Certainty equivalent citation: a generalized class of citation indexes

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28 July 2011

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Abstract

Citation indexes have attracted the interest of many researchers in the recent years. In this paper we propose a new class of citation indexes which is shown to generalize most of the citation indexes in the existing literature (h-, g-, f-, t-index). The class of indexes is obtained borrowing from the notion of "certainty equivalent income" or "equally distributed equivalent income" which has been largely implemented in the field of risk and inequality measurement. As a result citation orderings are shown to depend on a parameter of concentration/dispersion aversion capturing the value judgments of the decision-maker with respect to the distribution of citations. In order to verify the sensitivity of scientific productivity orderings with respect to concentration/dispersion aversion, an empirical application to a representative sample of Italian academic economists is presented.

Keywords: *citation index; h-index; research assessment; scientific impact measures*
JEL: *I23; D63; H52*

1 Introduction

Citation indicators have widely attracted the interest of the scientific community in the recent years. This is mostly motivated by (i) the increasing need for cost minimization in the allocation of public (and private) research funds (Segalat, 2009), (ii) the recognition of "some role" for citations when evaluating the scientific productivity of a research unit (scholar, department, institute), and (iii) the transparency, comparability of the assessments and criticisms of the *peer review* system (Weingart, 2005; Bornmann, 2011). Basically citation indexes may be intended as an improvement of purely quantitative measures (i.e., number of publications) through the addition of a qualitative perspective (i.e., citation requirements). This is evident itself when considering the h-index

which is *sic et simpliciter* the number of publications of a research unit satisfying a qualitative citation-based constraint each.
Despite of the increasing interest in this field, the existing literature is still far from being mature. Due to the recent history of citation indexes, a largely shared axiomatic framework does not exist yet. In addition the existing indexes may strongly differ to each other depending on the incidence of qualitative information (citations) with respect to quantitative ones (number of publications). On the one hand some indexes can be fully interpreted as a number of "accredited" publications. Here citations are exclusively used in order to establish quality requirements and, as a result, the score assigned to each research unit cannot be larger than the number of publications by construction. This is the case of the h-index (Hirsch, 2005), g-index (Egghe, 2006a), h^2-index (Kosmulski, 2006), w-index (Woenginger, 2008), f- and t-index (Tol, 2009).[1] On the other hand some other indexes have been recently proposed by which the qualitative perspective is enforced. Here the citation indexes do not necessarily identify a number of accredited publications since the score may be reasonably larger than the number of publications in the presence of few but highly cited scientific outputs. This is the case of the $\tilde{g}$-index (Egghe, 2006b) and the F^{max}-index (Woenginger, 2008).
In this paper a new class of citation indexes is proposed which, depending on a parameter of concentration/dispersion aversion of the decision-maker, is shown to generalize most of the common citation indexes. Basically, this class of indexes replicates the notion of "certainty equivalent income" or "equally distributed equivalent income" which has been largely implemented in the field of risk and inequality measurement (Markowitz, 1952; Atkinson, 1970).
More specifically, two different versions of the "certainty equivalent citation" are proposed. The former class (c-class), in line with the first group of citation indexes above, attributes to the qualitative perspective the sole constraint mission so that the maximum score of a research unit is inevitably restricted by the number of publications. Depending on the parameter of concentration/dispersion aversion, this class of indexes is shown to generalize the h-, g-, f-, and t-index. The second class ($\tilde{c}$-class), instead, is the immediate counterpart of the previous one when a major role is attributed to quality, that is, when the citation profile may determine a score higher than the number of publications. Depending on the parameter of concentration/dispersion aversion, this class is shown to generalize both the $\tilde{g}$-index and the F^{max}-index.
The paper is organized as follows. The pros and cons of the h-index are briefly discussed in section 2. Here some of the alternative citation indexes (g-, f-, t-, $\tilde{g}$-, and F^{Max}-index) are interpreted as plausible escapes from the main weaknesses of the h-index. In section 3 the two new classes of citation indexes are introduced. These are shown to generalize well known citation indexes in the existing literature. In section 4 an empirical application to a representative sample of Italian academic economists is also reported in order to highlight the

[1] In this paper we do not consider the indexes complementing the h-index (Jin et al., 2007; Zhang, 2009).

sensitivity of scientific productivity orderings to the concentration/dispersion aversion parameter. Section 5 concludes.

2 Citation indexes

2.1 The h-index: pros and cons

In this section we briefly discuss the main characteristics of the h-index with particular emphasis on its drawbacks. On our opinion, the focus on the drawbacks may be of help in order to highlight the main rationale behind most of the alternative indexes proposed in the recent literature.

