Controlling complex dynamics in a protected-area discrete-time model

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Abstract

This paper investigates how the introduction of user fees and defensive expenditures change the complex dynamics of a discrete-time model, which represents the interaction between visitors and environmental quality in a Open-Access Protected-Area (OAPA). To investigate this issue more deeply, we begin by studying in great detail the OAPA model and then we introduce the user fee (β) and the defensive expenditures ($\rho\beta$) specifically directed towards at the protection of the environmental resource. We observed that some values of β can generate a chaotic regime from a stable dynamic of the OAPA model. Finally, to eliminate the chaotic regime, we design a controller by OGY method, assuming the user fee as a controller parameter.

1 Introduction

Empirical analysis has shown that tourists are willing to pay more for environmental management, if they believe that the money they pay will be allocated for biodiversity conservation and protected area management (see [3] and [19]). Consequently, the funds for maintaining public goods can be increased by fees payed by visitors of the Protected Areas (PA).

Several works in economic literature analyze the effects on ecological dynamics generated by economic activity and environmental defensive expenditures. In particular, [1] and [2] analyze the stabilizing effect on ecological equilibria in an optimal control context in which ecological dynamics are represented by predator-prey equations

More recently economists, social and political scientists have started to develop and adapt chaos theory as a way of understanding human systems. Specifically, [8], [7], [6], [11], [16] and [17] have considered chaos theory as a way of understanding the complexity of phenomena associated with tourism.

In [18] a three-dimensional environmental defensive expenditures model with delay is considered. The model is based on the interactions among visitors V, quality of ecosystem goods E, and capital K, intended as accommodation and entertainment facilities, in PA. The visitors' fees are used partly as a defensive expenditure and partly to increase the capital stock.

Based on the continuous environmental model of [18], in this paper we analyze a discrete-time model with no capital stock and with no time delay. We aim at analyzing how the dynamics change when switching from OAPA (where, normally, there are no services or facilities) and PA with visitor fees to protection of the environmental resources.

This paper is organized as follows. In Section 2, we present the discrete-time model that embodies the user fees and defensive expenditures. In Section 3, the dynamics of an open-access protected area, i.e.

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without the user fee and defensive expenditures, is studied in great detail, including stable fixed point, periodic motion, bifurcation (flip-flop and Neimark-Sacker bifurcations) and chaos, using visitors' preferences on the environmental quality represented by the parameter σ. Section 4 deals with the control of chaotic motion and the process of control is achieved using a relative user fees and defensive expenditures.

2 The mathematical model with user fee and defensive expenditures

The model refers to a generic protected area and describes the interplay between two state variables: the size $V(t)$ of the population of visitors of the protected area at time t and an index $E(t)$ measuring the quality of environmental resources of the protected area. The dynamic of $V(\tau)$ is assumed to be described by the differential equation:

$$\frac{dV}{dt} = -b - cV + dE \tag{1}$$

According to such equation, the time evolution of $V(t)$ depends on three factors: i) $-b$ represents the negative effect of the fee that visitors have to pay to enter the protected area; ii) $-cV$ is the negative effect due to congestion; iii) dE (d is the parameter that presents attractiveness associated with high environmental quality) is the positive effect of environmental quality on visitors' dynamics. b, c and d are strictly positive parameters.

The dynamic of the environmental quality index $E(t)$ is assumed to be given by:

$$\frac{dE}{d\tau} = r_0(1-E)E - aV^2 + qbV \tag{2}$$

which assures that,the time evolution of environmental quality is described by a logistic equation (see [4]). According to equation (2), visitors generate a negative impact on environmental quality (this effect is represented by $-aV^2$); however visitors also generate a positive effect in that a share q of the revenues deriving from the fees is used for environmental protection (this effect is represented by qbV). r and a are strictly positive parameters while q is a parameter $0 \leq q \leq 1$.

Euler's difference scheme for the continuous system (1-2) takes the form (see [10]):

$$\begin{aligned} \frac{V(t+\Delta t)-V(t)}{\Delta t} =& -b - cV(t) + dE(t)) \\ \frac{E(t+\Delta t)-E(t)}{\Delta t} =& r_0(1-E(t))E(t) - aV^2(t) + bqV(t) \end{aligned} \tag{3}$$

Where Δt denotes the time step. As $\Delta t \to 0$, the discrete system converges to the continuous system. Roughly speaking, a discrete system can give rise to the same dynamics as a continuous system if the Δt is small. However, it may generate qualitatively different dynamics if Δt is large. In this sense, the discrete system with $\Delta t > 0$ generalizes the corresponding continuous system. In the following, we first simplify the discretised system (3) by changing variables [1] and posing $r = r_o\Delta t$, $\alpha = a\Delta t$, $\beta = b\Delta t$, $\gamma = c\Delta t$, $\rho = q\Delta t$ and $\sigma = d\Delta t$ (3) can be written as

$$\begin{aligned} x \to& x - \beta - \gamma x + \sigma y \\ y \to& y + r(1-y)y - \alpha x^2 + \beta\rho x \end{aligned} \tag{4}$$

where γ, σ, α are strictly positive parameters, while β and ρ are not negative. **UN RIFERIMENTO AL FATTO CHE I PARAMETRI SIANO UGUALI AL SISTEMA CONTINUO**

