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Abstract

I construct index-based weather insurance contracts with optimal hedging effectiveness for the insured or maximal profits for the insurer. In contrast to earlier work, I refrain from imposing functional form assumptions on the stochastic relationship between weather and yield and from restricting attention to (piecewise) linear contracts. Instead, I derive the shape of the optimal weather insurance contracts empirically by non-parametrically estimating yield distributions conditional on weather. I find that the optimal pay-off structure is non-linear for the entire range of weather realizations. I measure risk reduction of optimal weather insurance contracts for different weather indices and levels of risk aversion. Considering profit-maximizing contracts, I find that at modest levels of risk aversion (coefficient of relative risk aversion around 2), a loading factor of 10% of the fair premium is possible such that the insurance contract remains attractive for the insured. With higher levels of risk aversion, loading of more than 50% becomes possible.

Keywords: Agricultural insurance, optimal insurance design, weather derivatives, weather risk, hedging effectiveness, loading of premium

1 Introduction

Agricultural production and agribusiness are exposed to many weather-related influences that cause fluctuations in crop yields – so-called yield or production risk. The management of weather risk is of fundamental importance for weather-dependent sectors, and will become even more important with increasing risk of extreme weather events. Insurance has been an integral part in dealing with weather risk, as it helps reduce the residual risk that cannot be prevented through cost-effective on-site (on-farm) risk management strategies. Traditional crop insurance schemes provide farmers with coverage to manage weather-related yield risks. Insurance schemes where the insurance pay-offs are based on an assessment of the crop yields – as is the case with individual, or multi-peril crop insurance – are plagued by moral hazard, adverse selection and costly enforcement (Smith and Goodwin, 1994; Skees et al., 1997; Goodwin, 2001). The use of index-based weather insurance has recently emerged as an alternative as it avoids many of the problems associated with traditional insurance (Hazell and Skees, 2005). With index-based insurance, an exogenous, verifiable weather event is being insured, rather than a yield outcome. Problems of moral hazard and adverse selection are minimized, and administrative costs are reduced substantially since published (weather) data is used to settle a claim (Ibarra and Skees, 2007).[1]

The focus of this paper is on designing weather insurance for agricultural risk management with optimal hedging effectiveness. An expected utility framework is used to model the decision-making behavior of a representative farmer. The design of weather insurance can be decomposed into two separate problems: finding a weather index that correlates well with crop losses, and the derivation of the insurance contract for that given index. I first derive weather indices taking the varying sensitivity of crops during the growing season into account. Given the weather indices, I simulate optimal insurance contracts for different levels of risk aversion. Compared to existing work, I aim at characterizing and deriving the optimal pay-off structure by allowing for a non-linear stochastic relationship between weather and yield. Previous approaches, such as Vedenov and Barnett (2004), Osgood et al. (2007), and Musshoff et al. (2009) relied on specifying functional forms to characterize the weather-yield relationship. By assuming a given (linear) relationship between the index and yields, functional form assumptions are imposed on the optimiza-

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[1]The disadvantage of index-based weather insurance for the insured is that it comes at the cost of not being perfectly insured against weather related losses due to the imperfect correlation of yields and the weather index. The gap between the loss indicated by the index and the individual realized loss is known as basis risk. However, basis risk exists with farm-level multi-peril crop insurance as well. For a discussion, see Skees (2003).

tion problem, and the resulting pay-off structure reflects these assumptions. Instead, I estimate conditional probabilities of yield for different levels of the weather index using a fully non-parametric approach. The estimated conditional yield distributions are used to compute the expected utility maximizing insurance contract. Rather than restricting attention to piecewise linear contracts in the first place, I determine the classical parameters of a derivative contract (trigger[2] and limit) from the optimization problem and in addition derive the local slope of the pay-off function (tick size)[3] at each realization of the underlying index.

I implement the expected utility maximization problem numerically and apply our novel model to derive optimal weather insurance contracts for maize farmers in Schaffhausen (SHA), Switzerland. While the magnitude of the numerical results are crop- and location-specific, the insurance characteristics of the optimal weather insurance contract described in the following are crop- and location independent.[4] I propose a general method for deriving optimal weather insurance contracts that can be applied to any crop and to any location for which sufficient weather and yield data is available.

Examining the relationship between pay-offs and the frequency of payments, I find that the optimal insurance contract offers high levels of protection for catastrophic weather events that occur with very low probabilities. At the same time, the optimal insurance contract offers moderate payments for small deviations from average weather conditions. For all contracts considered, I find that the insured breaks-even, i.e. receives an indemnification that compensates for the premium, in 48% of the cases. An insured with moderate risk aversion (coefficient of relative risk aversion of around 2) benefits from the contract as his income without insurance would need to be increased by 2% to offer him the same expected utility as in the situation with optimal insurance. Significantly higher benefits (equivalent to an income increase of up to 10%) from hedging with an optimal weather insurance contract arise at higher levels of risk aversion.

Insurers are first assumed to make zero profits, which allows using the "burn rate" method to price insurance contracts. Subsequently, I relax this assumption to determine the maximum loading factor on fair premiums so that the optimal insurance contract remains attractive to farmers in the sense that it yields the same expected utility as the no-insurance scenario. Comparing the optimal weather insurance contract with the profit-

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[2]The trigger, or strike level, is the threshold level of the underlying meteorological index that triggers payments from the contract.

[3]The tick size is the incremental change in the payment for a change in the index.

[4]I also simulated optimal weather insurance contracts for wheat, potatoes, rape seed and sugar beet at two more locations in Switzerland, which confirmed the insurance characteristics of the optimal insurance contract described here. Results are available upon request.

maximizing contract, I find that the profit-maximizing contract displays the same non-linear shape as the optimal contract but has lower pay-offs. In our case study, loading factors of 10% to 50% are possible depending on the level of risk aversion.

The remaining paper is organized as follows. I relate our approach to the literature on index-based insurance in the remainder of this section. In section 2, I propose a theoretical model that yields the optimal weather insurance contract as a solution. The numerical implementation is explained in section 3.1, and the data used for simulating the weather insurance contracts is described in section 3.2. In section 4, suitable weather indices are derived, and the simulated optimal weather insurance contracts are presented in section 5 together with an evaluation of their hedging effectiveness. Profit-maximizing contracts are derived in section 6, and the maximal amount of loading on fair premiums is determined. Section 7 concludes and provides an outlook on further research.

1.1 Relation to the Literature

Weather-based insurance contracts were firstly proposed by Turvey (2000, 2001) and Martin et al. (2001). The proposed pay-off structures are similar to the (linear) weather derivatives that have been traded at the Chicago Mercantile Exchange since 1996. In these initial works, the tick size is determined by estimating the relationship between weather and yield, and the strike and limit parameters, which are needed to define an indemnity function, have to be specified by the insured. Martin et al. (2001) develop precipitation derivatives, and Turvey (2001) proposes derivative contracts based on cumulative precipitation and heat units.[5] Both authors demonstrate that (for different contracts and values of risk-aversion) the certainty equivalent gains from using weather insurance exceed the no-purchase situation, and that the extent of the certainty equivalent gains from using weather insurance depends on the chosen contracts.

To reduce farmers' exposure to weather-related shocks, pay-offs from the weather insurance contract have to closely match incurred losses. In this context, Goodwin and Mahul (2004) point out that the design of an efficient insurance contract depends on the relationship between the individual yield and the underlying weather index, and Vedenov and Barnett (2004) specifically emphasize the importance of the weather insurance parameters (tick size, strike, and limit) with respect to achieving hedging effectiveness,

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[5]Martin et al. (2001) estimate the probability density function of the underlying precipitation index parametrically, and use a quadratic function for estimating the weather-yield relationship in order to determine the tick size. Turvey et al. (2001) use a Cobb-Douglas production function with cumulative rainfall and heat units as inputs for their design. Based on the chosen contract parameters, the insurance contracts are priced using the "burn rate" method in both contributions.

i.e. the degree to which weather risk is being reduced by an insurance product. Since then, formal models to determine the buyer's optimal choice of the insurance parameters with respect to risk reduction have been developed, as outlined in the following.

Weather insurance pay-off functions have been designed by minimizing an aggregate measure of downside loss such as the semi-variance (Markowitz, 1991; Vedenov and Barnett, 2004).[6] Vedenov and Barnett (2004) derive the strike level by identifying the level of the index where predicted yields corresponded to the long time average. The remaining parameters are obtained by minimizing the semi-variance of loss assuming a linear relationship between index and insurance payments (between strike and maximum payout).

Another strand of literature maximizes the expected utility of the insured in order to derive the critical insurance parameters (tick size, strike, and limit), and then evaluates the insurance alternative based on their certainty equivalent gains[7] (Karuaihe et al., 2006; Berg et al, 2009; Leblois et al. 2011). These contributions share the assumption that the weather insurance contract is linear between the strike level and the maximum payout (limit). In contrast, I aim at deriving the entire pay-off structure optimally, without imposing a priori functional form assumptions on the contract.

Osgood et al. (2007, 2009) design index-based weather insurance contracts for several African countries (Malawi, Tanzania, Kenya, and Ethiopia) that are implemented by the World Bank and Oxfam America under pilot projects. The contract design chosen by Osgood et al. allows the authors to optimize over piecewise linear contracts by minimizing the variance. Osgood et al. (2007, 2009) are able to locally optimize contract parameters of a linear pay-off structure, thus making the piecewise contract more efficient. I derive instead the globally optimal, non-linear insurance contract without making use of a piecewise linear structure, and our objective is to maximize the expected utility of the insured.

Another design method is proposed by Musshoff et al. (2009), who use the weather-yield relationship to construct the payments from a weather put option by estimating a linear-limitational production function (for a given weather index). Based on the production function, Musshoff et al. (2009) calculate the revenue function, and then define the payoffs from the weather derivative by taking the inverse of the revenue function. Using a parametric approach to establish the relationship between weather and yield assumes

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[6]Minimization of the semi-variance instead of full variance is chosen because only downside losses are of major concern to crop producers (Miranda, 1991; Miranda and Glauber, 1997). This approach has been developed in the literature as an alternative to the traditional mean-variance analysis for situations where reduction of losses or failure to achieve a certain target is of importance (Hogan and Warren, 1972). It has also been shown to be consistent with the expected utility maximization (Selley, 1984).

[7]The certainty equivalent is the amount of certain income that a risk-averse individual finds equally desirable to an alternative random income with a known probability distribution.

that conditional yield distributions at different levels of the weather index are homogenous - an assumption that I do not find to be satisfied in our data. Similar to the piecewise pay-off structure assumed by Osgood et al. (2007, 2009), Musshoff et al.'s assumption about the functional form of the production function has an effect on the insurance parameters, and thus affects the hedging effectiveness of the contracts. Our objective is therefore to derive the insurance parameters without assuming a linear relationship between weather and yield, and without relying on a parametric production function or error term structure.

Our work is most closely related to Mahul (1999, 2000, 2001, 2003) who uses expected utility models to investigate theoretically the optimal design of agricultural insurance for different sources of risk (i.e. price risk, yield risk). While the focus of Mahul (1999) is on the design of area-yield crop insurance, his framework and findings can be translated to the design of index-based weather insurance. To characterize the optimal area-yield insurance, Mahul (1999) follows earlier work by Miranda (1991) and assumes a linear relationship between weather and yield.[8] Mahul (1999) demonstrates more generally than Miranda (1991) that the optimal coverage with area-yield insurance depends on the sensitivity of the individual yield to the area yield and is independent from risk aversion and premiums. Mahul (2001) expands this model to examine the implications of a weather-yield production function that is decomposed into two separate components. Weather and idiosyncratic risk enter as two separate inputs into an additive production function.[9] Through the choice of the production function, yields depend on weather in a linear way conditional on inputs. Under these assumptions, Mahul (2001) shows that the parameters of the optimal indemnity schedule (strike and coverage) depend on the stochastic relation between weather and idiosyncratic risk, and on the risk aversion of the insured. Mahul (2001) compares the optimal coverage for the situation where weather and idiosyncratic risk are independent, and when they interact. When the two sources of risk are correlated, Mahul (2001) finds that "... without further restrictions on the stochastic dependence and the producer's behavior, the indemnity schedule can take basically any form." (Mahul, 2001, p: 596). Empirically deriving an optimal weather-based indemnity schedule within this framework would require distinguishing the different cases outlined by Mahul (2001). I therefore propose a more general method for deriving optimal weather

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[8] Individual yields are the sum of a weather (systemic risk) and an independent idiosyncratic component. Mahul's (1999) model would be more general if the beta-coefficient, which measures the sensitivity of farm-yields to movements in the weather index, were allowed to depend on weather, but instead he assumes that the covariance between weather and the error term is zero.

