Exogenous and endogenous crashes as phase transitions in complex financial systems

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Abstract

In this paper we provide a unifying framework for a set of seemingly disparate models for exogenous and endogenous shocks in complex financial systems. Markets operate by balancing intrinsic levels of risk and return. This remains true even in the midst of transitory external shocks. Changes in market regime (bearish to bullish and bullish to bearish) can be explicitly shown to represent a phase transition from random to deterministic behaviour in prices. The resulting models refine the empirical analysis in a number of previous papers.

Keywords: Exogenous; Endogenous; Financial Crashes; Bubbles; Econophysics

1 Introduction

There is a burgeoning literature discussing endogenous/exogenous dynamics in complex systems [1]-[7]. This includes various applications to financial and social systems [4] including book sales [5]-[6], financial markets [8] and internet downloads [9]. This has occurred alongside various interrelated models for financial crash precursors [10]-[12]. This makes use of the oft-cited analogy between financial market crashes and phase transitions in critical phenomena, based upon lattice models in statistical physics [12]. The literature thus provides a series of essentially separate models for crash-precursors [10]-[12] and for associated after-shock patterns [8], [14].

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This paper provides a novel framework unifying this seemingly disparate set of models. Financial markets operate by balancing an intrinsic rate of return against an intrinsic level of risk. Further, the analogy between regime-changes in financial markets and phase transitions in critical phenomena is made explicit. Such changes are shown to represent a phase transition from random to deterministic behaviour in prices.

In terms of empirical applications our model for exogenous shocks refines similar analyses conducted in [8], [13]. Our model for speculative bubbles allows a more systematic treatment of log-periodic and related models for financial crashes and can be shown to lead to several interesting empirical applications discussed in [15]-[18].

The layout of this paper is as follows. Our mathematical model for exogenous and endoenous shocks is discussed in Section 2. Section 3 covers the empirical application of the model for exogenous shocks. Section 4 concludes and discusses further work.

2 The model

Markets are assumed to work by balancing the level of risk and the rate of return. The level of risk and return remain fixed even in the face of technological innovation or an influx of new investors [19]. These assumptions do not rely on complicated mathematics and avoid dubious assumptions such as the "riskless hedge" of the Black-Scholes model [20]. Our model makes several observable predictions for market crashes. Exogenous crashes result in a decrease in drift and changes in volatility. Speculation-induced crashes are preceded by an unsustainable super-exponential growth coupled with a detectable increase in market over-confidence.

Let P_t denote the price of an asset at time t and let $X_t = \log\ P_t$. The set up of the model is as follows

Assumption 1 (Intrinsic Rate of Return) *The intrinsic rate of return is assumed constant and equal to μ:*

$$E[X_{t+\delta}|X_t] = \mu\delta + o(\delta). \tag{1}$$

Assumption 2 (Intrinsic Level of Risk) *The intrinsic level of risk is assumed constant and equal to σ^2:*

$$Var[X_{t+\delta}|X_t] = \sigma^2\delta + o(\delta). \tag{2}$$

2.1 Endogenous crashes

Related work in [8] and [13] consider a series of dramatic world-news events such as the 9/11 terror attacks and the attempted coup against Soviet President Mikhail Gorbachev in 1991 and focusses upon the observed volatility response. In [8] volatility is shown to decay following the exogenous shock at a rate quicker that is both faster than after a corresponding endogenous shock (bubble-induced crash) and in such a way that can be distinguished from the background noise.

Were the model in [8] correct, we would anticipate that the integrated volatility would display concave growth of the form $\sim t^{1-\alpha}$, with $\alpha \approx 0.5$. Some evidence for this is presented in Figure 1 for the case of the French Cac 40 index. Following the 9/11 terror attacks the left panel of Figure 1 shows that this appears to be a reasonable assumption. However, in the aftermath of the attempted coup in 1991, the evidence provided by the right panel is rather less compelling.

