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Inflation Persistence and Optimal Positive Long-run Inflation*

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Abstract

Within New Keynesian economics, the optimality of a monetary policy that aims at zero inflation is surprisingly robust. Full price stability is optimal despite the inefficiency of the nonstochastic steady state and the existence of a positively sloped long-run Phillips-curve trade-off. Even under inflation persistence due to backward-looking price indexation by price setters, zero inflation remains optimal. We show how backward-looking rule-of-thumb behaviour by price setters results in optimal positive long-run inflation. Comparing theoretical explanations for structural inflation persistence, the features that seem capable of delivering an endogenously optimal positive inflation target are costly disinflation, long-run Phillips-curve trade-off, and steady-state distortions.

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It follows that the chances that a shock would push the nominal interest rate to zero are negligible. This result poses the challenge for future researchers of finding a theoretical explanation for the optimality of positive inflation targets. Schmitt-Grohé and Uribe (2005, p. 52)

1 Introduction

There is much debate among both economists and central bankers about the appropriate inflation target for monetary policy. This paper contributes to the debate by analytically deriving the optimal, under commitment, long-run inflation target when there is structural inflation persistence due to backward-looking rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003).

The problem of what constitutes optimal inflation in the long-run is not trivial as monetary policy cannot simultaneously eliminate steady-state distortions and distortions resulting from staggered price-setting[1]. Of course, discretionary conduct of monetary policy would result in the well-known inflation bias stressed by Kydland and Prescott (1977) and Barro and Gordon (1983).

The combination of inefficient nonstochastic steady state, from which stems the central bank's desire to stabilise output around a level that is higher than the inefficient *natural level of output* (Friedman, 1968), and long-run Phillips-curve trade-off makes positive inflation forever in principles desirable as it would result in positive output gap forever.

This paper owes a lot to the landmark contribution by Woodford (2003) as it builds upon the *basic neo-Wicksellian model*. Furthermore, we employ many of the techniques used in that work such as the utility-based framework for the evaluation of monetary policy and the concept of *optimality from a timeless perspective* (Woodford, 1999).

This paper makes two distinct contributions to the literature on structural inflation persistence and optimal monetary policy.

First, we show how extending an otherwise basic New Keynesian model to the case of inflation persistence due to backward-looking rule-of-thumb behaviour by price setters breaks the surprising robustness of zero long-run inflation target, namely backward-looking rule-of-thumb behaviour by price setters results in optimal positive long-run inflation.

[1] Long-run and steady-state are used interchangeably in this paper.

Second, comparing theoretical explanations for structural inflation persistence, which share the assumption of backward-looking behaviour, suggests that the features that seem capable of delivering an endogenously optimal positive inflation target are costly disinflation, long-run Phillips-curve trade-off, and steady-state distortions.

It is often argued that the *New Keynesian Phillips Curve* (Roberts, 1995) defies belief as it cannot explain inflation persistence: once the factors bringing about high inflation have passed, inflation can return immediately to target without incurring any loss in output. Since Fuhrer and Moore (1995) the literature has been concerned with providing theoretical explanations for structural inflation persistence.

A widely used explanation relies on the assumption that a subset of price setters behave in a backward-looking manner[2]. Fuhrer and Moore (1995) appeal to a relative contracting model where nominal wages are set so to match the relative wages of other workers. Christiano et al. (2005) and Smets and Wouters (2003) put forward a model with backward-looking price indexation where firms are continually indexing prices to past inflation between any two pricing decisions. Galì and Gertler (1999) and Steinsson (2003) propose a model with rule-of-thumb behaviour where some price setters abide to a simple backward-looking rule-of-thumb when resetting their prices.

Comparing these theoretical explanations for structural inflation persistence, the features that seem capable of delivering an endogenously optimal positive long-run inflation target are costly disinflation, long-run Phillips-curve trade-off, and steady-state distortions. Indeed, under backward-looking rule-of-thumb behaviour by price setters, optimal steady-state inflation is zero in the absence of backward-looking rule-of-thumb behaviour, in the absence of long-run Phillips-curve trade-off, and in the absence of steady-state distortions.

The importance of costly disinflation is established by comparing the optimal plan first-order condition for inflation implied by backward-looking rule-of-thumb behaviour with the one that obtains in the *basic neo-Wicksellian model* with backward-looking price indexation. The relevance of a long-run Phillips-curve trade-off is established by comparing the hybrid Phillips curve that obtains in Fuhrer and Moore's relative contracting model vis-a-vis the one reported in Walsh (2003).

The remainder of the paper is organised in three sections. Section 2 presents the theoretical economy. Section 3 studies the long-run inflation target under the optimal commitment policy. Section 4 provides concluding remarks.

[2]A second explanation hinges on inflation expectations not being formed rationally. See Woodford (2007) and the references therein.

2 The Model

The New Keynesian model laid out here is the *basic neo-Wicksellian model* in Woodford (2003). It shares the *basic neo-Wicksellian model*'s notation[3], assumptions, and general formalism. It integrates it with the hybrid Phillips curve and the central bank's objective that obtain under backward-looking rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003)[4].