Definition 2.1 (*h*-index)
Given the decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$, $h(x) = Max\{k : x_i \geq k \ \forall \ i \leq k\}$ *with* $1 \leq k \leq n$.[2]

The h-index is known to satisfy the two major normative requirements for citation indexes: citation monotonicity (CM) and publication monotonicity (PM). By the former, given a generic citation index $I(\cdot)$ and two citation vectors, $\{x_1, ..., x_n\}$ and $\{y_1, ..., y_n\}$ with $x, y \in \mathbb{N}_0^n$, if x is obtained from y adding $k \geq 1$ citations to the j-th publication, then $I(x) \geq I(y)$. By the latter, instead, given a generic citation index $I(\cdot)$ and the citation vector $\{y_1, ..., y_n\}$ with $y \in \mathbb{N}_0^n$, if the citation vector $x \in \mathbb{N}_0^{n+1}$ is obtained from y adding a new publication, then $I(x) \geq I(y)$.
For our purposes it is worth highlighting two additional features of the h-index. First, a citation requirement is established in order to "accredit" each single publication. Second, citations are exclusively used in order to constraint the number of accredited publications, that is, the h-index cannot be larger than the number of publications ($h(x) \in [0, n]$). As we will observe later on, citation indexes do not generally share these two features which, indeed, may strongly affect the evaluation of scientific outputs.
The pros and cons of the h-index have been widely discussed in the existing literature (Bornmann et al., 2007; Jin, 2006; Burgos, 2010). On the one hand, the main pros are computational simplicity, data availability, introduction of qualitative judgments, limited sensitivity to marginal increases of the scientific output (number of publications and citations), and suitableness for different definitions of the research unit (single or groups of researchers). On the other hand, the main cons of the h-index are (I) sensitivity to time passed by since the date of publication,[3] (II) invariance with respect to the number of co-authors,[4]

[2]Even if $1 \leq k \leq n$, the score of the index is assumed to be 0 if it cannot be greater or equal to one. This rule applies to the rest of the citation indexes discussed in this paper.

[3]Evidently the interval of time for an adequate assessment of the contribution of each publication may strongly differ depending on the social and cultural environment, the discipline and, within the same discipline, on the nature of the contribution.

[4]Co-authorship may assume very different characteristics depending on the nature of the collaboration (Bruno, 2010). As a result the need for a weighting procedure may be differently supported depending on the presence of opportunistic behaviors.

(III) excessive sensitivity to variations in the number of publications and/or citations for "poor" scientific profiles,[5] (IV) unreliability for the comparison of heterogeneous scientific outputs, (V) "depreciation" of highly cited publications (*seminal papers*),[6] (VI) "depreciation" of citation vectors with few but highly cited papers,[7] and (VII) lack of sensitivity with respect to the distributive value judgments of the decision-makers.[8]

Some of these cons have been partially or fully solved in the existing literature through the introduction of weighting procedures (I-II). This is the case, for instance, of the weights for years passed by since the original publication and the number of co-authors (Katsaros et al., 2006; Batista et al., 2006; Jin et al., 2007; Schreiber, 2008; Harzing, 2011). With respect to con III, Wu (2010) proposes the w^u-index which is defined as $10 \times h$: the score of a research unit is w^u if and only if there exists at least w^u publications cited at least $10 \times h$ times. This simple transformation allows to preserve the rationale of the h-index but posing a minor emphasis on the not-so-cited publications of any research unit.[9] Con IV emphasizes the centrality of the selection of the research units to be compared. Comparisons among research units working in different scientific fields are clearly unreliable due to different standards and objectives of the research activity. For this reason the existing literature generally agrees on the applicability of the citation-based comparisons to the sole research units working in the same discipline.

Finally, in order to overcome cons V-VI-VII of the h-index, alternative solutions have been proposed. On the one hand, some metrics have been promoted in order to complement the h-index (Jin et al., 2007; Zhang, 2009). On the other hand, new citation indexes have been defined in order to replace the h-index. In the next section we focus on the latter indexes, emphasizing the main differences with respect to the h-index.

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[5] Let's consider the two citation vectors $x := \{2, 1, 1\}$ and $y := \{2, 2, 1\}$ with $h(x) = 1$ and $h(y) = 2$. At the margin, in order to increase $h(x)$, an additional citation for the second publication is sufficient. Differently, in order to induce a marginal increase of $h(y)$ an additional citation for the first and the second publication as well as two additional citations for the third publication are required.

[6] Let's consider the two citation vectors $x := \{1000\}$ and $y := \{2, 2\}$ with $h(x) = 1$ and $h(y) = 2$. For the purposes of the h-index, 999 citations in x are not relevant at all (excess of citations).

[7] Let's consider two citation vectors $x := \{1000, 1000\}$ and $y := \{3, 3, 3\}$ with $h(x) = 2$ and $h(y) = 3$. Since the h-index cannot be larger than the number of publications by construction, x is under-estimated with respect to y independently of the (evident) larger impact of its scientific output.

[8] Let's consider two citation vectors $x := \{98, 2\}$ and $y := \{50, 50\}$ with $h(x) = h(y) = 2$. The h-index is independent of the distribution of citations within the same vector. Differently, one may observe that y has to be preferred to x since it allows for the same expected citations minimizing risk. On the contrary, one may observe that x has to be preferred to y since a scientific productivity index should be marginally increasing with respect to the citations received by each publication (Albarrán et al., 2011).