[1] A variable $w(t)$ in continuous time can be written by $w(t_n)$ in discrete time. Set $t_n = \Delta t \cdot n$ ($n = 1, 2 \ldots$). Then given $\Delta t > 0$, the variable can be expressed as follows; $w(t_n) = w(\Delta t \cdot n) = w_n$ and $w((t_n + \Delta t) = w(\Delta t \cdot (n+1)) = w_{n+1}$. Thus, by the same token, the discretized dynamic system (3) can be written as

$$\begin{aligned} x_{n+1} =& x_n - b\Delta t - c\Delta t x_n + d\Delta t y_n \\ y_{n+1} =& y_n + r_o\Delta t(1-y_n)y_n - a\Delta t x_n^2 + b\Delta t q x_n \end{aligned}$$

Length of one period is equal to Δt. For notational convenience, replacing n with t yields the following discrete-time system.

3 The dynamic behavior of an open-access PA model

In this section we analyze the dynamics of our model the assumption of free-access in the protected area; in this context, visitors have not to pay a fee to visit the area and system (4) becomes:

$$\begin{aligned} x \to & x - \gamma x + \sigma y \\ y \to & y + r(1-y)y - \alpha x^2 \end{aligned} \tag{5}$$

To compute the fixed points of (5) we have to solve the nonlinear system**:**

$$\begin{aligned} x =& x - \gamma x + \sigma y \\ y =& y + r(1-y)y - \alpha x^2 \end{aligned}$$

obtain from map (4) by $\beta = 0$.

Proposition 1 *The system* (5) *present always two fixed points:*

a) $P_1 = (x_1^*, y_1^*) = (0, 0)$

b) $P_2 = (x_2^*, y_2^*) = (\dfrac{r\dfrac{\gamma}{\sigma}}{\alpha + r(\dfrac{\gamma}{\sigma})^2}, \dfrac{\gamma}{\sigma}x^*)$

Now we study the stability of these fixed points.The local stability of a fixed point (x^*, y^*) (it denotes (x_1^*, y_1^*) or (x_2^*, y_2^*)) is determined by modules of eigenvalues of the characteristic equation at the fixed point.
The Jacobian matrix of the map (5) at positive point (x^*, y^*) is given by

$$J = \begin{pmatrix} -\gamma + 1 & \sigma \\ -\theta_1 & 1 + \theta_2 \end{pmatrix} \tag{6}$$

where $\theta_1(\sigma) = 2\alpha x^*$, $\theta_2(\sigma) = r(1 - 2y^*)$. The characteristic equation of the Jacobian matrix J can be written as

$$\lambda^2 + p(\sigma)\lambda + q(\sigma) = 0 \tag{7}$$

where $p(\sigma) = \gamma - \theta_2(\sigma) - 2$ and $q(\sigma) = (1 + \theta_2(\sigma))(1 - \gamma) + \sigma\theta_1(\sigma)$**.** In order to study the moduli the eigenvalues of the characteristic equation (7), we first give the following lemma, which can be easily proved

Lemma 1 *Let* $F(\lambda) = \lambda^2 + p\lambda + q$*. Suppose that* $F(1) > 0$*,* λ_1 *and* λ_2*are two roots of* $F(\lambda) = 0$*.*

(i) $|\lambda_1| < 1$ *and* $|\lambda_2| < 1$ *(sink) iff* $F(-1) > 0$ *and* $q < 1$

(ii) $|\lambda_1| < 1$ *and* $|\lambda_2| > 1$ *(or* $|\lambda_1| > 1$ *and* $(|\lambda_2| < 1$*)(saddle) iff* $F(-1) < 0$

(iii) $|\lambda_1| > 1$ *and* $|\lambda_2| > 1$ *(source) iff* $F(-1) > 0$ *and* $q > 1$

(iv) $\lambda_1 = -1$ *and* $|\lambda_2| \neq 1$ *(flip-flop bifurcation)iff* $F(-1) = 0$ *and* $p \neq 0, 2$

(v) λ_1 *and* λ_2 *are complex and* $|\lambda_1| = |\lambda_2| = 1$ *(Neimark-Sacker bifurcation) iff* $p^2 - 4q < 0$ *and* $q = 1$

From the Lemma 1 follows that:

