Hedonic Regressions, Matched Models and Economic Theory

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PRELIMINARY DRAFT

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ABSTRACT

Quality adjustment of price indexes affects the analysis of many sensitive economic issues, such as real growth, productivity, international competitiveness, real wages, per-capita consumption and poverty, other than inflation. Hedonic methods are often recommended and increasingly used in the compilation of consumer price indexes. Nevertheless many official statistical agencies continue adopting traditional methods considering only the dynamics of prices of products matching in two adjacent periods of time. Indeed, a number of studies have even recently remarked that hedonic methods sometimes provide results very similar to the traditional matching models approach, particularly when models included in price index sample are replenished frequently. This paper briefly surveys the economic theory behind hedonic and traditional quality adjustment methods, and demonstrates that average price changes estimated by hedonic regressions differ from matched models estimation only because of the sum of regression residuals associated to disappearing and new models included in the sample. Thus, hedonic regressions including among the explanatory variables some indicators of the novelty and oldness of models provide exactly the same results of traditional methods. This fact casts some doubt on the overall effectiveness of hedonic methods in quality adjustment. The paper also focuses on that some economic and statistical hypotheses underlying hedonic methods possibly conflict with the assumptions and practices embodied in compiling the harmonised index of consumer prices for European countries.

KEYWORDS: Consumer price index, Harmonised Index of Consumer Prices (HICP), Hedonic regressions, Matched models, Measurement of inflation, Quality adjustment.

J.E.L. CLASSIFICATION: C43, C82, E31.

Table of contents

1. Introduction [(*)]

In compiling consumer price indexes (CPI), proper adjustment of prices for quality changes of products is crucial, as has been recognised by the Boskin Report (1996) and the Schultze and Mackie Report (2002), among the others. Indeed, whatever over-evaluation of inflation, due to neglecting quality improvement of goods and services over time, produces a symmetrical under-estimation of economic growth. Thus quality adjustment of prices affects the analysis of many sensitive economic issues, such as real growth, productivity, international competitiveness, real wages, per-capita consumption and poverty, other than inflation.

Hedonic methods are often recommended in quality adjustment of price indices [1] and increasingly used in CPI compilation. Thus, the economic hypotheses underlying hedonic regressions should be carefully analysed, and compared to those already embodied in the practices adopted in compiling the European Harmonised Consumer Price Index (HICP), elaborated in the European Union. In addition, a number of studies (including Triplett, 2001, and Diewert, 2001) have even recently remarked that these methods sometimes provide results very similar to traditional matching models approach, particularly in connection with a frequent replenishment of models in the CPI sample (see Silver and Heravi, 2001, and Pakes, 2003).

It turns out, quite surprisingly, that adopting two different sets of hypotheses about quality adjustment may provide almost the same estimation of price index changes, even if the treatment of each model may differ considerably as well. It could be argued that data structure is usually such that both hedonic and matching approaches are virtually indistinguishable on empirical ground (as considered only as a special case by Triplett, 2004). As far as price data are the outcome of market functioning, this fact implies that market works in such a way that the prices dynamics of matched models does not differ too much from that of both new and disappearing models. A naïve economist would conclude that homogeneity in price changes supports the hypothesis that apparently the market is quite efficient, since a weak version of the "law of one price" ultimately holds. On the other hand, according to a more pessimistic view, both hedonic regressions and traditional procedures could be unable to

[(*)] The author is the only responsible for the opinions reported in this paper, which do not involve the institutions he is affiliated to under any respect. The author gratefully acknowledges the suggestions and criticisms come from some researchers who read the very first draft of this paper, even if they are not liable for the remaining errors.

capture some relevant quality changes occurring in goods and services, since the effect of very new characteristics included in appearing commodities are hard to be identified and estimated by using old data. If such factors were taken into account properly, hedonic adjustment should differ from traditional approach systematically.

The results of hedonic and traditional quality adjustment methods can be also explained on the pure statistical ground. It is easy to demonstrate that "pure" average price changes estimated by hedonic regressions differ from matching model estimation only for a (possibly small) fraction of the sum of regression residuals associated to non matching models (i.e.: disappearing and new product offers).