[9] An alternative solution to con III may be represented by the h^2-index (Kosmulski, 2006). By the latter a scientist has index k, if the top k of her n publications have at least k^2 citations each, whereas the remaining $(n - k)$ publications have at most $[(k + 1)^2 - 1]$ citations each.

2.2 Beyond the h-index

Since Hirsch's seminal paper (Hirsch, 2005), several citation indexes have been proposed in the existing literature. Despite of the common targets, the existing citation indexes may strongly differ to each other depending on the *definition* and the *incidence* of the citation requirement.

Definition 2.2 (*g*-index)
Given the decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$*,* $g(x) = Max \left\{k : \sum_{i=1}^{k} x_i \geq k^2\right\}$ *with* $1 \leq k \leq n$ *(Egghe, 2006a), or equivalently,* $g(x) = Max \left\{k : \frac{1}{k}\sum_{i=1}^{k} x_i \geq k\right\}$ *with* $1 \leq k \leq n$ *(Jin, 2006).*

The g-index is CM and PM. In addition, given $g(x) \in [0, n]$, it can be shown that $g(x) \geq h(x)$.
When comparing the g-index with the h-index, it is worth observing that the two indexes strongly differ to each other with respect to the definition of the citation requirement. Here, a research unit is valued k if the average citation of the top k publications is at least k, that is, each citation requirement is defined with respect to the citations received by a selected group of (g-core), not each single publication. Due to the different definition of the citation requirement, the g-index allows to overcome con V of the h-index, that is, this measure is not independent of the excess of citations received by each publication.

Definition 2.3 (*f*-index)
Given the decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$*,* $f(x) = Max \left\{k : \left(\frac{1}{k}\sum_{i=1}^{k} x_i^{-1}\right)^{-1}\right\}$ *with* $1 \leq k \leq n$ *(Tol, 2009).*

The score of the f-index is k if and only if the harmonic mean from the first k publications is k at least. The f-index is CM and PM with $f(x) \in [0, n]$. By construction, it cannot be larger than the number of publications with at least one citation and $h(x) \leq f(x) \leq g(x)$.
Basically the f-index replicates the main idea of the g-index and, as a result, it is not independent of the excess of citations (con V). However it also introduces a particular value judgment with respect to the distribution of the citations within the same vector (con VII). The g-index automatically implies the perfect substitutability of citations within the g-core; in contrast, the f-index automatically implies a precise degree of dispersion aversion for the distribution of citations within the f-core.

Definition 2.4 (*t*-index)
Given the decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$*,* $t(x) = Max \left\{k : \prod_{i=1}^{k} x_i^{\frac{1}{k}} \geq k\right\}$ *with* $1 \leq k \leq n$ *(Tol, 2009).*

The t-index is k if and only if the geometric mean of the first k publications is k at least. As the f-index, it replicates the main idea behind the g-index so that it overcomes con V of the h-index. The main difference with respect to the g-index (and f-index) consists of distributive value judgment: a precise degree of dispersion aversion, lower than for the f-index, is assumed. The t-index is PM and CM. In addition, given $t(x) \in [0, n]$ with $h(x) \leq f(x) \leq t(x) \leq g(x)$, the t-index cannot be higher than the number of publications with one citation at least.

Definition 2.5 ($\tilde{g}$-index)
Given the decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$, $\tilde{g}(x) = Max\left\{k : \sum_{i=1}^{k} x_i \geq k^2\right\}$ *with* $k \in \mathbb{N}$, *or equivalently,* $\tilde{g}(x) = Max\left\{k : \frac{1}{k}\sum_{i=1}^{k} x_i \geq k\right\}$ *with* $k \in \mathbb{N}$ *(Egghe, 2006b).*

A research unit is valued k if the average citation of the top k publications is at least k. Here, in contrast with the g-index, k is not upper bounded by the number of publications, e.g., a citation vector with two publications may be valued three if, once a fictitious publication with zero citations is added, the average citation is three, at least. The $\tilde{g}$-index is PM and CM. In addition, $\tilde{g}(x) \in [0, +\infty[$ with $\tilde{g}(x) \geq g(x)$ by construction.
As a major concern, it is worth observing that, in contrast with the h-index (and all citation indexes above), the $\tilde{g}$-index does not penalize research units with few but highly cited publications, that is, it allows to overcome con VI.

Definition 2.6 (F^{max}-index)
Given the decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$, $F^{max}(x) = x_1$ *(Woenginger, 2008).*

The F^{max}-index indicates the number of citations received by the top publication of a research unit. Basically it relies on the reasonable (but pessimistic) idea by which the most representative publication is the only reliable indicator of the scientific productivity of a research unit. Here, in contrast with all previous indexes, citations do not identify a quality constraint but fully determine the score of a research unit.
The F^{max}-index is PM and CM. In addition, like the $\tilde{g}$-index $F^{max}(x) \in [0, +\infty[$, that is, the F^{max} can be larger than the number of publications (con VI).