Proposition 2 *The fixed point* $P_1 = (0, 0)$ *is always unstable, while the fixed point* P_2*, varying* σ *can be a sink, a source or a saddle (see Figure 1).*

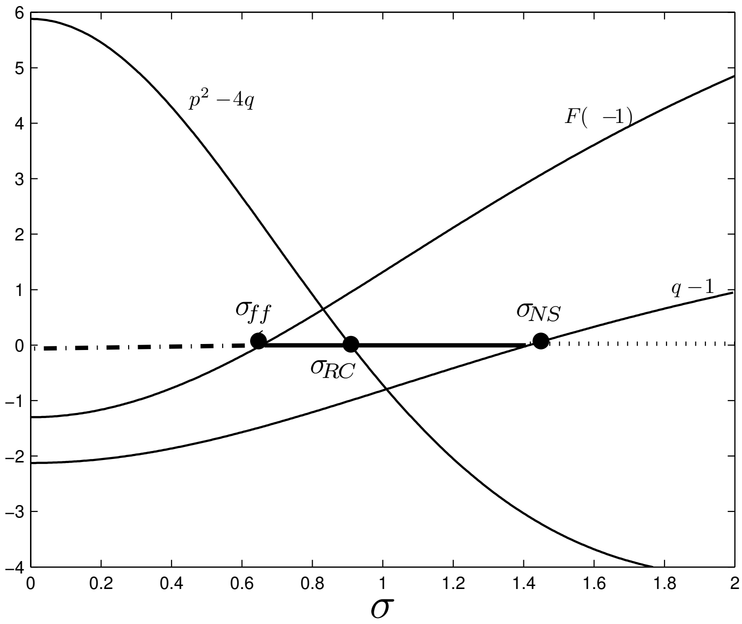


Figure 1: The parameter's are $\alpha = 0.12$, $\beta = 0$, $\gamma = 0.375$, $\rho = 0$, $r = 2.8$

The Figure 1 shows the values of $F(-1)$, $q - 1$, $p^2 - q$, defined in Lemma 1, as a function of the parameter σ.

We fix $\alpha = .12$,$\gamma = 0.375$, $r = 02.8$ and assume that σ can vary. Smaller values of *sigma* σ_{RC}(see Figure 1) are associated with real eigenvalues, while higher values of it are associated with complex eigenvalues.

According to Lemma 1, when the parameter σ belongs to the interval $(0, \sigma_{ff})$ (dash-dot line) we are in the situation described in point $ii)$ of Lemma 1,when $\sigma = \sigma_{ff} = 0.656407$ a flip-flop bifurcation occurs, when $\sigma \in (\sigma_{ff}, \sigma_{NS})$ we are in the situation described in $i)$ (solid line), at the value of $\sigma_{NS} = 1.416516$ a Neimark-Saker bifurcation takes place, finally for $\sigma > \sigma_{NS}$ the fixed point is unstable.

These various results will be discussed and illustrated in Figure 2 in the remaining part of this section.

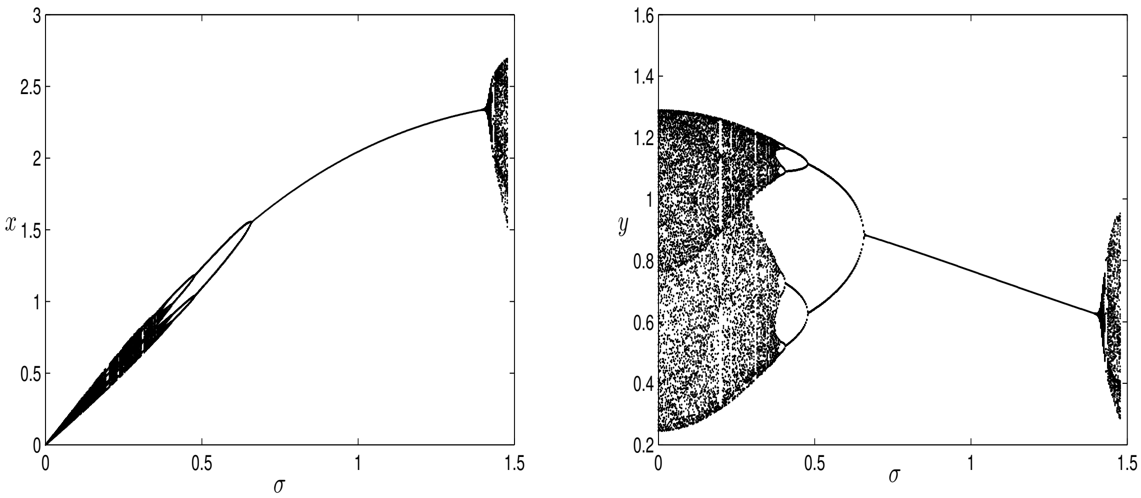
The bifurcation diagram with respect to x and y also shows all the remarkable phenomena that occur (see for instance Figure 2).