2.1 Households and market structure

There is a continuum of households of size one. The representative household seeks to maximize a discounted sum of utility of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \equiv E_0 \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t; \xi_t\right) - \int_0^1 v(h_t(i); \xi_t) di \right] \tag{1}$$

where $0 < \beta \leq 1$ is the discount factor, C_t is an aggregate of the household's consumption of each of the individual goods that are supplied (indexed by i over the unit interval), ξ_t is a vector of exogenous real shocks (i.e. exogenous shocks to household's impatience to consume and to the household's willingness to supply labour), and $h_t(i)$ is the supply of type i labour.

Following Dixit and Stiglitz (1977), the consumption aggregate is defined as

$$C_t = \left[\int_0^1 c_t(i)^{(\theta-1)/\theta} di \right]^{\theta/(\theta-1)} \tag{2}$$

where $c_t(i)$ is the consumption of good i and $\theta > 1$ is the constant elasticity of substitution between goods. For any given realisation of ξ_t, the period utility function, $u\left(C_t; \xi_t\right)$, is assumed to be concave and strictly increasing in C_t whereas the period disutility of supplying labour of type i, $v(h_t(i); \xi_t)$, is assumed to be convex and increasing in $h_t(i)$. Furthermore, we assume specific labour markets, namely type i labour is only used in the production of good i, and that the representative household simultaneously supplies all types of labour.

[3] This is precisely true for all variables and structural parameters but two. First, we denote with ω the degree of backward-looking rule-of-thumb behaviour by price setters rather than the elasticity of real marginal cost with respect to own output, which we denote with ϖ. Second, to avoid confusion with the Lagrangian multiplier associated with the period t hybrid Phillips Curve, φ_t, we denote with ϱ the parameter vector that indexes aspects of policy that determine steady-state values of inflation and output gap, $\overline{\pi}$ and $\overline{x}$.

[4] The hybrid Phillips curve and the central bank's objective in the case of backward-looking rule-of-thumb behaviour a là Steinsson (2003) correct the ones reported in Steinsson (2003). The hybrid Phillips curve and the central bank's objective in the case of backward-looking rule-of-thumb behaviour a là Galì and Gertler (1999) coincide (up to x^*) with the ones reported in Amato and Laubach (2003). A literally step-by-step derivation is available upon request.

We assume full financial markets, such that, through risk sharing, households face the same budget constraint, which is given by

$$\int_0^1 p_t(i)c_t(i)di + E_t\left[Q_{t,t+1}B_{t+1}\right] \leq B_t + \int_0^1 w_t(i)h_t(i)di + \int_0^1 \Pi_t(i)di - T_t \tag{3}$$

where $p_t(i)$ is the price of good i, B_t is the nominal value of financial wealth brought into the period, $Q_{t,t+1}$ is the stochastic discount factor for one period ahead payoff, T_t is net nominal tax collection by the Government, $w_t(i)$ is the nominal wage for labour of type i, and $\Pi_t(i)$ is the nominal profits from sales of good i. The budget constraint states that, in any period, financial wealth carried into the subsequent period plus consumption cannot be worth more than the value of financial wealth brought into the period plus after-tax nonfinancial income earned during the period. Note that we assume that every household owns an equal share of all the firms operating in the economy. The assumption of complete financial markets implies that the assumed firms' ownership and the fiction that the representative household supplies all types of labour directly are innocuous; dropping the assumptions would not change the conditions that determine equilibrium prices and quantities.

Optimal household's behaviour is described by three sets of requirements.

First, households face a decision in each period about how much to consume of each individual good. They adjust the share of a particular good in their consumption bundle so to exploit any difference in the relative price. Minimising the level of total expenditure, given the consumption aggregate in (2), yields the demand for each individual good

$$c_t(i) = \left(\frac{p_t(i)}{P_t}\right)^{-\theta} C_t \tag{4}$$

where the aggregate price level, P_t, is given by

$$P_t = \left[\int_0^1 p_t(i)^{1-\theta}di\right]^{1/1-\theta} \tag{5}$$

This specification of the price index has by construction the property that P_tC_t gives the minimum price for which an amount C_t of the aggregate consumption can be purchased.

Market clearing implies that the total nonfinancial income (i.e. the economy-wide sales revenues) can be written as P_tY_t where Y_t is an aggregate of the quantities supplied of the various differentiated goods, defined as in (2). The budget constraint can thus be rewritten as

$$P_tC_t + E_t\left[Q_{t,t+1}B_{t+1}\right] \leq B_t + P_tY_t - T_t \tag{6}$$

The absence of arbitrage opportunities implies that there exists a unique stochastic discount factor, $Q_{t,t+1}$. The riskless short-term nominal interest rate, i_t, has a simple representation in terms of the stochastic discount factor, namely $1/(1+i_t) = E_t\left[Q_{t,t+1}\right]$. A complete description of the household's budget constraint requires ruling out Ponzi schemes. The implied constraint for financial wealth carried into the subsequent period, B_{t+1}, is given by

$$B_{t+1} \geq -\sum_{T=t+1}^{\infty} E_{t+1}\left[Q_{t+1,T}\left(P_tY_t - T_t\right)\right] < \infty \tag{7}$$

with certainty, that is, in each state of the world that may be reached in the subsequent period. (7) implies that a household's debt in any state of the world is bounded by the present value of all future after-tax nonfinancial income, which is assumed to be finite. Furthermore, preventing unlimited consumption also requires that the nominal interest rate satisfies the zero lower bound, $i_t \geq 0$, at all times: a negative nominal interest rate would in fact allow to finance unbounded consumption by selling enough bonds. The entire infinite series of flow budget constraints and borrowing constraints in turn defines the lifetime budget constraint for the household

$$\sum_{t=0}^{\infty} E_0Q_{0,t}\left[P_tC_t\right] \leq B_0 + \sum_{t=0}^{\infty} E_0Q_{0,t}\left[\left(P_tY_t - T_t\right)\right] \tag{8}$$

