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The cost of counterparty risk and collateralization in longevity swaps*

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Abstract

Derivative longevity risk solutions, such as bespoke and indexed longevity swaps, allow pension schemes and annuity providers to swap out longevity risk, but introduce counterparty credit risk, which can be mitigated if not fully eliminated by collateralization. We examine the impact of bilateral default risk and collateral rules on the marking to market of longevity swaps, and show how longevity swap rates must be determined endogenously from the collateral flows associated with the marking-to-market procedure. For typical interest rate and mortality parameters, we find that the impact of collateralization is modest in the presence of symmetric default risk, but more pronounced when default risk and/or collateral rules are asymmetric. Our results suggest that the overall cost of collateralization is comparable with, and often much smaller than, that found in the interest-rate swaps market (as a result of the offsetting effects of interest rate and longevity risks), which may then provide the appropriate reference framework for the credit enhancement of both indemnity-based and indexed longevity risk solutions.

Keywords: longevity swap, counterparty risk, default risk, collateral, marking-to-market.

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1 Introduction

The market for longevity-linked securities and derivatives has recently experienced a surge in transactions in longevity swaps. These pure longevity hedges are agreements between two parties to exchange fixed payments against variable payments linked to the number of survivors in a reference population (see Dowd *et al.*, 2006). Table 1 presents a list of recent deals that have been publicly disclosed. So far, transactions have mainly involved pension funds and annuity providers wanting to hedge their exposure to longevity risk but without having to bear any basis risk. The variable payments in such longevity swaps are designed to match precisely the mortality experience of each individual hedger: hence the name bespoke longevity swaps. This is essentially a form of longevity risk insurance, similar to annuity reinsurance in reinsurance markets. Indeed, most of the longevity swaps executed to date have been bespoke, indemnity-based swaps of the kind familiar in reinsurance markets. This is true despite the fact that some of the swaps listed in table 1 have been arranged by investment banks: the banks have worked with insurance companies (in some cases insurance company subsidiaries) in order to deliver a solution in a format familiar to the counterparty.

A fundamental difference from other forms of reinsurance, however, is that longevity swaps are typically collateralized, whereas typical insurance/reinsurance transactions are not.[1] The main reason is that longevity swaps are often part of a wider de-risking strategy involving other collateralized instruments (interest-rate and inflation swaps, for example), and also the fact that hedgers have been increasingly concerned with counterparty risk[2] in the wake of the Global Financial Crisis of 2008-09. In this article, we

[1]One rationale for this is that reinsurers aggregate several uncorrelated risks and pooling/diversification benefits compensate for the absence of collateral (e.g., Lakdawalla and Zanjani, 2007; Cummins and Trainar, 2009). Insurers/reinsurers are still required by their regulators to post regulatory or solvency capital which plays a similar role to collateral.

[2]Basel II (2006, Annex 4) defines counterparty risk as 'the risk that the counterparty to a transaction could default before the final settlement of the transaction's cash flows'. The recent Solvency II proposal makes explicit allowance for a counterparty risk module in its 'standard formula' approach; see CEIOPS (2009).

provide a framework to quantify the trade-off between the exposure to counterparty risk in longevity swaps and the cost of credit enhancement strategies such as collateralization.

As there is no accepted framework yet for marking to market/model longevity swaps, hedgers and hedge suppliers look to other markets to provide a reference model for counterparty risk assessment and mitigation. In interest-rate swap markets, for example, the most common form of credit enhancement is the posting of collateral. According to the International Swap and Derivatives Association (ISDA) almost every swap at major financial institutions is 'bilaterally' collateralized (ISDA, 2010b), meaning that either party is required to post collateral depending on whether the market value of the swap is positive or negative.[3] The vast majority of transactions is collateralized according to the Credit Support Annex to the Master Swap Agreement introduced by ISDA (1994). The Global Financial Crisis highlighted the importance of bilateral counterparty risk and collateralization for over-the-counter markets, spurring a number of responses (e.g, ISDA, 2009; Brigo and Capponi, 2009; Assefa *et al.*, 2010; Brigo *et al.*, 2011). The Dodd-Frank Wall Street Reform and Consumer Protection Act (signed into law by President Barack Obama on July 21, 2010) is likely to have a major impact on the way financial institutions will manage counterparty risk in the coming years.[4] The recently founded Life and Longevity Markets Association (LLMA) [5] has counterparty risk at the center of its agenda, and will certainly draw extensively from the experience garnered in fixed-income and credit markets.

The design of collateralization strategies is intended to address the concerns aired by pension trustees regarding the efficacy of longevity swaps, but introduces another dimension in the traditional pricing framework used for insurance transactions. The 'insurance

[3]'Unlike a firm's exposure to credit risk through a loan, where the exposure to credit risk is unilateral and only the lending bank faces the risk of loss, counterparty credit risk creates a bilateral risk of loss: the market value of the transaction can be positive or negative to either counterparty to the transaction. The market value is uncertain and can vary over time with the movement of underlying market factors.' (Basel II, 2006, Annex 4).

[4]See, for example, 'Berkshire may scale back derivative sales after Dodd-Frank', *Bloomberg*, August 10, 2010.

[5]See `http://www.llma.org`.

premium' embedded in a longevity swap rate reflects not only the aversion (if any) of the counterparties to the risk being transferred and the cost of regulatory capital involved in the transaction, but also the expected costs to be incurred from posting collateral during the life of the swap. The fact that collateral is costly simply reflects the costs entailed by credit risk mitigation. To quantify the impact of collateral on swap rates, we must examine the relative sensitivity of the counterparties to its cost. Let us first take the perspective of a hedge supplier (reinsurer or investment bank) issuing a collateralized longevity swap to a counterparty (pension fund or annuity provider). Whenever the swap is sufficiently out-of-the-money, the hedge supplier is required to post collateral, which can be used by the hedger to mitigate losses in the event of default. Although interest on collateral is typically rebated, there is both a funding cost and an opportunity cost, as the posting of collateral depletes the resources the hedge supplier can use to meet her capital requirements at aggregate level as well as to write additional business. On the other hand, whenever the swap is sufficiently in-the-money, the hedge supplier will receive collateral from the counterparty, thus benefiting from capital relief in regulatory valuations and freeing up capital that can be used to sell additional longevity protection. The benefits can be far larger if collateral can be re-pledged for other purposes, as in the interest-rate swaps market.[6] The same considerations can be made from the viewpoint of the hedger, but the funding needs and opportunity costs of the two parties are unlikely to offset each other exactly. This is particularly relevant for transactions involving parties subject to different regulatory frameworks. In the UK and several other countries, for example, longevity risk exposures are more capital intensive for hedge suppliers, such as insurers, than for pension funds.[7]

In the absence of collateral, and ignoring longevity risk aversion, swap rates depend

[6] According to ISDA (2010b),the vast majority of collateral is rehypothecated for other purposes in interest-rate swap markets. Currently, collateral can be re-pledged under the New York Credit Support Annex, but not under the English Credit Support Deed (see ISDA, 2010a).

[7] This asymmetry is, in part, a by-product of rules allowing, for example, pension liabilities to be quantified by using outdated mortality tables or discount rates reflecting optimistic expected returns.

on best estimate survival probabilities for the hedged population and on the degree of covariation between the floating leg of the swap and the defaultable term structure of interest rates facing the hedger and the hedge supplier.[8] This means that a proper analysis of a longevity swap cannot disregard the sponsor's covenant when the hedger is a pension plan (see section 3 below). In the presence of collateralization, longevity swap rates are also shaped by the expected collateral costs, and swap valuation formulae involve a discount rate reflecting the cost of collateral. As a result, default-free valuation formulae are not appropriate even in the presence of full collateralization and the corresponding absence of default losses.[9]

We quantify collateral costs in two ways: i) in terms of funding costs that are incurred or mitigated when collateral is posted or received, and ii) as the opportunity cost of selling additional longevity protection. In both cases, we find that, for typical interest rate and mortality parameters, the impact of collateralization on swap rates is modest when default risk and collateral rules are symmetric. There are two opposing effects at play here:

i) On the one hand, the receiver of the fixed survival rate (the hedge supplier) posts collateral when mortality is lower and hence longevity exposures are more capital intensive. On the other hand, she receives collateral when mortality is higher and longevity protection less capital intensive. The overall effect is to push (fixed) swap rates higher, to compensate the hedge supplier for the positive dependence between collateral flows and capital costs.

ii) When the hedge supplier is out-of-the-money, collateral outflows are larger in low interest rate environments (i.e., when liabilities are discounted at a lower rate), hence there is a negative relationship between the amount of collateral posted and the hedge supplier's funding/opportunity costs. On the other hand, when the hedge supplier is

[8]Along the same lines, Inkmann and Blake (2010) show how the discount rate for the valuation of pension liabilities should reflect funding risk.

[9]See Johannes and Sundaresan (2007) for the case of symmetric default risk and full collateralization in interest-rate swaps.

in-the-money, collateral inflows are larger exactly when funding/opportunity costs are more significant. The opposite situation is faced by the hedger, who demands lower swap rates as a compensation for the positive dependence between collateral costs and collateral amounts.

When default risk and/or collateral rules are asymmetric, the offsetting effects are of different magnitudes and, as a result, the impact of collateral costs on longevity swap rates is larger. For example, we find that swap rates increase substantially when the hedger has a lower credit standing and the collateral rules are more favorable to the hedge supplier. Although collateralization introduces an explicit link between the individual risk exposures and the hedge supplier's funding risk (hence some of the pooling/diversification benefits used to substitute for collateralization in the standard insurance model may be lost), in our examples we find that the opposite effects of longevity and interest rate risk make the overall impact of collateralization comparable with, and typically lower than, that observed in fixed-income markets (e.g., Johannes and Sundaresan, 2007). An important implication is that the interest-rate swaps market might provide an appropriate framework for the collateralization of bespoke solutions, even though the latter lack of the transparency and standardization benefits associated with indexed-based instruments. Investment banks have sold index-based longevity swaps which have a structure that would be more familiar to capital markets investors, but they have so far been less popular than bespoke solutions. Nevertheless, for the longevity swaps market to really take off, it is necessary to expand beyond the limits of the reinsurance market and attract such new investors. We therefore also examine the costs of collateral in index-based swaps.

On the methodological side, we show how longevity swap rates must be determined endogenously from the dynamic marking to market[10] of the swap and the collateral

[10] Here and in what follows, by 'market value' and 'marking to market' we mean that assets and liabilities are assumed to be valued according to accounting/regulatory standards, all of which have now adopted a market-consistent valuation approach.

rules specified by the contract. To see why, note that the market value of the swap at each valuation date depends on the evolution of the relevant state variables (mortality, interest rates, credit spreads), as well as on the swap rate locked in at inception. On the other hand, the swap's market value will typically affect collateral amounts and, in a setting where collateral is costly, will embed the market value of expected future collateral flows. Hence, the swap rate can only be determined by explicitly taking into account the marking-to-market process and the dynamics of collateral posting. To avoid the computational burden of nested Monte Carlo simulations, we use an iterative procedure based on the Least-Squares Monte Carlo approach[11] (see Glassermann, 2004, and references therein). We provide several numerical examples showing how different collateralization rules shape longevity swap rates giving rise to margins in (best estimate) survival probabilities reflecting the cost of future collateral flows. Although our focus is on longevity risk solutions, the approach can be applied to other instruments, such as bespoke solutions for inflation and credit risk.