[9] As before, weather impacts yields directly, and in addition through its effect on the idiosyncratic component.

insurance contracts which also allows us to drop the assumption of a linear relationship between weather and yields, and of a specific error term structure.

2 Theoretical Framework

2.1 The Insurance Problem

I consider ex ante identical farmers who face a stochastic yield $y \in \mathcal{Y} \equiv [\underline{y}, \overline{y}]$. The distribution of output depends on the weather variable z, which can be cumulative rainfall in the growing season, average temperature during particular phases, or an index combining multiple such variables. Let the cdf of $z \in \mathcal{Z} \equiv [\underline{z}, \overline{z}]$ be denoted by $G(z)$, and the corresponding density function by $g(z)$. Then the dependence of the distribution of yields y on the weather index z is captured by the conditional cdf $F(y|z)$ and the corresponding conditional density $f(y|z)$.

Farmers have preferences defined over consumption c given by $u(c)$. I assume farmers to be risk-averse with $u'(c) > 0$, $u''(c) < 0$. There are risk-neutral insurers who offer insurance to the farmers. The key restriction is that insurance contracts cannot directly insure yields y, but only condition on the realization of the weather index z. Suppose the net insurance payout to the farmer if weather index z is realized is given by $p(z)$.[10] Then I require that the insurer does not make losses with the weather insurance contract $\{p(z)\}$ in expectation:

$$\int_{\mathcal{Z}} p(z)dG(z) \le 0. \tag{1}$$

Constraint (1) requires the average net payout to the farmer to be non-positive, which is equivalent to the insurer's profits to be non-negative. The constraint can also be interpreted as a mechanism for pricing the insurance contract, and is known as "burn rate" method.[11] The "burn rate" method is widely used as the standard basis for calculating insurance premiums due to its simplicity (Skees and Barnett, 1999; Mahul, 1999; Turvey, 2001; Martin et al., 2001 ; Vedenov and Barnett, 2004). More complicated cost functions for the insurers could be adopted without effect for the following results.[12] Administra-

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[10]One could separate the net payout to the farmer into the indemnity $I(z)$ paid by the insurer in case of weather realization z, and a fixed premium P, so that $p(z) = I(z) - P$ for all $z \in \mathcal{Z}$. Only the net payout $p(z)$, however, matters to the farmer and the insurer.

[11]Insurance actuaries calculate an expectation on future losses based on historical payouts for a given insurance contract. Expected losses are then considered as an expected breakeven premium rate. This method assumes that the underlying index has a stationary distribution. With climate change this assumption may no longer be valid, and alternative pricing mechanisms need to be considered.

[12]Pricing based on standard option valuation models is not possible since these models require that one be able to construct (at least conceptually) a riskless portfolio of both the option and the asset which forms

tive and transactions costs born by the insurer can be considered in this framework by adding a mark-up to the premium (see section 6 on profit maximizing loading factors).

Then the farmers' expected utility maximizing contract $\{p^*(z)\}$ solves

$$\max_{\{p(z)\}} \int_{\mathcal{Z}} \int_{\mathcal{Y}} u(y + p(z)) dF(y|z) dG(z) \tag{2}$$

subject to constraint (1).

2.2 Some Properties of Optimal Weather Insurance Contracts

Since Problem (2) is a strictly concave problem, it is immediate that there always exists a unique global optimum and first-order conditions are necessary and sufficient. In fact, setting up the Lagrangian

$$\mathcal{L} = \int_{\mathcal{Z}} \int_{\mathcal{Y}} u(y + p(z)) dF(y|z) dG(z) - \lambda \int_{\mathcal{Z}} p(z) dG(z) \tag{3}$$

yields the pointwise first-order conditions

$$\int_{\mathcal{Y}} u'(y + p(z)) dF(y|z) = \lambda \quad \forall z \in \mathcal{Z}, \tag{4}$$

where $\lambda > 0$ is the Lagrange multiplier on constraint (1). Optimality condition (4) requires the expected marginal utility of the farmer conditional on a realization of the weather index z to be equalized across all z at the optimal contract. This immediately leads to the following result:

Proposition 1. *Suppose $y \perp z$, i.e. yields y are independent from the weather variable z. Then $p(z) = 0$ for all $z \in \mathcal{Z}$.*

Proof. If y and z are independently distributed, then the conditional yield distribution $F(y|z)$ is in fact independent of z, so that the first order condition (4) becomes

$$\int_{\mathcal{Y}} u'(y + p(z)) dF(y) = \lambda \quad \forall z \in \mathcal{Z}.$$

Since $u''(.) < 0$, this can only be satisfied if $p(z)$ is independent of z and thus $p(z) = p$ where p is a constant. Then constraint (1) can only be satisfied if $p \leq 0$. Clearly the value of p that maximizes (2) subject to $p \leq 0$ is given by $p = 0$. □

the underlying index (Hull, 2000; Dischel, 1998). Given that there is no actively traded forward market for the underlying index alternative pricing mechanisms have evolved, e.g. stochastic pricing for heat-degree contracts (Turvey, 2001a), indifference pricing (Xu et al., 2007), equilibrium pricing method (Richards et al., 2004).

Proposition (1) demonstrates that weather insurance is ineffective if the weather index does not contain predictive power for yields. In contrast, it crucially relies on weather and yield being correlated. I next consider the opposite special case, in which weather is perfectly predictive of yield:

Proposition 2. *Suppose that each weather-yield observation (y_i, z_i) occurs only once in the data set with $i = 1, ..., n$ observations, i.e. $Pr(y_i, z_i) = 1/n$. Then p is a full insurance contract independent of weather.*

Proof. Under the assumptions in the proposition, the first order condition (4) becomes

$$\sum_{i=1}^{n} u'(y_i + p_i(z_i)) \frac{1}{n} = \lambda \quad \forall z \in \mathcal{Z}.$$

Since $u''(.) < 0$, this can only be satisfied if $y_i + p(z_i)$ is independent of z and thus $p(z_i) = -y_i + p$ where p is a constant. Given that the insurer can only make non-negative profits, the constant $p = \bar{y}$, such that $p(z) = -y_i + \bar{y}$ is a revenue insurance contract. $\square$

Intuitively, if the relationship between weather and yield is perfectly known, and weather is the only input to crop production, then the insurance contract can be written such that any difference between the realized yield, y_i, and the expected yield, $\bar{y}$ is perfectly reimbursed. In that case, the basis risk for the farmer is zero. In the real world, crop yields are influenced by a number of factors, such as management practices, fertilizer usage, soil quality. In addition, crop data is also subject to measurement errors so that the conditions for Proposition (2) are not fulfilled. Forecasting crop yields with just weather as input will therefore not fully explain the variation in yields. A special case of such a stochastic dependence is considered in the following proposition:

Proposition 3. *Suppose the stochastic dependence of yields on weather is given by*

$$y = \psi(z) + \varepsilon,$$

where ε is a stochastic shock such that $\varepsilon \perp z$ and with cdf $H(\varepsilon)$. Then

$$p(z) = -\psi(z) + p,$$

where p is some constant.

Proof. Under the assumptions in the proposition, the first order condition (4) becomes

$$\int_{\varepsilon} u'(\psi(z) + p(z) + \varepsilon) dH(\varepsilon) = \lambda \quad \forall z \in \mathcal{Z}.$$

Since $u''(.) < 0$, this can only be satisfied if $\psi(z) + p(z)$ is independent of z and thus $p(z) = -\psi(z) + p$ where p is a constant. $\qquad\square$

An example that would seem particularly natural for the case where z captures a measure of cumulative rainfall, for instance, would be a function $\psi(z)$ that is hump-shaped: yields tend to be low both for very high values of z (excessive precipitation) and very low values (droughts), and highest for intermediate values. Then Proposition (3) shows that the optimal weather insurance contract features a U-shaped pattern of net payouts that is inversely related to $\psi(z)$. Or, yields tend to increase in a non-linear way with the underlying index, reflecting plant and phenology specific sensitivities to weather. In either way, the intuition is clear: the contract is supposed to insure the farmer against low yield realizations. Thus, net payouts to the farmer should be high whenever $\psi(z)$ is low and vice versa.

The assumptions in Proposition (3) are still quite restrictive, however. Not only may extreme weather events as captured by high and low realizations of z reduce *expected* yields. They may also change the *distribution* of yields in a more general way. Notably, one may think of extreme weather events increasing the *yield risk* as captured by the variance of yields conditional on z. The following two results show how such more general relationships between yields and weather affect the shape of the optimal weather insurance contract (see also Mahul, 2001).

Proposition 4. *At some given $z \in \mathcal{Z}$, $p'(z) \le 0$ if $dF(y|z)/dz \le 0$ for all $y \in \mathcal{Y}$ (first order stochastic dominance). Conversely, $p'(z) \ge 0$ if $dF(y|z)/dz \ge 0$ for all $y \in \mathcal{Y}$.*

Proof. Since the first order condition (4) has to hold for all $z \in \mathcal{Z}$, it can be differentiated w.r.t. z to obtain

$$p'(z) = -\frac{\int_{\mathcal{Y}} u'(y + p(z))\frac{df(y|z)}{dz} dy}{\int_{\mathcal{Y}} u''(y + p(z))dF(y|z)}. \tag{5}$$

Integrating the numerator by parts yields

$$\int_{\mathcal{Y}} u'(y + p(z))\frac{df(y|z)}{dz} dy = u'(y + p(z))\frac{dF(y|z)}{dz}\Big|_{\underline{y}}^{\overline{y}} - \int_{\mathcal{Y}} u''(y + p(z))\frac{dF(y|z)}{dz} dy. \tag{6}$$

Note that the first term on the RHS is zero because $dF(\overline{y}|z)/dz = dF(\underline{y}|z)/dz = 0$. Hence substituting (6) in (5) yields

$$p'(z) = -\frac{\int_{\mathcal{Y}} u''(y + p(z))\frac{dF(y|z)}{dz} dy}{\int_{\mathcal{Y}} u''(y + p(z))dF(y|z)}. \tag{7}$$

Recall that the denominator is always negative since $u''(.) < 0$. Hence the sign of $p'(z)$ is equal to the sign of $dF(y|z)/dz$, which is the result in the proposition. $\qquad\square$

Proposition (4) is a generalization of Proposition (2). It shows that net insurance payouts to the farmer should be decreasing in the weather index if an increase in z induces higher yields in the first order stochastic dominance sense (FOSD), and vice versa. In the case of precipitation-based insurance, this again makes a non-linear shape of the optimal insurance contract plausible.

The following proposition captures the effect of changes in the *variability* of yields due to changes in the weather index on the shape of the optimal insurance scheme.

Proposition 5. *Suppose $u'''(c) > 0$ for all c. Then, at some given $z \in \mathcal{Z}$, $p'(z) \leq 0$ if $\int_{\underline{y}}^{y} \frac{dF(s|z)}{dz} ds \leq 0$ for all $y \in \mathcal{Y}$ (second order stochastic dominance). Conversely, $p'(z) \geq 0$ if $\int_{\underline{y}}^{y} \frac{dF(s|z)}{dz} ds \geq 0$ for all $y \in \mathcal{Y}$.*

Proof. Integrating the RHS of equation (6) by parts again yields

$$
\begin{aligned}
\int_{\mathcal{Y}} u''(y + p(z)) \frac{dF(y|z)}{dz} dy &= u''(y + p(z)) \int_{\underline{y}}^{y} \frac{dF(s|z)}{dz} ds \Big|_{\underline{y}}^{\overline{y}} - \int_{\mathcal{Y}} u'''(y + p(z)) \int_{\underline{y}}^{y} \frac{dF(s|z)}{dz} ds dy \\
&= u''(\overline{y} + p(z)) \int_{\underline{y}}^{\overline{y}} \frac{dF(s|z)}{dz} ds - \int_{\mathcal{Y}} u'''(y + p(z)) \int_{\underline{y}}^{y} \frac{dF(s|z)}{dz} ds dy. \quad (8)
\end{aligned}
$$

Substituting the RHS of (8) in the numerator of (5) and the assumption $u'''(.) > 0$ yields the result. □

Proposition (5) captures the case where an increase in z increases the riskiness of yields in the second order stochastic dominance (SOSD) sense. It shows that the optimal insurance net payout increases with z if the additional condition is satisfied that the farmer is *prudent*, as characterized by $u'''(.) < 0$. Conversely, the net payout decreases with z if a decrease in z makes yields riskier. This again pushes towards a non-linear shape of $p(z)$ given the realistic case that extreme weather occurrences not only reduce expected yields, but also increase yield variability.