The attractor in Figure 3(a) is a bounded region in the phase space to which all sufficiently close trajectories are asymptotically attracted for a long enough period of time. While every trajectory is chaotic, the chaotic attractor reveals information about the long-term trends of the system. The stretching causes orbits on the attractor to exhibit sensitive dependence on initial conditions (chaos) and the folding causes the fractal (strange) structure. The impressive structure appearing for $\sigma = 0.165$ is chaotic and is represented in Figure 3(a).

Continuing to increase the value of σ we arrive to a stable equilibrium point showed graphically in Figure 3(b). Both variables of the dynamic system converge towards a unique and stable point independently from the initial state. The equilibrium point is characterized by the values $x_2^* = 2.223, y_2^* = 0.6949$. The eigenvalues of the Jacobian matrix computed at the equilibrium point are $\lambda = 0.226651 \pm i0.7155$ with $|\lambda| = 0.7635$

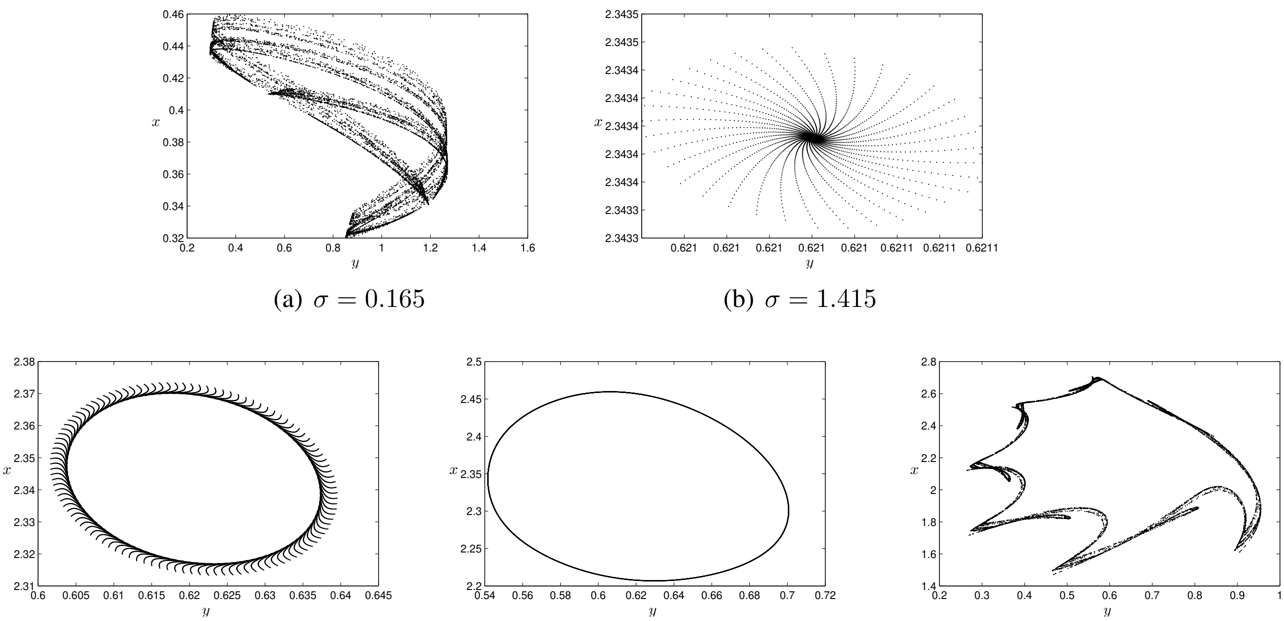
Continuing to increase the value of σ a Neimark-Sacker bifurcation takes place. For the parameter value $\sigma = 1.4165$, the equilibrium point occurs at $x_2^* = 2.3432$, $y_2^* = .6205$ and the associated pair of complex conjugate eigenvalues are $\lambda = .47498 \pm i.8799$ with $|\lambda| = 1.000$ this shows that the eigenvalues are belong to the unit circle and the stability properties of the equilibrium change through a Neimark-Sacker bifurcation. Figure 3(c) illustrates the phase plot for the bifurcation value of σ.