Our work contributes to the existing literature on longevity risk pricing in at least three ways: i) we introduce default risk in the pricing of longevity risk solutions, and properly address its bilateral nature; ii) we explicitly allow for collateralization rules, which are the backbone of any real-world hedging solution and materially affect the pricing of over-the-counter transactions; and iii) we introduce a 'structural' dimension in an otherwise reduced-form pricing framework, by allowing for funding/opportunity costs associated with longevity risk exposures held by hedgers and hedge suppliers. As there is essentially no publicly available information on swap rates, our approach[12] has the advantage of using publicly available information on credit markets and regulatory standards, without having to rely exclusively on calibration to primary insurance market prices, approximate hedging methods or assumptions on agents' risk preferences (e.g.,

[11] A similar approach is used by Bacinello *et al.* (2010) for surrender guarantees in life policies and by Bauer *et al.* (2010b) for the computation of capital requirements within the Solvency II framework.

[12] Similarly, Biffis and Blake (2010a) endogenize longevity risk premia by introducing asymmetric information and capital requirements in a risk-neutral setting.

Dowd *et al.*, 2006; Ludkovski and Young, 2008; Bauer *et al.*, 2010a; Biffis *et al.*, 2010; Chen and Cummins, 2010; Cox *et al.*, 2010, among others).

The article is organized as follows. In the next section, we introduce longevity swaps and formalize their payoffs. We consider the case of both bespoke and index-based swaps, but, in the latter case, we ignore the issue of basis risk[13] to keep the article focused. In section 2.1, we examine the marking to market of a longevity swap during its lifetime to demonstrate the impact of counterparty risk on the hedger's balance sheet. Section 3 introduces bilateral default risk in longevity swap valuation formulae. We identify the main channels through which default risk affects the market value of swaps and show why an iterative procedure is needed to compute swap rates. Section 4 introduces credit enhancement in the form of collateralization, and shows how longevity swap rates are affected even in the presence of full cash collateralization (and hence absence of default losses). We explain how swap rates can be computed by using an iterative procedure based on the Least-Squares Monte Carlo approach. In section 5, several stylized examples are provided to understand how different collateralization rules may affect longevity swap rates. Concluding remarks are offered in section 6. Further details and technical remarks are collected in an appendix.

< Table 1 about here >

2 Longevity swaps

We consider a hedger (insurer selling annuities, pension fund), referred to as party A, and a hedge supplier (reinsurer, investment bank), referred to as counterparty B. Agent A has the obligation to pay amounts $X_{T_1}, X_{T_2}, \ldots$, possibly dependent on interest rates and inflation, to each survivor at fixed dates $0 < T_1 \leq T_2, \ldots$ of an initial population

[13] See, for example, Coughlan *et al.* (2011), Salhi and Loisel (2010) and Stevens *et al.* (2010b) for some results related to this risk dimension.

of n individuals alive at time zero (annuitants or pensioners). We are clearly restricting our attention to homogeneous liabilities for ease of exposition, more general situations requiring obvious modifications. Party A's liability at a generic payment date $T > 0$ is given by the random variable $(n - N_T)X_T$, where N_T counts the number of deaths experienced by the population during the period $[0, T]$. Assuming that the individuals' death times have common intensity[14] $(\mu_t)_{t\geq 0}$, the expected number of survivors at time T can be written as $E^{\mathbb{P}}[n - N_T] = np_T$, with the survival probability p_T given by (see the appendix)

$$p_T := E^{\mathbb{P}}\left[\exp\left(-\int_0^T \mu_t \mathrm{d}t\right)\right]. \tag{2.1}$$

Here and in the following, $\mathbb{P}$ denotes the real-world probability measure. The intensity could be modeled by using, for example, any of the stochastic mortality models considered in Cairns *et al.* (2009). For our examples, we will rely on the simple Lee-Carter model.

Let us now consider a financial market and introduce the risk-free rate process $(r_t)_{t\geq 0}$. We assume that a market-consistent price of the liabilities can be computed by using a risk-neutral measure $\widetilde{\mathbb{P}}$, equivalent to $\mathbb{P}$, such that the death times have the same intensity process $(\mu_t)_{t\geq 0}$ (with different dynamics, in general, under the two measures; see Biffis *et al.*, 2010). The time-0 market value of the aggregate liability can then be written as

$$E^{\widetilde{\mathbb{P}}}\left[\sum_i \exp\left(-\int_0^{T_i} r_t \mathrm{d}t\right)(n - N_{T_i})X_{T_i}\right] = n\sum_i E^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^{T_i}(r_t + \mu_t)\mathrm{d}t\right)X_{T_i}\right].$$

For the moment, we take the pricing measure as given: we will give it more structure later on.

We consider two instruments which A can enter into with B to hedge its exposure: a

[14] Intuitively, μ_t represents the instantaneous conditional death probability for an individual alive at time t. As discussed more in detail in the appendix, for tractability we restrict our attention to the case of doubly stochastic (or Cox, conditionally Poisson) death times.

bespoke longevity swap and an index-based longevity swap. In these swaps, in contrast with interest rate swaps, the fixed leg will be a series of fixed rates each one pertaining to an individual payment date. The reason is that mortality increases substantially at old ages and a single fixed rate would introduce a growing mismatch between the cashflows provided by the swap and those needed by the hedger. However, as with interest rate swaps, we can treat a longevity swap as a portfolio of forward contracts on the underlying floating (survival) rate.[15] In this section, we ignore default risk and focus on individual payments at maturity $T > 0$. Throughout the article, we always assume the perspective of the hedger.

A **bespoke longevity swap** allows party A to pay a fixed rate $\overline{p}^N \in (0,1)$ against the realized survival rate experienced by the population between time zero and time T. Assuming a notional amount equal to the initial population size, n, the net payout to the hedger at time T is[16]

$$n\left(\frac{n - N_T}{n} - \overline{p}^N\right),$$

i.e., the difference between the realized number of survivors and the pre-set number of survivors $n\overline{p}^N$ agreed at inception. Letting S_0 denote the market value of the swap at inception, we can write

$$\begin{aligned} S_0 &= nE^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T r_t \mathrm{d}t\right)\left(\frac{n - N_T}{n} - \overline{p}^N\right)\right] \\ &= nE^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T (r_t + \mu_t)\mathrm{d}t\right)\right] - nB(0,T)\overline{p}^N, \end{aligned} \tag{2.2}$$

with $B(0,T)$ denoting the time-zero price of a zero-coupon bond with maturity T. By

[15] With a slight abuse of terminology, we use the term 'swap rate' for individual forward rates as well as for swap curves (a series of swap rates). We note that swap curves are often summarized by the improvement factor applied to the survival probabilities of a reference mortality table/model.

[16] For ease of exposition, here and in the following sections, we consider contemporaneous settlement only. Other settlement conventions (e.g. in arrears) have negligible effects, but make valuation formulae more involved when bilateral and asymmetric default risk is introduced.

setting $S_0 = 0$, we obtain the swap rate as

$$\overline{p}^N = \widetilde{p}_T + B(0,T)^{-1}\mathrm{Cov}^{\widetilde{\mathbb{P}}}\left(\exp\left(-\int_0^T r_t \mathrm{d}t\right), \exp\left(-\int_0^T \mu_t \mathrm{d}t\right)\right), \tag{2.3}$$

where the risk-adjusted survival probability $\widetilde{p}_T$ is defined as in (2.1) with expectations taken under $\widetilde{\mathbb{P}}$. Expression (2.3) shows that if the intensity of mortality is uncorrelated with bond market returns (a reasonable first-order approximation), the longevity swap curve just involves the survival probabilities $\{\widetilde{p}_{T_i}\}$ relative to the different maturities $\{T_i\}$. Several studies have recently addressed the issue of how to quantify risk-adjusted survival probabilities, for example, by calibration to annuity prices and books of life policies traded in secondary markets, or by use of approximate hedging methods (see references in Section 1). As there is essentially no publicly available information on swap rates, for our numerical examples we will suppose a baseline case in which $\widetilde{p}_{T_i} = p_{T_i}$ for each maturity T_i and focus on how counterparty default risk and collateral requirements might generate a positive or negative spread on best estimate survival rates. Although in what follows, we mainly concentrate on longevity risk, in practice, the floating payment of a longevity swap might involve a LIBOR component or survival indexation rules different from the ones considered above. To keep the setup general, we will at times consider instruments making a generic variable payment, P, and write the corresponding swap rate $\overline{p}$ as

$$\overline{p} = E^{\widetilde{\mathbb{P}}}[P] + B(0,T)^{-1}\mathrm{Cov}^{\widetilde{\mathbb{P}}}\left(\exp\left(-\int_0^T r_t \mathrm{d}t\right), P\right). \tag{2.4}$$

The setup can easily accommodate **index-based longevity swaps**, standardized instruments allowing the hedger to pay a fixed rate $\overline{p}^I \in (0,1)$ against the realized value of a survival index $(I_t)_{t\geq 0}$ at time T. The latter might reflect the mortality experience of a reference population closely matching that of the liability portfolio. Examples are represented by the LifeMetrics index developed by J.P. Morgan, the Pensions Institute and Towers Watson,[17] or the Xpect indices developed by Deutsche Boerse.[18] The relative advantages and disadvantages of index-based versus bespoke swaps are discussed, for example, in Biffis and Blake (2010b). Assuming that the index admits the representation $I_t = \exp(-\int_0^t \mu_s^I \mathrm{d}s)$, with $(\mu_t^I)_{t\geq 0}$ the intensity of mortality of a reference population, the swap rate $\overline{p}^I$ is given again by expression (2.3), but with the process μ replaced by μ^I, and with $\widetilde{p}_T$ replaced by the corresponding risk-adjusted survival probability $\widetilde{p}_T^I$.

2.1 The marking-to-market (MTM) process

Longevity swaps are not currently exchange traded and there is no commonly accepted framework for counterparties to mark to market/model their positions.[19] The presence of counterparty default risk and collateralization rules, however, makes the MTM procedure a very important feature of these transactions for at least three reasons. First, at each payment date, the difference between the variable and pre-set payment generates a cash inflow or outflow to the hedger, depending on the evolution of mortality. In the absence of basis risk (which is the case for bespoke solutions), these differences show a pure 'cashflow hedge' of the longevity exposure in operation. Second, as market conditions change (e.g., mortality patterns, counterparty default risk), the MTM procedure could result in the swap switching status in the hedger's balance sheet between that of an asset and that of a liability. This may have the implication that, even if the swap payments are expected to provide a good hedge against longevity risk, the hedger's position may still turn into a liability if, for example, deterioration in the hedge supplier's credit quality shrinks the expected present value of the variable payments. Third, for solvency requirements, it is important to value a longevity swap under extreme market/mortality scenarios ('stress testing'). This means, for example, that even if a longevity swap qualifies as a liability on a market-consistent basis, it might still provide considerable capital relief when valued

[17] See `www.lifemetrics.com`.

