Profitability in Cournot and Bertrand Mixed Markets under Endogenous Objectives

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Abstract. We examine both quantity and price competition between a number of profit-maximizing firms and a state-controlled enterprise (SCE). The objective function of the latter is strategically defined by a welfare-maximizing government which weighs the SCE's profits relative to consumer surplus and private profits. Different motives drive the government's optimal behavior in the two competitive settings and lead all firms in oligopoly to gain higher profits in Cournot than in Bertrand. The profit ordering is reverted, and social welfare is enhanced, with respect to the purely-mixed market examined by GHOSH AND MITRA [2010]. In duopoly, aggregate profits are equivalent in Cournot and Bertrand.

JEL Codes: D43, L13, L32

Keywords: Cournot, Bertrand, endogenous objectives, partial privatization

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1 Introduction

All literature comparing the Cournot and Bertrand outcomes in private markets points out the superiority of the Cournot model in terms of higher profits earned by competing firms.[1] This result, first established by SINGH AND VIVES [1984] in a symmetric duopoly with substitute goods, has been extended by HÄCKNER [2000] to an oligopoly with both horizontal and vertical differentiation where it has been shown to be conditioned on the existence of small quality differences. Indeed, that model highlights how, when quality differences are large, a profit ranking reversal occurs for high-quality firms which supply substitutes. Among others, a reversal of the Singh and Vives's ranking of profits is found in ZANCHETTIN (2006) which allows for a wide range of cost asymmetry across firms and shows that profits are higher under Bertrand when cost asymmetry is strong and/or products are weakly differentiated, in LÓPEZ AND NAYLOR [2004] in a wage bargaining with upstream and downstream firms, and in MOTTA [1993] in a vertical differentiation framework. Most recently, GHOSH AND MITRA [2010] have reconsidered the relative efficiency of Cournot and Bertrand equilibria in markets with public and private firms. In their model (hereafter GM model) they show that, in contrast to the standard result with private firms, price competition leads to higher profits with respect to quantity competition. The reversal result can be explained as follows. Indeed, the higher Cournot profitability in private markets arises as a consequence of the lower perceived elasticity of demand under Cournot which positively affects the ability of firms to set higher prices and gain higher profits. However, this ability is weakened by the presence of a public firm in a market which tends to reduce its own price and exert a competitive pressure on rivals' prices. This tendency towards a price reduction by all firms is higher when firms compete in quantities rather than in prices, due to the higher importance given in the Cournot setting by a pure-welfare-maximizing firm to consumers' surplus compared to industry profits, and can cause firms to set lower prices and gain lower profits under Cournot competition than under Bertrand, leading to the Ghosh and Mitra's result. This price-reduction effect, which prevails over the demand elasticity effect under Cournot, also implies that higher consumer surplus is enjoyed in the Cournot model relative to a Bertrand model.[2]

By developing a linear oligopoly model with substitute goods, the present paper challenges the Ghosh and Mitra's result, demonstrating that it does not hold in a mixed market when the objective of an *ex-ante* public firm is manipulated by the government in order to accomplish social welfare goals. Indeed, in our framework the objective of the state-controlled enterprise (SCE) is assumed to change according to the weight assigned by the government on a welfare-

[1] Whether firms earn higher profits competing under Cournot or under Bertrand is a relevant question in oligopoly theory, since it can affect market entry decisions, as well as firms' investment choices and the optimal policy settled by the government in regulated markets.

[2] Ghosh and Mitra also prove that the standard ranking implying higher welfare under Bertrand in a private market continues to hold in a pure mixed market.

maximizing basis to the different components of social welfare.[3] By assuming that this weight is optimally determined at a pre-play stage of both a quantity and a price game, we demonstrate that in oligopoly higher profitability is associated to Cournot relative to Bertrand, thus recovering the result obtained by Singh and Vives in a private market. This result derives from the endogenous choice of the government to give, in the balancing between industry profits and consumers' surplus, higher and lower weight to the SCE's profits respectively under Cournot and under Bertrand as compared to a purely-mixed market. This amounts to undertaking a partial privatization policy in the Cournot model,[4] which lets the different types of firms be more aligned with respect to a profit target, and a pro-consumer policy in the Bertrand model in which firms' optimal behavior allows for the attainment of higher consumers' surplus. With respect to a purely-mixed market, the government's optimal manipulation succeeds in enhancing social welfare by correcting the distortions that in a Cournot and a Bertrand purely-mixed market are respectively due to welfare detrimental quantity differences or excessive prices. By focusing on the way in which the presence of a SCE firm with endogenous objectives shrinks the price-reduction effect relative to the elasticity demand effect as compared to the pure-welfare maximization case, our model shows that in oligopoly the dominance of the latter makes Cournot more profitable than Bertrand, while in a duopoly the two effects are shown to be perfectly balanced. In the latter case, Cournot competition and Bertrand competition turn out to be equally profitable.