The following section briefly surveys the economic theory behind hedonic methods, only to point out some critical assumptions and their consequences in price index compilation. Section 3 briefly considers the procedures inspired to hedonic methods, adopted to adjust for quality changes the prices collected within the actual CPI surveys. The fourth section provides some economic background to the matched models procedure and analyses the statistical relationships between the latter and the estimation of hedonic price indices. Finally, section 5 compares some assumptions underlying hedonic methods with the rules accepted in HICP compilation. It turns out that most assumptions are neutral or consistent with HICP philosophy, excepted for rebasing and chaining. Indeed, the latter agrees with traditional matching models procedure.

2. The economics of hedonic methods

According to the hedonic methods, the price $P_{i,t}$ of the i-th "variety" (or "model") of the same good or service, observed at time t, depends on a vector of measurable characteristics, say $\mathbf{z'}_{i,t} = (z_{1,i,t}, \ldots, z_{K,i,t})$. Thus, in principle, the implicit price of each characteristic can be obtained by regressing a set of prices against the related corresponding vectors $\mathbf{z'}_{i,t}$. Of course, the relationship between $P_{i,t}$ and $\mathbf{z'}_{i,t}$ is possibly non linear.

At least two theoretic approaches seem to justify the practice of hedonic regressions. The first one dates back to the theory of implicit market for characteristics introduced by Rosen (1974) in his seminal paper, according to which a virtual market for characteristics exists rather than for goods. That is, consumer purchases a particular model only because it

[1] See Eurostat (1993), par. 16.126, among the others.

includes every desired characteristic in the preferred ratio. According to this view, the price of each model is simply a "bill", summing up the cost of each characteristic. That is

$$P_{it} = z_{1,i,t}c_{1,t} + \ldots + z_{K,i,t}c_{K,t} + u_{i,t} \qquad [1]$$

where $z_{h,i,t}$ is the (observable) quantity of the h-th characteristic included in i-th good at time t, and $c_{h,t}$ is the corresponding unit price, that is usually unobservable; $u_{i,t}$ is a stochastic disturbance that can be interpreted as the divergence between the actual market price and its "fair" value, justified by the characteristics of the i-th model. Thus, if $u_{i,t} < 0$ the i-th model can be considered a bargain (see also Griliches, 1961). If $c_{h,t}$ can be observed equation [1] provides the basis for option price quality adjustment, sometimes adopted by official statistical agencies. [2] Otherwise the vector of coefficients $c_{h,t}$ should be estimated from a set of prices and characteristics actually collected at time t, in order to evaluate the (virtual) quality adjusted price of models that cannot be observed at time t.

Hedonic regressions can be derived from the maximization of consumer utility under some special restriction as well. It is not by chance that hedonic methods were initially developed in the context of cost of living indices, as a tool to estimate quality-adjusted prices given a constant utility level (see Feenstra, 1995). According to this approach, derived from the traditional theory of consumption popularised by Muellbauer (1974), the price $P_{i,t}$ could be seen as the *product* of two components:

- the utility function $U(\mathbf{z}_i, \boldsymbol{\theta}, \boldsymbol{\Omega})$ related to the use of i-th model, which in turns depends on its characteristics $\mathbf{z}_i$, the associated $\boldsymbol{\theta}$ parameters, and on the influence of other goods $\boldsymbol{\Omega}$ included in the consumer basket;
- the price R_t of a unit of consumer utility, regardless to the good or service that increases consumer utility. Ideally, this price refers to a cost of living index (COLI), not to a pure price index, as considered by Eurostat (1999);

Thus, it reads

$$p_{i,t} = \rho_t + u(\mathbf{z}_{i,t}, \boldsymbol{\theta}, \boldsymbol{\Omega}) \qquad [2]$$

where $p_{i,t} = ln(P_{i,t})$, $\rho_t = ln(R_t)$, and $u(.) = lnU(.)$.