Continuing to increase the value of σ, we see what happens for $\sigma = 1.42$. The coordinates of the



(a) (b)

Figure 2: Bifurcation diagram for the a) x state coordinate, b) y state coordinate, varying σ. The parameter's are $\alpha = 0.12$, $\beta = 0$, $\gamma = 0.375$, $\rho = 0$, $r = 2.8$



(a) $\sigma = 0.165$ (b) $\sigma = 1.415$

(c) $\sigma = 1.4165$ (d) $\sigma = 1.42$ (e) $\sigma = 1.477$

Figure 3: Phase plot with the parameter's of Figure 1. a) chaotic trajectory, b) the stable fix point before the Neimark-Sacker bifurcation occurs, c) the Neimark-Sacker bifurcation, d) the stable invariant closed curve around the fixed point created after bifurcation, e) chaotic trajectory

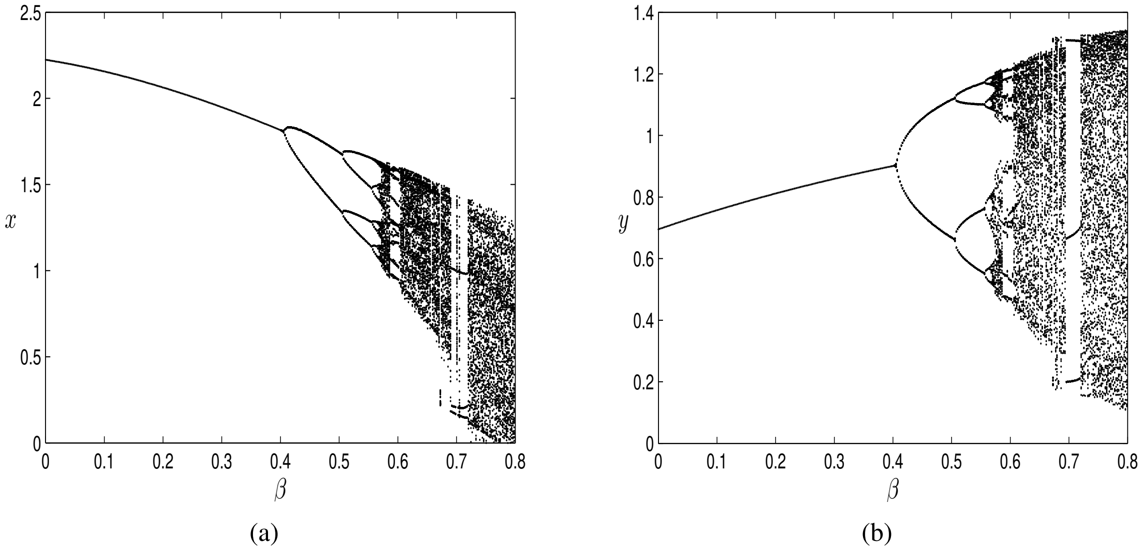


Figure 4: Bifurcation diagram for the a) x state coordinate, b) y state coordinate, varying β . The parameter's are $\alpha = 0.12$, $\sigma = 1.2$, $\gamma = 0.375$, $\rho = 0.2$, $r = 2.8$

equilibrium are $x_2^* = 2.3456$, $y_2^* = .61937$ and the associated eigenvalues are $\lambda = .4782 \pm i.8819$. The modulus of the complex conjugate eigenvalues is $|\lambda| = 1.0032$, and so we can conclude that the equilibrium became unstable and an invariant closed curve arises around the fixed point, which is shown in the Figure 3(d).

As σ is further increased a strange attractor is produced by successive stretching and folding. The equilibrium is $x_2^* = 2.3657$, $y_2^* = .6006$ and the eigenvalues are $\lambda = .53066 \pm i.88656$, with $|\lambda| = 1.0332$. The strange attractor is generated by the breaking of the invariant circles and the appearance of twelve chaotic (not shown in this figure) regions changes as they are linked into a single chaotic attractor.

4 Controlling through β by OGY method

We are interested in modifying the dynamic behavior of the $OAPA$ model, where we introduce the visitors fee β and the defensive expenditure $\rho\beta$.As it was shown in Figure 2, at the value $\sigma = 1.2$, the $OAPA$ model presents a stable fixed point. Figure 4 shows the bifurcation diagram of the system (3) where parameter β is varied in the interval $[0, 0.8]$ and the parameter ρ is posed equal to 0.2. We can achieve both stable dynamics and chaotic dynamics. In fact, starting from a stable fixed point of the $OAPA$ system, for values of $\beta \in [0, 0.42)$ the system (3) admits a stable fixed point, while for $\beta > 0.42$ the system exhibits chaotic dynamics.

In this section, we describes a method to stabilize this chaotic dynamic.

In order to achieve this goal the so-called OGY method (see [14])is used as main tool.

The OGY method was used successfully in several studies, both in economics and physics (see [5] and[9]). As it is summarized in [9], [15] and [13], the OGY method is based on the following assumption

a_1) A chaotic solution of a non linear dynamic system may have even an infinite number of unstable periodic orbits.

a_2) In a neighborhood of periodic solution the evolution of the system can be approximated by an appropriate local linearization of the equation of motion.

a_3) Small perturbations of the parameter p of the system can shift the chaotic orbit toward the so-called stable manifold of the chosen periodic orbit.

a_4) The points belonging to the stable manifold approach the periodic solution in the course of time.