In summary, for a given crop and a given weather index the shape of the optimal weather contract, $p(z)$, depends on the shape of the conditional cdf $F(y|z)$, and can be determined empirically by solving the nonlinear, constrained optimization problem (2) subject to (1) numerically. I use a non-parametric estimation procedure to determine $F(y|z)$ in order to to avoid that $p(z)$ may reflect assumptions contained in the functional form of the relationship between y and z. I describe how I derive the solution to this problem in the following section.

3 Implementation and Data

3.1 Implementation of the Optimization Problem

The optimization problem is implemented in its discrete form. A discrete weather variable, z_i, such as rainfall can take $i = 1, ..., N_z$ possible realizations. Then the optimization problem has $N_z + 1$ first-order-conditions in $N_z + 1$ unknowns and can be solved numerically (using a mathematical programming tool such as Matlab). For the weather index, N_z defines the number of points for which to produce a density estimate. Similarly, N_y defines the number of density estimates to be produced for the yield data. The size of the conditional yield density matrix $f(y|z)$ is thus $N_y \times N_z$. N_z and N_y can be set to any finite number, restricted by the size of the data. Increasing N_z or N_y implies that fewer yield observations fall within one kernel grid cell, which explains why a sufficiently large data set is needed for estimation. It follows that N_z has an effect on the precision of $p(z)$ since the number of first order conditions increases, and thus the number of payments $p(z_i)$ that are determined for different realizations of the index z_i. The effect on the hedging effectiveness of the optimal contract due to changes in either N_z or N_y is demonstrated in section 5.4.

The optimization problem is solved by defining a specific functional form for the farmer's preferences. I assume that farmers have preferences of the following form: $u(c) = c^{1-\sigma}/(1-\sigma)$, i.e. the utility function is characterized by constant relative risk aversion (CRRA).[13] I use a coefficient of relative risk aversion of $\sigma = 2$ as our benchmark[14], but demonstrate how our results change for different levels of risk aversion (see section 5.3).

Both the conditional yield density matrix, $f(y|z)$, and the density of the weather index, $g(z)$, are estimated using a Gaussian kernel density estimation procedure. Given that yield and weather data follow different statistical distributions, and differ in their range, I allow the conditional yield density matrix $f(y|z)$ to be non-quadratic (Härdle, 1991). Moreover, I allow for kernel bandwidths for the weather index, $bw(z)$, and for yields, $bw(y)$. Once all input parameters are determined, the optimization problem is solved numerically.[15]

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[13]CRRA preferences are supported in the literature by Pope and Just (1991), Chavas and Holt (1990), Martin et al. (2001), Zuniga et al. (2001), Wang et al. (2004), Karuaihe et al. (2006), Berg et al. (2009), Leblois and Quirion (2011).

[14]Arrow (1971) argues that the coefficient of relative risk aversion is near 1. Chetty (2006) shows that the coefficient of relative risk aversion is around 2 in his data. For agriculture, Pope and Just (1991) suggest that the relative risk aversion is constant.

[15]The nonlinear constrained optimization problem is solved by "shooting for lambda", i.e. I assume a starting value for λ and solve the first order conditions given this λ. Next, I check whether the output of

3.2 Description of Data

Historic yield time series data is often not sufficiently large for empirical applications as complex as the one I aim at. Following Torriani et al. (2007a, 2007b), I use a process-based crop simulation model to simulate $1,000$ yield observations.[16] For the purpose of this study, CropSyst, which is a multi-year, multi-crop, daily time step growth simulation model developed by Stöckle et al. (2003), is used to simulate crop development and total biomass accumulation of maize (*Zea mays L.*). The simulation of crop development is mainly based on the thermal time required to reach specific development stages. Thermal time is calculated as growing degree days (GDD) accumulated throughout the growing season (starting from sowing until harvest). With process-based crop models, the user can input management parameters such as the sowing date, cultivar genetic coefficients, soil profile properties (soil texture, depth), fertilizer and irrigation management, tillage and atmospheric CO_2 concentration, and the growing degrees needed to complete each phenological period.[17]

To simulate $1,000$ yield observations, daily weather data was obtained from a stochastic weather generator.[18] LARS-WG, a weather generator developed by Semenov (1997, 2002), was first calibrated for the current weather conditions (from 1981 to 2001) prevailing in at the weather station Schaffhausen (SHA) (latitude: 47.69, longitude: 8.62) in Switzerland.[19] For the time period from 1981 to 2001, $1,000$ years of daily weather observations were simulated and daily observations of minimum (T_{min}) and maximum temperatures (T_{max}) in Celsius, rainfall (R_i) in millimeters, and solar radiation (Sol_i), with $i = 1, ..., 365$ indicating the day of the year, were obtained.[20]

To derive the revenues from maize production per hectare, I use the average price

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the optimization problem $(p(z),z)$ satisfies the constraint (1). If the constraint is not satisfied, the procedure is repeated until a λ-value is found for which the constraint is fulfilled. The optimal values, $p^*(z)$, satisfy the necessary and sufficient FOCs as well as the constraint (1), and thus constitute a global optimum.

[16]Process-based crop models are deterministic models that simulate crop physiological growth for given environmental and management conditions. In theory, their precision in estimating crop yields is greater compared to regression models, but they require detailed time series data in order to calibrate model parameters and for evaluating the model's ability of predicting historic crop yields (Lobell et al., 2010).

[17]The calibrations of CropSyst for maize production in Switzerland have been carried out by Agroscope-Reckenholz-Taenikon Research Station (ART) in Switzerland. An overview of the calibration parameters for maize, winter wheat, and canola in Switzerland can be found in Torriani et al. (2007) together with a comparison of simulated yields with historical observations.

[18]Weather generators generate synthetic weather series which have statistical properties similar to the observed series. Means and variance of daily synthetic weather data are required to be not significantly different from those calculated from observed series, and the synthetic weather series should follow a probability distribution which is not statistically different from the observations.

[19] The weather data for SHA from 1981 to 2001 was obtained from MeteoSwiss.

[20]The calibration of LARS-WG for the weather measurement station SHA has been carried out by Agroscope-Reckenholz-Taenikon Research Station (ART) in Switzerland.

for maize from 2006 to 2009, which was 41.00 CHF/100kg (SBV, 2010). This allows us to derive the insurance contract in monetary units.[21]

4   Constructing a Suitable Weather Index

Finding a suitable weather index involves identifying the source of weather risk that the insurance contract is intended to hedge. One way of identifying the risks that cause crop losses is by interviewing farmers. I use instead a quantitative approach to identify the weather events that explain deviations in yields. The objective is to create weather indices that possess a high correlation with yields as this affects the hedging effectiveness (Miranda, 1991; Vedenov and Barnett, 2004; Musshoff et al., 2009).

While it is well known that the susceptibility of crops to meteorological stress (such as heat stress or shortage of available water) varies during the growing season (Meyer et al., 1993), the use of fixed calendar time periods for the construction of weather indices is common in the literature (Turvey, 2001; Martin et al., 2001; Musshoff et al. 2009). Hanks (1974) notes that there is a strong relationship between total water consumption by a plant over the growing season and final yields. However, Hanks (1974) also points out that "... it is essential for any study dealing with the estimation of future yield to take into account the element of time when referring to the effects of stress." In the same vein, Jensen (1968) and Nairizi et al. (1977) show that the yield response of a crop to soil moisture stress depends on the crop being studied, and the phenology during which the stress occurs. In the context of index construction, Turvey (2000) is the first to mention that " ... (it) would be advantageous to correlate weather events to specific phenological events." Leblois et al. (2011) also note that using the actual sowing date together with information about the different growing stages improves the predictive power of the index. In their study examining the potential of rainfall insurance in Niger, Leblois et al. (2011) simulate growth phases (following a method developed by Sivakumar, 1988), and create indices that weigh the effect of rainfall depending on the growing phase. The authors find that indices that account for growing phases improve the gains from weather insurance.

In my study, I measure weather events at each phenology stage considering year to year shifts in phenology stages due to to inter-annual weather variability. For the index design, I estimate the individual effect of weather events on yields to account for the differences in the weather susceptibility of maize during the growing period. Based on this

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[21]A higher (lower) price for maize yields will shift the insurance contract up (down), but will not change the shape of the contract due to the assumption of CRRA preferences. Production costs are not considered here.

information, I construct indices using the estimated coefficients obtained from multivariate weather-yield regression models as weights.

4.1 Identifying the Phenology Phases

The concept of Growing Degree Days (GDDs) is used to identify the different phenological phases during plant development. Growing degrees (GD), also referred to as heat units, are defined as the number of temperature degrees above a certain threshold temperature, T_{base}, and below an upper level, T_{cut}, and are frequently used to describe the timing of biological processes (Neild, 1982; McMaster, 1997).[22] I use the following formula to calculate GDDs, starting at the sowing date, with $i = 1$, until the end of the growing season, with $i = n$ at the harvest time:

$$GDD = \begin{cases} \sum_{i=1}^{n}\left(\frac{T_{max,i}+T_{min,i}}{2} - T_{base}\right) & \text{if} \frac{T_{max,i}+T_{min,i}}{2} < T_{cut} \\ \sum_{i=1}^{n}\left(T_{cut} - T_{base}\right) & \text{if} \frac{T_{max,i}+T_{min,i}}{2} > T_{cut} \end{cases}$$
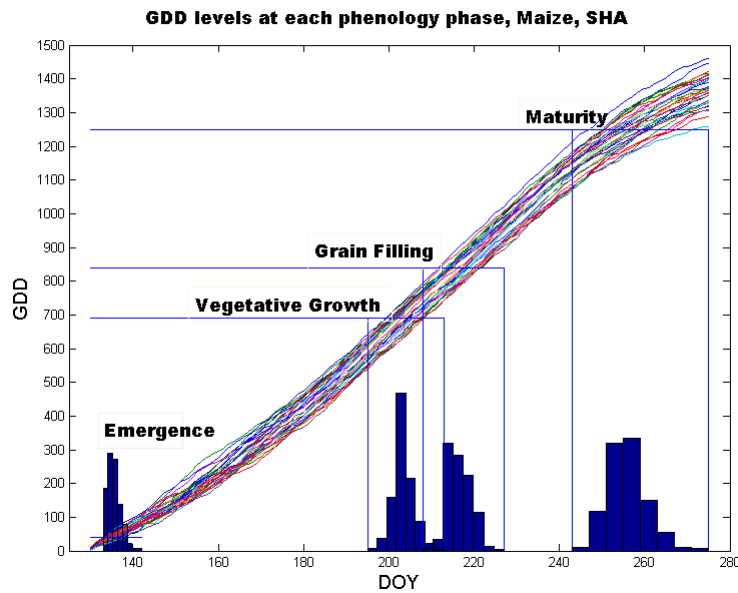

Figure 1: GDD during the growing phase

The day of the year (DOY) when sowing takes place is known from CropSyst, since a fixed planting mode is chosen. For maize in SHA, sowing takes place at $DOY = 130$. I use the daily GDDs throughout the entire growing period to find the day of the year

[22]T_{base} is the temperature below which development stops, and T_{cut} is the upper threshold which still contributes to plant growth.

when a given phenology stage ends. The number of GDDs needed to complete each phenology stage are also known from the CropSyst calibration.[23] For maize production, 4 phenology phases are distinguished: the emergence, the vegetative growth period with flowering, the grain filling period, and maturity. Table 1 summarizes the GDD levels that correspond to each phenology phase for maize in Schaffhausen until maturity is reached, and reports the range of days, as well as the mean DOY, when the phenological stages end. For example, the grain filling phase ends on average on $DOY = 217$. Depending on the particular weather conditions in a given year, I observe that grain filling came to an end 9 days before, or 10 days after the average end date (i.e. between $DOY = 208$ to $DOY = 227$). For maturity, the range of possible end dates is even wider. When using fixed calender periods for index construction (based on average start and end dates), the weather index may not adequately capture the weather deviations in extreme years, thus creating additional basis risk.