We can now complete the description of optimal household behaviour. Maximising utility (1) subject to the intertemporal budget constraint (8) delivers the familiar Euler equation for consumption

$$\beta E_t\left[\frac{u_c\left(C_{t+1};\xi_{t+1}\right)}{u_c\left(C_t;\xi_t\right)}\frac{P_t}{P_{t+1}}\right] = \frac{1}{1+i_t} \tag{9}$$

and the optimal supply of labour of type i

$$\frac{v_h(h_t(i);\xi_t)}{u_c\left(C_t;\xi_t\right)} = \frac{w_t(i)}{P_t} \tag{10}$$

where u_c and v_h denote respectively the partial derivative of u with respect to the level of consumption and the partial derivative of v with respect to the supply of labour. Rational consumers are attempting to smooth consumption over time such that the marginal utility of consumption is equal across periods.

2.2 Firms

We assume that each good i has the linearly homogeneous production function

$$y_t(i) = A_t h_t(i) \tag{11}$$

where A_t is an exogenous technology factor. It follows that the nominal marginal cost of supplying a quantity $y_t(i)$ of good i is given by

$$MC_t(i) = w_t(i) A_t^{-1} \tag{12}$$

Note that the assumption of specific labour markets does not imply that each price setter is a monopsonist in her labour market. The possibility of firms having any market power in their labour market is ruled out by assuming that price setters that change their prices at the same time also hire labour from the same market. Specifically, this is achieved by assuming a double continuum of differentiated goods, indexed by $(I\ ,\ j)$ with an elasticity of substitution of θ between any two goods. Goods belonging to the same industry (i.e. with the same index I) are then assumed to change their prices at the same time and to be produced using the same type of labour (type I labour)[5]. The fact that now a continuum of price setters demand type I labour eliminates the possibility of market power in their labour market without any change for the degree of market power of each price setter in her product market.

Substituting (10) in (12) yields the real marginal cost specification

$$mc(y_t(i); C_t; \widetilde{\xi}_t) \equiv \frac{MC_t(i)}{P_t} = \frac{v_h(y_t(i)/A_t; \xi_t)}{u_c\left(C_t; \xi_t\right) A_t} \tag{13}$$

where labour is expressed in terms of output and $\widetilde{\xi}_t$ denotes the vector of exogenous disturbances, which includes exogenous real shocks to technology, to household's impatience to consume, and to the household's willingness to supply labour.

2.3 Market clearing

Goods market clearing requires, for each good i and at all times

$$y_t(i) = c_t(i) + g_t(i) \tag{14}$$

[5] The Calvo lottery is over industries' prices rather than goods' prices.

equivalently, in aggregate terms

$$Y_t = C_t + G_t \tag{15}$$

where G_t, which is such that $G_t < Y_t$ at all dates, is the exogenous process that describes Government purchases of the aggregate good.

Substituting the market clearing condition into (9) and (13) yields the equilibrium conditions

$$\beta E_t \left[\frac{\widetilde{u}_c(Y_{t+1}; \widetilde{\xi}_{t+1})}{\widetilde{u}_c(Y_t; \widetilde{\xi}_t)} \frac{P_t}{P_{t+1}} \right] = \frac{1}{1 + i_t} \tag{16}$$

$$mc_t(i) = mc(y_t(i); Y_t; \widetilde{\xi}_t) = \frac{\widetilde{v}_y(y_t(i); \widetilde{\xi}_t)}{\widetilde{u}_c(Y_t; \widetilde{\xi}_t)} \tag{17}$$

where $\widetilde{u}(Y_t; \widetilde{\xi}_t) \equiv u(Y_t - G_t; \xi_t)$ and $\widetilde{v}(y_t(i); \widetilde{\xi}_t) \equiv v(y_t(i)/A_t; \xi_t)$ are the indirect utility functions. The former, which is increasing and concave in Y_t for each possible realisation of vector $\widetilde{\xi}_t$, indicates the utility flow to the representative household as a function of its aggregate demand for resources, where aggregate demand adds the household's share of Government purchases to the household's private consumption. Under the assumption of G_t being exogenously determined, variations in the level of Government expenditure are simply another source of exogenous variation in the Euler equation for consumption[6]. The latter, which is increasing and convex in $y_t(i)$ for each possible realisation of vector $\widetilde{\xi}_t$, converts the household's disutility of supplying labour used for the production of good i into the household's disutility of directly supplying good i. Accordingly, the model laid out here is identical to the one that obtains under the assumption of a single yeoman farmer (i.e. continuum of yeoman farmers).