Our goal is to find a "good" way to approach the periodic unstable orbit by proper changes of the parameter if the starting point is in a neighbourhood of the periodic unstable orbit.

Let us assume that the model can be described as

$$z_{n+1} = f(z_n, p) \tag{8}$$

where $n = 1, 2, ...$, p is real parameter, $z_n = (x_n, y_n) \in \Re^2$, $f = (f_1, f_2)$.

a_5) Suppose that we have a fixed point $z_0 = (x_0, y_0)$ belonging to a fixed parameter value p_0 such that

$$z_0 = f(z_0, p_0) \tag{9}$$

and this fixed point is unstable.

a_6) Assume that the Jacobian matrix has two eigenvalues λ_1, λ_2 satisfying $|\lambda_1| < 1 < |\lambda_2|$.

Then it follows from a_2) that, starting sufficiently close to z_0 and p_0, we can approximate the right hand side (8) by the first degree terms of its Taylor expansion around z_0 and p_0. Then by a_3), modifying p we try to shift the chaotic orbit toward a stable manifold.

Thanks to the OGY method the goal of approaching a stable manifold may be achieved as follows. Let z_n and p_n be closed enough to z_0 and p_0 as required in a_2). Then the next point of the orbit is determined by (8)

$$z_{n+1} = f(z_n, p_n) \tag{10}$$

Our aim is to determine p_n, i.e. how to control the system that orbit approaches the unstable fixed point.

From the above results we get the following theorem:

Theorem 1 *Under the assumptions $a_1) - a_6)$ there is a value for p_n such that trajectory of the recurrence map* (8) *is shifted towards to the stable manifold.*

We fix the parameters $\alpha = 0.12$, $\gamma = 0.375$, $\sigma = 1.2$, $\rho = 0.2$ and $r = 2.8$, $\beta = 0.745$ in such context the system exhibits a chaotic attractor. We consider that β is the control parameter which is available for external adjustment but is restricted to lying in some small interval $|\beta - \beta_0| < \delta$, $\delta > 0$ around the nominal value $\beta_0 = 0.745$. The system becomes:

$$\begin{aligned} x(t+1) &= x(t) - \beta - 0.375x(t) + 1.2y(t) \\ y(t+1) &= y(t) + 2.8(1 - y(t))y(t) - 0.12x^2(t) + 0.2\beta x(t) \end{aligned} \tag{11}$$

Following [12] we consider the stabilization of the unstable period one orbit $P_2 = (x^*, y^*) = (1.21738, 1.00126)$. The map (11) can be approximated in the neighborhood of the fixed point by the following linear map

$$\begin{pmatrix} x_{t+1} - x^\star \\ y_{t+1} - y^\star \end{pmatrix} \cong A \begin{pmatrix} x_t - x^\star \\ y_t - y{*}\star \end{pmatrix} + B \begin{pmatrix} \beta - \beta_0 \end{pmatrix} \tag{12}$$

where

$$A = \begin{pmatrix} \dfrac{\partial f(x^\star, y^\star)}{\partial x_t} & \dfrac{\partial f(x^\star, y^\star)}{\partial y_t} \\ \dfrac{\partial g(x^\star, y^\star)}{\partial x_t} & \dfrac{\partial g(x^\star, y^\star)}{\partial y_t} \end{pmatrix} \tag{13}$$

and

$$\begin{pmatrix} \dfrac{\partial f(x^\star, y^\star)}{\partial \beta} \\ \dfrac{\partial g(x^\star, y^\star)}{\partial \beta} \end{pmatrix} \tag{14}$$

are the Jacobian matrixes with respect to the control state coordinates $(x(t), y(t))$ and to the control parameter β. The partial derivatives are evaluated at the nominal value β_0 and at (x^*, y^*). In our case we get

$$\begin{pmatrix} x_{t+1} - 1.21738 \\ y_{t+1} - 1.00126 \end{pmatrix} \cong \begin{pmatrix} 0.625 & 1.2 \\ -0.14317 & -1.80708 \\ & \end{pmatrix} \begin{pmatrix} x_t - 1.21738 \\ y_t - 1.00126 \end{pmatrix} + \begin{pmatrix} -1 \\ 0.24347 \end{pmatrix} \begin{pmatrix} \beta - 0.75 \end{pmatrix} \tag{15}$$