The paper is structured as follows: Section 2 and Section 3 illustrate respectively the Cournot and the Bertrand model. A comparison across the two models and the main results are presented in Section 4 which also concludes.

2 The model with quantity competition

In an oligopolist market firm 0 is assumed to be the SCE firm and to compete against n private firms ($i = 1, .., n$) with respect to quantities. Firm 0's objective is endogenous and is defined by the generalized welfare (GW) function $M_0 = \alpha W + (1 - \alpha) \pi_0$, where W is social welfare (the sum of consumers' surplus CS and industry profits), π_0 the SCE firm's profits, and α (with $\alpha \geq 0$) the weight attached by the government to social welfare as opposite to the SCE's profits. Determining endogenously this weight amounts to choosing the SCE's optimal ownership structure or the optimal composition of its governing board. Indeed, by assuming $\alpha \geq 0$, the following cases may emerge at equilibrium: a) the SCE firm turns out to be partially privatized when $0 < \alpha < 1$, due to the higher concern for its own profits, or rather fully privatized ($\alpha = 0$) or nation-

[3] As shown in BENASSI, CHIRCO AND SCRIMITORE [2011], and following WHITE [2002], the optimal manipulation of the SCE's objectives enables the government to strategically alter the mix of components in a generalized welfare function, leaving room for further welfare improvements with respect to the pure-welfare maximization case.

[4] MATSUMURA [1998] first addressed partial privatization in a model with quantity competition. His analysis has been extended to a differentiated Cournot oligopoly by FUJIWARA [2007] and to price competition by OHNISHI [2010].

alized ($\alpha = 1$), b) the government allows for a pro-consumer composition of the SCE's governing board when $\alpha > 1$ as a consequence of the lower interest in the SCE's profits relative to the other components of social welfare.[5]

The linear demand function in each market is $p_s = 1 - q_s - \gamma \sum_{z \neq s} q_z$, with $s = (0, 1, .., n)$ and the parameter $\gamma \in (0, 1)$ capturing imperfect substitutability ($\gamma = 0$ implying independent goods and $\gamma = 1$ perfect substitutes).[6] We assume constant marginal costs c for all firms, which are normalized to zero without any loss of generality. A two-stage game runs as follows: the government determines the optimal α at the first stage, while all firms in the last stage compete simultaneously in quantities.

Starting from the last stage, the SCE firm solves the following optimization problem:

$$\max_{q_0} M_0 = \alpha W + (1 - \alpha)\,\pi_0 = \alpha \left(CS + \sum_{i=1}^{n} \pi_i \right) + \pi_0$$

where the social welfare function W is defined as the sum of consumer surplus $CS = 1/2 \left((1 - \gamma) \left(q_0^2 + \sum_{i=1}^{n} q_i^2 \right) + \gamma \left(q_0 + \sum_{i=1}^{n} q_i \right)^2 \right)$ and aggregate profits $\Pi = \pi_0 + \sum_{i=1}^{n} \pi_i$.

The solution of the first order condition gives the following reaction function for firm 0:

$$q_0 = \frac{1 - \gamma \sum_{i=1}^{n} q_i}{2 - \alpha} \tag{1}$$

At the same stage each private firm maximizes its own profits with respect to q_i, producing the output express by the following reaction function:

$$q_i = \frac{1 - \gamma q_0 - \gamma \sum_{j \neq i} q_j}{2} \tag{2}$$

By aggregating (2) over all the private firms and solving for the aggregate private output, we obtain:

$$\sum_{i=1}^{n} q_i = \frac{n\,(1 - \gamma q_0)}{2 + \gamma\,(n - 1)} \tag{3}$$

By solving simultaneously the reaction functions in (1) and (3) for q_0 and $\sum_{i=1}^{n} q_i$, and then substituting solutions in (2) in order to derive q_i, we find the optimal quantities q_0^C and q_i^C produced respectively by the SCE and each

[5] Indeed, by rewriting the GW function as $M_0 = \alpha \left(CS + \sum_{i=1}^{n} \pi_i \right) + \pi_0$, the assumption $\alpha > 1$ implies that the sum $CS + \sum_{i=1}^{n} \pi_i$ is taken into consideration to a larger extent than π_0.

[6] This demand derives from maximization of a semi-linear utility function of a representative consumer.

private firm as functions of α:

$$q_0^C = \frac{2-\gamma}{(2-\gamma)(2+\gamma n) - \alpha(2+\gamma(n-1))} \tag{4}$$

$$q_i^C = \frac{2-\gamma-\alpha}{(2-\gamma)(2+\gamma n) - \alpha(2+\gamma(n-1))} \tag{5}$$

At the first stage, the welfare-maximizing government chooses the optimal α, denoted by α_C, which satisfies the condition $\partial W/\partial \alpha_C = 0$. We find $\alpha_C = \frac{4(1-\gamma)+\gamma^2}{(1-\gamma)(4+\gamma n)+\gamma^2}$.[7] It can be easily checked that $0 < \alpha_C < 1$ in the interval $\gamma \in (0,1)$ and for any n, which reveals that the optimal choice is to partially privatize the SCE firm.