Figure 1 shows the relationship between GDD values for each DOY starting at emergence ($DOY = 130$) to harvest ($DOY = 243$-275) for maize in SHA for 30 different years. The histograms display the frequency of end dates for the 4 phenological stages and are derived from the entire data.

Table 1: Timing of phenology phases and corresponding GDDs

Phases	Emergence	Vegetative Period	Grain Filling	Maturity
GDD	40	700	840	1250
DOY	133-142	195-213	208-227	243-275
Expected DOY	137	204	217	259

Crop: maize, location: SHA, sowing date: DOY=130.

4.2 Measuring Weather Risks

I measure different weather risks using weather indicators from the literature. The simplest quantifiers of the prevailing weather conditions in a given time period are the averages of daily precipitation, and minimum and maximum temperature values. Average, or respectively cumulative, precipitation and temperature measures are often found in weather-yield models (Martin et al. 2001; Turvey, 2001; Musshoff et al, 2009; Berg et al., 2009; Leblois et al., 2011). Their use is however criticized on the basis that sub-seasonal variations, such as long dry spells or short heat waves, which are critical to crop growth,

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[23]For information about the CropSyst maize calibration used, see Torriani et al. (2007a). It should be noted that the approach recommended here for constructing phenology-specific weather indices can also be applied to historic yield data, since GDD requirements of plants at each phenology stage are often reported by breeding companies or agricultural extension services.

are not captured (Lobell and Burke, 2010). Since I divide the growing period into 4 sub-periods, the use of average measures at each growing stage can be justified.

When using precipitation averages, the water consumption by the plant is however not adequately reflected by the index since low precipitation may evaporate – especially on hot days – or run off with excess precipitation. To construct an index that takes the actual water availability to the plant into account, I calculate the daily values of potential evapotranspiration (ETo) using the Priestley-Taylor radiation-based method recommended by the FAO (1998), and a temperature-based method by Hamon (1963). This allows us to construct the Reconnaissance Drought Index (RDI) proposed by Tsakiris and Vangelis (2005, 2006), which is the ratio between the cumulated quantities of precipitation and ETo (given a time period), using both ETo methods as inputs. In addition, I construct a Moisture Deficit Index (MDI), which is the difference between daily precipitation and ETo, to approximate for the moisture deficit.

4.3   Index Construction

I use a multivariate regression framework to evaluate the effect of the above-stated weather variables on crop yields. While three different weather related sources of yield risk exist - drought, excess precipitation, and heat stress - I find that in Schaffhausen (SHA), weather events causing drought-like conditions explain the largest fraction of variation in maize yields. Some findings worth noting are that variations in maize yields are better explained by multivariate regression models which capture the effect of different weather events during the growing cycle, compared to bivariate models. Purely temperature based models were not further considered in our analysis since they can only explain a very small fraction of yield variation compared to precipitation-based models. Regression models that use measures of potential evapotranspiration in addition to precipitation perform the best, i.e. these models explain large fractions of yield variability. Including squared precipitation measures further increases the prediction accuracy of the model, and thus improves the quality of the weather index.

I select 4 multivariate regression models with different weather phenomena occurring at different phenology stages to construct weather indices. The 4 models vary in the time periods and weather events covered, and therefore in the complexity of communicating the index to the insured. For the contract design, working with 4 indices allows us to examine the effect of the goodness of fit of an index on risk reduction (see section 5.3 and 6.3). Table 2 provides an overview of which weather variables are used in each of the 4

models and shows the phenology phases at which these variables are measured.[24]

To construct Index 1, I only use mean precipitation during the vegetative period, which explains 37.1% of the variation in maize yields. Taking minimum temperatures during emergence and flowering, as well as the maximum temperatures during grain filling and maturity into account, Index 2 explains 49.3% of total maize yield variability.[25] Using the difference between mean precipitation and potential evapotranspiration to measure the water availability, I construct Index 3 with an adjusted R^2 of 47.1%. Considering the fact that precipitation has a nonlinear effect on crop growth, the squared mean precipitation is used in addition to the Reconnaissance Drought Index in Index 4. The rank correlation of Index 4 with yields is 78.9% and the adjusted R^2 is 62.5%. The goodness of fit of each of the 4 indices is described in Table 3, showing (Spearman) rank correlation, the adjusted R^2, and the Akaike Information Criterion (AIC).

Table 2: Description of weather indices

Index	Weather variable	No. of variables	Phenology Phase
Index 1	m.precip*	1	2
Index 2	m.precip, m.tmax**, m.tmin**	7	1-4
Index 3	P.ETo(Priest)***	3	2-4
Index 4	m.precip, m.precip^{2+}, RDI(Hamon)$^{++}$	9	2-4

Note: * m.precip is the mean of daily precipitation values. ** m.tmax and m.tmin are respectively the means of daily maximum and minimum temperatures. *** P.ETo(Priest) is the difference between daily precipitation and daily evapotranspiration (ETo), where ETo is measured using the Priestley-Taylor formula. $^{+}$ m.precip^2 are the squared daily mean precipitation values. $^{++}$ RDI(Hamon) is the Reconnaissance Drought Index derived using daily potential evapotranspiration, where ETo is measured using the Hamon formula.

Table 3: Goodness of fit of weather indices and yields

Index	Rank correlation	Adj.R^2	AIC
Index 1	64.7	37.1	16876
Index 2	74.1	49.3	16712
Index 3	71.3	47.1	16751
Index 4	78.9	62.5	16422

[24]Table 11 in Appendix shows the estimated coefficients and the t-statistics for the 4 regression models. Only models with weather variables that are significant at 10% or less are considered for constructing weather indices.

[25]I report the adjusted R^2 to make the comparison with other studies possible. The adjusted R^2 is however not the optimal measure for evaluating the quality of an index for non-linear regression models. Therefore, I also report the Akaike Information Criterion (AIC) and the rank correlation coefficients.
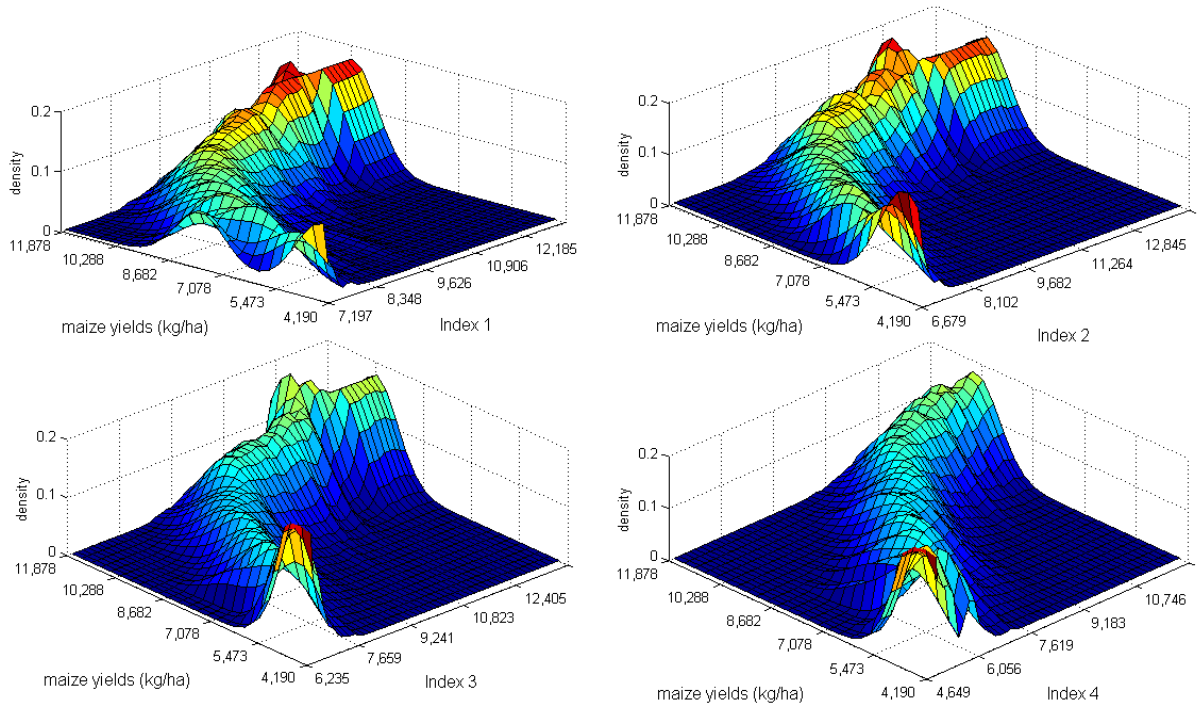
Figure 2: Conditional yield density for weather indices 1 to 4. Kernel estimates with $nz = 50, ny = 25, bw(z) = 300, bw(y) = 100$.

5 Results

5.1 Conditional Yield Distributions

In section 2, I showed that the optimal payoff structure has to reflect the information contained in the conditional yield distributions. Figure 2 shows the conditional yield densities for maize in SHA estimated with a two-dimensional kernel procedure for the four indices described in section 4. In particular, I observe that changes in the riskiness of yield production due to changes in the weather index have an effect on the local slope of the contract ($p'(z)$).

Maize yields in SHA range from 4,190 to 11,878 kilo per hectare (kg/ha), with an average of 9,241 kg/ha.[26] The weather index is measured in the same units (kg/ha) since the index has been constructed such that it possesses a high correlation with crop yields, and thus represents predicted yields for the given realizations of the weather index. Unless otherwise noted, the conditional yield densities are estimated using $ny = 25$ and $nz = 50$, and the Kernel bandwidth for the index, $bw(z)$, is set to 300, and for the yield data a Kernel bandwidth, $bw(y)$, of 100 is used.

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[26]Revenues from maize production thus range from 1,718 to 4,870 CHA per hectare.

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It can be seen that for all indices, low values of the index are associated with low expected yield levels. As the value of the index increases, maize yields tend to increase as well, albeit in a non-linear way. For Index 1 to 3, the conditional mean yield tends to increase rapidly for low values of the weather index. Once the weather index has reached its mean value, the increase in the conditional mean yield flattens out for a further increase in the underlying index. The conditional yield density for Index 4 behaves slightly different. For low values of Index 4, yields tend to increase only slowly, and almost linearly. The flattening of the conditional mean only occurs for very high values of the index (compared to Indices 1 to 3). Most notable is that the shape of the conditional yield distributions changes for different values of the weather index. This observations holds for all indices.[27] In particular, the riskiness of the conditional maize distributions may change in a non-continual way for small changes in the weather index. These changes in the riskiness explain why the optimal weather insurance contract is neither perfectly linear nor U-shaped, as will be demonstrated in the following.

5.2 Optimal Insurance Contract

The optimal weather insurance contract for Index 2 (Index 4) is shown together with the conditional yield density in Figure 3 (Figure 4).[28] The shape of both contracts reflects closely the changes in the respective conditional yield distributions. Both contracts pay out for low values of the index, and have negative net-payments for high values of the index. Net-payments for contract 2 decrease faster than for contract 4 since for low values of the weather index 4 the probability of getting high yields is low. The maximum net-payment from contract 2 is 1,660.60 CHF per hectare of insured maize production. The minimum of the indemnification curve is at -828.36 CHF, which can be interpreted as the premium (P), or the amount of money a farmer would have to pay to obtain the weather insurance contract. Assuming that the insured pays a premium of 828.36 CHF to purchase the contract at the beginning of the growing season, the gross-payments for a given weather realization are determined by adding the premium to each net-payment.[29]

In Figure 5, all 4 optimal weather insurance contracts are shown together. The weather insurance contracts for Indices 1 to 3 possess similar contract shapes and contract parameters.