Next, we check whether the system is controllable. A controllable system is one for which a matrix H can be found such that $J - BH$ has any desired eigenvalues. This is possible if $rank(C) = n$ where n is the dimension of the state space, and

$$C = (B : JB : J^2B : \cdots : J^{n-1}B) \tag{16}$$

In our case it follows that

$$C = (B : JB) = \begin{pmatrix} -1 & -0.3328 \\ 0.24347 & -0.29681 \\ & \end{pmatrix} \tag{17}$$

which obviously has rank 2 and so we are dealing with a controllable system. If we assume a linear feedback rule (control) for the parameter of the form

$$(\beta - \beta_0) = -H \begin{pmatrix} x(t) - x^* \\ y(t) - y^* \end{pmatrix} \tag{18}$$

where $H := [h1\ h2]$, then the linearized map becomes

$$\begin{pmatrix} x(t+1) - x^* \\ y(t+1) - y^* \end{pmatrix} \cong (J - BH) \begin{pmatrix} x(t) - x^* \\ y(t) - y^* \end{pmatrix} \tag{19}$$

that is

$$\begin{pmatrix} x_{t+1} - 1.21738 \\ y_{t+1} - 1.00126 \end{pmatrix} \cong \begin{pmatrix} 0.625 - h_1 & 1.2 - h_2 \\ -0.14317 + 0.2437h_1 & -1.80708 + 0.2347h_2 \\ & \end{pmatrix} \begin{pmatrix} x_t - 1.21738 \\ y_t - 1.00126 \end{pmatrix} \tag{20}$$

which shows that the fixed point will be stable provided that $A - BH$ is asymptotically stable, that is, all its eigenvalues have modulus smaller than one. The eigenvalues μ_1, μ_2 of the matrix $A - BH$ are called the "regulator poles" and the problem of placing these poles at the desired location by choosing with given is the "pole-placement problem". If the controllability matrix ' from equation (16) is of rank n, $n = 2$ in our case, then the pole-placement problem has a unique solution. This solution is given by

$$H = (\alpha_2 - a_2 \quad \alpha_1 - a_1)T^{-1} \tag{21}$$

where $T = CW$ and

$$W = \begin{pmatrix} a_1 & 1 \\ 1 & 0 \\ & \end{pmatrix} = \begin{pmatrix} 1.1820 & 1 \\ 1 & 0 \\ & \end{pmatrix} \tag{22}$$

Here a_1, a_2 are the coefficients of the characteristic polynomial of J 1.e.

$$|J - \lambda I| = \lambda^2 + a_1\lambda + a_2 = \lambda^2 + 1.1820\lambda - 0.9576 \tag{23}$$

and α_1, α_2 are the coefficients of the desired characteristic polynomial of $J - BH$, i.e.

$$\begin{aligned} ((J - BH) - \mu I) &= \mu^2 - \alpha_1\mu + \alpha_2 \\ &\Rightarrow \alpha_1 = -(\mu_1 + \mu_2) \\ &\Rightarrow \alpha_2 = \mu_1\mu_2 \end{aligned}$$

From equation (21) we get that

$$H = (\mu_1\mu_2 + 0.9576 \qquad (\mu_1+\mu_2) - 1.1820)\left(\begin{array}{cc} -0.64437 & -2.64657 \\ -0.02382 & 4.00933 \\ & \end{array}\right) \tag{24}$$

$$= (-.6444\mu_1\mu_2 - .5889 + .02382\mu_1 + .02382\mu_2 \qquad -2.647\mu_1\mu_2 - 7.274 - 4.009\mu_1 - 4.009\mu_2) \tag{25}$$

Since the $2-D$ map is nonlinear, the application of linear control theory will succeed only a sufficiently small neighborhood U around (x^*, y^*). Taking into account the maximum allowed deviation from the nominal control parameter β_0 and equation (18), we obtain that we are restricted to the following domain

$$D_H = \left\{(x(t), y(t)) \in \Re^2 : \left| H \left(\begin{array}{c} x(t) - x^* \\ y(t) - y^* \end{array} \right) \right| < \delta \right\} \tag{26}$$

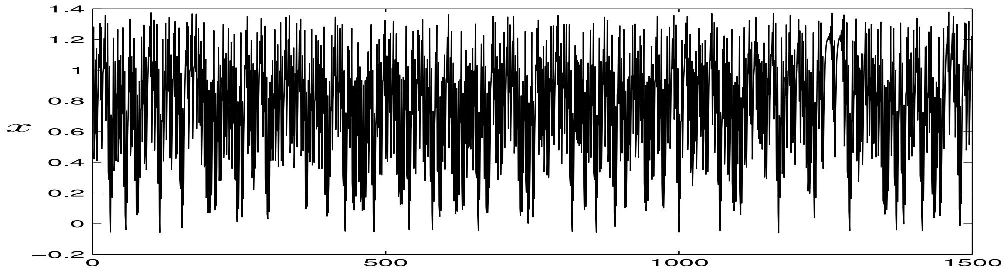
This defines a slab of width $\dfrac{2\delta}{|H|}$ and thus we activate the control (18) only for values of $(x(t), y(t))$ inside this slab, and choose to leave the control parameter at its nominal value when $(x(t), y(t))$ is outside the slab.