[18] See `www.xpect-index.com`.

[19] At the time of writing, LLMA was working on this issue.

on a regulatory basis.

To illustrate some of these points, let us consider the hypothetical situation of an insurer A with a liability represented by a group of ten thousand 65-year-old annuitants drawn from the population of England & Wales in 1980. We assume that party A entered a 25-year pure longevity swap in 1980 and we follow the evolution of the contract until maturity. The population is assumed to evolve according to the death rates reported in the Human Mortality Database (HMD) for England & Wales.[20] We assume that interest-rate risk is hedged away through interest rate swaps, locking in a rate of 5% throughout the life of the swap. The role of collateral is examined later on; here, we show how the hedging instrument operates from the point of view of the hedger. For this bespoke solution, the market value of each floating-for-fixed payment occurring at a generic date T can be computed by using the valuation formula

$$S_t = nE_t^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_t^T r_s \mathrm{d}s\right)\left(\frac{n-N_t}{n}\exp\left(-\int_t^T \mu_s \mathrm{d}s\right)\right)\right] - nB(t,T)\overline{p}^N, \tag{2.5}$$

for each time t in $[0,T]$ at which no default has yet occurred, with $B(t,T)$ denoting the market value of a zero-coupon bond with time to maturity $T-t$, and $E_t^{\widetilde{\mathbb{P}}}[\cdot]$ the conditional expectation given the information available at time t. As a simple benchmark case, we assume that market participants receive information from the HMD and use the Lee-Carter model to value longevity-linked cashflows. In other words, at each MTM date (including inception), longevity swap rates are based on Lee-Carter forecasts computed using the latest HMD information available.[21] Figure 1 illustrates the evolution of swap survival rates for an England & Wales cohort tracked from age 65 in 1980 to age 90 in 2005. It is clear that the systematic underestimation of mortality improvements by the Lee-Carter model in this particular example will mean that the hedger's position

[20] See `www.mortality.org`.

[21] See Dowd *et al.* (2010a,b); Cairns *et al.* (2011) for a comprehensive analysis of alternative mortality models; see also Girosi and King (2008) and Pitacco *et al.* (2009).

will become increasingly in-the-money as the swap matures. This is shown in Figure 2. In practice, the contract may allow the counterparty to cancel the swap or re-set the fixed leg for a nonnegative fee, but we ignore these features in this example. Figure 2 also reports the sequence of cash inflows and outflows generated by the swap, which are lower ex-post than what was anticipated from an examination of the MTM basis. As interest rate risk is hedged − and again ignoring default risk for the moment − cash inflows/outflows arising in the backtesting exercise only reflect the difference between the realized survival rates and the swap rates locked in at inception. On the other hand, the swap's market value reflects changes in market swap rates, which by assumption follow the updated Lee-Carter forecasts plotted in Figure 1 and differ from the realized survival rates. As is evident from Figure 2, the credit exposure of a longevity swap is close to zero at inception and at maturity, but may be sizable in between, depending on the trade-off between changes in market/mortality conditions and the residual swap payments (amortization effect). The credit exposure is quantified by the replacement cost, i.e., the cost that the nondefaulting counterparty would have to incur at the default time to replace the instrument at market prices then available. As a simple example which predicts the next section, let us introduce credit risk (but no default) and assume that in 1988 the credit spread of the hedge supplier widens across all maturities by 25 and 50 basis points. The impact of these two scenarios on the hedger's balance sheet is dramatic, as shown again in Figure 2, demonstrating how MTM profits and losses can jeopardize a successful cashflow hedge.

< Figure 1 about here >

< Figure 2 about here >

3 Counterparty default risk

The backtesting exercise of the previous section has demonstrated the importance of the hedge supplier's credit risk and the marking to market procedure in assessing the value of a longevity swap to the hedger. A correct approach, however, should allow for the fact that counterparty risk is bilateral. This is the case even when the hedger is a pension plan. Private sector defined benefit pension plans in countries such as the UK are founded on trust law and rely on a promise by (rather than a guarantee from) the sponsoring employer to pay the benefits to plan members. This promise is known as the 'sponsor covenant'. The strength of the sponsor covenant depends on both the financial strength of the employer and the employer's commitment to the scheme.[22] As a reasonable but imperfect proxy for the effect of the sponsor covenant, we use the sponsor's default intensity (party A's default intensity). For large corporate pension plans, the intensity can be derived/extrapolated from spreads observed in corporate bond and CDS markets. For smaller plans, an analysis of the funding level and strategy of the scheme is required.

Assume that both party A (the hedger) and B (the hedge supplier) may default at random times τ^A, τ^B, admitting default intensities[23] $(\lambda_t^A)_{t\geq 0}$, $(\lambda_t^B)_{t\geq 0}$. Defining by $\tau := \min(\tau^A, \tau^B)$ the default time of the swap transaction, we further assume that, on the event $\{\tau \leq T\}$, the nondefaulting counterparty, say party i, receives a fraction $\psi^j \in [0,1]$ ($i \neq j$, with $i, j \in \{A, B\}$) of the market value of the swap before default, $S_{\tau-}$, if she is in-the-money, otherwise she has to pay the full pre-default market value $S_{\tau-}$ to the defaulting counterparty. Following Duffie and Huang (1996), we can then

[22] In the UK, for example, The Actuarial Profession (2005, par. 3.2) defined the sponsor covenant as: "the combination of (a) the ability and (b) the willingness of the sponsor to pay (or the ability of the trustees to require the sponsor to pay) sufficient advance contributions to ensure that the scheme's benefits can be paid as they fall due." See also The Pensions Regulator (2009).

[23] For tractability and symmetry with the mortality model of section 2, we work with doubly stochastic default times (see the appendix). The main drawback is that the occurrence of default does not affect the conditional default probability of the surviving counterparty, thus limiting the extent to which close-out risk can be properly modelled.

write the market value of a swap with notional amount n as

$$S_0 = nE^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T (r_t + 1_{\{S_t<0\}}(1-\psi^A)\lambda_t^A + 1_{\{S_t\geq 0\}}(1-\psi^B)\lambda_t^B)\mathrm{d}t\right)\left(P - \overline{p}^d\right)\right], \tag{3.1}$$

where P denotes the variable payment, $\overline{p}^d$ the fixed rate, and the indicator function 1_H takes the value of unity if the event H is true, zero otherwise. To understand the above formula, note that, in our setting, the risk-neutral valuation of a defaultable claim involves the use of a default-risk-adjusted short rate $r_t + \lambda_t^A + \lambda_t^B$ and dividend payment $\lambda_t^A(\psi^A 1_{S_{t-}<0} + 1_{S_{t-}\geq 0}) + \lambda_t^B(\psi^B 1_{S_{t-}\geq 0} + 1_{S_{t-}<0})$ determined by the recovery rules described above. As a result, the valuation formula (3.1) entails discounting at a spread above the risk-free rate given by

$$\begin{aligned}\Lambda_t :=& \lambda_t^A + \lambda_t^B - \lambda_t^A(\psi^A 1_{S_t<0} + 1_{S_t\geq 0}) - \lambda_t^B(\psi^B 1_{S_t\geq 0} + 1_{S_t<0})\\ =& 1_{\{S_t<0\}}(1-\psi^A)\lambda_t^A + 1_{\{S_t\geq 0\}}(1-\psi^B)\lambda_t^B,\end{aligned}$$

showing a switching-type dependence on the characteristics of the counterparty that is out-of-the-money at each given time prior to default. The swap rate admits the representation

$$\overline{p}^d = E^{\widetilde{\mathbb{P}}}[P] + \frac{\mathrm{Cov}^{\widetilde{\mathbb{P}}}\left(\exp\left(-\int_0^T (r_t + \Lambda_t)\mathrm{d}t\right), P\right)}{E^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T (r_t + \Lambda_t)\mathrm{d}t\right)\right]}, \tag{3.2}$$

and hence depends in a complex way not only on the interaction between the variable payments and risk factors such as interest rates, default intensities and recovery rates, but also on the path of the swap's market value itself. When P does not include a demographic component, as in the case of interest-rate swaps, the covariance term is typically negative. To see this, consider the case of the standard swap valuation formula obtained by assuming that both counterparties have the same default intensity ($\lambda_t := \lambda_t^A = \lambda_t^B$) and there is no recovery conditional on default ($\psi^A = \psi^B = 0$). If the credit risk of the counterparties is equal to the average credit quality of the LIBOR panel, the discount rate in (3.2) is simply given by $r+\lambda$, where λ is just the LIBOR-Treasury (TED) spread. For a swap paying the LIBOR rate, we would then have a negative covariance term and hence $\overline{p}^d \leq E^{\widetilde{\mathbb{P}}}[P]$. When P only includes a demographic component, as in expression (2.3) for example, we may expect the covariance term also to be negative, as longevity-linked payments are likely to be positively correlated with the credit quality of hedge suppliers[24] and companies with significant pension liabilities. The case of floating payments linked to both mortality and interest rates would then suggest a swap rate satisfying $\overline{p}^d \leq E^{\widetilde{\mathbb{P}}}[P]$. In the next section, we will show that this is not necessarily the case. To understand why, consider, for example, the case of full recovery ($\psi^A = \psi^B = 1$): expression (3.2) reduces to a default-free risk-neutral valuation formula, irrespective of both the default intensities of the counterparties and the costs involved by the credit enhancement tools needed to ensure that full recovery is indeed achieved upon default! Counterparty risk can be mitigated in a number of ways, for example by introducing termination rights (e.g., credit puts and break clauses) or using credit derivatives (e.g., credit default swaps and credit spread options). We will focus on collateralization, a form of direct credit support requiring each party to post cash or securities when it is out-of-the-money. For simplicity, we consider the case of cash, which is by far the most common type of collateral (e.g., ISDA, 2010a) and allows us to disregard close-out risk, the risk that the value of collateral may change at default. In the interest-rate swaps market, Johannes and Sundaresan (2007) find evidence of costly collateral by comparing swap market data with swap values based on portfolios of futures and forward contracts. We cannot carry out a similar exercise for longevity swaps, because there are no publicly available data on these transactions. On the other hand, we can quantify the funding/opportunity costs associated with the collateral flows originating from the marking-to-market procedure. We will therefore work from the bottom up to 'synthesize'

[24]This is a reasonable assumption for monoline insurers such as pension buyout firms, but might be less so for well-diversified reinsurers.

the dynamics of collateral costs for a representative longevity-linked liability.