It should be remarked that the use of a unique utility function for all of the consumers implies the additivity of utility among individuals. That is, only private individual consumption should affect individual utility, and the quality adjustment as well, while

[2] See Hoven (1999) for a thoroughly analysis of the Dutch experience.

interaction among consumers does not matter. Thus, important issues such as externalities, rivalry in resources use, etc. must be disregarded. For instance, this assumption states that smoking increases smokers' utility without decreasing the welfare of non-smokers staying in the same room. Furthermore, in the case of Internet and other network services, additivity implies that the number of users connected at the same time does not influence consumer utility, contrarily to the common experience. Also the utility of a computer would be independent from the diffusion of standards in software, operating systems and data transmission.

In order to implement the hedonic approach empirically, the utility function is further decomposed in two multiplicative components:

- the sub-utility function $f(\mathbf{z}, \boldsymbol{\theta})$ associated to the consumption of i-th model, regardless to the rest of consumer basket;
- the utility function associated to the use of $\boldsymbol{\Omega}$, which, in turn, is held constant over time.

The latter decomposition is allowed if and only if an additional restriction on consumer behaviour is imposed: that is utility is *separable* among different goods. Separability means that only the characteristics of a single product determines its utility, regardless to possible interactions with other goods and services. It makes quality adjustment much simpler, since it enables CPI compilers to regard only characteristics observed within the sample of models. As a consequence, the sum of quality adjustments applied to the entire consumer basket turns out to be different from (and likely lower than) actual overall utility change. For instance, separability implies that utility (and quality adjustment) of computers does not depend on improvement in software and network connection. Thus, as computers, software and telephone lines improve together; their prices adjusted separately by using hedonic methods still overestimate both COLI and inflation. In a sense, separability assumption is linked to additivity: the former holds among products, the latter among consumers. Anyway, separability is almost indispensable to make hedonic approach practicable, since otherwise the dimension of $\mathbf{z}$ and $\boldsymbol{\theta}$ vectors included in $U(\mathbf{z}, \boldsymbol{\theta}, \boldsymbol{\Omega})$ would be intractable.

An additional set of assumptions concerns the functional form of $f(\mathbf{z}, \boldsymbol{\theta})$, which is not allowed to be too complicated in order to estimate the parameters $\boldsymbol{\theta}$ from real data. For instance, *translog* utility function, usually adopted in demand systems (Pollak and Wales, 1992), are hard to be utilised in hedonic regressions, since they take into account also the effect of every pairs of characteristics. It is worth noticing that, in such models, the marginal utility of each characteristic depends on the value of other characteristics as well, as it is usually the case. In a sense, excluding cross-relationships among characteristics extends the separability assumption from products to characteristics.

This further restriction imposed to f($\mathbf{z}$, $\boldsymbol{\theta}$) could be very binding in most cases. Namely, in the case of computers, it implies that the contribution to overall utility of a bigger hard disk is independent from the RAM size, the processor speed, etc. In the case of cars, simplifying f($\mathbf{z}$, $\boldsymbol{\theta}$) means that the utility of peak speed is not affected by brakes size.

In addition, f($\mathbf{z}$, $\boldsymbol{\theta}$) is usually assumed to be time invariant. This further hypothesis is almost necessary in order to deal with a tractable model, whose parameters are well identified and have a finite estimate covariance matrix. As far as $\boldsymbol{\theta}$ is allowed to change over time, it induces a strong relation between the estimates of $\boldsymbol{\theta}$ and the coefficients of time dummies.

In the very end, most of hedonic regressions assume that the logarithm of f($\mathbf{z}$, $\boldsymbol{\theta}$) is constant over time, and is linear or log-linear respect to the characteristics of goods. In fact, Diewert (2001) pointed out that the linear form of [1] is not fully compatible with the derivation of hedonic regressions from the consumer utility theory, recommending the logarithmic form. In the latter case, the coefficients $c_{h,t}$ should be viewed as complicated functions of the parameters of the utility function of the consumer, underlying his choices and the acceptance of price $P_{i,t}$.