Any choice of regular poles inside the unit circle serves our purpose. There are many possible choices of the matrix H. In particular, it is very reasonable to choose all the desired eigenvalues to be equal to zero and in this way the target would be reached at least alter n period, and, therefore, a stabile orbit is obtain out of the chaotic evolution of the dynamics.

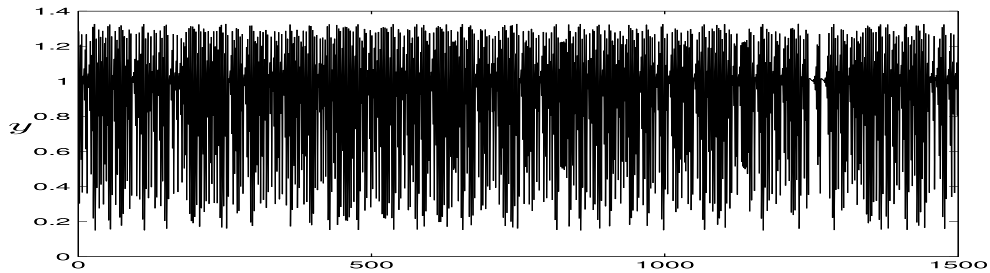
In Figure 5 (a)-(b) we show the time series of the chaotic trajectory initiated at point $(x_0, y_0) = (0.9, 0.8)$ which have chosen to control. In contrast Figure 5 (c)-(d) presents the controlled orbit converging to the stabilized fixed point when the feedback matrix H is chosen such that the eigenvalues of $(J - BH)$ are $\mu_1 = \mu_2 = 0$. This implies that $\mu_1 + \mu_2 = 0$, $\mu_1\mu_2 = 0$ and so $H = (-0.5889, -7.274)$. For this control strategy we have also chosen $\delta = 0.1$.

5 Conclusion

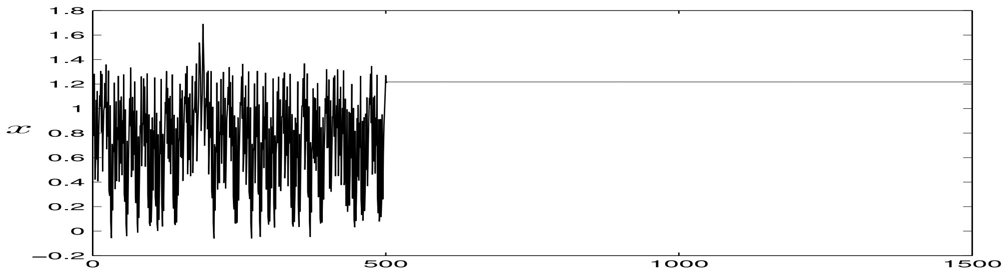
In this work we studied a discrete-time model that describes the interaction between visitors and the environment resource, in an Open Access Protected Area (OAPA). It was shown that by varying the parameter that indicates the preferences of visitors with reference to the environmental quality we can have complex dynamics (flip-flop bifurcation, Neimark-Sacker bifurcation and chaotic dynamics). Furthermore, we analyzed the impact that user fees and environmental defensive choices can have on the OAPA dynamics when it presents a stable equilibrium. Finally we have applied the OGY control technique (with user fee β as control parameter) and it was shown that the aperiodic and complicated motion that arises from the dynamics of the model can be easily controlled by small perturbations in their parameters and be turned into a stable steady state.



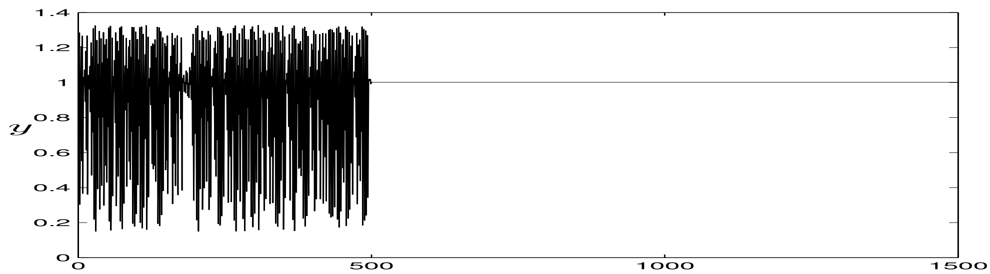
(a) Original chaotic orbit of the variable $x(t)$



(b) Original chaotic orbit of the variable $y(t)$



(c) Controlled chaotic orbit of the variable $x(t)$



(d) Controlled chaotic orbit of the variable $y(t)$

Figure 5: Compare between original and controlled orbit

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