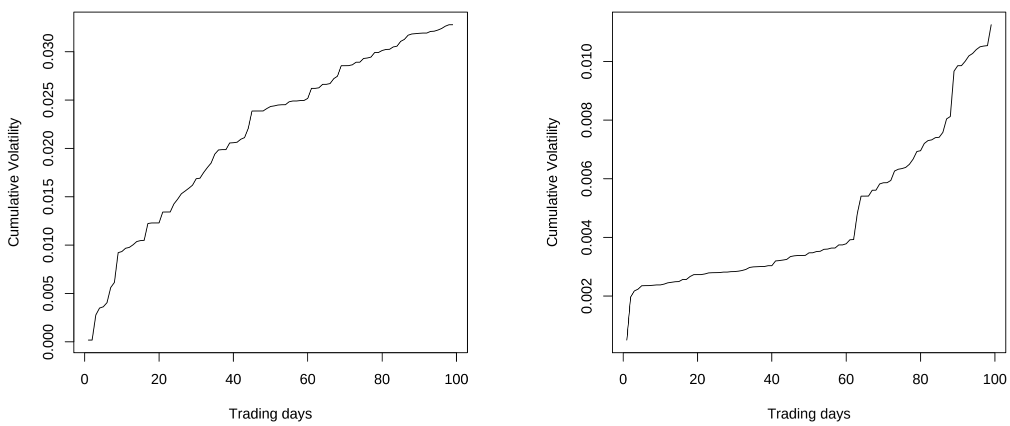

Figure 1: Cumulative "integrated" market volatility for the French CAC 40 index 100 trading days after putative exogenous shocks. Left panel: Market response to terror attacks of 9/11. Right panel: Market response to the attempted coup against President Gorbachev August 18th 1991.

Suppose that the market is exposed to an external shock. The shock is assumed to be completely unpredictable but as in [1]-[3] its affect is merely transitory. The shock occurs at time 0 and results in an initial decrease in drift by the amount μ_0 and an initial increase in volatility by the amount σ_0^2. At the random time t_0 the market recovers – the drift increases by μ_0 and volatility decreases by σ_0^2. The time t_0 of the market recovery is a random variable with hazard function $h(t)$. Since the effect of the external shock is transitory it follows that $h'(t) > 0$, since as time progresses a market rebound becomes increasingly likely. Also, since the shock is assumed to happen at $t = 0$ it follows that we must also have $h(0) = 0$.

$$h'(t) > 0; \quad h(0) = 0. \tag{3}$$

The price dynamics prior to the market recovery are described by the following equation

$$dX_t = \mu(t)dt + \sigma(t)dW_t + dj(t), \tag{4}$$

where $j(t)$ satisfies

$$dj(t) = \mu_0\delta(t - t_0)dt + i\sigma_0\delta(t - t_0)dW_t, \tag{5}$$

where $i = \sqrt{-1}$ and $\delta(\cdot)$ denotes Dirac's delta function. When a recovery happens, the effect is an increase in drift and a decrease in the variance, hence the introduction of $i = \sqrt{-1}$. Prior to the recovery we have that

$$E[X_{t+\delta}|X_t] = \delta(\mu(t) + \mu_0 h(t)) + o(\delta).$$

Thus, from equation (1) it follows that

$$\mu(t) + \mu_0 h(t) = \mu; \quad \mu(t) = \mu - \mu_0 h(t). \tag{6}$$

Equation (6) shows that an exogenous shock thus reduces the level of return. The risk (variance) associated with equation (5) is

$$\delta\left(\sigma^2(t) + \left(\mu_0^2 - \sigma_0^2\right)h(t)\right) + o(\delta).$$

Similarly, it follows from (2) that

$$\sigma^2(t) + \left(\mu_o^2 - \sigma_0^2\right)h(t) = \sigma^2; \quad \sigma^2(t) = \sigma^2 + \left(\sigma_0^2 - \mu_0^2\right)h(t). \tag{7}$$

If $\sigma_0^2 \geq \mu_0^2$ the external shock affects volatility more than it does the drift. The shock thus results in an increase in market volatility alongside a decrease in drift. If $\sigma_0^2 \leq \mu_0^2$ the external shock actually results in a reduction in volatility. However, irrespective of the effect upon market volatility the shock decreases the rate of return so is still likely to remain bad news for investors. If $\sigma_0^2 = \mu_0^2$ market volatility remains unaffected.