4 Collateralization

Collateral agreements reflect the amount of acceptable credit exposure that each party agrees to take on. We will consider simple collateral rules capturing the main features of the problem. Formally, let us introduce the pre-default collateral process[25] $(C_t)_{t\geq 0}$, which indicates how much cash, C_t, to post at each time t prior to default in response to changes in market conditions, including, in particular, the MTM value of the swap (we provide explicit examples below). Again, we develop our analysis from the point of view of the hedger, so that $C_t > 0$ ($C_t < 0$) means that party A is holding (posting) collateral. Using the notation $a^+ := \max(a, 0)$ and $a^- := \max(-a, 0)$, the recovery rules take the following form:

- On the event $\{\tau^A \leq \min(\tau^B, T)\}$ (hedger's default), party B (the hedge supplier) recovers any collateral received by the hedger an instant prior to default, $C^-_{\tau^A-}$, and pays the full MTM value of the swap to party A if $S_{\tau^A-} \geq 0$. The net flow to party A is then $S^+_{\tau^A-} - C^-_{\tau^A-}$.
- On the event $\{\tau^B \leq \min(\tau^A, T)\}$ (hedge supplier's default), party A (the hedger) pays the full MTM value of the swap to party B if $S_{\tau^B-} < 0$, and recovers any collateral received by B an instant prior to default, $C^+_{\tau^B-}$. The net flow to party A can then be written as $-S^-_{\tau^B-} + C^+_{\tau^B-}$.
- Whenever the nondefaulting counterparty, say A, is out-of-the-money, payment of the full MTM value of the swap is accomplished by party A recovering the extra amount $(S_{\tau-} - C_{\tau-})^+$ in case of overcollateralization, or by party A paying the extra amount $(C_{\tau-} - S_{\tau-})^+$ in case of undercollateralization. In case of full

[25]In other words, the actual collateral process supporting the transaction is $(1_{\{\tau>t\}}C_t)_{t\geq 0}$; hence, we are not concerned with the value taken by C_t after default.

collateralization, party A simply loses any collateral posted with B.

To obtain neater results, it is convenient to express the collateral before default of either party as a fraction of the MTM value of the swap,

$$C_t = \left(c_t^B 1_{\{S_{t-}\geq 0\}} + c_t^A 1_{\{S_{t-}<0\}}\right) S_{t-}, \tag{4.1}$$

where c^A, c^B are two nonnegative left-continuous processes giving the fraction of the MTM value of the swap that is posted as collateral by party A or B, respectively. Note that representation (4.1) comes at a cost: we cannot encompass the case when collateral is posted by a counterparty at inception (a form of overcollateralization), which may be the case for some transactions. Finally, we introduce a nonnegative continuous process $(\delta_t)_{t\geq 0}$ representing the yield on collateral, in the sense that holding/posting collateral of amount C_t yields/costs instantaneously the net amount $\delta_t C_t$ (after rebate). Instead of capturing simultaneously the perspective of both counterparties with δ, it may be convenient to introduce some asymmetry by considering $\delta_t = \delta_t^A 1_{\{S_{t-}<0\}} + \delta_t^B 1_{\{S_{t-}\geq 0\}}$, so that δ_t^A (δ_t^B) can be interpreted as the cost of posting collateral for party A (B) when it is out-of-the-money. Denoting by $\overline{p}^c$ the swap rate available in case of collateralization, we can write the MTM value of the swap as in (3.1), but with the spread Λ now replaced by (see the appendix for a proof)

$$\Gamma_t = \lambda_t^A (1 - c_t^A) 1_{\{S_t<0\}} + \lambda_t^B (1 - c_t^B) 1_{\{S_t\geq 0\}} - \left(\delta_t^A c_t^A 1_{\{S_t<0\}} + \delta_t^B c_t^B 1_{\{S_t\geq 0\}}\right). \tag{4.2}$$

In the above expression, we recognize the typical features of valuation formulae for credit-risky securities (e.g., Bielecki and Rutkowski, 2002): the first two terms account for the fractional recovery of the swap MTM value in case of default of the counterparty, the third one for the costs incurred when posting collateral before default. We now examine simple special cases to understand better the role of collateral in shaping swap rates.

4.1 Full collateralization

Consider the collateral rule obtained by setting $c^A = c^B = 1$ and $\delta^A = \delta^B = \delta$, meaning that the full MTM value of the swap is received/posted as collateral depending on whether the marking-to-market process results in a positive/negative value for S_t. As we consider cash collateral, default is immaterial. In contrast with section 3, however, the expression for the swap MTM value does not reduce to the usual default-free, risk-neutral valuation formula in general, unless collateral costs are zero:

$$\overline{p}^c = E^{\widetilde{\mathbb{P}}}[P] + \frac{\mathrm{Cov}^{\widetilde{\mathbb{P}}}\left(\exp\left(\int_0^T(\delta_t - r_t)\mathrm{d}t\right), P\right)}{E^{\widetilde{\mathbb{P}}}\left[\exp\left(\int_0^T(\delta_t - r_t)\mathrm{d}t\right)\right]}. \tag{4.3}$$

If the cost of collateral is positively dependent on interest rates, we expect the swap rate to be higher than $\overline{p}^d$ in expression (3.2), even if it is only the floating payments that are linked to interest rates, reflecting the fact that (costly) collateralization requires a premium to be paid to the payer of the fixed rate (see Johannes and Sundaresan, 2007). The intuition is that the party paying the floating rate will have to both post collateral and incur higher funding costs when the floating rate increases. As was emphasized in the introduction, in longevity space, the cost of collateral is positively dependent on mortality improvements, but longevity-linked liabilities are more capital intensive in low interest rate environments (due to lower discounting of future cashflows). The combined impact of these two effects is ambiguous, and we may have situations in which $\overline{p}^c \geq E^{\widetilde{\mathbb{P}}}[P]$ even if $E^{\widetilde{\mathbb{P}}}[P] \geq \overline{p}^d$.

4.2 Partial collateralization

According to ISDA (2010a), it is typical for collateral agreements to specify collateral triggers based on the market value of the swap or other relevant variables (credit ratings, credit spreads, etc.) crossing pre-specified threshold levels. The following are relevant (if somewhat stylized) examples that are useful for our case:

a) Consider the collateral rule obtained by setting $c_t^B = 1_{\{S_{t-} \geq \underline{s}(t)\}}$ and $c_t^A = 1_{\{S_{t-} \leq \overline{s}(t)\}}$ (for continuous functions $\underline{s}, \overline{s}$ defined on $[0, T]$ and satisfying $\underline{s} \leq \overline{s}$), meaning that the hedge supplier (hedger) is required to post full collateral if the swap's MTM value is above (below) the appropriate time-dependent threshold. More general collateral rules can be obtained by setting $c_t^B = \gamma_t^B 1_{\{S_{t-} \geq \underline{s}(t)\}}$ and $c_t^A = \gamma_t^A 1_{\{S_{t-} \leq \overline{s}(t)\}}$, for suitable processes γ^A, γ^B depending on prevailing market conditions, such as the credit standing of the counterparties.

b) In longevity swaps, however, it is more common to define collateral thresholds in terms of mortality forecasts based on a model agreed at contract inception, and monitor the deaths in the hedger's population instead of the market value of the swap. This is due to both the re-estimation risk affecting any given mortality model and the presence of substantial model risk, which most likely would prevent the counterparties from agreeing on a common model at future dates. We can set $c_t^B = 1_{\{N_{t-} \leq \alpha(t)\}}$ and $c_t^A = 1_{\{N_{t-} \geq \beta(t)\}}$, for continuous functions α and β satisfying $0 \leq \alpha \leq \beta \leq n$, meaning that the hedge supplier (hedger) is required to post full collateral if realized deaths are below (above) the relevant threshold.

c) For an index-based swap, it may be more convenient to work with the mortality intensity μ^I of the reference population (see section 2) and set $c_t^B = 1_{\{\int_0^t \mu_s^I \mathrm{d}s \leq a(t)\}}$ and $c_t^A = 1_{\{\int_0^t \mu_s^I \mathrm{d}s \geq b(t)\}}$ for (say) continuous functions a, b satisfying $0 \leq a \leq b$. This means that collateral posting is triggered at each time t if the realized value of the longevity index, $\exp(-\int_0^t \mu_s^I \mathrm{d}s)$, falls outside the open interval $(\exp(-b(t)), \exp(-a(t)))$.

d) As was emphasized in section 2.1, the severity of counterparty risk depends on the credit quality of the counterparties. This is why collateralization agreements may set collateral thresholds that explicitly depend on credit ratings or CDS spreads. A simple example of this practice can be obtained as a special case of (a) by setting

$c_t^B = 1_{\{N_{t-} \leq \alpha(t)\} \cup \{\lambda_t^B \geq \underline{\lambda}\}}$, $c_t^A = 1_{\{N_{t-} \geq \beta(t)\} \cup \{\lambda_t^A \geq \underline{\lambda}\}}$, meaning that, at each time t, the hedger (hedge supplier) receives collateral when either realized deaths fall below the level $\alpha(t)$ (respectively $\beta(t)$) or the hedge supplier's (respectively hedger's) default intensity overshoots a given threshold $\underline{\lambda} \geq 0$. Note that both c^A and c^B can be non zero at the same time (for example on the event $\{N_{t-} \leq \alpha(t)\} \cap \{\lambda_t^A \geq \underline{\lambda}\}$), but expression (4.1) ensures that only the party out-of-the-money will have to post collateral.

4.3 Computing the swap rate

The recursive nature of swap valuation formulae in the case of bilateral and asymmetric counterparty risk has already been noted by Duffie and Huang (1996). By modeling the recovery rates and the difference in counterparties' credit spreads in reduced form, however, they could use a simple iterative procedure to determine swap rates.[26] Here, we explicitly allow for the impact of collateral and the marking-to-market process in the pricing functional, and hence need a different approach. Working in a Markov setting, we let X denote the state variable process and use a Least-Squares Monte Carlo approach. Exploiting the properties of the doubly stochastic setup (see the appendix), we do not model death/default times explicitly, but just rely on the mortality/default intensities (see algorithm 2 in Bacinello *et al.*, 2010, for example). The procedure involves the following steps:

Step 1. For an arbitrary fixed swap rate $\overline{p}_i^c$, generate M simulated paths of X under $\widetilde{\mathbb{P}}$ along the time grid $\mathcal{T} := \{0 < t_1, t_2, \ldots, t_n = T\}$. Denote by $S_{t_j}^{m,i}$ the MTM value of the swap and by $f_{t_j}^{i,m}$ the cashflows originating from the swap (collateral flows and swap payments) at time t_j on path m and for given swap rate $\overline{p}_i^c$.