3 A generalized-class of citation-based indexes

In this section a new class of citation measures is introduced by which (i) the citation indexes above are generalized, and (ii) the preferences of the decision-maker with respect to the distribution of the citations are endogenized.

Definition 3.1 (c-class of citation indexes) *Given the decreasingly ordered* *citation vector* $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$,

$$c(x) = Max\left\{k : \left(\frac{1}{k}\sum_{i=1}^{k} x_i^{\varepsilon}\right)^{\frac{1}{\varepsilon}} \geq k\right\} \forall\ \varepsilon \neq 0 \ with\ 1 \leq k \leq n,\ and$$
$$c(x) = Max\left\{k : \prod_{i=1}^{k} x_i^{\frac{1}{k}} \geq k\right\} for\ \varepsilon = 0\ with\ 1 \leq k \leq n \tag{1}$$

The c-index (c-class of indexes), proposed in this paper, replicates the main idea behind the "certainty equivalent income" or "equally distributed equivalent income" which has been widely implemented in the field of inequality and risk measurement (Markowitz, 1952; Atkinson, 1970). Basically, given $\varepsilon \in \Re$, the c-index is k if the "certainty equivalent citation" for the first k publications is k at least, where the certainty equivalent citation indicates the number of citations $\tilde{x}_k = \left(\frac{1}{k}\sum_{i=1}^{k} x_i^{\varepsilon}\right)^{\frac{1}{\varepsilon}}$ so that, given the citation vector $\{x_1, ..., x_k\} \in \mathbb{N}_0^k$ and the citation vector $\{\tilde{x}_k, ..., \tilde{x}_k\} \in \Re_+^k$, $\left(\frac{1}{k}\sum_{i=1}^{k} x_i^{\varepsilon}\right)^{\frac{1}{\varepsilon}} = \left(\frac{1}{k}\sum_{i=1}^{k} \tilde{x}^{\varepsilon}\right)^{\frac{1}{\varepsilon}}$ $\forall\ k$.

Proposition 3.1 *The c-class of citation indexes is PM and CM for all* $\varepsilon \in \Re$. *In addition*

$$c(x) = \begin{cases} g(x) & if \quad \varepsilon = 1 \\ t(x) & if \quad \varepsilon = 0 \\ f(x) & if \quad \varepsilon = -1 \\ h(x) & if \quad \varepsilon \to -\infty \end{cases} \tag{2}$$

Proof 3.1 *c(x) is strictly increasing with respect to citations and non-decreasing with respect to publications. For the generalization of the h-index it is worth observing that if* $\varepsilon \to -\infty$ *then* $c(x) = Max\{k : Min\{x_1, ..., x_k\} \geq k\}$ *with* $1 \leq k \leq n$*, that is just equivalent to* $h(x)$.

In line with the g-, e-, f-, t-index this class of indexes is not independent of the excess citations for all $\varepsilon \neq -\infty$ (con V). In addition, the c-class of indexes endogenizes the value judgments of the decision-maker with respect to the distribution of citations within the same vector (con VII).

Property 3.1 (ε-sensitivity to transfers) *Given a citation vector* $\{y_1, ..., y_n\} \in \mathbb{N}_0^n$*, if x is obtained from y through a non re-ranking citation transfer among publications within the c-core so that, given* $y_j > y_i$, $x_i = y_i + \delta$ *and* $x_j = y_j - \delta$ *with* $\delta \geq 1$*, then* $c(x) \gtreqless c(y)$ *if and only if* $\varepsilon \gtreqless 1$.

The decision-maker is concentration averter for all $\varepsilon > 1$ (Albarrán et al., 2011), while she is dispersion averter for all $\varepsilon < 1$. In the case of infinite aversion to dispersion the c-index is $f^{max}(x) = Max\{k : Max\{x_1, ..., x_k\} \geq k\}$ with $1 \leq k \leq n$, while the c-index coincides with the h-index in the case of infinite aversion to concentration. Finally, given $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$ with $c(x) \in [0, n]\ \forall\ \varepsilon \in \Re$, the following property holds.