In empirical work we choose

$$h(t) = \lambda[1 - (1+t)^{-\alpha}]. \tag{8}$$

Not only does $h(t)$ in (8) satisfy (3) but the special case $\alpha = 0.5$ in (8) recreates both the empirical power-law reported in [8] and related phenomenology in [1]-[3]. Equation (8) also ensures that $h(t)$ is bounded – an important facet of empirical work on related models in [15]-[18] – and provides a natural test for the presence of an exogenous shock (see below). From (8) it follows that

$$\sigma^2(t) = \sigma^2 + \beta[1 - (1+t)^{-\alpha}], \tag{9}$$

where $\beta = \lambda(\sigma_0^2 - \mu_0^2)$. Equations (8-9) provide a natural way of testing for an exogenous shock in empirical data. The case $\alpha = 0$ corresponds to the case of an efficient market where price changes are completely unpredictable and we are left with the classical random walk or Black-Scholes model:

$$dX_t = \mu dt + \sigma dW_t.$$

We have that

$$\frac{\partial \sigma^2(t)}{\partial t} = \alpha\beta(1+t)^{-\alpha-1}. \tag{10}$$

The interpretation depends on the sign of $\alpha\beta$ and hence upon the sign of β in (10) since it is assumed $\alpha > 0$. If $\beta > 0$ then then $\sigma^2(t)$ increases without bound. This does not appear to be physically reallistic. In contrast, if $\beta < 0$ the market recovery becomes the inevitable phase transition between random and deterministic behaviour with

$$\lim_{t\to\infty} \sigma^2(t) = 0. \tag{11}$$

This suggests that

$$\sigma^2 + \beta = 0; \quad \sigma^2 = -\beta. \tag{12}$$

2.2 Endogenous crashes

In this section we show how the framework laid out by the equations (1-2) can be used to generate a model for financial bubbles discussed in [15]-[18] extending a deterministic version of the same model in [12] and a series of later papers [21]-[33] which all omit a critical second-order condition given by equation (18). Let $P(t)$ denote the price of an asset at time t. Our starting point is the equation

$$P(t) = P_1(t)(1-\kappa)^{j(t)}, \tag{13}$$

where $P_1(t)$ satisfies

$$dP_1(t) = \mu(t)P_1(t)dt + \sigma(t)P_1(t)dW_t,$$

where W_t is a Wiener process and $j(t)$ is a jump process satisfying

$$j(t) = \begin{cases} 0 & \text{before the crash} \\ 1 & \text{after the crash.} \end{cases}$$

When a crash occurs $\kappa\%$ is automatically wiped off the value of the asset. Prior to a crash $P(t) = P_1(t)$ and $X_t = \log(P(t))$ satisfies

$$dX_t = \tilde{\mu}(t)dt + \sigma(t)dW_t - vdj(t), \tag{14}$$

where $\tilde{\mu} = \mu(t) - \sigma^2(t)/2$ and $v = -\ln[(1-\kappa)]$ with $v > 0$. If a crash has not occurred by time t, we have that

$$\begin{aligned} E[j(t+\delta) - j(t)] &= \delta h(t) + o(\delta), && (15) \\ \mathrm{Var}[j(t+\delta) - j(t)] &= \delta h(t) + o(\delta), && (16) \end{aligned}$$

where $h(t)$ is the hazard rate. Hence it follows from (1) and (15) that

$$\tilde{\mu}(t) - vh(t) = \mu; \quad \tilde{\mu}(t) = \mu + vh(t). \tag{17}$$

Similarly, from (2) and (16) that

$$\sigma^2(t) + v^2h(t) = \sigma^2; \quad \sigma^2(t) = \sigma^2 - v^2h(t). \tag{18}$$

This model characterises bubbles as a dramatic super-exponential price rise shown by equation (17) and market over-confidence shown by (18). As $\sigma^2(t)\downarrow 0$ (18) shows that speculative bubbles are characterised by a phase transition between deterministic and random behaviour.

