to maximize the present discounted value of the sum of current and future dividends by setting  $y$ , the control variable, optimally during each period. The law of motion of the state is such that the future state  $x'$  is a (random) function of the state variable in the past  $x$ , and the value of control variable  $y$ .

In this generic setup, the solution of the model consists of the optimal policy functions and the value function of the firm. These policy function  $h(x)$  and value function  $V(x)$  are the functions that satisfy the following system:

$$h(x) = \operatorname{argmax}_y [D(x, y) + E[\beta V(x)]]$$

and

$$V(x) = D(x, h(x)) + E[\beta V(x)]$$

The rest of this section describes an example of a model in this literature:

## 2.1 The firm's cash flow

The firm's cash flow ( $CR$ ) is the difference between its income from sales of output ( $A_t K_t^\alpha$ ), where  $K$  is capital and  $A$  is a productivity shock, plus its net debt issuance ( $\Delta B$ ), minus its net expenditure on investment ( $C(I_t)$ ) and its interest payments on debt ( $B_t r_t^B$ ):

$$CF_t = A_t K_t^\alpha - C(I_t) + \Delta B_t - B_t r_t^B \quad (1)$$

$$(2)$$

## 2.2 Dividends and equity issuance

The firm's dividends and equity issuance are defined in terms of the firm's cash flows. A positive cash flow implies the firm's optimal decision is to pay dividends ( $Div = CF$ ) to its stockholders, a negative cash flow implies that the firm's optimal decision is to set dividends at 0 and instead obtain funds from the equity market ( $X = -CF$ ). Here  $\lambda$  stands for the (proportional) cost of issuing equity.

$$CF_t > 0 \Rightarrow Div_t = CF_t, X_t = 0 \quad (3)$$

$$CF_t \leq 0 \Rightarrow Div_t = 0, X_t = (1 + \lambda)(-CF_t) \quad (4)$$

## 2.3 The firm's optimization problem

The firm's problem is to maximize the discounted value of dividends for current owners of the firm. With this objective the firm chooses investment and net debt issuance. These variables feed into their respective laws of motion, for capital ( $K$ ) and debt ( $B$ ), where  $\delta$  is the rate of depreciation. Also, the firm's choices are restricted by the firm's cash flow equation described further below. Finally, the firm receives log-normally distributed

productivity shocks ( $A$ ).

$$V_t = \max_{I_t, \Delta B_t} \left[ (D_t + E_t[\beta V'_{t+1}]) \left( \frac{V_t}{V_t + X_t} \right) \right] \quad (5)$$

$$K_{t+1} = K_t(1 - \delta) + I_t \quad (6)$$

$$B_{t+1} = B_t + \Delta B_t \quad (7)$$

$$A_t = \bar{A} + \rho A_{t-1} + \sigma \epsilon_t \epsilon_t \sim Z(0, 1) \quad (8)$$

## 2.4 Interest rate on debt

The interest rate on debt is a function of leverage:

$$r_t^B = R^{rf} + R^{rp}((B_t - \phi K_t)/E[A_t K_t])^2 \quad (9)$$

## 2.5 Real sector frictions

The firm faces a set of frictions. Consistent with most of the literature it faces convex costs of investment as well as some degree of investment irreversibility. The firm's cost of investment function includes the cost of purchasing the capital, plus a convex cost of investment and plus an investment irreversibility term.

$$C(I_t) = I_t + \gamma^{cc} K_t (I_t/K_t)^2 - \gamma^{Ir} I_t * (I_t < 0), \quad (10)$$

where  $\gamma^{CC}$  represents the magnitude of convex costs of investment and  $\gamma^{Ir}$  represents the magnitude of investment irreversibility.

## 3 Empirical Policy Functions

In order to obtain empirical policy function estimates it is necessary to have estimates of (a) the state in which firms are and of (b) the policies that they follow in each state. We use a simple process inspired in the portfolio formation frequently used in the asset pricing literature.

An empirical equivalent of the function  $h(x)$  can be found by estimating a function

$$h(\hat{x}) = (h_1(\hat{x}), h_2(\hat{x}), \dots, h_N(\hat{x})).$$

where  $h_i(\hat{x})$  represents the average behavior along control dimension  $i$  for firms that observe a value  $x$  for the state variable. However, with continuous state variables in  $x$ , there are generally no two firms with the same  $x$ . Therefore we estimate  $h(\hat{x})$  by splitting the state space into percentile bins for each of the state variables and then estimating the average choice for the control variables for all firms within a particular bin.