Property 3.2 (Relative ε-based co-domains) *Given a decreasingly ordered citation vector* $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$*, then*

$$\begin{array}{ll} h(x) \le f(x) \le t(x) \le g(x) \le c(x) = f^{max}(x) & if\ \varepsilon \to +\infty \\ h(x) \le f(x) \le t(x) \le g(x) \le c(x) \le f^{max}(x) & if\ \varepsilon \in]1, +\infty[\\ h(x) \le f(x) \le t(x) \le g(x) = c(x) \le f^{max}(x) & if\ \varepsilon = 1 \\ h(x) \le f(x) \le t(x) \le c(x) \le g(x) \le f^{max}(x) & if\ \varepsilon \in]0, 1[\\ h(x) \le f(x) \le t(x) = c(x) \le g(x) \le f^{max}(x) & if\ \varepsilon = 0 \\ h(x) \le f(x) \le c(x) \le t(x) \le g(x) \le f^{max}(x) & if\ \varepsilon \in]-1, 0[\\ h(x) \le f(x) = c(x) \le t(x) \le g(x) \le f^{max}(x) & if\ \varepsilon = -1 \\ h(x) \le c(x) \le f(x) \le t(x) \le g(x) \le f^{max}(x) & if\ \varepsilon \in]-\infty, -1[\\ h(x) = c(x) \le f(x) \le t(x) \le g(x) \le f^{max}(x) & if\ \varepsilon \to -\infty \end{array} \quad (3)$$

The c-class of indexes is defined in such a way to attribute the sole constraint mission to the qualitative perspectives. However, in order to enforce the mission of the qualitative constraint, the c-class can be reformulated as follows.

Definition 3.2 (The $\tilde{c}$-class of indexes) *Given the decreasingly ordered citation vector* $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$*,*

$$\tilde{c}(x) = Max\left\{k : \left(\frac{1}{k}\sum_{i=1}^{k} x_i^{\varepsilon}\right)^{\frac{1}{\varepsilon}} \ge k\right\}\ \forall\ \varepsilon > 0\ with\ k \in \mathbb{N} \quad (4)$$

The $\tilde{c}$-class of indexes is still defined as the certainty equivalent citation in the $\tilde{c}$-core (class of indexes) but, in contrast with the c-class, is defined for $\varepsilon > 0$ only.[10] In addition, like the $\tilde{g}$-index, k is not upper bounded by the number of publications.
Mostly, the $\tilde{c}$-class allows to overcome cons V-VI-VII of the h-index. Indeed, it endogenizes the distributive judgment of the decision-maker (con VII). Like the $\tilde{g}$-index, it is not upper bounded by the number of publications, that is, it does not undervalue citation vectors with few but highly cited publications (con VI). Finally, since the qualitative requirements is defined with respect to the unweighted aggregation of the citations for publications in the $\tilde{c}$-core, this index is not independent of the excess citations (con V).

Proposition 3.2 *The $\tilde{c}$-class of citation-based indexes is PM and CM for all* $\varepsilon > 0$*. In addition*

$$\tilde{c}(x) = \begin{cases} F^{max}(x) & if \quad \varepsilon \to +\infty \\ \tilde{g}(x) & if \quad \varepsilon = 1 \end{cases} \quad (5)$$

[10] Alternatively, if the neutrality of zero-cited publications is assumed *a priori* (which is the case of all indexes discussed in this paper), then one may consider the same class of indexes $\forall\ \varepsilon \in \Re$ whenever this index is calculated with respect to the extended citation vector $\hat{x} := \{x_1, ..., x_n, 0, 0, ...\}$ in place of $x := \{x_1, ..., x_n\}$.

Proof 3.2 *$\tilde{c}(x)$ is strictly increasing with respect to citations and non-decreasing with respect to publications. For the generalization of the $F^{max}(x)$, it is worth observing $F^{max}(x) = Max\{k : Max\{x_1, ..., x_k\} \geq k\}$ with $k \in \mathbb{N}$, that is $F^{max}(x) = x_1$*

Property 3.3 (ε-sensitivity to transfers) *Given a citation vector $\{y_1, ..., y_n\} \in \mathbb{N}_0^n$, if x is obtained from y through a non re-ranking citation transfer among publications within the core so that, given $y_j > y_i$, $x_i = y_i + \delta$ and $x_j = y_j - \delta$ with $\delta \geq 1$, then $\tilde{c}(x) \gtreqless \tilde{c}(y)$ if and only if $\varepsilon \gtreqless 1$ (with $\varepsilon > 0$).*

Property 3.4 (Relative ε-based co-domains) *Given a decreasingly ordered citation vector $\{x_1, ..., x_n\} \in \mathbb{N}_0^n$, then $\tilde{c}(x) \geq c(x)$ with $\tilde{c} \in [0, +\infty[$ and*

$$\begin{array}{ll} h(x) \leq f(x) \leq t(x) \leq \tilde{g}(x) \leq \tilde{c}(x) = F^{max} & \varepsilon \to +\infty \\ h(x) \leq f(x) \leq t(x) \leq \tilde{g}(x) \leq \tilde{c}(x) \leq F^{max} & \varepsilon \in]1, +\infty[\\ h(x) \leq f(x) \leq t(x) \leq \tilde{g}(x) = \tilde{c}(x) \leq F^{max} & \varepsilon \in 1 \\ h(x) \leq f(x) \leq t(x) \leq \tilde{c}(x) \leq \tilde{g}(x) \leq F^{max} & \varepsilon \in]0, 1[\end{array} \quad (6)$$

4 An empirical assessment of the certainty equivalent citations

This section aims to validate empirically the properties 2 and 4 (Relative ε-based co-domains) for the two versions of certainty equivalent citation indexes $c(x)$ and $\tilde{c}(x)$. The analysis is carried out by examining the bibliometric records of the 1,355 economists, distributed across 77 Italian Universities. This sample includes all economists that held tenured positions (assistant, associate and full professors) in the fields of Economics, Economic Policy and Public finance[11] during the 2010/2011 academic year.