At the subgame perfect equilibrium the optimal quantities are:

$$q_0^C = \frac{(1-\gamma)(4+n\gamma)+\gamma^2}{\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2}$$

$$q_i^C = \frac{(1-\gamma)(2+\gamma(n-1))}{\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2}$$

The prices clearing the market at equilibrium are the following:

$p_0^C = \frac{n\gamma(1-\gamma)}{\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2}$
$p_i^C = q_i^C = \frac{(1-\gamma)(2+\gamma(n-1))}{\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2}$

Firms' profits are reported in Appendix 1B. It is worth noting that, by evaluating quantities in (4) and (5) when $\alpha = 1$, we obtain the equilibrium quantities found in a mixed oligopoly with pure-welfare maximization by Ghosh and Mitra. We also calculate social welfare under partial privatization and compare it with that at the GM equilibrium. The analysis allows us to introduce the following remark.

Remark 1 *Under endogenous objectives, partial privatization arises at equilibrium, with the SCE firm producing less and each private firm producing more than that they would do in the pure-welfare maximization case. With respect to the latter, we find on the one hand a more equally distributed production between the two types of firms which causes a positive impact on social welfare, on the other hand a market quantity reduction which negatively affects social welfare. The net effect is positive, as a result of the optimal SCE's objective manipulation, and yields social welfare improvements regardless of the degree of product differentiation and the total number of firms.*

By weighing to a larger extent its own profits in the GW function, the government lets the SCE firm be partially owned by the private sector and behave

[7] See Appendix 1A for a proof of the second order conditions of this maximization problem.

less aggressively. An alignment of objectives towards profits between the SCE and the private firms is realized due to the government's optimal manipulation, which induces a lower production by the SCE firm and a higher production by the private firms, leading both firms' profits to increase. While the quantity differential between the two types of firms decreases with a positive effect on welfare *via* higher profits, the consumers' surplus decreases as a consequence of market quantity contraction. The latter, indeed, is caused by an output reduction by the SCE firm which is not outweighed by the private firms' output expansion. The re-balancing between the different welfare components yields welfare improvements with respect to a purely-mixed market, since it allows for the positive effect *via* profits to dominate the negative effect *via* consumer surplus.[8]

3 The model with price competition

In the price competition framework we keep the assumptions of the above model and solve backwards the two stage game, identifying the optimal prices at the last stage and the optimal α at the first one. The direct demand function faced by firm s (with $s = (0, 1, .., n)$) is:

$$q_s = \frac{1 - \gamma - (1 + \gamma (n - 1)) p_s + \gamma \sum_{z \neq s} p_z}{(1 - \gamma)(1 + \gamma n)}$$

As regards the SCE firm, we identify the price which maximizes the function $M_0 = \alpha W + (1 - \alpha) \pi_0$. That price, namely the reaction function of firm 0, is the following:

$$p_0 = \frac{(1 - \alpha)(1 - \gamma) + \gamma \sum_{i=1}^{n} p_i}{(2 - \alpha)(1 + \gamma (n - 1))} \tag{6}$$

For each private firm i $(i = 1, .., n)$ we find that the reaction function $p_i = \frac{1-\gamma+\gamma\left(p_0+\sum_{j\neq i} p_j\right)}{2(1+\gamma(n-1))}$ maximizes its own profits. By aggregating the first order conditions over all the private firms and solving for $\sum_{i=1}^{n} p_i$ we obtain:

$$\sum_{i=1}^{n} p_i = \frac{n (1 - \gamma + \gamma p_0)}{2 + \gamma (n - 1)} \tag{7}$$

The solutions to the simultaneous equations in (6) and (7), and their substitution in the private firm's reaction function, yields the optimal prices p_0^B and

[8] As stated by GHOSH AND MITRA [2010] p.74, social welfare depends positively on the aggregate quantity and negatively on the quantity differential between the SCE and each private firm. This amounts to saying that any strategy reducing the difference $q_0 - q_i$ contributes to enhancing welfare. The objective to reduce $q_0 - q_i$ in the Cournot framework of our model, however, is realized by contracting market quantity and thus hurting social welfare. The optimal manipulation of the SCE firm, indeed, solves this trade-off leading social welfare to rise.