Step 2. Compute recursively the value of the swap at time t_j (for $j = n-1, \ldots, 0$ with $t_0 = 0$) as $S_{t_j}^{m,i} = \beta_j^* \cdot e(X_{t_j}^m)$, where $e(x) := (e_1(x), \ldots, e_H(x))^\mathsf{T}$ and $\{e_1, \ldots, e_H\}$

[26] Johannes and Sundaresan (2007) sidestep recursivity issues by considering full collateralization and symmetric default risk and collateral costs.

is a finite set of functions taken from a suitable basis, and β_j^* is given by

$$\beta_j^* = \arg \min_{\beta_j \in \mathbb{R}^H} \sum_{m=1}^{M} \left(S_{t_{j+1}}^{i,m} + f_{t_{j+1}}^{i,m} - \beta_j \cdot e(X_{t_j}^m) \right).$$

At each time t_j, use $S_{t_j}^{m,i}$ to check whether the collateral thresholds are triggered and determine the corresponding amount of collateral and associated costs.

Step 3. Iterate the above procedure over different values for $\overline{p}_i^c$ until a candidate swap rate $\overline{p}_{i^*}^c$ is found, such that the initial price of the swap, $\frac{1}{M} \sum_{m=1}^{M} S_{t_0}^{m,i^*}$, is close enough to zero. Set $\overline{p}^c = \overline{p}_{i^*}^c$.

Of course, the procedure relies on knowledge of the dynamics of the state variable process under the pricing measure. To this end, in the next section, we outline a calibration approach based on the joint use of fixed-income data and funding costs / capital requirements for longevity-linked liabilities.

5 Examples

We use a continuous-time model for the risk-free yield curve, the LIBOR and mortality rates, as well as for the cost of collateral. The credit risk of party B (the hedge supplier) is assumed to be equal to the average credit quality of the LIBOR panel, so that the TED spread would be party B's default intensity if there were zero recovery upon default (see section 3). We then set $\lambda^A = \lambda^B + \Delta$ and consider two cases: party A is either of the same credit quality as party B ($\Delta = 0$) or is more credit-risky ($\Delta > 0$).

We consider a Markov setting, and describe the evolution of uncertainty by a six-dimensional state variable vector X with the Gaussian dynamics reported in appendix B. The first four components are: the short rate, $r = X^{(1)}$, assumed to revert to the long-run central tendency factor $X^{(2)}$, representing the slope of the risk-free yield curve; the TED spread $X^{(3)}$, so that the LIBOR rate is given by $X^{(1)} + X^{(3)}$; and the net yield on collateral in the interest-rate swap market, $X^{(4)}$. The remaining two components describe the yield on collateral attached to longevity risk business, $X^{(5)}$, and the log-intensity of mortality of a given population, $\log \mu = X^{(6)}$. Under the assumption of independence between the interest rate and mortality rates, we can estimate separately the dynamics of the two groups of factors $(X^{(1)}, X^{(2)}, X^{(3)}, X^{(4)})$ and $X^{(6)}$. For the first group, we rely on the estimates of Johannes and Sundaresan (2007), who use weekly Treasury and swap data from 1990 to 2002 to obtain the parameter values reported in table 2. For the intensity $\exp(X_t^{(6)})$, we use a continuous-time version of the Lee-Carter model; see appendix B for details. As we do not have any publicly available transaction data from the longevity swap market to proxy $X^{(5)}$, we use information on credit markets (funding costs) and regulatory requirements (capital charges). In particular, as a first example, we focus on funding costs and simply take $\delta^B = X^{(3)}$ and $\delta^A = X^{(3)} + \Delta$, meaning that net collateral costs coincide with each party's borrowing rate net of the risk-free rate (assuming it is rebated). In the case of asymmetric default risk, we consider values of 100 and 200 basis points for Δ. In a second example, discussed in detail below, we focus on the opportunity cost of selling additional longevity protection and simulate the capital charges arising from holding a representative longevity-linked liability to estimate the dynamics of $X^{(5)}$. In both cases, we compute the longevity swap rates for a 25-year swap written on a population of 10,000 US males aged 65 at the beginning of 2008.

< Table 2 about here >

In figure 3, we plot the swap curves obtained for different collateralization rules against the percentiles of survival rate improvements based on Lee-Carter forecasts. We see that margins are positive and increasing with payment maturity in the case of symmetric default risk, for both uncollateralized and fully collateralized transactions. As soon as we introduce asymmetry in default risk ($\Delta > 0$), however, margins widen in the case of no collateralization, reflecting the fact that the hedger needs to pay an additional premium on account of its higher credit risk. In the case of full collateralization, the hedge supplier benefits from the negative dependence between funding costs and collateral amounts discussed before: equilibrium swap rates are pushed lower and produce a negative margin on best estimate swap rates. In figure 4, we examine the swap margins induced by one-sided collateralization in the case of asymmetric default risk. When only the hedge supplier has to post full collateral, swap rates are higher than best estimate survival probabilities, meaning that the hedger has to compensate the hedge supplier for bearing both the cost of risk mitigation and the hedger's default risk. The opposite is true when it is the hedger who has to post full collateral when out-of-the money. In this case, swap margins are clearly negative, and decreasing in payment maturity. These effects are amplified when the asymmetry in counterparties' credit quality is greater, as can be seen from the swap spreads reported in table 3 for some key maturities and collateralization rules.

Plotting the swap rate *margins* against best estimate mortality improvements allows one to interpret the swap rates as outputs of a pricing functional based on adjustments to a reference mortality model (which is common practice in longevity space). On the other hand, longevity swap *spreads* are easier to compare with those emerging in other transactions. In table 4, we make a comparison with the interest-rate swap spreads implied by our parameterization of the state vector $(X^{(1)}, X^{(2)}, X^{(3)}, X^{(4)})$. In particular, we report the difference between interest-rate futures prices (obtained by considering full collateralization and setting the cost of collateral equal to the risk-free rate) and interest-rate swap rates for collateralized transactions with collateral costs equal to the funding costs of the counterparties. Spreads are negative, in line with the intuition that interest rate risk leads to a discount for the payer of the fixed rate, as discussed in the introduction, and are of a magnitude consistent with the findings of Johannes and Sundaresan (2007). The results show that longevity swap spreads are comparable with, and often much smaller in absoulte value than, those found in the interest-rate swap market. For example, in the case of bilateral full collateralization, longevity swap rates for 15- to 25-year maturities embed a spread substantially smaller than that of interest-rate swaps of corresponding maturity. In the case of one-sided collateralization on the hedger's side, in interest-rate swap rates we find a discount (negative spread) that turns into a premium (positive spread) of comparable size in the corresponding longevity swap, due to the additional and opposite effect of longevity risk on swap rates. Our findings are robust to the choice of maturity, collateralization rules, and counterparty credit quality, and are mainly driven by the fact that interest rate risk and longevity risk impact longevity swap margins in opposite directions, thus diluting the overall effect of collateralization on longevity swap rates.

< Table 3 about here >

< Table 4 about here >

< Figure 3 about here >

< Figure 4 about here >

In a second example, we 'synthesize' the dynamics of $X^{(5)}$ by using information on regulatory requirements to quantify the capital charges accruing to the counterparties during the life of the swap. In particular, we use the following bottom-up procedure:

Step 1: We simulate several paths of the factors $X^{(1)}, \ldots, X^{(4)}$ and $X^{(6)}$ along a time grid $\hat{\mathcal{T}} := \{t_1, t_2, \ldots, t_k\}$ (with $t_k = \hat{T} > t_1 > 0$) and under the pricing measure $\widetilde{\mathbb{P}}$. Again, for our example, we focus on the baseline case of $\widetilde{p}_T = p_T$, and hence assume the $\widetilde{\mathbb{P}}$-dynamics of $X^{(6)}$ to be the same as under the physical measure.

Step 2: The paths simulated in the previous step are used to compute, at each date $t \in \hat{\mathcal{T}}$, the regulatory capital needed by an insurer to hold the liability $n - N_{t+\overline{T}}$, where $\overline{T} < \hat{T}$ is a representative maturity proxying the average duration of longevity-linked liabilities in the longevity swap market. We use $\overline{T} = 15$ and $\hat{T} = 40$ (years) for our example. To compute the capital requirements, we use the Solvency II framework, which is based on the 99.5% value-at-risk of the net assets over a one-year horizon. For simplicity, we assume holders of longevity exposures to be invested in cash. The distribution of the one-year-ahead market-consistent value of the liability usually requires nested simulation, unless a simplified approach is adopted. In our setting, market-consistent discount factors can be computed analytically based on the one-year-ahead simulated realizations. We use a Least-Squares Monte Carlo approach (see section 4.3) to determine the expected number of survivors.[27]

Step 3: We use the simulated capital charges obtained in the previous step to compute the gains/costs incurred to reduce/increase capital at each time step along each simulated path. We assume that capital charges are funded at the counterparties' funding cost, plus a spread of 6%[28] to reflect the opportunity cost of diverting to an individual liability funds that could be used to support insurance business at aggregate level. The simulated realizations of the opportunity cost of capital are used to estimate the dynamics of $X^{(5)}$ reported in the appendix. The parameter estimates are included in table 2.

In the case of symmetric collateralization, we find results comparable with those obtained by using the counterparties' funding costs for the process δ. However, figure 5 shows that margins increase (decrease) considerably when one-sided collateralization on the hedge supplier's (hedger's) side is considered. This is because the party required to post collateral explicitly takes into account tail events in computing collateral costs, whereas in figure 4 funding costs where computed on the basis of the market value of

[27]See Stevens *et al.* (2010a) for other approximation methods in the context of Lee-Carter forecasts.

[28]This is a reasonable, conservative value for the cost of internal capital: anecdotal evidence suggests that this cost may be twice as large for longevity swap dealers.

the longevity swap.

Finally, we study the sensitivity of longevity swap spreads to the volatility of the net collateral cost $X^{(5)}$. To close off the interest-rate risk channel, we fix the factors $X^{(1)}, X^{(2)}$ equal to their long-run means. Table 5 reports the results obtained for different values of the volatility parameter σ_5 in the case of symmetric default risk and bilateral full collateralization. We see that spreads increase dramatically for large values of the volatility parameter, but are comparable with those found in the previous examples for reasonable volatility levels (i.e., below 5%).

< Table 4 about here >

< Figure 5 about here >

6 Conclusion

In this study, we have provided a framework for understanding and quantifying the cost of bilateral default risk and collateral strategies on longevity risk solutions. The results address the concerns aired by potential hedgers regarding how to measure the trade-off between the hedge effectiveness of longevity-linked instruments and the counterparty risk they involve. We have described a methodology for pricing longevity swaps that explicitly takes into account the dynamics of the marking-to-market process, the collateral flows it generates, and the costs associated with the posting of collateral. We have shown how collateral strategies can mitigate if not eliminate counterparty risk, but inevitably introduce an extra cost that must be borne by the hedge supplier or by the hedger, depending on whether it is longevity risk or interest rate risk that has a stronger impact on the cost of collateral. Our most significant and useful finding is that the overall cost of the collateralization strategies in the longevity swap market is comparable with, and often smaller than, that found in the much more liquid interest-rate swap market. Hence, there is no reason to suppose that counterparty risk will provide an insurmountable barrier to the further development of the longevity swap market. Our analysis accordingly provides a robust framework for comparing the costs of credit enhancement in bespoke longevity swaps with the benefits offered by competing solutions such as securitization and indexed swaps.