Many restrictions on [1] suggested by the economic theory could be ignored if hedonic regressions are considered as a pure statistical tool to project the N-dimensional vector of prices on the K-dimensional characteristics space. Notably, if K=N the projection turns out to be a pure linear algebraic transformation of prices. If K>N the result is undetermined, and if, as it is usual, K<N the coefficients $c_{h,t}$ can be estimated by using suitable statistical techniques. However, if this is the case, CPI should be defined conformingly as an index of characteristics prices, instead of products prices. Hence, classification should be revised, in order to take into account products attributes instead of product offers. For instance, statistical agencies should compile a hedonic index of “speed” price, based on bus services, private cars, etc.

3. Quality adjustment procedures based on hedonic regressions

Regardless to the economic theory behind hedonic regressions, at least three basic procedures are commonly adopted by researchers and statistical agencies: the time dummy variables adjustment, the characteristics price index, and the imputation method.

The time dummy variables adjustment, introduced by Griliches (1961),[3] "pools" in the same regression [1] the prices of models observed in different points of time, including among the regressors a set of T-1 time dummies D_t ($t = 1, \dots ,T$), whose value is 1 if the observation relates to time t and 0 otherwise. Usually, only two time periods are concerned, so that t = [T-1, T], therefore, only one time dummy, say D_T, is defined. If π_t is the (unknown) average price change during the time span (T-1, T), then an "augmented" version of the log-linear version of [1] reads [4]

$$p_{i\,t} = \pi_t D_t + z_{1,i,t}c_1 + \dots + z_{K,i,t}c_K + u_{i,t} \qquad [3]$$

Thus, the estimate of π_t provides the "pure" average price changes of the good, excluding the effect of quality changes, if any. By summing member by member the equations [3] related to prices observed at time t=T and subtracting those related to time t=T-1, the average price changes between the time periods T-1 and T reads

$$\Delta p_T^a = \pi_T = \Delta p_T - \left(\frac{1}{N_T} \sum_j c_j \sum_i z_{j,i,T} - \frac{1}{N_{T-1}} \sum_j c_j \sum_i z_{j,i,T-1} \right) \cong$$

$$\cong \Delta p_T - \frac{1}{N} \sum_j c_j \left(\sum_i z_{j,i,T} - \sum_i z_{j,i,T-1} \right) \qquad [4]$$

where $\Delta p_T = \frac{1}{N_T} \sum_i p_{i,T} - \frac{1}{N_{T-1}} \sum_i p_{i,T-1}$ is the unadjusted average price change, computed on the full sample of prices (often referenced as "unit value"); N_t is the number of prices collected at time t and N lies between N_T and N_{T-1}. The second row of [4] holds exactly if the sample size for CPI estimation is almost constant over time.

It is worth noticing that, in order to make the econometric estimation being workable, model [3], at variance with [1], imposes a strong constraint to the coefficients associated to

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[3] In his invaluable historical digression on hedonic methods, Triplet (2004) goes back to the thirties' finding out examples of the (implicit) application of hedonic regressions.

[4] Only for sake of notational simplicity, henceforth I use the subscript i in referring to different models both at time T and T-1, even if it is intended that, strictly speaking, i identifies the same model only for matched models.

the characteristics, that is they are time invariant between T-1 and T. The last row of [4] does not implies that $\Delta p_T^a = \Delta p_T$ holds only if every characteristic of each model is exactly the same over time, that is if $z_{h,i,t} = z_{h,i,t-1}$, but simply if the difference between $\sum_i z_{j,i,T}$ and $\sum_i z_{j,i,T-1}$ is null. Thus, a possible compensation among the changes in the characteristics of products is considered.