p_i^B set respectively by the SCE and the private firm and expressed as functions of α:

$$p_0 = \frac{(1-\gamma)(2+\gamma(2n-1)-\alpha(2+\gamma(n-1)))}{(2(1+n\gamma)-\gamma)(n\gamma+2(1-\gamma))-\alpha(2+\gamma(n-1))(1+\gamma(n-1))} \quad (8)$$

$$p_i = \frac{(1-\gamma)(2+\gamma(2n-1)-\alpha(1+n\gamma))}{(2(1+n\gamma)-\gamma)(2(1-\gamma)+n\gamma)-\alpha(2+\gamma(n-1))(1+\gamma(n-1))} \quad (9)$$

At the first stage of the game the SCE firm solves the social welfare maximization problem by choosing $\alpha_B = \frac{(2+\gamma(2n-1))^2}{3n^2\gamma^2+\gamma n(7-4\gamma)+(2-\gamma)^2}$.[9] The latter, in contrast to the quantity competition case, is higher than 1 for any $\gamma \in (0,1)$ and any n. The solution $\alpha_B > 1$ reveals that partial privatization is not optimal under Bertrand, as also shown in ONHISHI [2010]. In this regard it should be stressed that this solution is consistent with our assumptions on SCE's objective manipulation which allow for $\alpha > 1$. From this perspectve our analysis extends that carried out by GHOSH AND MITRA [2008] under the hypothesis of partial privatization and, by allowing to take into account the SCE's optimal behavior in the comparison between quantity and price competition, puts forward new insights with respect to it.[10]

The optimal prices at the subgame perfect equilibrium are:

$$p_0^B = \frac{\gamma n(1-\gamma)(\gamma(n-1)+1)}{\gamma^3 n^3 + (1-\gamma)\left(5\gamma^2 n^2 + (2-\gamma)^2 + \gamma n(8-5\gamma)\right)}$$

$$p_i^B = \frac{(1-\gamma)(n\gamma(n\gamma+3(1-\gamma))+(1-\gamma)(2-\gamma))}{\gamma^3 n^3 + (1-\gamma)\left(5\gamma^2 n^2 + (2-\gamma)^2 + \gamma n(8-5\gamma)\right)}$$

The output produced by firms at equilibrium are:

$$q_0^B = \frac{\gamma^3 n^3+(1-\gamma)\left(6n^2\gamma^2+\gamma n(9-5\gamma)+(2-\gamma)^2\right)}{(1+\gamma n)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)}$$

$$q_i^B = \frac{(1+\gamma(n-1))\left(\gamma^2 n^2+(1-\gamma)(\gamma(3n-1)+2)\right)}{(1+\gamma n)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)}$$

Firms profits are calculated and reported in Appendix 1B. By calculating prices in (8) and (9) when $\alpha = 1$, we obtain the equilibrium prices of the GM model. We also compare social welfare at the GM equilibrium with social welfare under endogenous objectives. This analysis allows us to introduce the following remark.

[9] The proof of the second order conditions for α_B to be a maximum is in Appendix 1A.

[10] Indeed, in their paper of 2008 Ghosh and Mitra search for the optimal degree of privatization in the interval $(0,1)$ and assess the Cournot-Bertrand ordering comparing market variables at the Cournot optimal interior solution - which entails that the public firm is partially privatized - and at the upper bound solution in Bertrand, which implies that no weight is given to the SCE profits. However, for a comparison to be meaningful, the optimal incentives in the two competitive regimes must be taken into account.

Remark 2 *Under endogenous objectives, the government assigns a lower weight to the SCE's profits relative to the other components of social welfare, which causes both the SCE and the private firms to set lower prices than in the welfare maximization case. With respect to the latter, we find a market quantity expansion which positively impacts on social welfare via consumers' surplus, and an aggregate profit reduction. The former positive effect affects social welfare to a larger extent than the latter negative effect, regardless of the degree of product differentiation and the total number of firms.*

Through strategic manipulation, the government chooses to give higher weight to the outsider components of social welfare, thus pushing the SCE firm towards a more aggressive behavior than in the welfare maximization case. Due to higher aggressiveness, the SCE firm sets a lower price which induces a price reduction by the private firms and favors an aggregate quantity's expansion. Clearly the government's strategic behavior and the optimal firms' reactions create at equilibrium a competitive environment which is more favorable to consumers, with reduced profits for all firms. We finally highlight that the optimal balancing between consumers' welfare and profits allows for welfare improvements compared to the GM case, since the welfare increase *via* consumer surplus dominates the welfare reduction due to lower profits.

4 A comparison between Cournot and Bertrand

In this section we compare the market variables derived in the Cournot and Bertrand models in the previous section. An inspection of the optimal parameter $\alpha_C(\gamma, n)$ and $\alpha_B(\gamma, n)$ respectively under Cournot and Bertrand allows us to capture the different forces at work in the balancing between the different components of social welfare.