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A Details on the setup

We take as given a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\in[0,T]}, \mathbb{P})$, and model the death times in a population of n individuals (annuitants or pensioners) as stopping times $\tau^1, \ldots, \tau^n$. This means that at each time t the information carried by $\mathcal{F}_t$ allows us to state whether each individual has died or not. The hedger's liability is given by the random variable $\sum_{i=1}^{n} 1_{\{\tau^i > T\}}$, which can be equivalently written as $n - \sum_{i=1}^{n} 1_{\{\tau^i \leq T\}} = n - N_T$ (recall that the indicator function 1_H takes the value of unity if the event H is true, zero otherwise). We assume that death times coincide with the first jumps of n conditionally Poisson processes with common random intensity of mortality $(\mu_t)_{t\geq 0}$ under both $\mathbb{P}$ and an equivalent martingale measure $\widetilde{\mathbb{P}}$ (see Biffis *et al.*, 2010, for details). The expected number of survivors over $[0,T]$ under the two measures can then be expressed as $E^{\mathbb{P}}\left[\sum_{i=1}^{n} 1_{\{\tau^i > T\}}\right] = np_T$ and $E^{\widetilde{\mathbb{P}}}\left[\sum_{i=1}^{n} 1_{\{\tau^i > T\}}\right] = n\widetilde{p}_T$, with p_T and $\widetilde{p}_T$ given by the expectation (2.1) computed under the relevant probability measure.

Consider any stopping time τ^i satisfying the above assumptions, an integrable random variable $Y \in \mathcal{F}_T$ and a bounded process $(X_t)_{t\in[0,T]}$ such that each X_t is measurable with respect to $\mathcal{F}_{t-}$, the information available up to, but not including, time t. Then a security paying Y at time T in case $\tau^i > T$ and X_{τ^i} at time τ^i in case $\tau^i \leq T$ has time-zero price

$$E^{\widetilde{\mathbb{P}}}\left[\int_0^T \exp\left(-\int_0^s (r_t + \mu_t)\mathrm{d}t\right) X_s \mu_s \mathrm{d}s + \exp\left(-\int_0^T (r_t + \mu_t)\mathrm{d}t\right) Y\right].$$

Consider now two stopping times τ^i, τ^j, with intensities μ^i, μ^j, jointly satisfying the above assumptions (i.e., they are the first jump times of the components of a bivariate conditionally Poisson process). A security paying Y at time T in case neither stopping time has occurred (i.e., $\min(\tau^i, \tau^j) > T$) and X_t in case the first occurrence is at time $t \in (0,T]$ (i.e., $t = \min(\tau^i, \tau^j)$) has time-zero price given by the same formula, with μ_t replaced by $\mu_t^i + \mu_t^j$. This follows from the fact that the stopping time $\min(\tau^i, \tau^j)$ is the first jump time of a conditionally Poisson process with intensity $(\mu_t^i + \mu_t^j)_{t\geq 0}$ (e.g., Bielecki and Rutkowski, 2002). The expressions presented in sections 2-4 all follow from these simple results.

Proof of expression (4.2). Let $(\delta_t^A, \delta_t^B)_{t\geq 0}$ denote the opportunity costs of collateral for the two parties, meaning that holding collateral of amount C_t provides an instantaneous yield equal to $\delta_t^B C_t^+ - \delta_t^A C_t^-$ (we use the notation $a^+ := \max(a,0)$, $a^- := -\min(a,0)$). We assume that collateral is bounded and C_t is measurable with respect to $\mathcal{F}_{t-}$ for all $t \in [0,T]$. Parties A and B are assumed to have death (default) times satisfying the properties reviewed above, in particular having intensities λ^A, λ^B. Recalling the recovery rules described in section 4, we can then write:

$$
\begin{aligned}
S_0 =& E^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T (r_t + \lambda_t^A + \lambda_t^B)\mathrm{d}t\right)\left(P - \overline{p}^d\right)\right] \\
&+ E^{\widetilde{\mathbb{P}}}\left[\int_0^T \exp\left(-\int_0^s (r_t + \lambda_t^A + \lambda_t^B)\mathrm{d}t\right)\left(\lambda_s^A(S_s^+ - C_s^-) + \lambda_s^B(C_s^+ - S_s^-)\right)\mathrm{d}s\right] \\
&+ E^{\widetilde{\mathbb{P}}}\left[\int_0^T \exp\left(-\int_0^s (r_t + \lambda_t^A + \lambda_t^B)\mathrm{d}t\right)(\delta_s^B C_s^+ - \delta_s^A C_s^-)\mathrm{d}s\right].
\end{aligned}
$$

Using representation (4.1), the amount recovered by the nondefaulting counterparty at time $\tau = \min(\tau^A, \tau^B) \le T$ is

$$
1_{\{\tau=\tau^A\}} S_{\tau-}(c_\tau^A 1_{\{S_{\tau-}<0\}} + 1_{\{S_{\tau-}\ge 0\}}) + 1_{\{\tau=\tau^B\}} S_{\tau-}(c_\tau^B 1_{\{S_{\tau-}\ge 0\}} + 1_{\{S_{\tau-}<0\}}),
$$

where we see that c^A, c^B replace the recovery rates ψ^A, ψ^B introduced in section 3. We can then write

$$
\begin{aligned}
S_0 =& E^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T (r_t + \lambda_t^A + \lambda_t^B)\mathrm{d}t\right)\left(P - \overline{p}^d\right)\right] \\
&+ E^{\widetilde{\mathbb{P}}}\left[\int_0^T \exp\left(-\int_0^s (r_t + \lambda_t^A + \lambda_t^B)\mathrm{d}t\right)\left(\lambda_s^A + (\lambda_s^B + \delta_s^B)c_s^B)S_s^+ - (\lambda_s^B + (\lambda_s^A + \delta_s^A)c_s^A)S_s^-\right)\mathrm{d}s\right] \\
=& E^{\widetilde{\mathbb{P}}}\left[\exp\left(-\int_0^T (r_t + \Gamma_t)\mathrm{d}t\right)\left(P - \overline{p}^d\right)\right],
\end{aligned}
$$

which is nothing other than the usual risk-neutral valuation formula for a security with terminal payoff $S_T = P - \overline{p}^d$ paying continuously a dividend equal to a fraction

$$
(\lambda_s^A + (\lambda_s^B + \delta_s^B)c_s^B)1_{\{S_{t-}\ge 0\}} + (\lambda_s^B + (\lambda_s^A + \delta_s^A)c_s^A)1_{\{S_{t-}<0\}}
$$

of the security's market value an instant before each $t \in [0, T]$. Subtracting the dividend rate from $\lambda^A + \lambda^B$ and rearranging terms we obtain expression (4.2) for Γ.

B Details on the numerical examples

The numerical examples are based on a six-dimensional state variable process $X = (X^{(1)}, \ldots, X^{(6)})^\mathsf{T}$ having $\widetilde{\mathbb{P}}$-dynamics

$$
\begin{aligned}
\mathrm{d}X_t^{(1)} &= \Big(k_1(X_t^{(2)} - X_t^{(1)}) - \eta^1\Big)\,\mathrm{d}t + \sigma_1 \mathrm{d}W_t^1 \\
\mathrm{d}X_t^{(2)} &= \Big(k_2(\theta_2 - X_t^{(2)}) - \eta^2\Big)\,\mathrm{d}t + \sigma_2 \mathrm{d}W_t^2 \\
\mathrm{d}X_t^{(3)} &= \Big(\kappa_3(\theta_3 - X_t^{(3)}) + \kappa_{3,1}(X_t^{(1)} - \theta_2) + \kappa_{3,4}(X_t^{(4)} - \theta_4) - \eta_3\Big)\,\mathrm{d}t + \sigma_3 \mathrm{d}W_t^3 \\
\mathrm{d}X_t^{(4)} &= \Big(\kappa_4(\theta_4 - X_t^{(4)}) + \kappa_{4,1}(X_t^{(1)} - \theta_2) + \kappa_{4,2}(X_t^{(2)} - \theta_2) - \eta_4\Big)\,\mathrm{d}t + \sigma_4 \mathrm{d}W_t^4 \\
\mathrm{d}X_t^{(5)} &= \big(\kappa_5(\theta_5 - X_t^{(5)}) + \kappa_{5,1}(X_t^{(1)} - \theta_2) + \kappa_{5,2}(X_t^{(2)} - \theta_2) + \kappa_{5,3}(X_t^{(3)} - \theta_3) \\
&\qquad + \kappa_{5,4}(X_t^{(4)} - \theta_4) + \kappa_{5,6}(X_t^{(6)} - E_0[X_t^{(6)}]) - \eta_5\big)\mathrm{d}t + \sigma_5 \mathrm{d}W_t^5 \\
\mathrm{d}X_t^{(6)} &= \Big(A(t) + B(t)(X_t^{(6)} - a(t))\Big)\,\mathrm{d}t + \sigma_6(t)\mathrm{d}W_t^6,
\end{aligned}
$$

where $W = (W^1, \ldots, W^6)^\mathsf{T}$ is a standard $\widetilde{\mathbb{P}}$-Brownian motion, the constants η^i represent market prices of risk and the functions $A(\cdot), B(\cdot), \sigma_6(\cdot)$ are defined below. The $\mathbb{P}$-dynamics are obtained by removing the market prices of risk from the drifts of the relevant factors and replacing the innovations with the corresponding $\mathbb{P}$-Brownian innovations. We assume that $X^{(6)}$ has the same dynamics under the physical and the pricing probability measures, consistent with our baseline case of a swap rate equal to p_T for each T in the absence of collateral. The Brownian innovations are uncorrelated, with the exception of the pair W^1, W^2, whose instantaneous correlation is denoted by $\rho_{1,2}$.

For the first four factors, we use data from Johannes and Sundaresan (2007) who rely on a two-stage maximum likelihood procedure based on weekly data sampled on Wednesdays, from 1990 to 2002, and set the long-run mean of $X^{(3)}$ equal to the average of the 3-month TED spread over the sampling period. For the log-intensity $X^{(6)}$, we use the mortality model described below, and assume that the Brownian component W^6 is uncorrelated with the other ones. The intensity of mortality is modeled using a continuous-time version of the Lee-Carter model (see Biffis *et al.*, 2010). We first use the annual central death rates $\{m_{y,s}\}$ for US and UK males from the Human Mortality Database to estimate the model $m_{y,s} = \exp(\alpha(y) + \beta(y)K_s)$ for dates $s = 1961, 1962, \ldots, 2007$ and ages $y = 20, 21, \ldots, 89$ with Singular Value Decomposition. The resulting estimates for K are then fitted with the process $K_{s+1} = \delta_K K_s + \sigma_K \varepsilon$, with $\varepsilon \sim N(0, 1)$. For fixed age $x = 65$, the estimates for $\{\hat{\alpha}(x + h), \hat{\beta}(x + h)\}_{h=0,1,...}$ are interpolated with differentiable functions $a(t), b(t)$. The functions A, B, σ_6 are finally obtained by setting $A(t) = a'(t) + b(t)\delta_K$, $B(t) = b'(t)b(t)^{-1}$ and $\sigma_6(t) = b(t)\sigma_K$. The expectation appearing in the drift of $X^{(5)}$ ensures that the longevity capital charges react to departures of realized mortality from the term structure of survival rates estimated at inception.