Another hedonic method, usually preferred by statistical agencies, is the characteristics price index method, already discussed by Griliches (1971). It is based on a set of separate regressions of the form [1], one for each time period. This approach removes the strong assumption on the time invariance of the coefficients, allowing for each characteristic being evaluated differently in each period of time. Once two sets of coefficients have been estimated, it is possible to evaluate the overall price change of a given amount of characteristics at the prices prevailing during two subsequent periods of time. Namely, considering the characteristics of the models actually available at time T-1, according to the the Laspeyres index philosophy, [5] the average price changes between the time periods T-1 and T is

$$\Delta p_T^a = \Delta p_T - \left(\frac{1}{N_T} \sum_j c_{j,T-1} \sum_i z_{j,i,T} - \frac{1}{N_{T-1}} \sum_j c_{j,T-1} \sum_i z_{j,i,T-1} \right) \cong$$

$$\cong \Delta p_T - \frac{1}{N} \sum_j c_{j,T-1} \left(\sum_i z_{j,i,T} - \sum_i z_{j,i,T-1} \right) \qquad [5]$$

Thus, the final result of adopting the characteristics price approach is almost similar to that of the time dummy variable adjustment showed by [4]. Formally, the only difference is the use of the set of coefficients $c_{j,T-1}$ instead of the average estimates c_j. Notably, [5] depends on the assumption that the vector **z** consists of exactly the same elements at time T-1 and T, namely no feature can be considered if it is completely new and was not present also in the products available at time T-1.

The hedonic imputation method admits several variants, all based on the idea that equation [1] allows to estimate the (virtual) price of any model available only in one period

[5] Also, a Paasche type index can be defined, considering the characteristics of the models available at time T, instead of T-1, or even a Fischer type formula combining the Laspeyres and the Paasche indexes.

but not in the others. For instance, at time T-1, the price of a model introduced into the market at time T, but unavailable at time T-1 yet, is

$$p^*_{i,T-1} = z_{1,i}\, c_{1,T-1} + \ldots + z_{K,i}\, c_{K,T-1} \qquad [6]$$

The estimates $p^*_{i\,t}$ can be utilised in compiling the usual consumer price indexes in place of the missing prices, if any. [6]

From [4], [5] and [6], it is easy to demonstrate that the various methods based on hedonic regressions provides a "generalised" estimator for the quality adjusted price changes that reads

$$\Delta p_T^a \cong \Delta p_T - \frac{1}{N}\sum_j c_j \sum_{i\in M}\left(z_{j,i,T} - z_{j,i,T-1}\right) -$$

$$-\frac{1}{N}\sum_j c_j \left(\sum_{i\notin M} z_{j,i,T} - \sum_{i\notin M} z_{j,i,T-1}\right) -$$

$$-\frac{1}{N}\left(\alpha \sum_{i\notin M} u_{i,T} - \beta \sum_{i\notin M} u_{i,T-1}\right) \qquad [7]$$

where c_j is a set of coefficients amid $c_{j,T-1}$ and $c_{j,T}$, and M is the set of matched models, whose prices are collected both at time T-1 and T; and the parameters α and β may be 1 or 0.

Since, by definition, the features of matching models do not change over time, the first summation in [7] disappears for whatever set of coefficients c_j, thus [7] gives

$$\Delta p_T^a \cong \Delta p_T - \frac{1}{N}\sum_j c_j \left(\sum_{i\notin M} z_{j,i,T} - \sum_{i\notin M} z_{j,i,T-1}\right) -$$

$$-\frac{1}{N}\left(\alpha \sum_{i\notin M} u_{i,T} - \beta \sum_{i\notin M} u_{i,T-1}\right) \qquad [8]$$

The summation in first row of [8] measures the difference between the average value of new and replenished models in case market prices exactly match the quality of goods. The second row of [8] includes the regression residuals for non matched models.

[6] Silver and Heravi (2002) and Pakes (2003) suggest different procedures in order to take into account the influence of regression residuals, that signal if the estimated price is a bargain price or not.

Of course the terms $u_{i,t}$ in [8] are observed only if p_{it} is observed and is included in the hedonic regression, otherwise it is assumed $u_{i,t} = 0$. If $\alpha = 0$, every actual (observed) price of a new model enters directly in the computation of the CPI and an hedonic estimate for its (virtual) value at time T-1 is needed, hence equation [8] exactly replicates [4] or [5] depending on the value assigned to the coefficients c_j. If $\beta = 0$ also the prices of models available at time T-1 and disappeared at time T continue to be considered in the index, and an estimation for their current values is needed. If both α and β are set to 1, only the "virtual" prices of unmatched prices enter the index, in accordance with a version of hedonic imputation.