Proposition 1 *The optimal parameter $\alpha_i(\gamma, n)$ $(i = C, B)$ decreases in the number of private firms n in the Cournot model and increases in n in the Bertrand model. When the private firms' number tends to infinite, $\alpha_C(\gamma, n)$ entails full privatization, while $\alpha_B(\gamma, n)$ implies that the lowest weight is attached to SCE's profits. The pattern of the optimal parameter with respect to the number of firms in each setting is shown in Figure 1.*

Proof:

The monotonic decreasing pattern of $\alpha_C(\gamma, n)$ derives from the negative sign of its first derivative with respect to n:

$$\frac{\partial(\alpha_C(\gamma, n))}{\partial n} = -\frac{(1-\gamma)(2-\gamma)^2}{\left((2-\gamma)^2+\gamma n(1-\gamma)\right)^2} < 0$$

In contrast, the monotonic increasing pattern of $\alpha_B(\gamma, n)$ derives from the positive sign of its first derivative with respect to n:

$$\frac{\partial(\alpha_B(\gamma, n))}{\partial n} = \frac{\gamma(2+\gamma(2n-1))(2\gamma n(1-\gamma)+(2-\gamma))}{\left(\gamma n(\gamma(3n-4)+7)+(2-\gamma)^2\right)^2} > 0.$$

The following limit results hold: $\lim_{n\to\infty} \alpha_C(\gamma, n) = 0$; $\lim_{n\to\infty} \alpha_B(\gamma, n) = 4/3$.

As discussed in the previous section, in the Cournot model the incentives for partial privatization imply a higher weight assigned by the government to π_0 relative to the outsider components of social welfare $CS + \sum_{i=1}^{n} \pi_i$ (firm 0 cares to some extent about π_0 besides W), while in the Bertrand model the optimal manipulation of the SCE's objective function leads the government to assign a lower weight to π_0 relative to $CS + \sum_{i=1}^{n} \pi_i$. These different incentives in the two settings explains the decreasing pattern of α_C and the increasing pattern of α_B when n increases (see Figure 1). Indeed, in the Cournot setting, welfare maximization by the government requires that, following an increase of the number of firms which reduces individual firm's' output, the SCE firm produces decreasing quantities being increasingly privatized, consistently with the aim of reducing quantity differentials and aligning firms' objectives towards a profit target. This argument implies that full privatization emerges when $n \to +\infty$. By contrast, when the number of firms increases in the Bertrand setting, welfare maximization by the government requires a progressively lower price set by the SCE firm as a result of the decreasing weight put on its own profits. This is in line with the reduction of private firms' prices caused by increased competition and the aim of raising market quantity rather than profits in the optimal welfare balancing. It is worth noting that, in the limit, the outcomes of the two model coincide, independently of the mode of competition and the manipulation strategy. Indeed, when the number of firms increases infinitely, competition among firms guarantees the achievement of the first-best allocation, with marginal cost pricing by all firms, zero profits and maximum consumer surplus. In this respect, our analysis highlights the irrelevance of a strategic manipulation of the SCE's objectives in competitive markets.

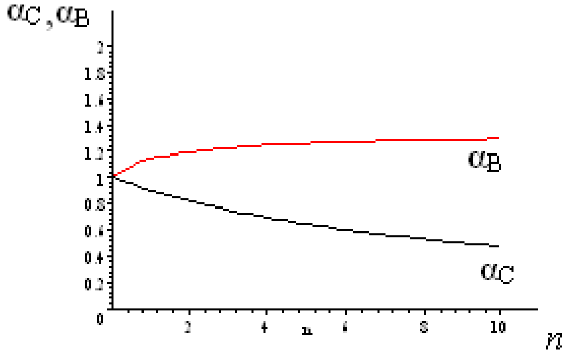

Figure 1. The optimal weights α_C and α_B as functions of n ($\gamma = 1/2$)

A comparison of equilibrium prices in the two models allows us to introduce the following proposition.

Proposition 2
- $p_0^C(\gamma,1) = p_0^B(\gamma,1)$; $p_0^C(\gamma,n) > p_0^B(\gamma,n)$ for $n > 1$ $\quad (a)$
- $p_i^C(\gamma,n) > p_i^B(\gamma,n)$ for any n $\quad (b)$