We now turn to the description of pricesetting behaviour. Following Calvo (1983), we assume that only a fraction $1 - \alpha$ of industries' prices are reset in each period. The probability of not resetting the price in each period, $0 < \alpha < 1$, is independent of both the time that has gone by since the last price revision and the misalignment between the actual price and the price that would be optimal to charge, namely pricing decisions in any period are independent of past pricing decisions. Furthermore, we assume that profits are discounted using a stochastic discount factor that equals on average β. Firms allowed to change their price at time t set it so to maximise expected future profits subject to the demand they face.

[6] Henceforth, the vector $\widetilde{\xi}_t$ includes exogenous real shocks to technology, to Government purchases, to household's impatience to consume, and to the household's willingness to supply labour.

The price setter's objective is given by

$$E_t \sum_{s=0}^{\infty} (\alpha\beta)^s \Pi(p_t(i), p_{t+s}^I, P_{t+s}, Y_{t+s}, \widetilde{\xi}_{t+s}) \tag{18}$$

The price setter's nominal profit function, Π, is linearly homogeneous in its first three arguments (i.e. good's price, industry's price, p_t^I, and aggregate price level) and, for any value of the industry price and the aggregate price level, single-peaked for some positive value of the good's price[7].

We now depart from full rationality by introducing backward-looking rule-of-thumb behaviour by price setters. Following Galì and Gertler (1999), we assume that only a fraction $1-\omega$ of industries behave optimally (i.e. in a forward-looking manner) when setting the price, the remaining fraction of industries use the same backward-looking rule-of-thumb when revising their prices. The degree of backward-looking rule-of-thumb behaviour, $0 \leq \omega < 1$, is thus constant over time and price setters cannot switch between backward-looking and forward-looking behaviour.

If follows that in each period all forward-looking price setters will set the same price, which we denote with p_t^f, and all backward-looking price setters will as well charge a common price, which we denote with p_t^b. The common forward-looking reset price, p_t^f, is implicitly defined by the relation

$$E_t \sum_{s=0}^{\infty} \alpha^s Q_{t,t+s} \Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s}, \widetilde{\xi}_{t+s}) = 0 \tag{19}$$

where $\Pi_1(p_t^f, p_t^f, P_{t+s}, Y_{t+s}, \widetilde{\xi}_{t+s}) = 0$ (i.e. the first-order condition for optimal pricing by all the suppliers of good i, which belongs to industry I) implicitly defines what Woodford (2003) labels the *notional Short-Run Aggregate Supply* curve. The common rule-of-thumb backward-looking reset price, p_t^b, is specified as in Steinsson (2003)

$$p_t^b = p_{t-1}^* \frac{P_{t-1}}{P_{t-2}} \left(\frac{Y_{t-1}}{Y_{t-1}^n} \right)^{\delta} \tag{20}$$

where $0 \leq \delta \leq 1$ is the degree of indexation to past demand conditions. Rule-of-thumb price setters thus index the previous period overall reset price, p_{t-1}^*, to past inflation (fully) and past demand conditions

[7] Under (11), the nominal profit function is given by

$$\Pi(p_t(i), p_t^I, P_t, Y_t, \widetilde{\xi}_t) \equiv p_t(i) y_t(i) - w_t^I h_t(i) \equiv p_t(i) \left(\frac{p_t(i)}{P_t} \right)^{-\theta} Y_t - \frac{v_h((p_t^I/P_t)^{-\theta} Y_t/A_t); \xi_t)}{u_c(C_t; \xi_t)} P_t \left(\frac{p_t(i)}{P_t} \right)^{-\theta} \frac{Y_t}{A_t}$$

(according to δ). The aggregate price level hence evolves according to

$$P_t = \left\{(1-\alpha)(p_t^*)^{1-\theta} + \alpha P_{t-1}^{1-\theta}\right\}^{\frac{1}{1-\theta}} \tag{21}$$

where

$$p_t^* = (1-\omega)p_t^f + \omega p_t^b \tag{22}$$

denotes the overall reset price.

2.4 Log-linearised model

Profit-maximising behaviour under perfectly flexible prices (i.e. all industries adjust prices optimally each period, taking the path of aggregate variables as given) implies that firms will operate at the point at which the relative price is a mark-up over the real marginal cost

$$\frac{p_t(i)}{P_t} = mc(y_t(i); Y_t; \widetilde{\xi}_t)\mu \tag{23}$$

where $\mu = \theta/1-\theta > 1$ is the desired constant mark-up. The relative supply of good i must satisfy

$$\left(\frac{y_t(i)}{Y_t}\right)^{-1/\theta} = mc(y_t(i); Y_t; \widetilde{\xi}_t)\mu \tag{24}$$

In a symmetric equilibrium, each good is supplied in the same quantity, which we denote with Y_t. Equilibrium output is then given by $Y_t = Y_t^n(\widetilde{\xi}_t)$, where the *natural level of output*, $Y_t^n(\widetilde{\xi}_t)$, is implicitly defined by

$$mc(Y_t^n; Y_t^n; \widetilde{\xi}_t) = \mu^{-1} \tag{25}$$

In the case of fully flexible prices, equilibrium output equals the *natural level of output* at all times. The *natural level of output* in turn depends only on the exogenous real shocks, namely equilibrium output under perfectly flexible prices is completely independent of monetary policy.