**Step 1:** Obtain de-meaned (at firm level) values for each state and control variable. The models described and exemplified above are typically models where firms are ex-ante homogenous. Also, we are concerned mainly with the dynamics of the different variables. For these reasons we estimate the state of nature of firms by de-meaning all variables at the level of the individual firm. An reasonable alternative specification would consist of de-meaning the variables at the industry level.

**Step 2:** Generate bins across each of the state variables of the model. We define these bins as the 0%-20%, 20%-40%, 40%-60%, 60%-80% and 80%-100% percentiles of each (de-meaned) state variable. the Classify each observation of each state variable as belonging into one variable-specific bin. This is done independently for each variable, i.e. it is 'non-sequential' sorting. Each firm-period observation is therefore given a classification as  $b(x_{i,t}) = (b_1, b_2, \dots, b_N)$ , with each  $b_i$  representing state variable  $i$ 's classification for that firm in that period.

**Step 3:** Estimate the average value of the control variable as the average observed choice within each of the (composite) bins:

$$h_i(B) = \text{mean}_{\{x_{j,t} | b(j,t)=B\}} H_i(j,t)$$

where  $H_i(x_{j,t})$  represents the observed choice of the control variable  $i$  by



firm  $j$  at time  $t$ .

**Step 4:** Summarize  $h(\hat{x})$  by describing its linear and quadratic properties. I.e., describe the difference between the policies of the first and the last bin across each state variable, keeping the other state variables constant, and describe whether the relation is monotonous, and/or concave or convex.

**Step 5:** Replicate Steps 1-4 for simulated data from the model and compare the model and the data, focusing on the benchmarks described in the tables below.

## 4 Benchmarks

### 4.1 Data

We use the well known accounting data from Compustat from 1970 to 2010. We drop all firms without at least 10 observations in the annual files. We drop any observations where any of the variables are missing. We define the following variables to be used in the rest of the analysis:

- Market Value ( $MV$ ): Book Value of Liabilities + Market Value of Equity
- (Book) Leverage ( $L$ ): Total Liabilities / Total Assets
- Investment Rate ( $I$ ):  $\% \Delta$  (Property Plant and Equipment) / Total Assets
- Debt Issuance ( $D$ ): (Debt Issuance - Debt Repurchasing) /  $MV$
- Equity Issuance ( $X$ ): Common and Preferred Shares Net Sales /  $MV$
- Profitability ( $\pi$ ): Operating Income Before Depreciation / Total Assets

### 4.2 State Variable Variation

The variation of the state variables is an essential quantitative feature of the data. Table 1 describes the distribution of the (de-meaned) state variables

Table 1: Empirical Estimate of State Variables' Range

Variable	Quantile					5-1
	1	2	3	4	5	
Profitability	-0.118	-0.022	0.003	0.029	0.120	<b>0.238</b>
Leverage	-0.178	-0.052	-0.007	0.036	0.173	<b>0.351</b>
Market to Book	-1.222	-0.262	-0.039	0.182	1.646	<b>2.868</b>

in the data. It shows that the bottom quintile of profitability is about 12% lower than the mean, while the top quintile is about 12% higher, so that the 5-1 inter-quantile range is about 24% of average profitability. With respect to leverage it shows that the inter-quantile range is about 35% of total assets, evenly distributed above and below the mean. with respect to market to book it shows that the inter-quantile range is (an astonishing) 287% of the book value of the firm, somewhat skewed towards to top of the distribution.

### 4.3 Benchmarks for Models - One State Variable

Tables 2 through 4 describe the empirical estimates of the firm's optimal policies as a function of the values of different state variables: Leverage, Profitability and the Book to Market ratio. They are the benchmarks that correspond directly to models with a single state variable. They can also be used for models with more state variables by integrating out (though simulation) the other state variables. These tables also describe the differences between the observed policy in the first and last quintile and the corresponding difference between the first and last quintile of the control variable when classified in bins on its own. The last column contains the ratio of these two numbers. We have called it the 'potential  $R^2$ ' in the sense that it tells us how much of the total variation in the control variable can be potentially traced to differences in the state variable of the model.