4. Hedonic regressions and matched models procedure

Many statistical agencies compute average price changes by using only data of matched models collected during two succeeding periods. In doing so, they implicitly adopt the following estimator of Δp_T^a

$$m_t = \frac{1}{K} \sum_{i \in M} (p_{i,T} - p_{i,T-1}) \qquad [9]$$

where K is the number of matched models.

The economic hypothesis underlying such estimator is that the dynamics of the price of matched models do not differ significantly from the one of disappearing and new goods. This would be the case in an ideal perfect competitive market, where arbitrage and competition make it impossible for a firm to sell a new model at a price higher than the one justified by special characteristics included in that model. If perfect market hypothesis is rejected, weak consumer rationality, or preferences consistency, is required. [7] Anyway, in this ideal world, consumers are purchasing baskets of characteristics instead of single good and services. The latter assumption exactly matches the hypothesis of "implicit markets" underlying the hedonic approach.

Diewert (2001) and Triplett (2001), among the others, have pointed out that estimator m_t in [9] coincides with hedonic estimate of Δp_T^a in [4] and [5] under special, but not

[7] See D'Elia (2000) among the others.

unlikely, circumstances. Here I present a slightly different result, almost simplified in a sense, since formally it refers only to the time dummy variables adjustment, but more general in some other respects, because it considers any arbitrary quality adjustment function.

Let $h(\mathbf{z}_{it}, \theta)$ be a quality adjustment function for the i-th model purchased at time t, so that

$$p_{i,t} = \pi d_t + h(\mathbf{z}_{i,t}, \theta) + e_{i,t} \qquad [10]$$

where θ is a vector of parameters conformable to the set of characteristics $\mathbf{z}_{it}$. For sake of simplicity, let consider only the two period problem, in which π is a scalar and d_t equals 1 if the price $p_{i,t}$ refers to the time T, and 0 otherwise. If the estimator of π in [10] is unbiased it turns out that the residuals $e_{i,t}$ must be such that

$$\sum_{i,t} e_{i,t} = \sum_i (p_{i,T} - \pi - h(\mathbf{z}_{i,T},\theta)) + \sum_i (p_{i,T-1} - h(\mathbf{z}_{i,T-1},\theta)) = 0 \qquad [11]$$

Another property commonly required to the estimators is that explanatory variables in the model do not convey any information about residuals and vice versa. This is essentially a normalization of residuals, which may not be fulfilled if explanatory variables are endogenous, that is if some qualitative feature of products depends on $p_{i,t}$ in turn, as assumed, for instance, by Stiglitz (1987).

In [10], the independence of explanatory variables must hold, first of all, for the time dummy d_t. A weak version of this assumption is orthogonality, that reads $\sum e_{i,t}\ d_t = 0$, and implies

$$\sum_i (p_{i,T} - \pi - h(\mathbf{z}_{i,T},\theta)) = 0 \qquad [12]$$

It should be noted that, by definition, d_t is a fully pre-determined variable, thus the possible bias related to endogeneity is excluded in [12]. Ordinary least square and other estimators of π assume unbiasedness of residuals and orthogonality between residuals and explanatory variables.

The conditions [$unbiased13

$$\sum_i (p_{i,T-1} - h(\mathbf{z}_{i,T-1},\theta)) = 0 \qquad [14]$$

It is worth noticing that the condition [12] and [14] are much more stronger than unbiasedness [$unbiased15.

Distinguishing between matching and non matching models in [12] and [14], and taking into account that $\mathbf{z}_{i,T} = \mathbf{z}_{i,T-1}$ for matching models, it reads

$$\sum_{i \in M} (p_{i,T} - \pi - h(\mathbf{z}_{i,T},\theta)) = - \sum_{i \notin M} e_{i,T} \qquad [16]$$

and

$$\sum_{i \in M} (p_{i,T-1} - h(\mathbf{z}_{i,T},\theta)) = - \sum_{i \notin M} e_{i,T-1} \qquad [17]$$

The difference between [16] and [17], divided by the number of matching models, gives

$$\pi = \frac{1}{K} \sum_{i \in M} (p_{i,T} - p_{i,T-1}) + \frac{1}{K} \sum_{i \notin M} e_{i,T} - \frac{1}{K} \sum_{i \notin M} e_{i,T-1} \qquad [18]$$

Thus, the estimation of π equals the one based on matching models, i.e. m_t, as defined by [9], plus the difference between the residuals attached to non matched models at time T and T-1, both divided by K.