The natural steady-state level of output is the equilibrium level of output that obtains in the absence of sticky prices and in the absence of exogenous real shocks (i.e. $\widetilde{\xi}_t = 0$ at all times). The natural steady-state level of output, $\overline{Y}$, is implicitly defined by

$$mc(\overline{Y};\overline{Y};0) = \mu^{-1} \tag{26}$$

Henceforth, we log-linearise the structural equations around the natural steady-state level of output, $\overline{Y}$. If $\widetilde{\xi}_t = 0$ and $Y_t = \overline{Y}$ at all times, (21) has a solution with zero inflation at all times (i.e. $P_t = p_t^* = p_t^f = p_t^b = P_{t-1} = \overline{P}$ at all times). In the case of small enough fluctuations in $\widetilde{\xi}_t$ and Y_t around 0 and $\overline{Y}$ respectively, the solution to the log-linear approximate model is one in which any variable's log-deviation from its natural steady-state value (for instance, $\widehat{P}_t \equiv \log(P_t/\overline{P}\,))$ remains always close to 0[8].

Log-linearising (16) yields the intertemporal expectational IS relation

$$\widehat{Y}_t = E_t\widehat{Y}_{t+1} - \sigma\left[\widehat{i}_t - E_t\pi_{t+1} - \sigma^{-1}(g_t - E_t g_{t+1})\right] \tag{27}$$

where $\widehat{i}_t \equiv \log[(1+i_t)/(1+\bar{i})]$, $\pi_t \equiv \widehat{P}_t - \widehat{P}_{t-1} \equiv \log(P_t/P_{t-1})$, $\sigma \equiv -\widetilde{u}(\widetilde{u}_{cc}\overline{Y})^{-1} > 0$ measures the constant intertemporal elasticity of substitution of aggregate expenditure, and the disturbance term $g_t \equiv -\widetilde{u}_{c\xi}\xi_t(\widetilde{u}_{cc}\overline{Y})^{-1}$ indicates the percentage variation in output required to keep the marginal utility of expenditure at its natural steady-state level (given shocks to Government purchases and to household's impatience to consume).

Log-linearising (17) yields

$$\widehat{mc}_t(i) = \varpi\left(\widehat{y}_t(i) - q_t\right) + \sigma^{-1}(\widehat{Y}_t - g_t) \tag{28}$$

where $\widehat{mc}_t(i) \equiv \log(mc_t(i)/\mu)$, $\varpi \equiv \widetilde{v}_{yy}\overline{Y}\widetilde{v}_y^{-1} > 0$ measures the constant elasticity of real marginal cost with respect to own output, and the disturbance term $q_t \equiv -\widetilde{v}_{y\xi}\widetilde{\xi}_t(\widetilde{v}_{yy}\overline{Y})^{-1}$ indicates the percentage variation in output required to keep the marginal disutility of labour supply at its natural steady-state level (given shocks to technology and to the household's willingness to supply labour).

Under perfectly flexible prices, (28) reduces to

$$\log\left(\frac{\mu^{-1}}{\overline{\mu}^{-1}}\right) = \varpi\left(\widehat{Y}_t^n - q_t\right) + \sigma^{-1}(\widehat{Y}_t^n - g_t) \tag{29}$$

Solving for $\widehat{Y}_t^n \equiv \log(Y_t^n/\overline{Y})$ yields

$$\widehat{Y}_t^n = \frac{\varpi q_t + \sigma^{-1}g_t}{\varpi + \sigma^{-1}} \tag{30}$$

[8] Henceforth, a variable's log-deviation from its natural steady-state value, which is denoted with a bar, is denoted with a hat.

In the presence of a constant elasticity of substitution, percentage fluctuations in the *natural level of output* are equal to the percentage fluctuations in the efficient level of output, namely the equilibrium level of output under perfect competition and perfectly flexible prices. The efficient level of output, $Y_t^*(\widetilde{\xi}_t)$, is implicitly defined by

$$mc(Y_t^*; Y_t^*; \widetilde{\xi}_t) = 1 \tag{31}$$

Accordingly, the efficient steady-state level of output, $\overline{Y}^*$, is implicitly defined by

$$mc(\overline{Y}^*; \overline{Y}^*; 0) = 1 \tag{32}$$

Using (28), percentage fluctuations in the efficient level of output are then given by

$$\widehat{Y}_t^* = \frac{\varpi q_t + \sigma^{-1} g_t}{\varpi + \sigma^{-1}} \tag{33}$$

which equals percentage fluctuations in the *natural level of output* (i.e. (30)).