It is well known that bibliographical metrics are highly dependent on the quality and completeness of the bibliographical databases. To take this issue into account authors' citation data were collected from both Scopus[12] and Google Scholar[13]. These archives have the widest bibliographic coverage for Social Sciences among open-access and non-open access (peer-reviewed) citation databases respectively (Meho et al., 2007; Bar-Ilan, 2008; Bornmann et al., 2009). However, there are at least three significant limitations to reliability of citation data

[11]In the Italian University system these disciplines are coded as SECS-P/01, SECS-P/02 and SECS-P/03.

[12]It is the largest citation database containing both peer-reviewed research literature and quality web sources. It is edited by Elsevier. Details are available from http://www.scopus.com/home.url. The search for author name is limited to the area: "*Social Sciences & Humanities*".

[13]Data included on Google Scholar are articles, theses, books, abstracts, from academic publishers, professional societies, online repositories, universities and other web sites. Details are available from http://scholar.google.com/intl/en/scholar/about.html. Raw data are extracted by Publish or Perish published by Harzing (2011). The search of publications is limited to the area of "*Business, Administration, Finance, Economics*".

collected from Google Scholar. Since there is no quality control by peer review on the publications, the citation statistics can be easily manipulated by opportunistic behaviour (e.g. self-citations). As pointed out by Baneyx (2008), the list of publications obtained by Google Scholar is often biased by the presence of duplicates[14] (i.e. two publications with the same title) and publications authored by scholars with similar names. Therefore, it needs of a manual checked by excluding the articles incorrectly attributed to the author and by merging the duplicates entries. This post-process may cause inaccuracies in the citation data. On the contrary, the main advantage of Google Scholar is the greater coverage of scientific documents in economics. It is a key issue to analyse scientists with slight articles published in peer-review Journals. Since we focus on the top-twenty Italian economists, the citation statistics calculated by Scopus dataset are deemed as preferable for our analysis.
From these datasets, the indexes exposed in the paragraph 3 of each of 1,355 academics are calculated. They are special cases of the certainty equivalent citation index (including h-, f-, t-, g-, $\tilde{g}$-, F^{max}-index).
To single a top scholar out in the sample, we ranked all scholars respect to each citation index - in parenthesis are shown the relative position of each academics among his/her Italian colleagues - and estimate the mean over the 11 epsilon values used in our computation. The top-twenty economists are defined as the twenty scientists with the highest averages of their 11 ranking. This score is reported in the column "Rank" of Table 1 and 2. The last three rows report means, medians and standard deviations calculated on the entire sample.

[14]Merging implies to sum the citations so that the overall sum of citations is maintained and the number of publications is reduced.

Table 1: Ranking of the top 20 Italian Economists (Scopus database)