Proof: See Appendix 2A

A comparison with prices in the GM model case explains the above results. Indeed, as highlighted by Ghosh and Mitra, and in contrast to a private market in which the Cournot price always dominates the Bertrand price, pure-welfare maximization by the public firm induces it to set a price under Bertrand that is always higher than the equilibrium price under Cournot, the latter being equal to marginal cost. The same price ordering applies to private firms, the prices of which coincide only under duopoly. Two effects can be distinguished: a 'welfare maximization effect' which lowers to a larger extent p_0^C relative to p_0^B and creates a downward pressure on the private firm's price which lowers p_i^C with respect to p_i^B; a 'perceived-demand-elasticity effect' which in Cournot keeps up p_i^C with respect to p_i^B. A perfect balancing of the two effects generates the equivalence $p_i^C = p_i^B$ in a duopoly, while in oligopoly competition among private firms leads the second effect to prevail. In contrast to the pure-welfare maximization case, the strategic manipulation of the SCE's objective function, aimed to enhance welfare by reducing the quantity differentials under Cournot and by raising market quantity under Bertrand, shrinks the difference between p_0^C and p_0^B and reduces the extent of the 'welfare maximization effect' which never dominates the 'perceived-demand-elasticity effect' in our framework. The aforementioned effects exactly compensate in a duopoly, which causes the equivalence $p_0^C = p_0^B$ whatever the degree of product differentiation, while in oligopoly the inequality $p_0^C > p_0^B$ always holds due to the prevailing second effect, the magnitude of which crucially depends on the number of competing firms. This second effect always prevails for the private firms and leads to the ranking $p_i^C > p_i^B$, regardless of market structure and the degree of product substitutability.

The results of a comparison between the output produced by each firm and the aggregate output in the Cournot and the Bertrand market are summarized in the following proposition.

Proposition 3
- $q_0^C(\gamma,n) > q_0^B(\gamma,n)$ $\quad (a)$
- $q_i^C(\gamma,n) < q_i^B(\gamma,n)$ $\quad (b)$
- $q_0^C + nq_i^C < q_0^B + nq_i^B$ $\quad (c)$
- $q_0^B - q_i^B < q_0^C - q_i^C$ $\quad (d)$

Proof: See Appendix 2B

In line with Ghosh and Mitra, we show that quantity competition induces the SCE firm to produce more and each private firm to produce less than price competition, irrespective of number of firms and the degree of product substitutability. Indeed, notwithstanding in the Cournot market the SCE firm is induced to produce less than in the GM case, it keeps on producing relatively more than in the Bertrand market, due also to the positive effect of higher prices set by the private firms under Cournot relative to Bertrand, and despite a lower perceived elasticity of demand. This causes the difference $q_0^C - q_0^B$ to shrink, but its ordering is never reverted. The combined effect of a higher SCE's production in a Cournot market and the lower perceived elasticity of demand induces a lower production by the private firms. In contrast to the GM case, the SCE's output dominance under Cournot turns out not to be large enough to outweigh the output contraction of private firms under this regime, so that the market output is lower in Cournot compared to Bertrand. As a consequence of the higher market quantity, consumers' surplus under Bertrand always dominates consumers' surplus under Cournot: - $S^C(\gamma, n) < S^B(\gamma, n)$. Moreover, despite the welfare-enhancing reduction of the quantity differential between the SCE's and the private firms under Cournot, it turns out to be higher under Cournot than under Bertrand. Both the inequalities (c) and (d) ensure that the standard welfare ranking $W^C(\gamma, n) < W^B(\gamma, n)$ always holds in our setting.

The inspection of prices and quantities allows us to interpret the results concerning the profits' ranking which are presented in the following proposition.

Proposition 4
- $\pi_0^C(\gamma, n) > \pi_0^B(\gamma, n)$ for any n
- $\pi_i^C(\gamma, 1) < \pi_i^B(\gamma, 1)$; $\pi_i^C(\gamma, n) > \pi_i^B(\gamma, n)$ for $n > 1$

Proof: See Appendix 2C

In oligopoly, higher Cournot profitability applies to both the SCE firm and the private firms, as standard in a private market. While the result for private firms derives from a positive effect of higher prices under Cournot which always dominates the negative effect of lower quantities compared to Bertrand, for the SCE firm it emerges as a consequence of its attitude to produce more in a Cournot market than in a Bertrand market and is sustained by equivalent prices in the two settings under duopoly and higher Cournot prices under oligopoly. In a duopoly, however, a reversal of the profits' order with respect to the oligopoly case occurs for the private firm which gains higher profits under Bertrand. The role of market structure on equilibrium profits is highlighted in the following corollary.

Corollary 1 *Aggregate profits in Cournot and Bertrand markets are respectively* $\Pi^C = \frac{p_0^C}{\gamma}$ *and* $\Pi^B = \frac{p_0^B}{\gamma}$. *For any degree of product substitutability, Bertrand and that Cournot aggregate profits are never lower than Bertrand profits: indeed,* $\Pi^C \geq \Pi^B$ *for* $n \geq 1$, *with the equality holding for* $n = 1$.

The above corollary clearly shows that firm 0's equilibrium prices defines the extent of aggregate profits under Cournot and Bertrand. The latter are shown to be equivalent in a duopoly, due to the equivalence of the SCE's prices in the two competition regimes under duopoly that follows from Proposition $2a$, which amounts to proving that at equilibrium the contribution of aggregate profits to social welfare is independent of the mode of competition. The mode of competition matters in defining a different profits' contribution to welfare when the market is populated by a higher number of private firms, competition among which lowers the equilibrium prices in Bertrand to a larger extent than in Cournot, and consequently lowers p_0^B more than p_0^C. In oligopoly, indeed, aggregate profits are higher under Cournot since the inequality $p_0^C > p_0^B$ holds in this case, as stated in Proposition $2b$.