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[27] For further empirical research, it is worth noting that when estimating the relationship between weather and yields, one should account for the fact that the assumption of homogenously distributed error terms is not valid, as is obvious from Figure 2.

[28] For Index 1 and 3, the optimal contracts together with the conditional yield densities are shown in the Appendix (Figures 11 and 12).

[29] In the Appendix (see Figure 13), I show the gross-payments from the insurance contract for Index 4.
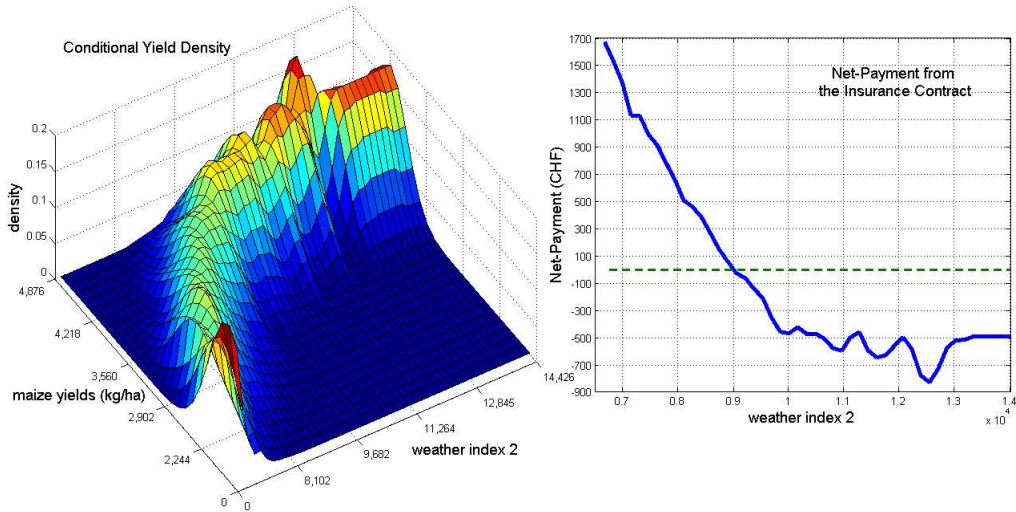
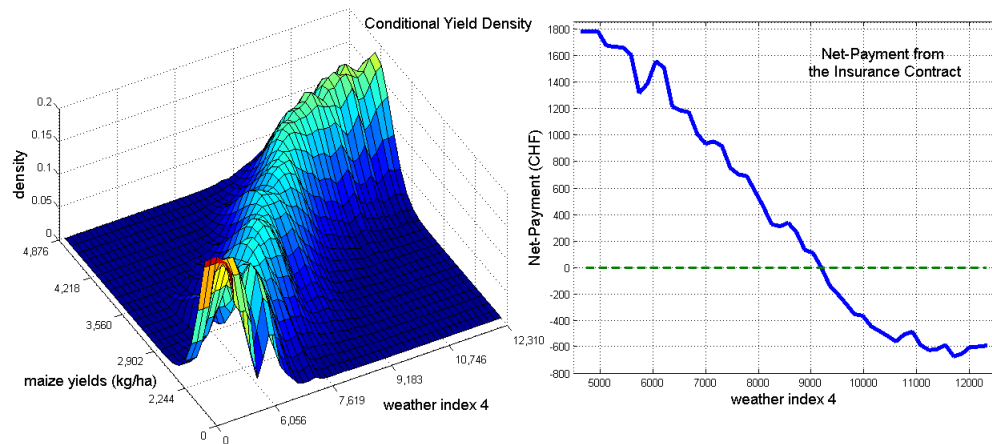Figure 3: Conditional yield density and insurance contract for Index 2

Figure 4: Conditional yield density and insurance contract for Index 4

At the point where the net-payment is equal to zero, the purchaser of the contract recovers the premium. Once the weather index has reached values smaller than the "recovery point", the contract is "in the money" or has positive net-payments. The probability of this event ("recovery probability") can be derived from the probability density function of the underlying weather index. As shown in Table 4, the probability of the weather index being equal to or lower than the recovery point is between $46 - 49\%$ (depending on the index), i.e. the contract pays out (in net terms) almost every second year.[30] Fur-

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[30]The frequency of (historical) pay-outs has been found to be a critical factor influencing farmers' decision to purchase protection against adverse weather conditions (Patt et al., 2009). The optimal weather insurance contract should thus be quite attractive to farmers given its high recovery probability.
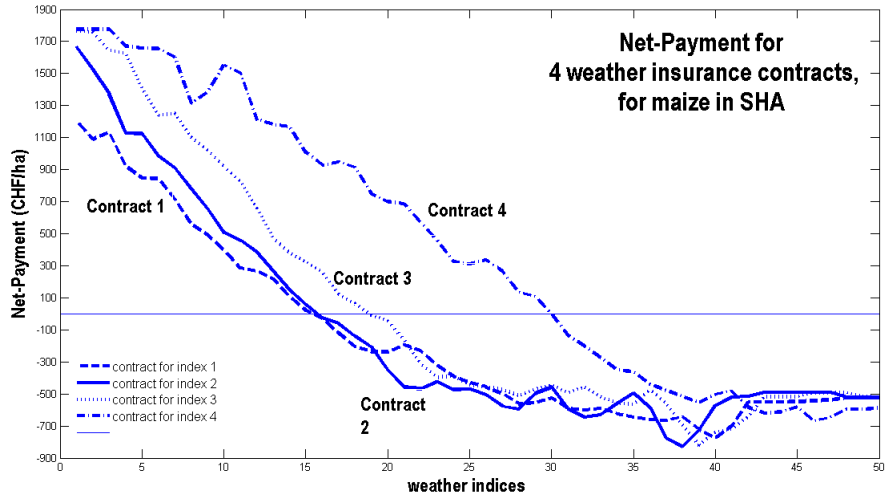
Figure 5: Weather insurance contracts for Index 1 to 4 and sigma=2

thermore, the premium and maximum payout, and the realization of the weather index at which the insured recovers the premium are shown in Table 4. Contract 4 has the smallest premium with 668.90 CHF, and at the same time the highest maximum net-payment with 1,775.60 CHF and in 49% of the cases the insured fully recovers the premium.

Table 4: Contract parameters

Index	Premium	max. Payout	Recovery Point	Recovery Probability
Index 1	776.30 CHF	1,207.20 CHF	9.081	0.48
Index 2	828.36 CHF	1,666.60 CHF	8.971	0.46
Index 3	821.10 CHF	1,762.50 CHF	8.003	0.47
Index 4	668.90 CHF	1,775.60 CHF	9.183	0.49

Note: Premium and maximum payout are measured in CHF. The recovery point is in the same units as the index. Recovery probability is the probability of realizing index values equal or smaller than the recovery point. Crop: Maize; Location: SHA; Contract parameters: $\sigma = 2$, $nz = 50$, $ny = 25$, $bw(z) = 300$, $bw(y) = 100$.

To describe the insurance properties of an optimal insurance contract, the information contained in the recovery point (trigger), maximum payout (cap) and premiums is insufficient. For a complete picture of the insurance coverage inherent in an optimal weather insurance contract, the relationship between the pay-out probabilities at different levels of the weather index and the respective net-payments has to be analyzed. For that purpose, I compare the net-payment curve with the probability distribution of the underlying weather index. In Figure 6, these two functions are shown for index 4. I find that very high net-payments only occur with low probabilities, and at the same time the probabil-

ity of having to pay the full premium is also very low. The intuition for this observation is clear: the insured receives very high-payments for catastrophic weather events such as droughts that cause substantial losses. These events however only occur with a very low probability. Similarly, perfect growing conditions (as indicated by high values of the weather index) also occur only with a low probability, and therefore the insured faces a very low probability of paying the full premium. The optimal contract provides moderate payments between $0 - 500$ CHF for very likely deviations from the recovery point, and therefore comes in most years at moderate costs of 0 to -500 CHF for Index 1 and 2, which are even lower for Index 3 and 4.
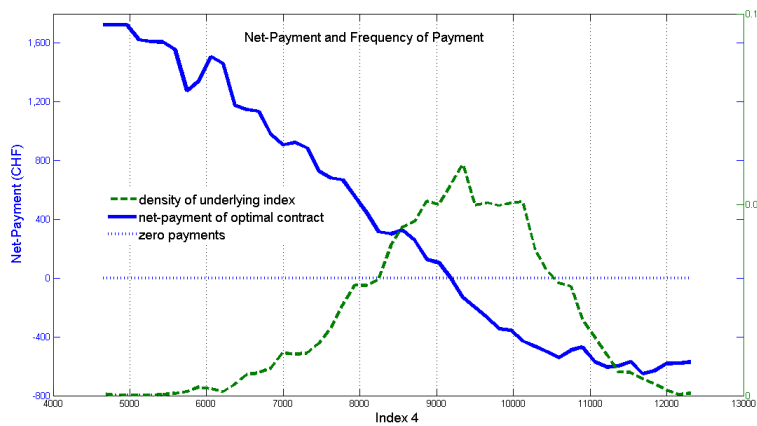

Figure 6: Probabilities of net-payments from insurance contract 4

To characterize the optimal insurance contract in terms of pay-out frequencies, I computed the probabilities of realizing net-payments that range between the maximum pay-out (limit) and 500 CHF, between 500 and 0 CHF, 0 and -500 CHF, and between -500 and the premium. Table 5 summarizes the pay-out probabilities for all 4 insurance contracts. In the case of contract 2, the probability of net-payments between 500 and 0 CHF is 39.60%, and together with the probability of 43.30% for net-payments between 0 and -500 CHF, this contract has net-payments between 500 CHF and -500 with a probability of 82.80%. At the same time, this contract offers high indemnities in times of severe weather events. The extended coverage comes at a cost of having to pay between -500 CHF and the premium when weather conditions are excellent, which occurs in 7.40% of the cases. Overall, all optimal insurance contracts are characterized by moderate net-payments of 500 to -500 CHF occurring with a probability of 78.60% (Index 4) to 86% (Index 1 and 3). Thus, optimal weather insurance offers protection for catastrophic, infrequent-high-loss events and compensates the insured on a regular basis for moderate fluctuations of yields.

Table 5: Pay-out probabilities of optimal insurance contracts

Net-Payment (in CHF)	500 to max.payout	0 to 500	-500 to 0	premium to -500
Index 1 probability	6.60%	39.20%	47.20%	7.10%
Index 2 probability	9.80%	39.60%	43.20%	7.40%
Index 3 probability	11.00%	39.60%	46.80%	2.60%
Index 4 probability	14.60%	34.50%	44.10%	6.80%

Note: Payments are measured in CHF. Crop: maize, location: SHA, contract parameters: $\sigma = 2$, $nz = 50$, $ny = 25$, $bw(z) = 300$, $bw(y) = 100$.

5.3 Evaluation of Hedging Effectiveness

The risk-reduction that can be achieved from using an optimal weather insurance contract can be evaluated by comparing the revenue distribution without insurance to the situation where the farmer hedges the weather exposure by buying insurance. In the situation without insurance, the income from maize production (per hectare) is equal to the revenues from maize production, i.e. maize yields y_i (kg/ha) in a given year i multiplied by the respective price p_m, which is 0.41 (CHF/kg). The income per hectare of maize production in SHA thus ranged from 1,718 to 4,870 CHF, with mean revenues of 3,696 CHF and standard deviation of 576.90 CHF. To derive the income of a maize farmer in SHA for the situation with insurance, the net-payments in each year are added to the revenues from selling maize. If the farmer hedges the weather risk, the lowest income realizations range from 2,162 to 2,376 CHF depending on the contract. By hedging, the farmer thus receives 25 to 38% more income in the worst possible year (depending on the contract) than without hedging. At the upper end of the income distribution, incomes of 4,821 to 4,959 CHF/ha are possible (depending on the contract).

Table 6 summarizes the statistical properties (mean, standard deviation, skewness, and the 10%, 25%, 50%, 75%, and 90% quantiles) of the income distributions with and without insurance. As expected, the mean income is the same in all scenarios since insurance reduces the risk of realizing low incomes, but does not cause a change in the mean income due to the zero profit condition (1).[31] Without insurance, the risk of realizing an income that is lower than 2,865 CHF is 10%. With insurance, in 10% of all cases the income falls below 3.109 to 3.268 CHF depending on the chosen contract, i.e. the risk of low incomes (in the 10% quantile) is substantially reduced.