To estimate the dynamics of $X^{(5)}$, the component of collateral costs related to longevity risk, we implement the procedure discussed in section 5, setting the duration $\overline{T}$ of the representative liability equal to 15. We simulate forward all of the other state variables, and at each time step we compute the opportunity cost of capital arising from the capital charges accruing to the hedge supplier based on the simulated mortality and market conditions. We assume that funding occurs at the LIBOR rate plus a fixed spread of 6%, a reasonable value for the cost of internal capital. To obtain the net cost of collateral, we take into account the rebate of the risk-free rate. We estimate the dynamics of $X^{(5)}$ on each simulated path. We set the parameter θ_5 equal to the average of $X^{(5)}$ along the simulated path. The parameter estimates are computed for each simulated path and then averaged across all simulations. The estimates are reported in table 2.

C Tables and figures

Date	Hedger	Size	Term (yrs)	Type	Interm./supplier
Jan 08	Lucida	Not disclosed	10	indexed	JP Morgan ILS funds
Jul 2008	Canada Life	GBP 500m	40	bespoke	JP Morgan ILS funds
Feb 2009	Abbey Life	GBP 1.5bn	run-off	bespoke	Deutsche Bank ILS funds / Partner Re
Mar 2009	Aviva	GBP 475m	10	bespoke	Royal Bank of Scotland
Jun 2009	Babcock International	GBP 750m	50	bespoke	Credit Suisse Pacific Life Re
Jul 2009	RSA	GBP 1.9bn	run-off	bespoke	Goldman Sachs (Rothesay Life)
Dec 2009	Berkshire Council	GBP 750m	run-off	bespoke	Swiss Re
Feb 2010	BMW UK	GBP 3bn	run-off	bespoke	Deutsche Bank Paternoster
Dec 2010	Swiss Re (Kortis bond)	USD 50m	8	indexed	ILS funds
Feb 2011	Pall (UK) Pension Fund	GBP 70m	10	indexed	JP Morgan

Table 1: Publicly announced longevity swap transactions 2008-2011.

κ_1	0.969	η_1	-0.053	σ_1	0.008	UK	
κ_2	0.832	η_3	-0.014	σ_2	0.155	δ_K	-0.888
κ_3	1.669	η_4	0.007	σ_3	0.009	σ_K	1.156
κ_4	0.045	η_5	0.055	σ_4	0.010	US	
κ_5	0.990	$\kappa_{5,1}$	0.147	σ_5	0.690	δ_K	-0.761
$\kappa_{3,1}$	-0.163	$\kappa_{5,2}$	1.340	θ_2	0.046	σ_K	1.078
$\kappa_{4,1}$	0.114	$\kappa_{5,3}$	2.509	θ_3	0.003		
$\kappa_{3,4}$	0.804	$\kappa_{5,4}$	-0.133	θ_4	0.007		
$\kappa_{4,2}$	-0.038	$\kappa_{5,6}$	-0.002	θ_5	0.115	$\rho_{1,2}$	-0.036

Table 2: Parameter values for the dynamics of X given in Appendix B. The estimates for $X^{(5)}$ are based on the assumption that capital increases are funded by counterparties at 6% plus the LIBOR rate.

	Maturity payment (yrs)	$c^A = 0$ $c^B = 0$ (bps)	$c^A = 0$ $c^B = 1$ (bps)	$c^A = 1$ $c^B = 0$ (bps)	$c^A = 1$ $c^B = 1$ (bps)
$\lambda^{A,B} = \lambda,$	15	0.03	11.34	-11.76	0.05
$\delta^{A,B} = \delta,$	20	1.11	19.93	-17.94	0.86
$\delta = \lambda$	25	1.50	21.25	-18.35	1.24
$\lambda^A = \lambda^B + \Delta,$	15	5.45	16.79	-17.29	-5.84
$\delta^i = \lambda^i,$	20	10.16	28.95	-27.08	-8.23
$\Delta = 0.01$	25	10.96	30.75	-27.76	-9.19
$\lambda^A = \lambda^B + \Delta,$	15	11.30	22.29	-22.90	-11.25
$\delta^i = \lambda^i,$	20	19.26	38.06	-36.16	-17.42
$\Delta = 0.02$	25	19.46	40.27	-37.02	-18.38

Table 3: Second example in section 5: swap spreads $\bar{p}^c_{T_i} - p_{T_i}$ (in basis points) for different collateralization rules, maturities and credit spread $\Delta \in \{0, 0.01, 0.02\}$. The LSMC procedure uses 5000 paths over a quarterly grid with polynomial basis functions of order 3, and is repeated for 100 seeds.

		IRSs			longevity swaps		
	Maturity payment (yrs)	$c^A = 0$ $c^B = 1$ (bps)	$c^A = 1$ $c^B = 0$ (bps)	$c^A = 1$ $c^B = 1$ (bps)	$c^A = 0$ $c^B = 1$ (bps)	$c^A = 1$ $c^B = 0$ (bps)	$c^A = 1$ $c^B = 1$ (bps)
$\lambda^{A,B} = \lambda,$	15	-7.96	-44.97	-52.86	11.34	-11.76	0.05
$\delta^{A,B} = \delta,$	20	-12.68	-42.64	-56.22	19.93	-17.94	0.86
$\delta = \lambda$	25	-17.94	-40.98	-58.92	21.25	-18.35	1.24
$\lambda^A = \lambda^B + \Delta,$	15	-8.00	-67.87	-75.23	16.79	-17.29	-5.84
$\delta^i = \lambda^i,$	20	-12.65	-63.84	-77.42	28.95	-27.08	-8.23
$\Delta = 0.01$	25	-17.65	-60.63	-77.64	30.75	-27.76	-9.19

Table 4: Second example in section 5: comparison of interest-rate swaps (IRSs) with longevity swaps. The IRS spreads represent the difference betweeen the futures prices (the opportunity cost of collateral coincides with the risk-free rate for both parties) and the swap rate for the collateralized IRS (for different collateralization rules, maturities, and credit risk).

σ_5	p_{25}	$\overline{p}^c$	spread (bps)
0.0005	0.201425	0.201469	2.15
0.0100	0.201425	0.201822	19.68
0.0150	0.201425	0.202009	28.96
0.0200	0.201425	0.202196	38.26
0.1000	0.201425	0.205237	189.24
0.1500	0.201425	0.207184	285.90

Table 5: Sensitivity with respect to parameter σ_5: we compute 25-year swap rates and spreads (in basis points) under full collateralization by setting $X^{(1)}, X^{(2)}$ equal to their long run means. The baseline estimated parameter values for the dynamics of $X^{(5)}$ are $\theta_5 = 0.000254$, $\kappa_5 = 1.005073$, $\sigma_5 = 0.000542$, $\eta_5 = 0.000269$, $\kappa_{53} = 0.003648$, $\kappa_{54} = 0.000018$, $\kappa_{56} = 0.000261$.

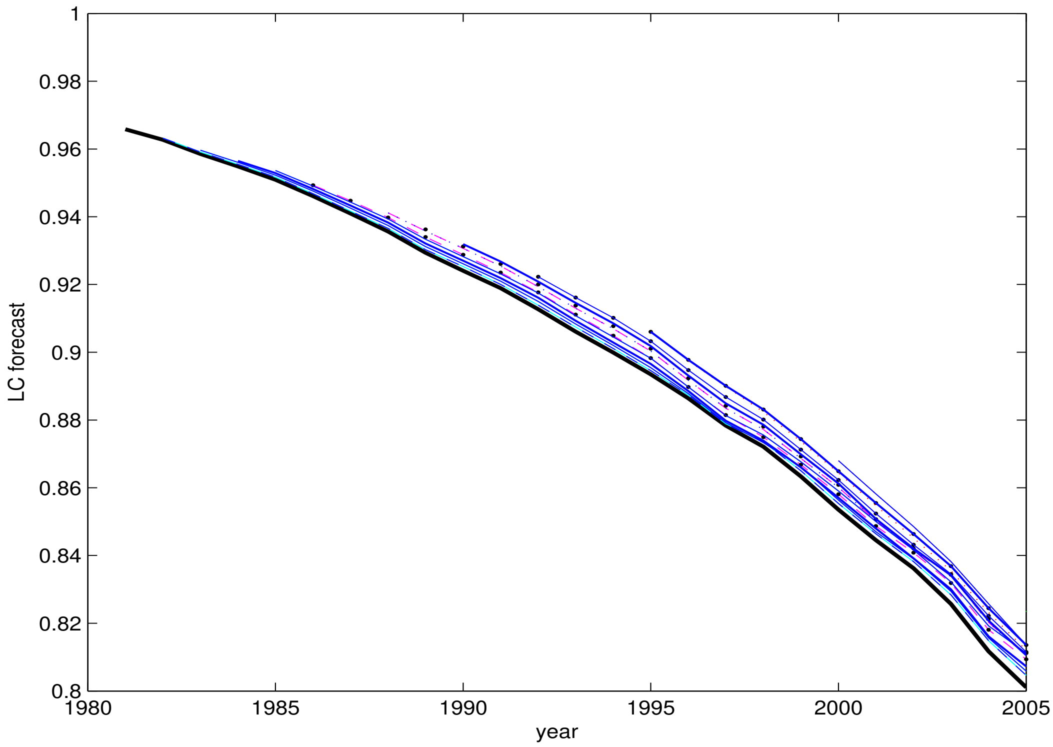

Figure 1: Survival curves computed at the beginning of each year $t = 1980, \ldots, 2004$ for England & Wales males aged $65 + t - 1980$ in year t. Forecasts are based on the Lee-Carter model using the latest Human Mortality Database data available at the beginning of each year t.