4.1 Concluding remarks

This paper revisits the standard comparison between Cournot and Bertrand focusing on firms' profitability in mixed markets when the objective function of a state-controlled firm is optimally defined by a welfare-interested government. This manipulation strategy is shown to enhance social welfare with respect to the pure-welfare maximization case, being consistent with a privatization policy under Cournot and a pro-consumer policy under Bertrand. Our work basically highlights how the presence of firms with heterogeneous objectives on a market affects the conditions for firms' profitability in the two settings of Cournot and Bertrand competition. While Ghosh and Mitra have proved that the presence of a welfare-maximizing firm on the market leads quantity competition to be more beneficial than price competition for consumers and less beneficial for firms, we have shown how a behavioral alignment between the public and the private firms, realized through a SCE objective's manipulation by the government, restores in oligopoly the conditions for higher Cournot profitability that is typical of private markets. Larger aggregate output in Bertrand markets is also shown to translate into higher consumers' surplus. By underlining the impact of the government's strategic choices on the optimal behavior of private firms in the two settings, and moreover the role of competition among private firms on the market outcomes, the paper identifies the market forces moving towards higher Bertrand or higher Cournot profitability, showing how in a duopoly these forces are perfectly balanced. In this regard, our study offers an interesting example of market in which firms share the same amount of profits, regardless of whether they compete in quantities or prices.

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APPENDIX 1

Appendix 1a

The second order conditions for welfare maximization with respect to α_C and α_B are satisfied:

$$\frac{\partial^2 W}{\partial^2 \alpha_C} = -\frac{\left(\gamma^2+(1-\gamma)(n\gamma+4)\right)^4}{(\gamma^2+(1-\gamma)(n\gamma^2(n-1)+4(1+\gamma n)))\left((2-\gamma)\left(\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)\right)^2} < 0$$

$$\frac{\partial^2 W}{\partial^2 \alpha_B} = -\frac{(1-\gamma)^3\left(\gamma^2\left(3n^2+1\right)+\gamma n(7-4\gamma)+4(1-\gamma)\right)^4}{(1+n\gamma)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)^3(2(1+n\gamma)-\gamma)^2} < 0$$

Appendix 1b

SCE's profits and private firm's profits under Cournot:

$$\pi_0^C = \frac{n\gamma(1-\gamma)\left((1-\gamma)(4+n\gamma)+\gamma^2\right)}{\left(\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)^2}$$

$$\pi_i^C = \frac{(1-\gamma)^2(2+\gamma(n-1))^2}{\left(\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)^2}$$

SCE's profits and private firm's profits under Bertrand:

$$\pi_0^B = \frac{n\gamma(1-\gamma)(1+\gamma(n-1))\left(\gamma^3 n^3+(1-\gamma)\left(6\gamma^2 n^2+\gamma n(9-5\gamma)+(2-\gamma)^2\right)\right)}{(1+\gamma n)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)^2}$$

$$\pi_i^B = \frac{(1-\gamma)(1+\gamma(n-1))\left(\gamma^2 n^2+(1-\gamma)(2+\gamma(3n-1))\right)^2}{(1+\gamma n)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)^2}$$

APPENDIX 2

Appendix 2a

(a) The price set by the SCE firm under Cournot is never lower than the same price under Bertrand:

$$p_0^C - p_0^B =$$
$$= \frac{n^2\gamma^4(1-\gamma)(n-1)(2+\gamma(n-1))}{\left(n\gamma(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)} \geq 0$$

with the equality holding when $n = 1$.

(b) The price set by a private firm under Cournot is always higher than the same price under Bertrand:

$$p_i^C - p_i^B =$$
$$= \frac{n\gamma^2(1-\gamma)\left(\gamma n(5n\gamma-13\gamma+8)+\gamma^3 n(n-2)^2+(1-\gamma)(2-\gamma)^2\right)}{\left(n\gamma(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)} > 0$$

Appendix 2b

(a) The SCE's output under Cournot is always higher that the same output under Bertrand:

$$q_0^C - q_0^B = $$
$$= \frac{n^2\gamma^3\left((1-\gamma)(2(2-\gamma)+n\gamma(6-\gamma))+\gamma^2 n^2(2-\gamma)\right)}{(1+n\gamma)\left(n\gamma(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)} > 0$$

(b) As regards each private firm, the output under Cournot is always lower that the same output under Bertrand:

$$q_i^C - q_i^B = $$
$$= -\frac{n\gamma^2\left(n\gamma(1-\gamma)\left(\gamma^2\left(1+n^2\right)+8-5\gamma\right)+n^2\gamma^2\left(5+2\gamma^2-6\gamma\right)+(1-\gamma)(2-\gamma)^2\right)}{(1+n\gamma)\left(n\gamma(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)} < 0$$