All weather insurance contracts greatly reduce the standard deviation and skewness

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[31]The fact that the reported mean incomes for the situations with insurance are by 5-7 CHF smaller than in the situation without insurance is due to numerical imprecision from solving the optimization problem.

Table 6: Income with and without insurance for 4 weather insurance contracts

	Not Insured	Index 1	Index 2	Index 3	Index 4
mean	3696	3691	3691	3691	3689
std	576.9	436.0	369.0	379.3	338.2
skw	-0.73	-0.65	-0.54	-0.41	-0.50
10%	2865	3109	3211	3203	3268
25%	3337	3448	3478	3471	3501
50%	3815	3749	3725	3713	3708
75%	4147	4000	3946	3946	3908
90%	4349	4193	4138	4135	4094

Note: Units: CHF/ha, crop: maize, location: SHA, model parameters: $\sigma = 2$, $ny = 25$, $nz = 50$, $bw(1) = 100$, $bw(2) = 300$.
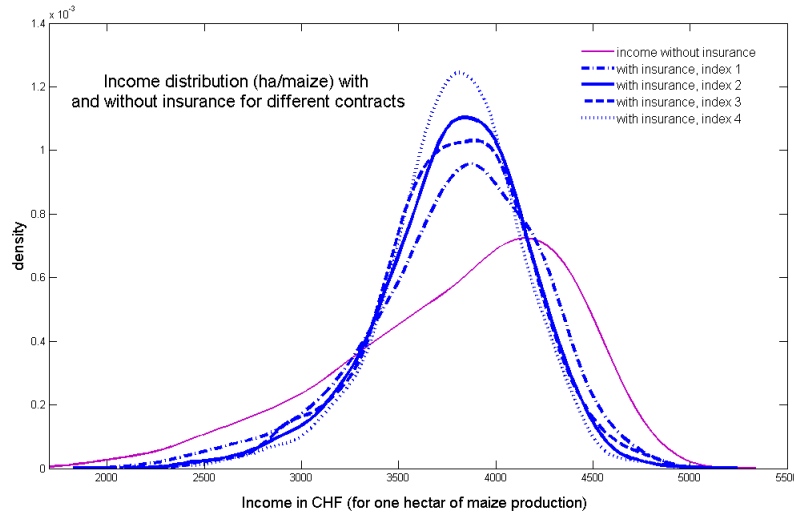

Figure 7: Income distribution without insurance and for 4 weather insurance contracts

of the income distribution. Overall, farmers face less risk of obtaining low incomes.[32] The contract based on Index 4 almost halfs the standard deviation. While the incomes for the 10% and 25% quantiles increase with insurance, the incomes at the 75% and 90% quantiles decrease. As a result of facing less risk at the lower end of the income distribution, the insured faces now lower probabilities for realizing extremely high incomes (compared to the situation without insurance). The compression of the income distribution with insurance can also be seen in Figure 7, which shows the income distributions for the 4 insurance contracts, and for the scenario without hedging. Insurance contract 4 performs

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[32]An Ansari-Bradley test has been performed, and for all weather insurance contracts we can reject at the 5% level the hypothesis that the income distribution with insurance has the same dispersion as the income distribution without insurance.

the best.

To measure the effect of hedging, I compute the percentage increase (of all income realizations) in the situation without insurance that makes the farmer equally well-off as in the situation with insurance. In equation (2.9), the farmers expected utility from insurance is set equal to the expected utility in the situation without insurance when all income realizations are multiplied by $(1 + \delta)$. I solve the expression in the following equations (2.10-2.11) for δ.

$$\int_z g(z) \int_y f(y|z) \frac{(p(z) + y)^{1-\sigma}}{1 - \sigma} dydz = \int_z g(z) \int_y f(y|z) \frac{((1+\delta)y)^{1-\sigma}}{1 - \sigma} dydz \qquad (9)$$

$$\Leftrightarrow \int_z g(z) \int_y f(y|z)(p(z) + y)^{1-\sigma} dydz = (1 + \delta)^{1-\sigma} \int_z g(z) \int_y f(y|z)y^{1-\sigma} dydz \qquad (10)$$

$$\Leftrightarrow \delta = \left(\frac{\int_z g(z) \int_y f(y|z)(p(z) + y)^{1-\sigma} dydz}{\int_z g(z) \int_y f(y|z)y^{1-\sigma} dydz} \right)^{\frac{1}{1-\sigma}} - 1 \qquad (11)$$

The percentage increase (of all income realizations) in the situation without insurance that would lead to the same level of expected utility as in the situation with insurance, δ, is a measure of the value of weather insurance. I compute δ for the 4 contracts described in section 5.2 with risk aversion of $\sigma = 2$. Buying optimal weather insurance is equivalent to increasing the insured's income (in all states of the world) by 1.25 to 1.95 % depending on the contract. The benefit from hedging with weather insurance can reach considerable values ($\delta > 10\%$) for higher levels of risk aversion ($\sigma > 5$).[33] Table 7 shows δ (in percent) for the 4 indices and different levels of risk aversion. I also observe that δ tends to be higher for contracts for which the quality of the weather index is high (see Table 3). For Index 1, which has a (Spearman) rank correlation coefficient of 64.7%, a δ of 0.57% to 10.2% can be achieved. For Index 4, which possesses the highest rank correlation of 78.9%, I find δ's in the range of 0.89% to 16.92% depending on the level of risk aversion.

The contracts shown in Figure 3 to 5 are derived for a risk aversion of $\sigma = 2$. For Figure 8, I computed the optimal contract for Index 4 for different levels of risk aversion ($\sigma = 1, 2, 5, 7, 10$). The more risk averse the insured, the more protection is being sought in the optimum, which can be seen in a shift in the recovery point, and higher compensation (in the form of positive net-payments) for medium deviations of the weather index from its mean. To compensate for the increased protection, a higher premium has to be charged

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[33]The observation that benefits from risk reduction increase with risk aversion is in line with related findings in the literature. For instance, in an empirical analysis of the incentives to participate in the U.S. multi-peril crop insurance scheme Just et al. (1979) find that risk-avers farmers generally have larger risk premiums (compared to risk-neutral farmers).

Table 7: δ (in %) for different levels of risk aversion

	Index 1	Index 2	Index 3	Index 4
$\sigma = 1$	0.57	0.80	0.79	0.89
$\sigma = 2$	1.25	1.76	1.72	1.95
$\sigma = 4$	3.06	4.29	4.23	4.75
$\sigma = 5$	4.22	5.94	5.88	6.56
$\sigma = 7$	6.78	9.69	9.68	10.7
$\sigma = 10$	10.2	15.31	15.47	16.92

Note: δ is the percentage increase of all income realizations without insurance compared to the situation with insurance. Crop: maize, location: SHA, model parameters: $ny = 25$, $nz = 50$, $bw(1) = 100$, $bw(2) = 300$.

since moderate deviations tend to occur quite frequently. The hedging effectiveness as expressed by δ increases by factor 8 when sigma increases by factor 5 (from $\sigma = 2$ to $\sigma = 10$). For low levels of risk-aversion ($\sigma = 1, 2$), the optimal contract focuses on providing high payments in times of catastrophic weather events, as soon as yields tend to increase, net-payments tend to decrease sharply. The reduced protection for moderate deviations from the mean, which a farmer with low risk aversion seeks, is at the same time available at a lower premium.
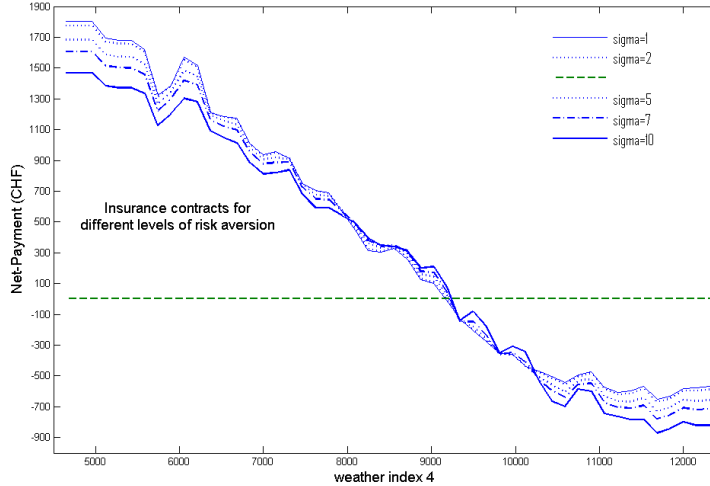

Figure 8: Optimal weather insurance contract for different levels of risk aversion

5.4 Effect of Kernel Density Estimation Parameters

The robustness of the optimal insurance contract, $p(z)$, is finally evaluated for changes in the parameter choice of the nonparametric kernel density estimation. It turns out that the choice of the kernel function does not have an effect on our results, so I use a standard

bivariate normal kernel for all our analysis. The choice of the bandwidth in both dimensions ($bw(z)$ and $bw(y)$) is however an important factor affecting our estimates since it controls the amount (and orientation) of smoothing induced (Wand and Jones, 1995). It can be shown that increasing the kernel bandwidth for both the weather index ($bw(z)$) and yields ($bw(y)$) has an effect on the smoothness of the contract, and consequently on the measure of risk reduction δ. The smoother the contract $p(z)$ is, due to high values for $bw(z)$ and $bw(y)$, the lower the risk reduction as measured by δ. With less smoothing, the net-payment curve responds more to small changes in the weather index, i.e. the noise in the data receives more attention. At the same time, the income distribution with insurance becomes smoother. When over-smoothing the contract, less weather-related variability in the income is hedged. Consequently, the income distribution becomes less smooth.

Table 8: δ (in %) for different kernel estimation parameters

	Index 1	Index 2	Index 3	Index 4
specification 1	1.25	1.76	1.72	1.95
specification 2	1.57	2.19	1.84	2.31
specification 3	1.31	1.83	1.75	2.01
specification 4	1.40	1.92	2.03	2.03

Note: All results are for a risk aversion of $\sigma = 2$.
specification 1: $nz = 50, ny = 25, bw(z) = 300, bw(y) = 100$
specification 2: $nz = 50, ny = 25, bw(z) = 100, bw(y) = 40$
specification 3: $nz = 20, ny = 15, bw(z) = 300, bw(y) = 100$
specification 4: $nz = 20, ny = 15, bw(z) = 100, bw(y) = 40$

Increasing ny increases the number of yield density estimates that are derived for a given value of z. It can be shown that changes in ny have only small effects on risk reduction. In contrast, the choice of nz has a bigger effect on δ.[34] I find that increasing nz decreases the risk reduction as measured by δ. Overall, changes in either of the estimation parameters have a small impact on risk reduction as can be seen in Table 8. I derived δ for different specifications of the estimation parameters for all contracts considered. In particular, when comparing specification 1 with 2 (and 3 with 4), the effect of over-smoothing can be seen.[35] Increasing the kernel bandwidth from $bw(z) = 100$ and $bw(y) = 40$ to $bw(z) = 300$ and $bw(y) = 100$ decreases δ by about 10% on average. Comparing the specification 1 with 3 (and 2 with 4), the effect of increasing nz on δ can be seen.

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[34]In general, when estimating the conditional yield densities at more points (higher nz) – while holding ny constant – fewer weather-yield observations are available at the various evaluation points. To derive the density estimates, the kernel procedure has to interpolate more.

[35]Specification 1 constitutes the baseline scenario which is used throughout the paper.

6 Optimal Insurance Contract for the Insurer

6.1 The Profit-Maximization Problem

Contrary to the assumption stated in the theoretical model (see section 2), expected profits from offering weather insurance are in reality not found to be zero. The insurer requires a positive expected-return to cover his administrative and transaction costs. Depending on the nature of the transaction and the degree of systemic risk in the insured pool, the insurer must factor costs for re-insurance in the premium. Therefore, weather insurance contracts must sell at a price above the expected pay-outs so that location-specific weather insurance coverage can be provided in the long-run. For the insured, the difference between the fair premium, which reflects his expected losses, and the additional costs represent the price for transferring weather-related risks. For the insured to be willing to buy insurance, this cost must not be excessive compared to bearing the weather risk himself.