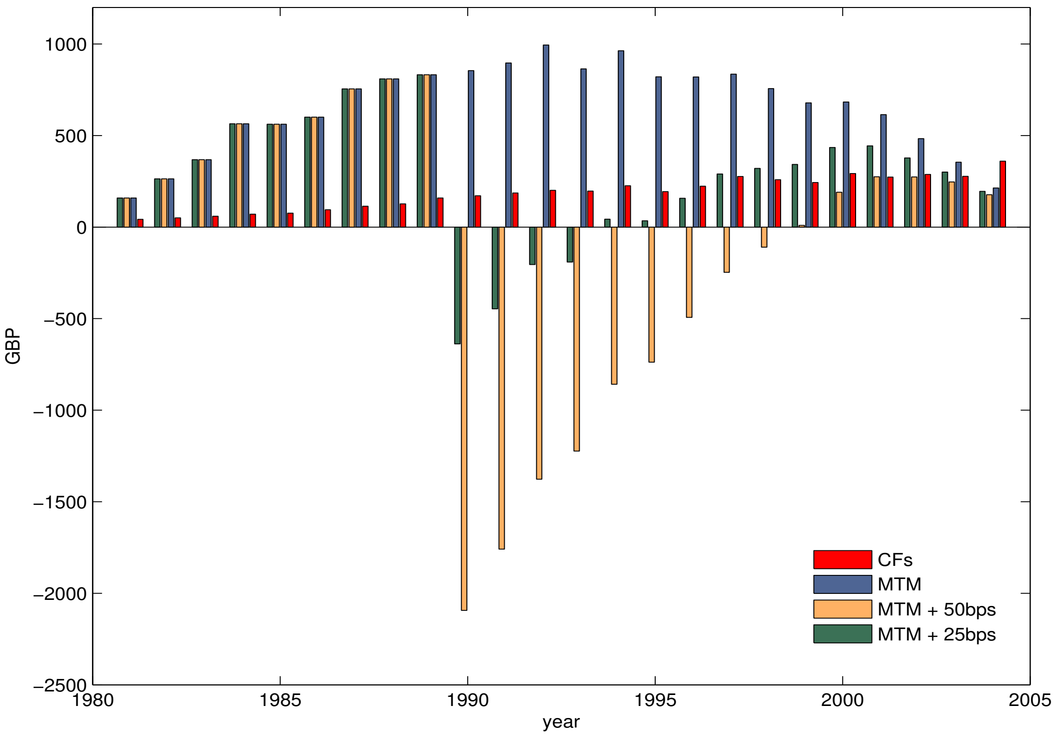

Figure 2: Mark-to-market value of the longevity swap (MTM) and stream of cashflows with no credit risk (CFs), and with counterparty B's credit spreads widening by 25 and 50 basis points over 1988-2005.

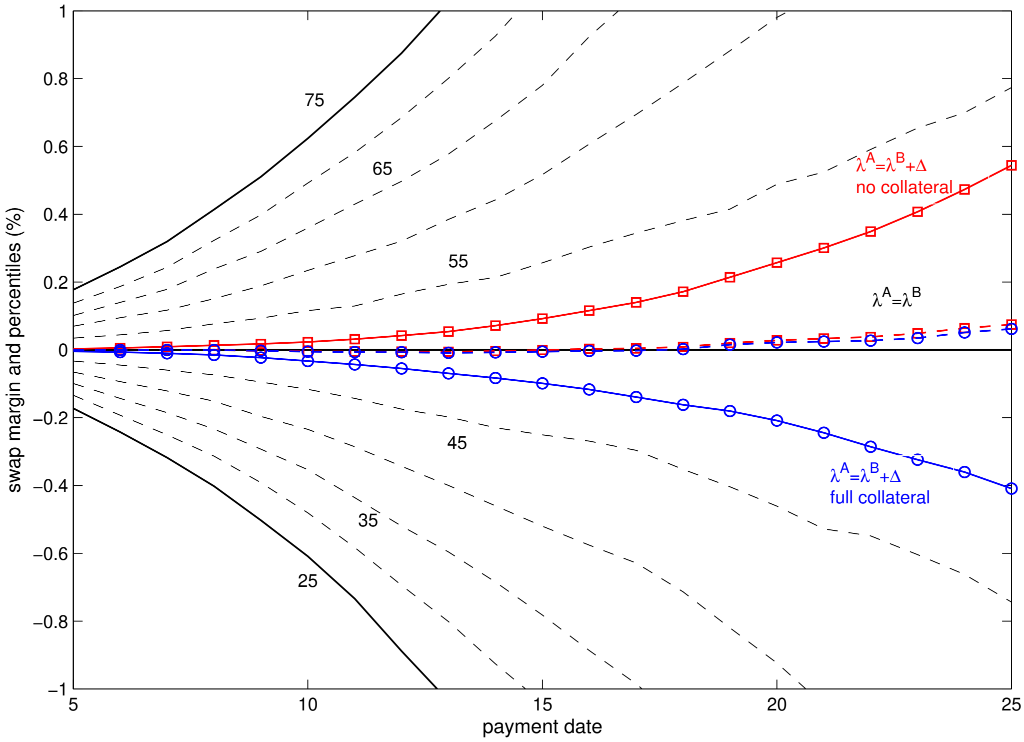

Figure 3: Swap margins $\bar{p}^c_{T_i}/p_{T_i} - 1$ computed for different maturities $\{T_i\}$ and collateral rules, with $\delta^A = \lambda^A$ and $\delta^B = \lambda^B$: no collateral (squares), full collateralization (circles); $\lambda^A = \lambda^B + \Delta$, with $\Delta = 0$ (dashed lines) and $\Delta = 0.01$ (solid lines). The underlying is a cohort of 10,000 US males aged 65 at the beginning of 2008. Swap rates are plotted against the percentiles of improvements in survival rates based on Lee-Carter forecasts.

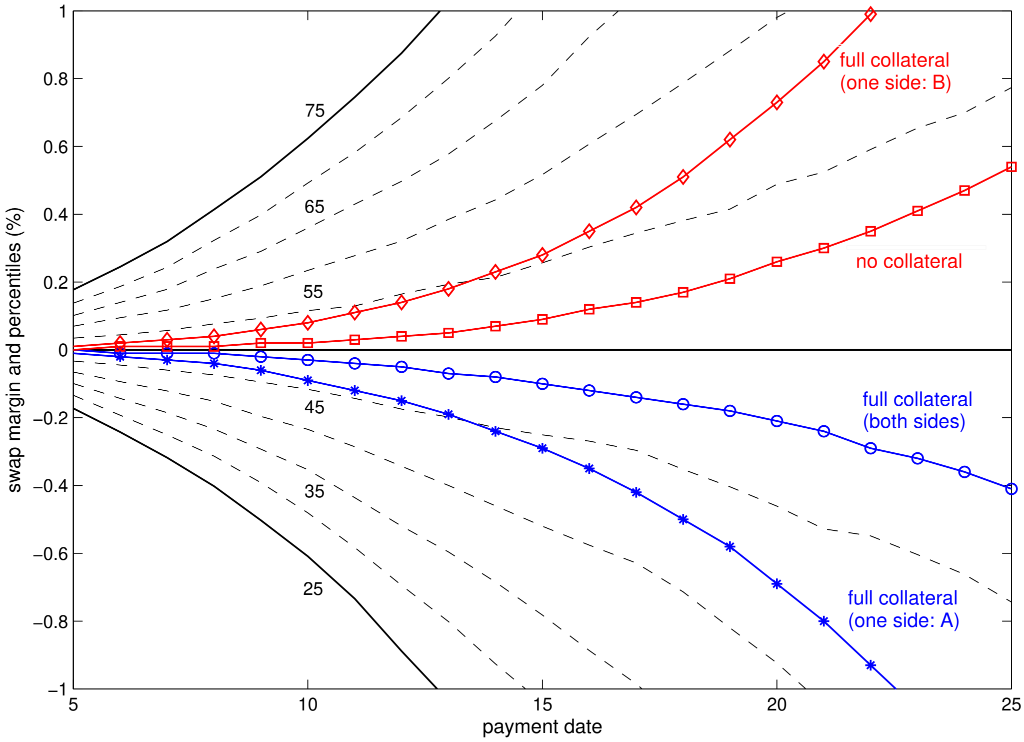

Figure 4: Swap margins $\bar{p}^c_{T_i}/p_{T_i} - 1$ computed for different maturities $\{T_i\}$ and collateral rules, with $\delta^A = \lambda^A = \lambda^B + 0.01$ and $\delta^B = \lambda^B$: no collateral (squares), full collateralization (circles), full collateral posted only by party A (stars) or party B (diamonds). The underlying is a cohort of 10,000 US males aged 65 at the beginning of 2008. Swap rates are plotted against the percentiles of improvements in survival rates based on Lee-Carter forecasts.

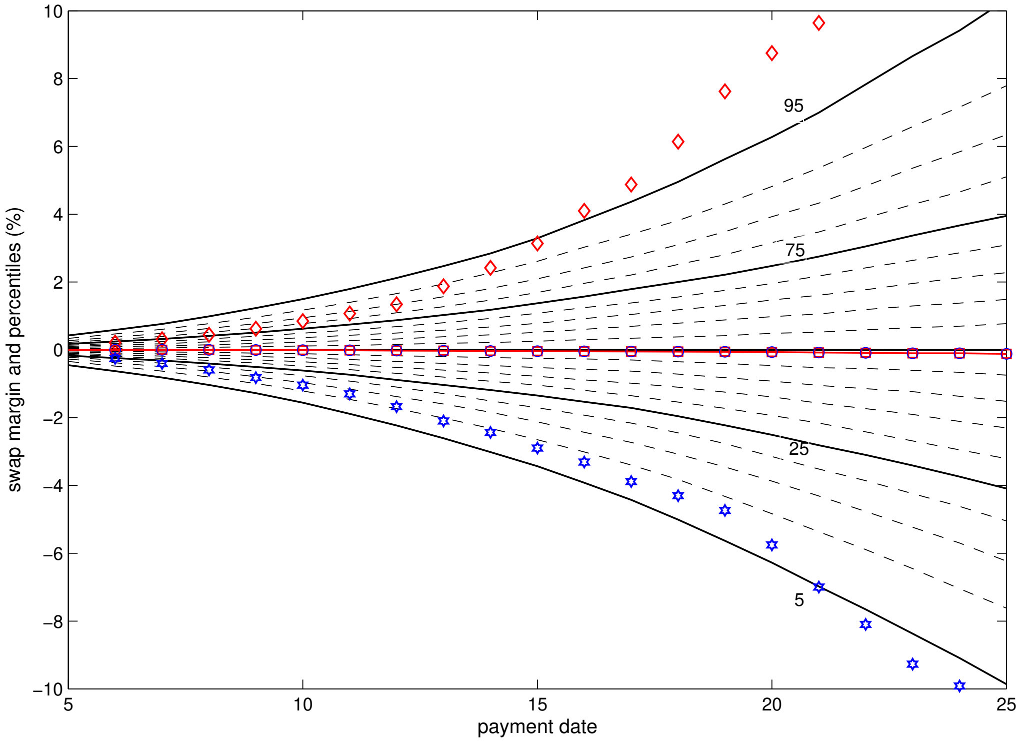

Figure 5: Swap margins $\overline{p}^{c}_{T_i}/p_{T_i} - 1$ computed for different maturities $\{T_i\}$ and collateral rules, with $\lambda^A = \lambda^B$ and $\delta = X^{(5)}$, where the parameter estimates of X are given in table 2. Collateral rules: no collateral (squares), full collateralization (circles), full collateral posted only by party A (stars) or B (diamonds). Swap rates are plotted against the percentiles of improvements in survival rates based on Lee-Carter forecasts.