3 Empirical Application

Several works have looked at the empirical implementation of the endogenous bubble model in Section 2.2 [15]-[18]. This sits alongside a number of related papers, see e.g. [21]-[31] exploring the empirical implementation of the deterministic version of this model – omitting the second-order condition shown in equation (18). Here, in contrast, we fit the model for exogenous crashes in Section 2.1.

Following methodology in [8], [13] we fit equations (6-12) to data on real financial markets following putative exogenous shocks – namely the attempted coup against President Gorbachev and the terror attacks of 9/11. From equations (6-12) it follows that under this model the log-returns $\Delta X_t = X_{t+1} - X_t$ are independent and normally distributed:

$$\begin{aligned} \Delta X_t &\sim N\left(\mu_t, \sigma_t^2\right), \\ \mu_t &= \mu - \mu_0\lambda + \lambda\left[\frac{(2+t)^{1-\alpha}}{1-\alpha} - \frac{(1+t)^{1-\alpha}}{1-\alpha}\right], \\ \sigma_t^2 &= \beta\left[\frac{(2+t)^{1-\alpha}}{1-\alpha} - \frac{(1+t)^{1-\alpha}}{1-\alpha}\right] \end{aligned} \tag{19}$$

The model in equation (19) can thus be estimated by maximum likelihood. Evidence for an exogenous shock is found if we reject the hypothesis $\alpha = 0$ in favour of the hypothesis $\alpha > 0$.

Results in Table 1 show that the effects of the 1991 coup are largely muted. For the CAC 40, S& P 500 and the FTSE we retain the hypothesis $\alpha = 0$ at the 5% level. Any effects present are indistinguishable from the background noise and the hypothesis of an exogenous shock is rejected. In contrast, on the Nikkei the null hypothesis $\alpha = 0$ is rejected in favour of the hypothesis $\alpha < 0$. This result contravenes the model for exogenous shocks in Section 2 and in contrast seems to confirm evidence of an endogenous antibubble in the Nikkei identified in [32]-[33]. This interpretation also coincides with Japan's "lost decade" and recession in the early 1990s [33]-[35], with the observed price fluctuations a harbinger of future stock market falls still to come.

	$\hat{\alpha}$	e.s.e α	t-value	p-value
Nikkei	-0.288	0.125	2.294	0.024$*$
Cac 40	-0.075	0.107	0.708	0.481
FTSE	-0.223	0.131	1.702	0.092
S&P 500	0.024	0.133	0.179	0.858

Table 1: Maximum likelihood estimates of α in equation (8): 100 trading days after the attempted coup against President Gorbachev (August 18th 1991)

In contrast the results in Table 2 strongly support the hypothesis of an exogenous shock for the terror attacks of 9/11. In all cases the hypothesis $\alpha = 0$ is rejected in favour of the hypothesis $\alpha > 0$. Estimated values of α are in reasonable agreement with the estimate of $\alpha = 0.5$ obtained in [8]. However, our results suggest slightly higher values of α with $\alpha = 0.62 \pm 0.5$.

	$\hat{\alpha}$	e.s.e α	t-value	p-value	95 % Confidence Interval
Nikkei	0.502	0.150	3.350	0.001$**$	$(0.202, 0.801)$
Cac 40	0.588	0.178	3.308	0.001$**$	$(0.232, 0.944)$
FTSE	0.787	0.172	4.586	0.000$***$	$(0.457, 1.117)$
S&P 500	0.417	0.149	2.807	0.006$**$	$(0.120, 0.715)$

Table 2: Maximum likelihood estimates of α in equation (8): 100 trading days after the terrorist attacks of 9/11

4 Conclusions and further work

This paper provides a unifying framework for a seemingly disparate set of models for endogenous [12] and exogenous crashes [8], [13]. Financial markets function by balancing an intrinsic rate of return against an intrinsic level of risk. This feature is easy to lose sight of – particularly during bubbles and the introduction of new technologies/influx of new investors [19]. Our model also makes explicit the oft-cited link between financial crashes and phase transitions and critical phenomena. In particular, changes of market regime (bearish to bullish; Section 2.1) and (bullish to bearish; Section 2.2) are identified with a transition from random to deterministic behaviour in prices. Our model for exogenous shocks refines the analysis in [8], [13]. Similarly, the speculative bubble model in Section 2.2 allows for a more systematic treatment of log-periodic and related models for bubbles and crashes, whilst retaining the explicit analogy between physical and financial systems.