One mechanism to examine whether a profit-making (loaded) weather insurance contract is attractive for the insured is by comparing the risk reduction achieved from a fair insurance contract with the risk reduction from a contract that includes a mark-up, i.e. factors additional costs into the premium. Berg et al. (2009) add a mark-up of around 10% to the premium to address this question. Vedenov and Barnett (2004) evaluate the effect of transaction costs on risk reduction by adding different loading factors on the premium and comparing the changes in the insured's income under these loaded contracts.

I derive instead the optimal weather insurance contract, $p_m(z)$, that maximizes profits of the insurer subject to the constraint that the insured is indifferent between buying the contract and remaining uninsured. This allows us to numerically determine the maximum loading factor on fair premiums that the insured is still willing to bear. Hence, the insurer is faced with the following constraint:

$$\int_{\mathcal{Z}} \int_{\mathcal{Y}} u(y + p_m(z)) dF(y|z) dG(z) \geq \int_{\mathcal{Z}} \int_{\mathcal{Y}} u(y) dF(y|z) dG(z). \tag{12}$$

The insured's expected utility in the situation with the profit-maximizing contract has to be equal to or greater than his expected utility without insurance. Otherwise, the insured would not be willing to buy the insurance contract. The net-payments $p_m(z)$ constitute liabilities for the insurer. Thus, the insurer maximizes his expected profits by selecting a contract, $\{p_m^*(z)\}$, for which the expected net-payments are as small as possible given the

constraint (12). The insurer's profit-maximizing contract $\{p_m^*(z)\}$ solves

$$\max_{\{p_m(z)\}} - \int_{\mathcal{Z}} p_m(z)dG(z) \tag{13}$$

subject to constraint (12). The Lagrangian can be written as follows:

$$\mathcal{L} = - \int_{\mathcal{Z}} p_m(z)dG(z) + \lambda_m \left\{ \int_{\mathcal{Z}} \int_{\mathcal{Y}} u(y + p_m(z))dF(y|z)dG(z) - \int_{\mathcal{Z}} \int_{\mathcal{Y}} u(y)dF(y|z)dG(z) \right\} \tag{14}$$

which yields the pointwise first-order conditions

$$\int_{\mathcal{Y}} u'(y + p_m(z))f(y|z)dy = \frac{1}{\lambda_m} \quad \forall z \in \mathcal{Z}, \tag{15}$$

where $\lambda_m > 0$ is the Lagrange multiplier of constraint (12). Optimality condition (15) requires that the expected marginal profit of the insured conditional on a realization of the weather index z is equalized across all z by the optimal contract. I implement the profit-maximization problem by assuming CRRA preferences for the insured with σ=2. The implementation is analogous to the one described in section 3.

6.2 The Profit-Maximizing Insurance Contract

Comparing the profit-maximizing insurance contract with the optimal (zero-profit) insurance contract in Figure 9 (top panel), we find that the profit-maximizing contract $p_m(z)$ displays the same shape as the optimal insurance contract $p(z)$. As pointed out in section 5.1, the shape of $p(z)$ is influenced by the changes in the riskiness of the conditional yield distributions. With respect to the shape of $p_m(z)$, Propositions 1 to 5 apply analogously. While both contracts possess the same shape, they differ in their absolute amount of net-payments. For the entire range of the weather index, net-payments are lower for the profit-maximizing contract. In Figure 9 (bottom panel), the difference between the net-payments from the optimal contract and the profit-maximizing contract are shown (for Index 4). The optimal insurance contract pays between 69 to 71.50 CHF more depending on the realized value of the index. The insurer can capture the absolute difference in net-payments because the profit-maximizing contract makes the insured as well-off (in expected utility terms) as in the situation without insurance.
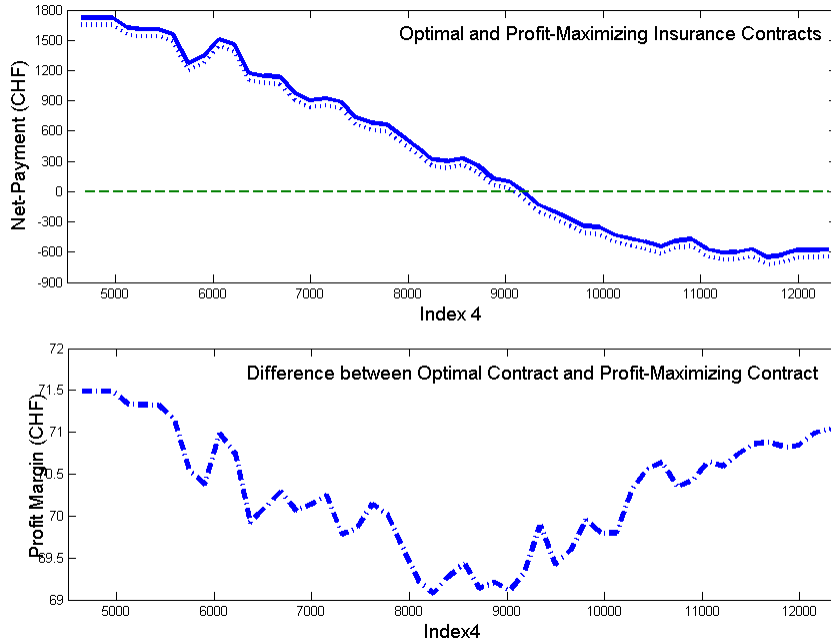
Figure 9: Optimal and profit-maximizing insurance contract *(top panel)* and profit margin *(bottom panel)* for Index 4 and $\sigma = 2$.

6.3 Evaluation of the Profit-Maximizing Insurance Contract

The profits that an insurer can expect to earn by offering the profit-maximizing insurance contract are calculated by

$$\prod = -\int_{\mathcal{Z}} p_m^*(z) dG(z). \tag{16}$$

Expected profits for the 4 insurance contracts are derived for different levels of risk aversion and are summarized in Table 9. For low levels of risk aversion, such as $\sigma = 2$, the insurer can expect to earn between 43.30 to 69.70 CHF per hectare of insured maize production (depending on the contract offered). Expected profits increase to substantial values (123 to 207 CHF) the higher the coefficient of relative risk aversion ($\sigma = 5$).

Profits are found to be positively correlated with the goodness of fit of the underlying weather index. The higher the correlation coefficient of an index (see Table 3), the better the hedging effectiveness as measured by δ (see section 5.3). With rational insurers, I expect an insurance contract similar to contract 4 to be offered since it possesses the highest expected profits. At the same time, insurance contract 4 delivers the highest risk reduction, as measured by δ, and is therefore also the most attractive risk management tool for the insured.

As I set out to determine the maximum amount of loading on fair premiums, I com-

Table 9: Expected profits for different levels of risk aversion

	Index 1	Index 2	Index 3	Index 4
$\sigma = 1$	20.10	30.60	29.10	34.90
$\sigma = 2$	43.30	57.80	59.90	69.70
$\sigma = 4$	100.20	141.80	139.78	154.33
$\sigma = 5$	123.00	182.80	181.20	207.90
$\sigma = 7$	179.30	267.60	267.30	298.10
$\sigma = 10$	221.90	349.60	355.70	389.40

Note: Profits are measured in CHF/ha. Crop: maize, location: SHA, model parameters: $ny = 25$, $nz = 50$, $bw(1) = 100$, $bw(2) = 300$.

pare the premium of the profit-maximizing contract to the premium of the fair (optimal) contract. The loading factors (in percent) of fair premiums are presented in Table 10 for different levels of risk aversion. I find that at moderate risk aversion, it is possible to add a 10% mark-up on the fair premium (for Index 4). As before, with a higher levels of risk aversion, loading factors of 30 to 50% become possible (for Index 4).

Table 10: Loading of fair premium for different levels of risk aversion

	Index 1	Index 2	Index 3	Index 4
$\sigma = 1$	2.89	4.02	3.89	5.63
$\sigma = 2$	5.93	7.72	7.75	10.88
$\sigma = 4$	12.90	17.32	17.18	23.24
$\sigma = 5$	15.41	21.90	21.88	30.72
$\sigma = 7$	(32.63)	31.03	31.16	41.77
$\sigma = 10$	23.88	36.65	39.32	53.38

Note: The loading factor is expressed in % of the optimal premium. Crop: maize, location: SHA, model parameters: $ny = 25$, $nz = 50$, $bw(1) = 100$, $bw(2) = 300$.

Finally, I evaluate the benefits for the insured from hedging with a profit-maximizing insurance contract. The income distribution of the situation without insurance is compared to the income situation where the insured bought a profit-maximizing contract, and to the situation with a fair contract.[36] Figure 10 shows the different income distributions for Index 4. As can be seen, the income distribution from a profit-maximizing contract is less risky (compared to the un-hedged situation), but has a lower mean income. With profit-maximizing insurers, the income distribution from the optimal contract is shifted to the left and the difference between the two mean incomes is captured by the insurer.

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[36] The changes in the statistical properties (mean, standard deviation, skewness and the 10%, 25%, 50%, 75%, and 90% quantiles) are reported in the Table 12 in the Appendix.
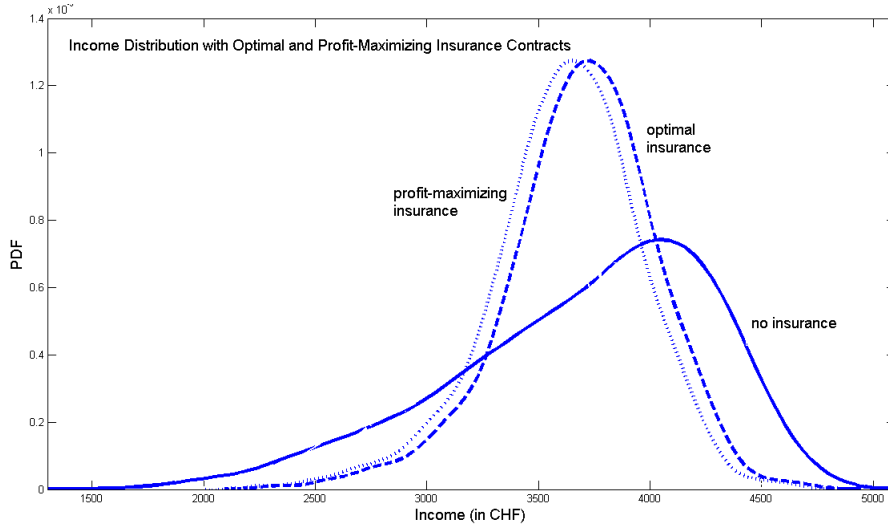
Figure 10: Income distributions for optimal and profit-maximizing weather insurance contracts for Index 4

7 Conclusion

7.1 Summary and Outlook

I propose a new method to derive an index-based weather insurance contract with optimal hedging effectiveness. To illustrate it, I apply it to simulated crop and weather data of maize production in Switzerland, and derive nonparametrically the shape of the optimal contract, and demonstrate that the slope of an optimal weather insurance contract is characterized by changes in the estimated conditional yield distributions.

I show how to quantify the risk reduction from hedging weather-related production risk with our novel approach. By comparing the income distribution without insurance to a hedged situation, I find that income without insurance can be increased (depending on the underlying index) by 1.8 to 2.2% for our benchmark level of risk aversion ($\sigma = 2$), and that benefits can become quite substantial (equivalent to a more than 10% increase in incomes) for higher levels of risk aversion ($\sigma = 5$). Furthermore, I demonstrate the robustness of the optimal contract to changes in the parameters used to derive the conditional yield densities.

In an extension of our model, I show how to derive a profit-maximizing insurance contract in order to determine the maximum amount of loading on fair premiums so that the contract remains attractive to the insured. I find that loading factors can become quite substantial (120 to 200 CHF/ha, and, respectively, 15 to 30%) depending on the weather index and the level of risk aversion ($\sigma = 5$). The question of how the efficiency gains

from hedging farmers' weather exposure are shared between the insurer and the insured is a matter of market power in the weather insurance sector, and depends on the cost of obtaining re-insurance. Our analysis provides the bonds between which insurance contracts will be located for any distribution of bargaining power.