(c) Market quantity is always higher in a Bertrand market:

$$\left(q_0^B + nq_i^B\right) - \left(q_0^C + nq_i^C\right) = $$
$$= \frac{(1-\gamma)(2-\gamma)^4+2\gamma^5 n^5(1-\gamma)+n^2\gamma^2 A(1-\gamma)+n\gamma B(1-\gamma)(2-\gamma)^2+\gamma^3 n^3 C+n^4\gamma^4 D}{(1+n\gamma)\left(n\gamma(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)} > 0$$

where:
$A = 68 - 6\gamma^3 + 46\gamma^2 - 101\gamma$
$B = 13 + \gamma^2 - 10\gamma$
$C = 45 - 12\gamma^3 + 62\gamma^2 - 94\gamma$
$D = 15 + 9\gamma^2 - 23\gamma$
with $A, B, C, D > 0$

(d) The quantity differential between the SCE and a private firm is always higher under Cournot than under Bertrand:

Let us pose $\Psi^C = q_0^C - q_i^C$ and $\Psi^B = q_0^B - q_i^B$. We find the following quantity differentials in the two regimes:
$$\Psi^C = \frac{2-\gamma}{\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2}$$
$$\Psi^B = \frac{2-\gamma(3-\gamma-2n(1-\gamma))}{\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)}$$

with the difference $\Phi = \Psi^C - \Psi^B$ always positive:
$$\Phi = \frac{n\gamma^2\left((1-\gamma)(2-\gamma)^2+\gamma n((1-\gamma)(8-3\gamma)+\gamma n(3-2\gamma))\right)}{\left(\gamma^3 n^3+(1-\gamma)\left(5\gamma^2 n^2+(2-\gamma)^2+\gamma n(8-5\gamma)\right)\right)\left(\gamma n(1-\gamma)(\gamma(n-1)+4)+(2-\gamma)^2\right)} > 0$$

Appendix 2c

In a duopoly, the SCE firm always gets higher profits under Cournot. Indeed, calling $f(n,\gamma)$ the difference $\pi_0^C(n,\gamma) - \pi_0^B(n,\gamma)$, it is easy to verify that when $n = 1$ that difference is strictly positive:

$f(1,\gamma) = \left(\gamma^4(1-\gamma)\right)/(\gamma+1)\left(4-3\gamma^2\right)^2 > 0$. In contrast, the private firms earn higher profits under Bertrand, as shown by the negative sign of the function $g(n,\gamma) = \pi_i^C(n,\gamma) - \pi_i^B(n,\gamma)$ when $n = 1$. Indeed $g(1,\gamma) = -\left(\gamma^4(1-\gamma)\right)/(\gamma+1)\left(4-3\gamma^2\right)^2 < 0$.

Since $f(1,\gamma) = -\,g(1,\gamma)$, the two differences perfectly compensate. Under oligopoly the higher Cournot profitability for the SCE firm continues to hold and also applies to private firms. The ranking $\pi_0^C(\gamma,n) > \pi_0^B(\gamma,n)$, namely the inequality $f(n,\gamma) > 0$, is shown to hold by numerical simulation (see Figure A1).

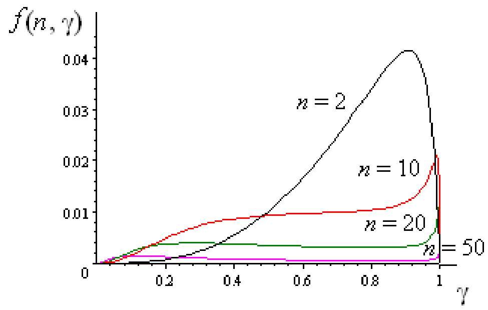

Figure A1

In contrast, for a private firm the oligopolist market structure matters in reverting the order of profits with respect to the duopoly case. Numerical simulations indicate that the inequality $\pi_i^C(\gamma,n) > \pi_i^B(\gamma,n)$ - equivalent to $g(n,\gamma) > 0$ - holds for any $n > 1$ and any γ, as shown in Figure A2.

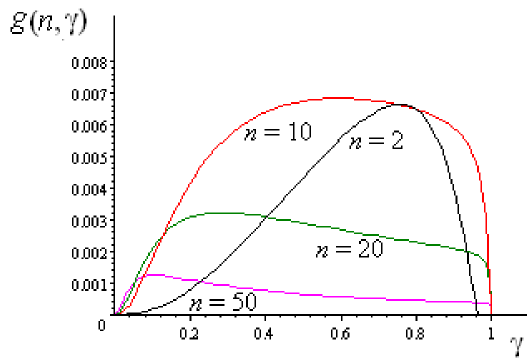

Figure A2