		ε=	*-∞*	*-1.5*	*-1*	*-0.5*	*0*	*0.5*	*1*	*1.5*	*1*	*1.5*	*∞*
		Rank	*h-*		*f-*		*t-*		*g-*		*g̃-*		*F-max*
1	Tabellini G	1.36	25(1)	32(1)	33(1)	34(1)	37(1)	40(1)	47(2)	51(1)	47(2)	60(2)	607(2)
2	Dosi G	1.45	21(2)	28(2)	29(2)	31(2)	34(2)	40(1)	51(1)	51(1)	52(1)	77(1)	1042(1)
3	Pagano M	3.55	20(3)	23(3)	24(3)	25(3)	26(4)	29(3)	29(4)	29(4)	39(4)	47(4)	231(4)
4	Perotti R	3.91	16(6)	22(4)	24(3)	25(3)	28(3)	28(4)	28(5)	28(6)	40(3)	50(3)	338(3)
5	Guiso L C G	5.64	18(4)	22(4)	23(5)	25(3)	26(4)	27(5)	27(6)	27(7)	33(5)	39(5)	122(14)
6	Ottaviano G I P	5.64	17(5)	21(6)	21(6)	23(6)	25(6)	27(5)	31(3)	36(3)	31(6)	36(6)	149(10)
7	Jappelli T	7.82	14(7)	18(7)	19(7)	19(7)	21(7)	23(7)	25(7)	29(4)	25(7)	29(9)	106(17)
8	Bertola G	8.36	14(7)	18(7)	19(7)	19(7)	20(8)	21(8)	24(8)	25(8)	24(10)	28(11)	148(11)
9	Boeri T M	10.64	14(7)	18(7)	18(9)	19(7)	19(9)	21(8)	22(9)	22(9)	22(11)	24(12)	80(29)
10	Antonelli C	12.36	13(10)	17(10)	17(10)	18(10)	19(9)	20(10)	21(10)	22(9)	21(12)	22(13)	77(33)
11	Santarelli E	13.45	13(10)	16(11)	17(10)	17(11)	18(11)	19(11)	20(11)	22(9)	20(13)	22(13)	71(38)
12	De Fraja G	14.55	12(12)	14(13)	14(14)	15(13)	15(14)	16(14)	18(13)	20(12)	18(17)	20(20)	102(18)
13	Lambertini L	15.55	12(12)	16(11)	16(12)	16(12)	17(12)	18(12)	19(12)	20(12)	19(15)	20(20)	63(41)
14	Giavazzi F	15.73	10(16)	13(16)	13(16)	14(16)	14(18)	15(18)	15(24)	15(26)	25(7)	31(8)	153(8)
15	Cigno A	17.36	10(16)	13(16)	13(16)	14(16)	15(14)	16(14)	18(13)	20(12)	18(17)	20(20)	72(37)
16	Geuna A	17.45	10(16)	13(16)	13(16)	14(16)	15(14)	16(14)	17(16)	17(20)	19(15)	22(13)	73(36)
17	Marengo L	19.00	9(23)	12(22)	12(22)	13(19)	14(18)	15(18)	17(16)	17(20)	17(20)	21(16)	120(15)
18	Gallegati M	20.36	11(14)	14(13)	15(13)	15(13)	16(13)	17(13)	18(13)	19(15)	18(17)	19(24)	40(76)
19	Ichino A	21.64	10(16)	12(22)	12(22)	13(19)	13(23)	15(18)	17(16)	18(17)	17(20)	20(20)	59(45)
20	Elia S	22.55	10(16)	13(16)	13(16)	13(19)	14(18)	14(23)	16(20)	17(20)	16(23)	17(34)	61(43)
	Mean*	-	**1.6**	**1.84**	**1.88**	**1.98**	**1.95**	**2.21**	**2.36**	**2.45**	**2.7**	**3.08**	**10.61**
	Median*	-	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**2**
	Standard Dev*	-	**2.4**	**2.98**	**3.07**	**3.23**	**3.45**	**3.71**	**4.07**	**4.31**	**4.54**	**5.57**	**39.62**

Note: *Calculated over entire sample (1355 academics).

Table 2: Ranking of the top 20 Italian Economists (Google Scholar database)

		ε=	*-∞*	*-1.5*	*-1*	*-0.5*	*0*	*0.5*	*1*	*1.5*	*1*	*1.5*	*∞*
		Rank	*h-*		*f-*		*t-*		*g-*		*$\tilde{g}$-*		*F-max*
1	Tabellini G	1.45	62(1)	80(1)	84(1)	90(1)	99(1)	116(1)	147(2)	195(2)	147(2)	195(2)	2395(2)
2	Dosi G	1.55	56(2)	73(2)	76(2)	81(2)	90(2)	109(2)	152(1)	232(1)	152(1)	232(1)	4158(1)
3	Pagano M	3.64	47(3)	63(3)	67(3)	72(3)	80(3)	93(3)	115(3)	144(4)	115(3)	144(4)	1126(8)
4	Perotti R	4	38(5)	54(4)	57(4)	62(4)	70(4)	86(4)	113(4)	150(3)	113(4)	152(3)	1296(5)
5	Giavazzi F	5.18	38(5)	51(6)	53(7)	57(5)	62(6)	73(5)	92(5)	125(5)	92(5)	125(5)	1595(3)
6	Bertola G	5.73	40(4)	51(6)	54(5)	57(5)	63(5)	73(5)	89(6)	112(6)	89(6)	112(6)	1070(9)
7	Corsetti G	8.09	37(8)	47(9)	49(9)	52(9)	57(10)	64(10)	79(8)	105(7)	79(8)	105(7)	1342(4)
8	Guiso L C G	9.09	35(10)	46(10)	49(9)	52(9)	58(8)	67(7)	83(7)	104(8)	83(7)	104(8)	642(17)
9	Jappelli T	9.18	36(9)	48(8)	50(8)	53(8)	58(8)	67(7)	79(8)	95(9)	79(8)	95(9)	536(19)
10	Boeri T M	10.27	38(5)	52(5)	54(5)	57(5)	60(7)	65(9)	71(10)	78(11)	71(10)	78(13)	318(33)
11	Ottaviano G I P	11.55	29(12)	37(12)	38(12)	41(12)	45(12)	52(12)	65(11)	84(10)	65(11)	84(10)	787(13)
12	Antonelli C	14.09	30(11)	40(11)	42(11)	45(11)	48(11)	54(11)	61(12)	70(14)	61(12)	70(16)	296(35)
13	Ichino A	14.09	22(18)	31(15)	33(15)	35(15)	39(13)	45(13)	57(13)	75(12)	57(13)	75(14)	777(14)
14	Marengo L	16.73	25(15)	31(15)	32(16)	33(16)	36(16)	40(14)	47(16)	54(16)	47(18)	58(21)	429(21)
15	Monacelli T	19.82	18(31)	25(24)	27(23)	28(25)	31(24)	37(18)	49(14)	54(16)	49(16)	69(17)	863(10)
16	Benigno P	20	19(25)	25(24)	27(23)	29(22)	32(21)	38(17)	48(15)	62(15)	48(17)	62(19)	428(22)
17	Vivarelli M	20.82	26(13)	33(13)	34(13)	36(13)	38(14)	40(14)	44(19)	49(21)	44(21)	49(28)	209(60)
18	Coricelli F	21	26(13)	33(13)	34(13)	36(13)	38(14)	40(14)	44(19)	48(23)	44(21)	48(29)	210(59)
19	Schivardi F	23.73	21(21)	26(23)	27(23)	29(22)	32(21)	36(19)	43(21)	52(19)	43(23)	52(25)	251(44)
20	Lippi F	24.36	19(25)	23(32)	24(32)	26(28)	30(25)	35(23)	46(17)	49(21)	46(19)	59(20)	342(26)
	Mean*	**-**	**4.7**	**5.76**	**5.98**	**6.3**	**6.69**	**7.48**	**8.43**	**9.43**	**8.83**	**10.48**	**52.87**
	Median*	**-**	**3**	**4**	**4**	**4**	**5**	**5**	**5**	**6**	**6**	**6**	**15**
	Standard Dev*	**-**	**5.39**	**7.06**	**7.39**	**7.83**	**8.57**	**9.76**	**11.91**	**15.1**	**12.04**	**15.81**	**177.02**