Several novel empirical applications of the models in Sections 2 are possible. Here, in our empirical application, we restrict attention to exogenous crashes. We examine the market response to putative exogenous shocks; the attempted coup against USSR's President Gorbachev and the terror attacks of 9/11. The 1991 coup does not constitute a significant exogenous effect upon global financial markets. Results are either indistinguishable from background noise or in the case of Japan's Nikkei index appear linked instead to the recession of the early 1990s and Japan's "lost decade". In contrast, the terror attacks of 9/11 *are* seen to represent an exogenous shock. Evidence of a power-law decay in volatility is found, with estimated exponent $\alpha = 0.62 \pm 0.5$ slightly higher than the value of 0.5 suggested in [8].

The models for endogenous and exogenous market crashes suggest a number of applications to risk management and several interesting avenues for further research. This paper sits alongside wider work on financial aspects of societal resilience [36] and future work will examine the implications for economic policy [37] and towards a more systematic treatment of market psychology and related themes discussed in [38].

References

[1] Crane, R. and Sornette, D. (2008) Robust dynamic classes revealed by measuring the response function of a social system. *PNAS* **105** 15649-15653.

[2] Saichev, A. and Sornette, D. (2010) Generation-by-generation dissection of the fesponse function in long memory epidemic processes. *European Physical Journal B* **75**.

[3] Sornette, D., and Helmstetter, A. (2003) Endogenous versus exogenous shocks in systems with memory. *Physica A* **318** 577-591.

[4] Sornette, D. (2008) Inderdisciplinarity in Socio-economics, mathematical analysis and predictability of complex systems. *Socio-Economic Review* **6** 27-38.

[5] Deschatres, F. and Sornette, D. (2005) The Dynamics of Book Sales: Endogenous versus Exogenous Shocks in Complex Networks. *Physical Review E* **72** 016112.

[6] Sornette, D., Deschatres, F., Gilbert, T. and Ageon, Y. (2004). Endogenous Versus Exogenous Shocks in Complex Networks: an Empirical Test Using Book Sale Ranking. *Physical Review Letters* **93** 228701.

[7] Sornette, D. (2005) Endogenous versus exogenous origins of crises. In Albeverio, S., Jentsch, V., and Kantz, H. (eds) *Extreme events in nature and society*. Springer, Berlin pp. 95-119.

[8] Sornette, D., Malevergne, Y., and Muzy, J. F. (2003) What causes crashes? *Risk* **16** 67-71.

[9] Johansen, A. and Sornette, D. (2000) Download relaxation dynamics on the WWW following newspaper publication of URL.*Physica A* **276** 338345.

[10] Johansen, A. and Sornette, D. (1999) Critical Crashes. *Risk***12** 91-94.

[11] Johansen, A. and Sornette, D. (1999). Modeling the stock market prior to large crashes. *European Physical Journal B* **9** 167-174.

[12] Johansen, A., Ledoit, O. and Sornette, D. (2000) Crashes as critical points *International Journal of Theoretical and Applied Finance*, **3**, 219-255.

[13] Johansen, A., and Sornette, D. (2010) Shocks, crashes and bubbles in financial markets. *Brussells Economic Review* **53** 201-253.

[14] Lillo, F. and Mantegna, R. (2003) Power law relaxation in a complex system: Omori law after a financial market crash. *Physical Review E* **016119**.