The three tables show large variation in firms' investment as a function of the state variable. The investment rate is 13% lower for firms with high debt than for firms with low debt. It is 16% higher for firms with high profitability relative to firms with low profitability and it is a full 25% larger for firms with high Market to Book than form firms with low Market to

Book .

In contrast to the findings for investment, we find relatively small variation in debt and equity issuance as a function of any of the state variables, however, this variation is still a relatively high fraction of the total variation in debt and equity issuance in the sample. As we'd expect leverage leads firms to issue less debt about 4% less as a fraction of market value, and to issue more equity, about 2% more as a fraction of market value. Higher profitability leads to more debt issuance and to less equity issuance. A higher Market to Book ratio leads to more debt and more equity issuance.

Table 2: Empirical Policy Functions,  $x = \{\text{Leverage}\}$

Variable	Leverage Sorts (t-1)					Policy's q5-q1	Variable's q5-q1	Potential $R^2$
	1	2	3	4	5			
Investment	0.055	0.002	-0.025	-0.041	-0.075	<b>-0.130</b>	0.900	<b>0.144</b>
Debt I.	0.020	0.007	0.000	-0.007	-0.020	<b>-0.039</b>	0.157	<b>0.248</b>
Equity I.	-0.010	-0.006	-0.003	0.001	0.009	<b>0.019</b>	0.105	<b>0.182</b>

Table 3: Empirical Policy Functions,  $x = \{\text{Profitability}\}$

Variable	Profitability Sorts (t-1)					Policy's q5-q1	Variable's q5-q1	Potential $R^2$
	1	2	3	4	5			
Investment	-0.102	-0.051	-0.017	0.021	0.065	<b>0.166</b>	0.900	<b>0.185</b>
Debt I.	-0.007	-0.006	0.000	0.006	0.007	<b>0.013</b>	0.157	<b>0.084</b>
Equity I.	0.004	-0.002	-0.003	-0.003	-0.006	<b>-0.010</b>	0.105	<b>0.094</b>

Table 4: Empirical Policy Functions,  $x = \{\text{Market to Book}\}$

Variable	M/B Sorts (t-1)					Policy's q5-q1	Variable's q5-q1	Potential $R^2$
	1	2	3	4	5			
Investment	-0.142	-0.066	-0.022	0.029	0.117	<b>0.259</b>	0.900	<b>0.288</b>
Debt I.	-0.006	-0.007	-0.003	0.007	0.009	<b>0.015</b>	0.157	<b>0.097</b>
Equity I.	-0.007	-0.003	-0.002	-0.001	0.003	<b>0.010</b>	0.105	<b>0.097</b>

### 4.3.1 Relative Variation: State and Control Variables

Table 5: Variation of Control Variables Relative to State Variables

Leverage		
Variable	q5-q1	Control Var.(q5-q1) / State Var.(q5-q1)
Investment	-0.130	<b>-0.37</b>
Debt I.	-0.039	<b>-0.11</b>
Equity I.	0.019	<b>0.054</b>
Profitability		
Variable	q5-q1	Control Var.(q5-q1) / State Var.(q5-q1)
Investment	0.166	<b>0.72</b>
Debt I.	0.013	<b>0.057</b>
Equity I.	-0.010	<b>-0.0434</b>
Market to Book		
Variable	q5-q1	Control Var.(q5-q1) / State Var.(q5-q1)
Investment	0.259	<b>0.090</b>
Debt I.	0.015	<b>0.0052</b>
Equity I.	0.010	<b>0.0034</b>

An essential measure of whether our theories explain the data is one that compares the relative variation of explanatory and dependent variables. The figures in table 5 show this comparison. These are one set of benchmarks for dynamic model evaluation: they describe the relative variation in the data between state variables and control variables. Under this measure a quantitatively good dynamic model of investment and debt issuance that as a function of changing profitability and current leverage is one that replicates the relative variation in investment and debt issuance with respect to leverage and profitability.

Table 5 describes these relative variations by combining the information in the empirical policy function estimates in tables 2 through 4 and that in table 1. With respect to leverage it shows that a 10% change in debt relative to total assets leads in the data to a 3.7% decrease in investment relative to total assets, to a -1.1% decrease in debt issuance relative to market value and to a 0.5% increase in equity issuance relative to the firm's market value. With respect to profitability it shows that a 10% change in profits relative

to total assets leads in the data to a 7.2% increase in investment relative to total assets, to a 0.6% increase in debt issuance relative to market value and to a 0.4% decrease in equity issuance relative to the firm's market value. Finally, with respect to market to book it shows that a 10% change in the market value relative to total assets leads in the data to a 0.9% increase in investment relative to total assets, to a 0.05% increase in debt issuance relative to market value and to a 0.034% increase in equity issuance relative to the firm's market value.