It is worth noticing that equation [18] is very general, since it holds regardless to the form of quality adjustment function h(.). In particular, the latter can derive either from a subjective judgement or a high sophisticated hedonic method. In addition, residuals orthogonality is imposed only respect to the time dummy d_t but not to the vector of characteristics $\mathbf{z}$, thus a very weak hypothesis about estimation of θ is made here. Indeed, the only strong requirement for [18] concerns the time invariance of h(.), that can be violated when consumer preferences are fast changing, such as in the case of products strictly related to fashion. Finally, equation [18] can be easily generalised to the case of weighted observations, e.g. by using data on models sales.

Equation [18] has a number of interesting consequences. First of all, if the number of non-matching models is much less than K, and the residual size is small enough, the correction factor $\frac{1}{K}\left(\sum_{i \notin M} e_{i,T} - \sum_{i \notin M} e_{i,T-1} \right)$ is negligible compared to π. Thus, the more the quality adjustment is accurate, the lesser residuals size is expected, and thus a very sophisticated hedonic regression tends to produce results very similar to a rude matching models comparison. Nevertheless, if new models tend to have higher prices compared to the disappearing ones, even after having adjusted prices for quality changes, then the correction factor should be positive, and an estimator base don matching models would underestimate π.

The opposite happens if new models enter the market at a lower price in order to crowd out old models.

In any case, the correction factor is always null if products characteristics include some indicators of novelty and obsolescence. The latter result derives directly from introducing in the hedonic model [10] two explanatory variables defined as follow: a “novelty” dummy which is 1 if the model enters the sample for the first time, and zero otherwise; an “oldness” dummy which is 1 if the model disappeared at time T, and 0 otherwise. In fact, the orthogonality assumed between residuals and explanatory variable ensures that

$$\sum_{i \notin M} e_{i,T} = \sum_{i \notin M} e_{i,T-1} = 0 \qquad [19]$$

The result [19] holds approximately if some characteristics included in hedonic regressions actually are strongly correlated with “newness” an “oldness”. For instance, in the case of cars, hybrid technology is a good proxy to identify new models entering the sample for the first time at time T but unavailable at time T-1, while only old models, possibly disappearing at time T, have less than four valves per cylinder and are not compliant to the latest ecological standard.

Notably, the condition [19] also makes the last row of [8] null, thus the imputation method gives

$$\Delta p_T^a \cong \Delta p_T - \frac{1}{N}\sum_j c_j \left(\sum_{i \notin M} z_{j,i,T} - \sum_{i \notin M} z_{j,i,T-1} \right) =$$

$$= \frac{1}{N}\sum_j c_j \left(\sum_{i \in M} z_{j,i,T} - z_{j,i,T-1} \right) \cong$$

$$\cong \frac{1}{N}\sum_{i \in M} \left(p_{i,T} - p_{i,T-1} \right) \qquad [20]$$

where the last quasi–equivalence assumes that the sum of regression residuals of the prices of matching models is negligible.

Equations [18] and [20] show that, when the special qualitative features of new entries and replenished products are fully taken into account, the results of hedonic adjustment converge to that of the traditional matching model adjustment. On the other hand, the difference between the two procedures are larger as the regression residuals are bigger and unevenly distributed among new, old and matching models, that is if the regression models fit worse the data and the residuals are heteroschedastic. This fact casts some doubt on the overall effectiveness of hedonic methods in quality adjustment.

Even if "newness" and "oldness" of products are disregarded, the correction factor in [18] and its corresponding item in [8] are almost null if statistical agencies tend to include in the sample new goods whose price diverges from the hedonic value in the same direction and size of excluded ones. That is if they apply implicitly a "smart" one-to-one replacement of old models.