The natural steady-state level of output, $\overline{Y}$, can be rewritten as

$$mc(\overline{Y}; \overline{Y}; 0) = \overline{\mu}^{-1} \equiv 1 - \Phi_y \tag{34}$$

where the parameter Φ_y summarises the distortions in the natural steady-state level of output due to monopolistic competition. When Φ_y is small enough, the steady-state (i.e. constant over time) efficiency gap, $x^* \equiv \log(\overline{Y}^*/\overline{Y}) = O\left(\|\Phi_y\|\right)$, can be log-linearised as

$$\log(\overline{Y}^*/\overline{Y}) = \frac{\Phi_y}{\varpi + \sigma^{-1}} \tag{35}$$

Output gap, $x_t \equiv \widehat{Y}_t - \widehat{Y}_t^n \equiv \log(Y_t/Y_t^n)$, is the deviation of actual output from the *natural level of output*. (27) can be expressed in terms of output gap as

$$x_t = E_t x_{t+1} - \sigma \left(\widehat{i}_t - E_t \pi_{t+1} - \widehat{r}_t^n \right) \tag{36}$$

where

$$\widehat{r}_t^n = \sigma^{-1} \left[(g_t - \widehat{Y}_t^n) - E_t(g_{t+1} - \widehat{Y}_{t+1}^n) \right] \tag{37}$$

is the natural rate of interest, namely the real interest rate consistent with output equalling the *natural level of output* at all times. The interest rate gap, $\widehat{r}_t - \widehat{r}_t^n$ (with $\widehat{r}_t = \widehat{i}_t - E_t\pi_{t+1}$), thus indicates the effects on the actual level of output due to sticky prices.

We can now turn to the aggregate supply function. The aggregate inflation rate, π_t, and the aggregate output gap, x_t, in any period satisfy an aggregate supply relation of the form[9]

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_1 x_t + \kappa_2 x_{t-1} \tag{38}$$

with

$$\begin{aligned} \phi &= \alpha + \omega - (1-\beta)\omega\alpha; \chi_f = \frac{\alpha}{\phi}; \chi_b = \frac{\omega}{\phi}; \kappa_2 = \frac{(1-\alpha)\omega\delta}{\phi} \\ \kappa_1 &= \frac{(1-\omega)\alpha\kappa - (1-\alpha)\alpha\beta\omega\delta}{\phi}; \kappa = \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1}+\varpi)}{(1+\varpi\theta)\,\alpha} \end{aligned} \tag{39}$$

If $\omega = 0$, (38) and (39) collapse to Woodford (2003, 2.12 and 2.13, p. 187). If the fraction ω is reset according to backward-looking rule-of-thumb behaviour à la Galì-Gertler (1999) (i.e. $\delta = 0$), (38), standing (39), collapses to

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_1 x_t \tag{40}$$

2.5 Central bank's loss function

In the case of small enough fluctuations in the production of each good around the natural steady-state level of output, small enough exogenous real shocks, and small enough steady-state distortions, the period utility U_t can be approximated to second order as in Woodford (2003, 2.13, p. 396)

$$U_t = -\frac{\overline{Y}\widetilde{u}_c}{2}\left[(\sigma^{-1}+\varpi)(x_t - x^*)^2 + (1+\varpi\theta)\theta var_i \log p_t(i)\right] + t.i.p + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right) \tag{41}$$

where $var_i \log p_t(i)$ is a measure of the degree of price dispersion across industries (i.e. goods), $t.i.p$ collect terms that are independent of monetary policy (i.e. irrelevant to the welfare ranking of alternative equilibria), and ϱ is the parameter vector that indexes aspects of policy that determine the steady-state values of inflation and output gap, $\overline{\pi}$ and $\overline{x}$. In addition to stabilising output gap, around a level that exceeds the inefficient *natural level of output* by the steady-state efficiency gap, it is also appropriate for monetary policy to aim to curb price dispersion. This is achieved by stabilising the aggregate price level,

[9] See Appendix A.

but how fluctuations in the general price level affect price dispersion, hence welfare, depend upon the details of the pricesetting[10].

The discounted sum of utility of the representative household can then be approximated to second-order by

$$\sum_{t=0}^{\infty}\beta^t U_t = -\Omega\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c}\pi_t^2+\lambda_1(x_t-x^*)^2\\ +\lambda_2\left[\pi_t-(\pi_{t-1}+(1-\alpha)\delta x_{t-1})\right]^2\end{array}\right]+t.i.p+O\left(\left\|\Phi_y,\widetilde{\xi},\varrho,\Delta_{-1}^{1/2}\right\|^3\right) \tag{42}$$

The definition of κ in (39) holds. The constant Ω is given by $\Omega = \overline{Y}\widetilde{u}_c(\sigma^{-1}+\varpi)\theta/2\kappa$. The relative weight on output fluctuations is given by $\lambda_1 = \kappa/\theta$. The relative weight on $[\pi_t - (\pi_{t-1} + (1-\alpha)\delta x_{t-1})]^2$ is given by $\lambda_2 = \omega/\left[(1-\omega)\alpha\right]$. If $\omega = 0$, (42) collapses to Woodford (2003, 2.21 and 2.22, p. 400). In the presence of backward-looking rule-of-thumb behaviour a là Galì-Gertler (1999) (i.e. $\delta = 0$), (42) collapses to

$$\sum_{t=0}^{\infty}\beta^t U_t = -\Omega\sum_{t=0}^{\infty}\beta^t\left[\pi_t^2+\lambda_1(x_t-x^*)^2+\lambda_2(\pi_t-\pi_{t-1})^2\right]+t.i.p+O\left(\left\|\Phi_y,\widetilde{\xi},\varrho,\Delta_{-1}^{1/2}\right\|^3\right) \tag{43}$$

Interestingly, in the presence of backward-looking rule-of thumb behaviour by price setters, the utility-based central bank's loss function can now be seen as penalising variations in inflation as well as variations in the difference between general inflation and rule-of-thumb price increases.