#### 4.4 Benchmarks for Models - Two State Variables

Tables 6 through 8 describe the estimated empirical policy functions for models that have two state variables, either Leverage and Profitability, or Leverage and Market to Book or Market to Book and Profitability. As in the one state variable benchmarks they can also be used for evaluating moments with more state variables by integrating out the extra dimensions. For purposes of readability we present and focus only the differences in behavior for firms at the 'edges' of the sample.

Table 6 shows that as before leverage and profitability are strong determinants of investment. However this table also shows that Profitability has a stronger impact on investment when leverage is high: the investment of highly profitable firms is 16% larger as a fraction of total assets than the investment of low profitability firms, when leverage is high but it is only 11% higher when leverage is low.

It also shows that the impact of profitability on debt issuance changes sign depending on whether the firm is highly levered or not: the debt issuance of more profitable firms is higher than that of less profitable firms when leverage is low, but it is lower than that of unprofitable firms when leverage is high.

The behavior of equity issuance as a function of profitability is the mirror image of that for debt issuance. However, the difference in equity issuance between the highly levered and the un-levered firms is most notorious for firms that are highly profitable. These firms seem to 'readjust' their capital

Table 6: Empirical Policy Functions,  $x = \{\text{Profitability, Leverage}\}$

		Variable: Investment					
Profitability		1	2	3	4	5	5-1
Leverage	1	-0.03	0.00	0.04	0.09	0.12	<b>0.11</b>
	2	-0.09	-0.03	0.01	0.03	0.06	
	3	-0.11	-0.05	-0.02	0.01	0.04	
	4	-0.11	-0.06	-0.03	-0.01	0.03	
	5	-0.14	-0.09	-0.06	-0.04	0.02	<b>0.16</b>
	5-1	<b>-0.11</b>				<b>-0.10</b>	
		Variable: Debt Issuance					
Profitability		1	2	3	4	5	5-1
Leverage	1	0.01	0.02	0.02	0.03	0.02	<b>0.00</b>
	2	0.00	0.00	0.01	0.01	0.01	
	3	0.00	0.00	0.00	0.01	0.00	
	4	-0.01	-0.01	-0.01	0.00	0.00	
	5	-0.02	-0.02	-0.02	-0.02	-0.01	<b>0.01</b>
	5-1	<b>-0.03</b>				<b>-0.03</b>	
		Variable: Equity Issuance					
Profitability		1	2	3	4	5	5-1
Leverage	1	0.00	-0.01	-0.01	-0.01	-0.01	<b>0.00</b>
	2	0.00	-0.01	-0.01	-0.01	-0.01	
	3	0.00	0.00	0.00	0.00	0.00	
	4	0.00	0.00	0.00	0.00	0.00	
	5	0.01	0.01	0.01	0.01	0.01	<b>-0.01</b>
	5-1	<b>0.01</b>				<b>0.02</b>	

structure faster than do unprofitable firms.

Table 8 describes the behavior of firms across investment, debt issuance and equity issuance, as a function of the Market to Book ratio and Profitability. It shows that, firms with high market to book invest substantially more as a fraction of total assets than firms with low Market to Book ratios. However, it also shows that this effect is more pronounced for firms with high leverage.

The table also shows that the difference between the debt issuance of high and low market to book firms is concentrated on the high leverage firms. High market to book firms issue more debt than low market to book firms only within the sample of firms for which leverage is already relatively high.