Note: *Calculated over entire sample (1355 academics).

Table 1 and 2 validate the ability to the certainty equivalent citation index to replicate Hirsch (2005),[15] Tol (2009), Egghe (2006a,b) and Woenginger (2008) indexes.
By comparing citation scores between Table 1 and Table 2, we find a relatively large difference between the metrics calculated by Scopus and Google Scholar. First, the lists of the top scientists count the same scholar at the 65% (13 on 20). Second, Scopus archive has lower coverage of economic literature. It is largely expected result since Scopus includes in the database only the articles and citations published in peer review sources. However, it is significant to see that the coverage of Google Scholar is about three times than Scopus.[16]
With reference to the proposed generalized class of index, we find evidence that the absolute values and the standard deviations of the $c(x)$-index increases by rising of ε. This feature makes possible for decision maker to calibrate ε in order to increase the discriminatory power of citation analysis among units of research.[17] Calibration of ε became much more relevant when citation metrics were used to compare research units with low scores.[18]

5 Conclusive remarks

We draw a comparison between the notion of "certainty equivalent income" implemented in the field of risk and income inequality measurement (Markowitz, 1952; Atkinson, 1970) and the citation indexes proposed by (Hirsch, 2005; Egghe, 2006a,b; Tol, 2009; Woenginger, 2008). As a result we propose a class of indexes, which, depending on the value of a parameter of concentration/dispersion aversion, is shown to replicate well known citation orderings (h-, f-, t-, g-, $\tilde{g}$, F^{max}-index). In line with the existing literature, the new class of indexes is shown to satisfy the axioms of citation and publication monotonicity. Additionally, the certainty equivalent citation indexes have also the properties of ε-sensitivity and relative ε-based co-domains. These properties have two main operative advantages with respect to the aforementioned special cases. The decision maker can fine-tune the parameter ε (i.e. ε-sensitivity) in order to set a citation index consistent with decision-maker's preference on (1) dispersion/concentration of citations among publications and (2) the level of discrimination among research units.
In this sense we wish to contribute to the ongoing debate on the use of citation-

[15] For ε equal to -50 and -150 $c(x)$ scores are equal to the h-indexes for Scopus and Google Scholar, respectively.

[16] This relationship roughly holds for the other economists in the sample.

[17] For instance from Table 1, Jappelli, Bertola and Boeri have the same score of h-index (14) therefore they are ranked in the same position. By assuming that the decision maker requires to discriminate among the three scholars, it may fix $\varepsilon = 0.52$. This calibration makes $c(x)$ able to rank with the units: Jappelli (23), Bertola (22) and Boeri (21).

[18] In the Italian sample the h-index (Scopus data) can not distinguish among 307 scholars with $h = 1$ (or 220 with $h = 2$). These two groups count about the 40% of Italian economists. This evidence makes h-index unable to rank these academics. It may be useful to improve discriminatory power to increase ε. For instance, if we fix $\varepsilon = 2$, scholars are classified into seven categories instead of the previous 2 groups based on h-index $\varepsilon = -50$.

based indicators for assessing the impact of research findings by pointing out that, also behind the apparent "impartiality" of citation statistics, there are subjective decision-maker's preferences on the dispersion/concentration of citations among publications (here defined ε-sensitivity).

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