3 The Optimal Long-run Inflation

Following the theoretical literature on optimal monetary policy, we assume that the central bank's policy instrument is the short-term nominal interest rate. The assumption reflects the actual practice of monetary policy by large central banks such as the European Central Bank, the Federal Reserve, and the Bank of England. The combination of cashless economy (i.e. there are no costs associated with varying the nominal interest rate) and central's bank control of the nominal interest rate implies that the intertemporal expectational IS relation imposes no real constraint on the central bank. Given the central bank's optimal choices for inflation and output gap, the expectational IS equation simply determines the path of nominal interest rate necessary to achieve the optimal path for the output gap. As a consequence, it is more convenient to treat output gap as if it were the central bank's policy instrument. The analysis is conducted in a purely deterministic setting, certainty equivalence guarantees that the results obtained hold in the presence of random disturbances.

[10] See Appendix A.

Under the optimal commitment policy, the central bank chooses paths for inflation and output gap to minimise the future discounted sum of losses from date 0 (i.e. when the policy is implemented) onward subject to the constraint that the paths must satisfy the aggregate supply relation each period. In the *basic neo-Wicksellian model*, the hybrid Phillips curve, namely a log-linear approximation to the model structural equations, suffices for a correct linear approximation to the optimal commitment policy only in the case of small steady-state distortions (i.e. x^* is small enough). Given the assumed deterministic setting, the solution for the optimal paths of inflation and output is accurate up to a residual that is only of second order. This is enough for a characterisation of the first-order consequences of allowing for the empirically realistic case of steady-state distortions (i.e. for inefficiency of the natural rate of output).

Precisely, we analytically derive the unique long-run inflation targets that are *optimal from a timeless perspective*, $\overline{\pi}$.

> *A constant inflation target $\overline{\pi}$ is optimal from a timeless perspective if the problem of minimising* the discounted sum of losses *subject to the constraint that the bounded sequences, $\{\pi_t, x_t\}_{t=0}^{\infty}$, satisfy the* aggregate supply curve *for each $t \geq 0$, and the additional constraint that $\pi_0 = \overline{\pi}$, has a solution in which $\pi_t = \overline{\pi}$ for all t.* Woodford (2003, p. 475).

The two commitment policies (i.e. timeless-perspective and zero-optimal) in the literature differ as the requirement that $\pi_0 = \overline{\pi}$ under timeless-perspective is replaced by the initial condition $\varphi_{-1} = 0$ (i.e. no fulfilment of expectations existing prior to the policy implementation) in the case of zero-optimal commitment policy. The two commitment policies in the literature thus share the same target[11]. Accordingly, we also assume that both inflation and output gap in the period before policy is implemented (i.e. date -1) are at their values of zero (i.e. the optimal paths for inflation and output gap are flat at their respective long-run optimal targets). As long as inflation at date -1 is nonzero, and/or output gap at date -1 is nonzero under Steinsson's rule-of-thumb, backward-looking rule-of-thumb behaviour implies that the optimal commitment policy, either zero-optimal or timeless-perspective, involves transition paths for inflation and the output gap to their respective long-run targets.

Under backward-looking rule-of-thumb behaviour by price setters a là Steinsson (2003), a central bank acting under commitment faces the problem of choosing bounded deterministic paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise (42) subject to the constraint that the sequences must satisfy (38) each period. We form the following Lagrangian.

[11] There is a unique optimal long-run inflation target. Hence, we can refer to it as the optimal long-run inflation (i.e. optimal steady-state inflation).

$$L=\sum_{t=0}^{\infty}\beta^t\left\{\begin{array}{c}\frac{1}{2}\pi_t^2+\frac{\lambda_1}{2}(x_t-x^*)^2+\frac{\lambda_2}{2}\left[\pi_t-(\pi_{t-1}+(1-\alpha)\delta x_{t-1})\right]^2\\ +\varphi_t\left[\pi_t-\chi_f\beta\pi_{t+1}-\chi_b\pi_{t-1}-\kappa_1x_t-\kappa_2x_{t-1}\right]\end{array}\right\} \tag{44}$$

where φ_t is the Lagrangian multiplier associated with the hybrid Phillips Curve. Differentiating with respect to π_t and x_t, we get the two first-order conditions

$$\pi_t+\lambda_2\left[\pi_t-(\pi_{t-1}+(1-\alpha)\delta x_{t-1})\right]-\beta\lambda_2\left[\pi_{t+1}-(\pi_t+(1-\alpha)\delta x_t)\right]+\varphi_t-\chi_f\varphi_{t-1}-\beta\chi_b\varphi_{t+1}=0 \tag{45}$$

$$\lambda_1(x_t-x^*)-\beta\lambda_2(1-\alpha)\delta\left[\pi_{t+1}-(\pi_t+(1-\alpha)\delta x_t)\right]-\kappa_1\varphi_t-\beta\kappa_2\varphi_{t+1}=0 \tag{46}$$

for each $t\geq 0$.