[15] Fry, J. M. (2010a) Bubbles and crashes in finance: A phase transition from random to deterministic behaviour in prices. *Journal of Applied Research in Finance* **2** 131-137.

[16] Fry, J. M. (2010b) *Modelling bubbles and crashes in economics: With an application to English house prices.* VDM Publishing, Saarbrucken.

[17] Fry, J. M. (2011) Gaussian and non-Gaussian models for financial bubbles via econophysics. *Hyperion International Journal of Econophysics and New Economy* **4** 7-22.

[18] Fry, J. M., and Masood, O. (2011) Modelling over-pricing and speculative bubbles: An application to emerging stock markets. *Risk Governance and Control: Financial Markets and Institutions* (accepted)

[19] Zeira, J. (1999) Informational over-shooting, booms and crashes. *Journal of Monetary Economics* 43 237-257.

[20] Bouchaud, J-P., and Potters, M. (2003) *Theory of financial risk and derivative pricing. From statistical physics to risk management,* second edition. Cambridge University Press, Cambridge.

[21] Wanfeng, Y., Woodard, R., Sornette, D. (2012) Leverage Bubble. *Physica A* **391** 180-186.

[22] Zhou, W-X. and Sornette, D. (2009) A case study of speculative financial bubbles in the South African stock market 2003-2006. *Physica A* **388** 869-880.

[23] Zhou, W-X. and Sornette, D. (2008) Analysis of the real estate market in Las Vegas: Bubble, seasonal patterns, and prediction of the CSW indexes. *Physica A* **387** 243-260.

[24] Jiang, Z-Q., Zhou, W-X., Sornette, D. Woodard, R., Bastiaensen, K. and Cauwels, P. (2010) Bubble Diagnosis and Prediction of the 2005-2007 and 2008-2009 Chinese stock market bubbles. *Journal of Economic Behavior and Organization* **74** 149-162.

[25] Feigenbaum, J. A. (2001a) A statistical analysis of log-periodic precursors to financial crashes *Quantitative Finance* **1** 346-360.

[26] Feigenbaum, J. (2001b) More on a statistical analysis of log-periodic precursors to financial crashes *Quantitative Finance* **1** 527-532.

[27] Chang, G., and Feigenbaum, J. (2006) A Bayesian analysis of log-periodic precursors to financial crashes *Quantitative Finance* **6** 15-36.

[28] Chang, G., and Feigenbaum, J. (2008) Detecting log-periodicity in a regime-switching model of stock returns *Quantitative Finance* **8** 723-738.

[29] Bree, D., Challet, D., and Peirano, P. P. (2010) *Prediction accuracy and sloppiness of log-periodic functions.* Preprint. `http://arxiv.org/abs/1006.2010.`

[30] Bree, D. and Joseph, N. L. (2010) *Fitting the log periodic power law to financial crashes: a critical analysis.* Preprint. `http://arxiv.org/abs/1002.2010.`

[31] Geraskin, P. and Fantazzini, D. (2011) Everything you always wanted to know about log periodic power laws for bubble modelling but were afraid to ask. *European Journal of Finance* (forthcoming)

[32] Johansen, A., and Sornette, D. (1999) Financial "anti-bubbles": log-periodicity in gold and Nikkei collapses. *International Journal of Modern Physics C* **10** 563-575.

[33] Sornette, D. (2003) *Why stock markets crash: critical events in complex financial systems.* Princeton University Press, Princeton.

[34] Roubini, N., and Mihm, S. (2010) *Crisis economics: A crash course in the future of finance.* Penguin, New York.

[35] Reinhart, C. M., and Rogoff, K. (2009) *This time its different: Eight centuries of financial folly.* Princeton Univesity Press, Princeton.

[36] Coaffee, J. (2003) *Terrorism, risk and the city.* Ashgate, Aldershot.

[37] Carnot, N., Koen, V., and Tissot, B. (2011) *Economic forecasting and policy,* Second Edition. Palgrave Macmillan, Basingstoke New York.

[38] Plummer, T. (2006) *Forecasting financial markets. The psychology of successful investing.* Kogan Page London Philadelphia.