Table 7: Empirical Policy Functions,  $x = \{\text{Market to Book, Leverage}\}$

Variable: Investment						
Market/Book Leverage	1	2	3	4	5	5-1
1	-0.10	0.00	0.05	0.10	0.18	<b>0.18</b>
2	-0.13	-0.05	0.01	0.04	0.11	
3	-0.14	-0.07	-0.02	0.01	0.09	
4	-0.15	-0.08	-0.05	0.01	0.07	
5	-0.18	-0.12	-0.08	-0.01	0.08	<b>0.27</b>
5-1	<b>-0.08</b>				<b>-0.10</b>	
Variable: Debt Issuance						
Market/Book Leverage	1	2	3	4	5	5-1
1	0.01	0.02	0.02	0.03	0.02	<b>0.00</b>
2	0.00	0.00	0.01	0.01	0.01	
3	0.00	-0.01	0.00	0.00	0.01	
4	-0.01	-0.01	-0.01	0.00	0.00	
5	-0.02	-0.03	-0.03	-0.01	0.00	<b>0.02</b>
5-1	<b>-0.03</b>				<b>-0.02</b>	
Variable: Equity Issuance						
Market/Book Leverage	1	2	3	4	5	5-1
1	-0.02	-0.01	-0.01	-0.01	-0.01	<b>0.00</b>
2	-0.01	-0.01	-0.01	-0.01	0.00	
3	-0.01	0.00	0.00	0.00	0.00	
4	0.00	0.00	0.00	0.00	0.01	
5	0.00	0.01	0.01	0.01	0.02	<b>0.02</b>
5-1	<b>0.02</b>				<b>0.03</b>	

With respect to equity issuance the table shows that high market to book firms issue more equity than low market to book firms, but this is also only true within the sub-sample that has relatively high leverage. It also shows that the extra equity issuance of highly levered firms is higher for firms with high market to book.

Table 8 describes firms' average control variable choices as a function of the market to book ratio and of profitability. It shows that the higher investment of market to book firms is more pronounced for firms with high profitability. It also shows that the higher investment of more profitable firms is more pronounced for firms with higher market to book ratios.

With respect to debt issuance, it shows that the higher debt issuance of firms with higher market to book values is more pronounced for firms

Table 8: Empirical Policy Functions,  $x = \{\text{Market to Book, Profitability}\}$

Variable: Investment						
Market/Book Profitability	1	2	3	4	5	5-1
1	-0.18	-0.10	-0.05	-0.05	0.05	<b>0.14</b>
2	-0.13	-0.07	-0.04	0.00	0.08	
3	-0.13	-0.06	-0.02	0.03	0.07	
4	-0.10	-0.04	-0.01	0.04	0.11	
5	-0.10	-0.02	0.01	0.07	0.16	<b>0.26</b>
5-1	<b>0.08</b>				<b>0.11</b>	
Variable: Debt Issuance						
Market/Book Profitability	1	2	3	4	5	5-1
1	-0.01	-0.01	-0.01	0.00	0.01	<b>0.02</b>
2	-0.01	-0.01	-0.01	0.00	0.01	
3	-0.01	-0.01	0.00	0.01	0.01	
4	0.00	0.00	0.00	0.01	0.01	
5	0.00	0.01	0.01	0.01	0.01	<b>0.01</b>
5-1	<b>0.01</b>				<b>0.00</b>	
Variable: Equity Issuance						
Market/Book Profitability	1	2	3	4	5	5-1
1	0.00	0.00	0.00	0.01	0.02	<b>0.02</b>
2	-0.01	0.00	0.00	0.00	0.00	
3	-0.01	0.00	0.00	0.00	0.00	
4	-0.01	-0.01	0.00	0.00	0.00	
5	-0.02	-0.01	-0.01	0.00	0.00	<b>0.02</b>
5-1	<b>-0.02</b>				<b>-0.02</b>	

with low profitability. Along the profitability dimension it shows that the higher debt issuance of more profitable firms is concentrated on those with low market to book values.

With respect to equity issuance, it shows that it is homogenously 2% lower as a fraction of market to book for high profitability firms than for low profitability firms, irrespective of the value of the market to book ratio. Also, the table shows that equity issuance is homogenously higher for firms with higher market to book, also about 2% as a fraction of market value, irrespective of the level of profitability.



## 5 Conclusions

This paper describes a set of benchmarks that can be used for the quantitative evaluation of dynamic corporate finance models and the methodology with which to construct them for any given model, based on its state and control variables. The benchmarks are the empirical counterparts of the optimal policy functions of the firm, defined over the state space for each of the control variables. We presented these benchmarks for models where investment and/or debt issuance and/or equity issuance are the control variables and where profitability, leverage and/or market to book are the state variables. Three key numbers that we suggest quantitative dynamic models have match closely are that within firms, for every 10% increase in debt relative to assets investment relative to assets declines 3.7%, debt issuance relative to market value decreases 1.1% and equity issuance relative to market value increases 0.5%. Also,

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