As far as the accuracy of estimator π is concerned, it is obvious that, in principle, m_t is less efficient than the hedonic estimator, owing to the effect of correction term in [18]. Efficiency loss is small under the same conditions that make bias negligible. Nevertheless, in small samples, it is likely that the uncertainty on estimate of parameters θ of quality adjustment function exceeds the potential gain in efficiency.

5. The hypotheses underlying HICP compilation and hedonic methods

The previous sections have pointed out that hedonic regressions are based on some economic and statistical hypotheses. Some of them possibly conflict with those commonly accepted in the compilation of the HICP and many other national CPIs'.

First of all, strictly speaking, periodical rebasing and chaining of HICP seems to mimic a quality adjustment procedure based matching models. As matter of facts, chaining two indexes including different models implies that quality differences are completely disregarded during the chaining period. The change in weights from one basis to the following one may attenuate the effect of quality changes only assuming that consumers are so rational and reactive that they choose every time the basket of goods that provides them with the same utility as in the previous period of time. Nevertheless, if consumers are so "smart" to achieve a higher utility, then the implicit quality adjustment due to rebasing would overestimate inflation. Actually, chaining turns out to be a general and comprehensive matching models procedure applied to the consumer basket as a whole. As matter of facts, during chaining, all the price difference between the two baskets involved is regarded as quality. The same fact happens when only matched models are utilised in computing inflation monthly, since the effect of non-matching component in each month is completely disregarded.

Also the procedure agreed for the inclusion of new countries in the HICP for the Monetary Union (MUICP)[8] is consistent with periodical rebasing and chaining, since during the month J in which the new countries join, two indices are computed: the one excluding that countries and the other including. The former is utilised to compute MUICP changes until J, the latter later. Quality difference between the two baskets is disregarded again, and only matching countries (and related goods and models) are considered every month.

HICP is meant explicitly to be a pure inflation index and not a cost of living index (COLI),[9] while a strict interpretation of hedonic regressions implies that hedonic indices measure the changes in the price of one utility unit. On the other hand, this assumption is not necessary if regressions are regarded as simple mapping of models prices on the characteristics price space. However, in this case the definition of HICP changes slightly from a goods and services price index to a characteristics price index.

The hypothesis that consumer's utility can be summed up is quite binding, but is essentially consistent with price index compilation. The weighting system is computed by adopting such assumption implicitly, since individuals' consumption expenditures are summed up as well. In other words, it is excluded the possibility that somebody's consumption may worsen other people's condition, albeit this is a key issue of economic theory.

Utility separability among products is another questionable assumption underlying hedonic regressions. It implies that purchases are mutually independent. However, this hypothesis seems almost neutral for price index compilation practices, since weights of different products are determined considering only actual consumption shares, disregarding the interaction between consumer choices. For instance, purchase of music CDs depends surely on CD players owned by consumers, but HICP compilers do not need to analyse this relationship in order to establish the relevant weights, since they simply collect data on actual sales of CDs and CD players, which already embody the effects on the structure of consumer expenditure of the interactions between the utility of the two goods. In addition, separability is consistent with the fact that HICP tends to measure prices and not user costs. The former is the amount of money due to purchase a product, the latter depends on goods and services

[8] See Eurostat (1999), pp. 87-91.
[9] See Astin (1999).

necessary to utilise the product. For instance, cars price index does not depend on gasoline and garage prices, highway tolls, etc. The latter are accounted in HICP as separate issues.

On its turn, matching models procedure relies heavily on market equilibrium hypothesis. If prices do not reflects such a situation, quality adjustment of single non-matching models might be unreliable. For instance, the average price change of matching models could be higher than those of disappearing (outdated) products. However, if the current price sample includes both newly introduced and disappearing models, then individual quality adjustment distortions may compensate on average. On the other hand, if a new model has completely new features, it is very hard to estimate the implicit value of such characteristics from a sample of old products. Thus, also hedonic methods are fully safe from the possible bias due to different price policies for new and disappearing goods.

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