Due to the spatial correlation of weather, small (localized) insurers are faced with systemic risk, i.e. they face high pay-outs in the situation of extreme weather events, which requires them to obtain re-insurance. The presence of systemic risk can therefore be an obstacle of insurability (Quiggin, 1991). While we cannot answer the question whether the optimal loading factors are large enough to cover re-insurance (and administrative costs), our approach characterizes the entire set of mutually-feasible insurance contacts (by deriving both the optimal and the profit-maximizing contract).

By construction, the proposed optimal weather insurance contract implies that no other insurance contract can achieve more risk reduction. In future work, this advantage remains to be quantified. In particular, using the same weather and yield data – ideally for different crops – weather insurance contracts can be derived using the classical derivative structure proposed by Turvey (2000, 2001) and Martin et al. (2001), the semi-optimization approaches described by Berg et al. (2009) and Leblois et al. (2011), and the parametric method proposed by Musshoff et al. (2009). Differences in the hedging effectiveness from these contracts are then solely attributable to our new methodology used for the pay-off structure design.

I realize that optimizing an insurance contract and evaluating its hedging effectiveness on the same data exposes our results to the risk of over-fitting. One way of dealing with over-fitting is by conducting a cross-validation analysis (Vedenov and Barnett, 2004). Given the size of our data, I rate this risk as rather small. In contrast, the risk of over-fitting from using simulated crop yield data should be analyzed. I therefore propose to compare the hedging effectiveness from an insurance contract, which has been derived using simulated yield data, to the risk reduction of contracts which were designed using historical yield and weather data (for the same region and crop). I leave this to be demonstrated in future applications.

It is well known that the wealth level of the insured has an effect on the risk reduction sought. In our study, I implicitly assume that the wealth of the insured is entirely earned from the production of the insured crop. I leave the question of how initial wealth affects the optimal contract to future research.

To our knowledge, I am the first to propose a method for implementing weather insurance contracts based on phenology-driven weather indices. While I found that these more complex indices outperform indices based on fixed time windows, a more thorough

investigation of this observation with data from different crops may be insightful.

Weather insurance is by construction a specific-peril insurance. I account for the influence of different weather events (occurring throughout the growing season) by constructing indices that weight these events and their respective impacts on crops. I then use the statistical distribution of the index to design and price the insurance contracts. This procedure may not adequately account for the fact that the probability of a given weather phenomena occurring at the beginning of the season impacts the likelihood of other weather events occurring later in the season. Future research could explore the design of multi-peril index-based weather insurance where the conditional probabilities of sequential weather events are explicitly modeled.

7.2   Practical Considerations for Implementing Optimal Weather Insurance

I conclude with some remarks on how the proposed optimal insurance contracts could be implemented. Weather insurance is intended to hedge against production risk rather than price risk, therefore choosing the crop price for converting net-payments in monetary units is a critical aspect. I recommend to use the crop price at the end of the growing season to determine the insurance payments. Using the end-of-season crop price helps reduce price uncertainty, and farmers' decisions to obtain weather insurance are independent from price variability and thus speculation. Farmers' individual yields will usually not be correlated with the crop price if the commodity sector is engaged in international trade and the total production supplied is small relative to world production, i.e. the country is a price taker.[37] Farm revenues are therefore subject to price volatilities. Using a crop price different from the end-of-season price exposes the farmer unnecessarily to price risk. In practice, the insured and the insurer sign a contract before the growing season, which stipulates the index and pay-off structure in yield units, and agree that yield units are converted into monetary units using the end-of-season crop price. Alternatively, if future-markets exist for the crop to be insured, the future price can be used.

I argued that the use of weather indices that consider the phenological timing improves the goodness of fit between index and yields. As shown, the insurer and the insured benefit from using more accurate weather indices since both profits and risk reduction are enhanced. To derive phenology-sensitive weather indices, the farmer has to correctly report the sowing date of the insured crop to the insurer. The insured has a

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[37]Otherwise, a "natural hedge" exists, which compensates negative yield variations through higher prices.

self-interest in correctly reporting the sowing date as this information affects the correct measurement of the index, and thus insurance payments. Based on the sowing date, the insurer can determine the start and end dates of the phenology phases with the help of a process-based crop simulation model – as demonstrated in this paper – or by using the GDD levels corresponding to the crop's phenology phases.

8 Appendix

Table 11: Weather-yield regression outputs

Weather variable	Index 1	Index 2	Index 3	Index 4	Phenology Phase
m.precip.2	969.8***	13.6***		4446.2***	2
m.precip.3		224.5***		430.2**	3
m.precip.4		304.2**		975.5**	4
m.tmin.1		-13.3*			1
m.tmin.2		214.2**			2
m.tmax.3		34.6*			3
m.tmax.4		108.6**			4
P.ETo.2			12.8***		2
P.ETo.3			15.5***		3
P.ETo.4			6.9**		4
RDI.2				-111810.6***	2
RDI.3				-3949.2*	3
RDI.4				-2447.5**	4
m.precip.2^2				-282.7***	2
m.precip.3^2				-16.2***	3
m.precip.4^2				-51.8**	4
Adj.R^2	37.1	49.3	47.1	62.5	

Note: $***$ significant at the 1% level. $**$ significant at the 5% level. $*$ significant at the 10% level.
m.precip is the mean of daily precipitation values.
m.tmax and m.tmin are, respectively, the means of daily maximum and minimum temperatures.
P.ETo(Priest) is the difference between daily precipitation and daily evapotranspiration (ETo), where ETo is measured using the Priestley-Taylor formula. m.precip^2 are the squared daily mean precipitation values.
RDI(Hamon) is the Reconnaissance Drought Index derived using daily potential evapotranspiration, where ETo is measured using the Hamon formula.
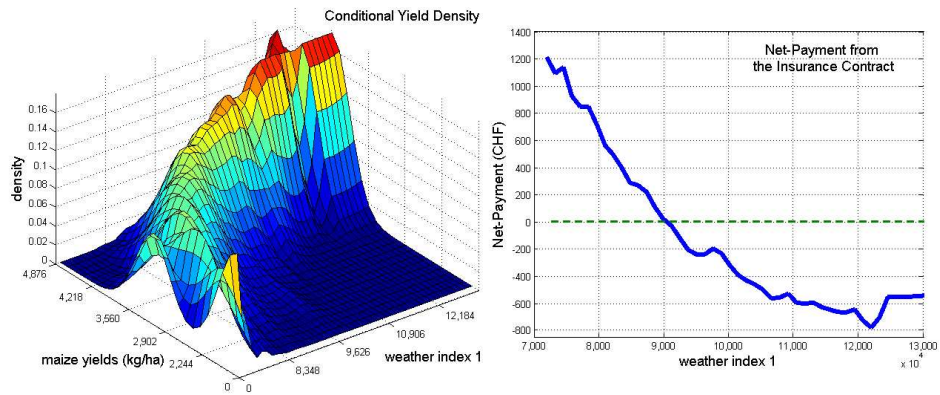
Figure 11: Conditional yield density and insurance contract for index 1
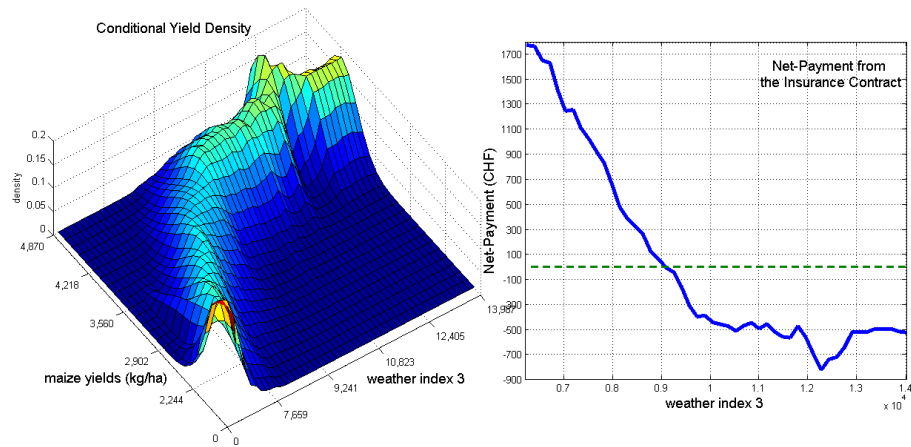

Figure 12: Conditional yield density and insurance contract for index 3
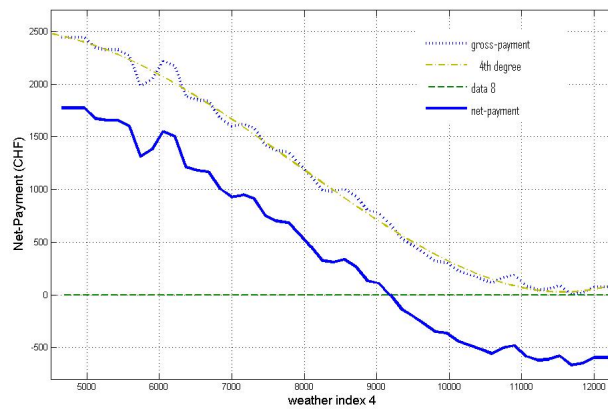

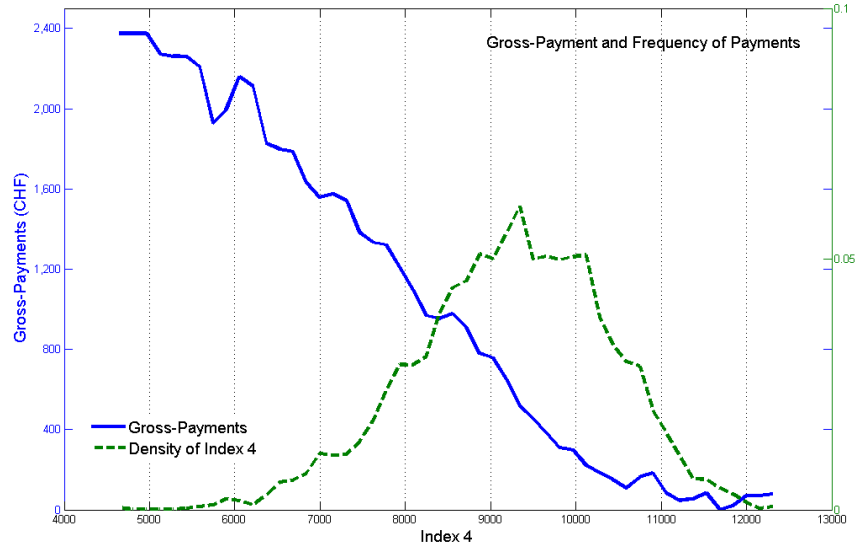Figure 13: Gross and net-payments for insurance contract 4

Figure 14: Gross-payments with payment probabilities for insurance contract 4

Table 12: Comparison of income distributions for the optimal and profit-maximizing insurance contracts

	Not Insured	Index 1	Index 2	Index 3	Index 4
mean	3696	3691	3691	3691	3689
	-	*(3648)*	*(3630)*	*(3630)*	*(3619)*
std	576.9	436.0	369.0	379.3	338.2
	-	*(436.2)*	*(369.0)*	*(379.4)*	*(338.2)*
skw	-0.73	-0.65	-0.54	-0.41	-0.50
	-	*(-0.65)*	*(-0.54)*	*(-0.41)*	*(-0.50)*
10%	2865	3109	3211	3203	3268
	-	*(3066)*	*(3151)*	*(3143)*	*(3199)*
25%	3337	3448	3478	3471	3501
	-	*(3404)*	*(3417)*	*(3411)*	*(3431)*
50%	3815	3749	3725	3713	3708
	-	*(3706)*	*(3665)*	*(3653)*	*(3638)*
75%	4147	4000	3946	3946	3908
	-	*(3956)*	*(3885)*	*(3886)*	*(3839)*
90%	4349	4193	4138	4135	4094
	-	*(4150)*	*(4078)*	*(4075)*	*(4024)*

Note: In brackets are the results for the profit-maximizing contract. Units: CHF/ha, crop: maize, location: SHA, model parameters: $\sigma = 2, ny = 25, nz = 50, bw(1) = 100, bw(2) = 300$.

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