Proposition 1 *Consider a cashless economy with flexible wages, Calvo pricing, backward-looking rule-of-thumb behaviour a là Steinsson (*2003*) by price setters, and no real disturbances. Assume that the initial price dispersion of prices $\Delta_{-1}\equiv var\left\{\log_{-1}(I)\right\}$ is small, initial inflation is zero $\pi_{-1}=0$, initial output gap is zero $x_{-1}=0$, and steady-state distortions (measured by Φ_y) are small as well, so that an an approximation to the welfare of the representative household of the form (42) is possible, with the steady-state efficiency gap, x^*, a small parameter ($x^*=O(\|\Phi_y\|)$). Then, at least among inflation paths in which inflation remains forever in a certain interval around zero, there is a unique policy that is optimal from a timeless perspective. Under this policy, the positive optimal long-run inflation is given by*

$$\overline{\pi}=\frac{(1-\alpha)(1-\beta)\kappa\theta^{-1}\omega\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]}{\left\{\begin{array}{c}(1-\omega)(1-\alpha)(\theta^{-1}-\delta)(1-\beta)^2\alpha\omega\kappa+\\ \left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]\end{array}\right\}}x^*+O(\left\|\Delta_{-1}^{1/2},\Phi_y\right\|^2) \tag{b}$$

*Under backward-looking rule-of-thumb behaviour by price setters a là Galì and Gertler (*1999*) (i.e. $\delta=0$), the positive optimal long-run inflation is given by*

$$\overline{\pi}=\frac{(1-\alpha)(1-\beta)\omega\kappa}{(1-\omega)\alpha\theta\kappa+(1-\alpha)(1-\beta)^2\omega}x^*+O(\left\|\Delta_{-1}^{1/2},\Phi_y\right\|^2) \tag{a}$$

Under backward-looking rule-of-thumb behaviour by price setters, optimal steady-state inflation is zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega=0$), in the absence of long-run Phillips *curve trade off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$).*

Proof. See appendix B ■

The combination of steady-state distortions, from which stems the central bank's desire to stabilise output around a level that is higher than the inefficient *natural level of output,* and long-run Phillips-curve trade-off makes positive inflation forever in principles desirable as it would result in positive output gap forever.

Positive inflation forever obtains if and only if there is a steady-state incentive for positive inflation, namely the stimulative effect of inflation on output is not offset by the output cost of inflation. In all the variants of the *basic neo-Wicksellian model*, the optimal plan first-order condition for output gap determines a positive relationship between the long-run value of the Lagrange multiplier, $\overline{\varphi}$, and the long-run value of the output gap, $\overline{x}$. Precisely, $\overline{\varphi}$ is found to be a positive function of the difference between the long-run value of the output gap and the steady-state efficiency gap, x^*. Analysing the absence/presence of long-run incentive for positive inflation thus amounts to consider whether there is a steady-state relationship between inflation and the Lagrange multiplier. If the stimulative effect of higher inflation on output is greater than the output cost of higher inflation, $\overline{\pi}$ would then be negatively related to $\overline{\varphi}$. Hence, optimal long-run inflation would be found to be a positive function of the steady-state efficiency gap. In what follows, we are analysing the optimal plan first-order condition for inflation so to check whether the coefficients on the Lagrange multipliers add up to zero.

In the *basic neo-Wicksellian model* with backward-looking rule-of-thumb behaviour a là Galì and Gertler (1999) the optimal plan implies that inflation evolves according to (94)[12]. Substituting for χ_f and χ_b in terms of structural parameters yields

$$\pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \beta\lambda_2(\pi_{t+1} - \pi_t) + \varphi_t - \frac{\alpha}{\phi}\varphi_{t-1} - \frac{\beta\omega}{\phi}\varphi_{t+1} = 0 \tag{47}$$

Higher inflation in any period results in output increase in the same period, φ_t, and reduction in output in both the previous period as a result of expected higher inflation, $(\alpha/\phi)\varphi_{t-1}$, and the subsequent period, $(\beta\omega/\phi)\varphi_{t-1}$. Recalling that $\phi \equiv \alpha + \omega\left[1 - \alpha(1-\beta)\right]$, the absolute value of the overall output cost of higher inflation in any period is given by

$$\frac{\alpha + \beta\omega}{\alpha + \omega\left[1 - \alpha(1-\beta)\right]} \tag{48}$$

[12] See appendix B.

Checking the relationship between the stimulative effect of higher inflation on output and the output cost of higher inflation amounts to solve the inequality

$$1 \geq \frac{\alpha + \beta\omega}{\alpha + \omega\left[1 - \alpha(1 - \beta)\right]} \tag{49}$$

The solution is given by

$$\omega(1 - \beta)(1 - \alpha) \geq 0 \tag{50}$$

Note that (50) equally applies to the *basic neo-Wicksellian model* with backward-looking rule-of-thumb behaviour a là Steinsson (2003) as the Lagrange multipliers enter the optimal plan first-order condition for inflation in the same way. Backward-looking rule-of-thumb behaviour results in the stimulative effect of higher inflation being generally greater than the output cost of higher inflation. Not surprisingly, the stimulative effect of higher inflation equals the output cost of higher inflation in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$) or in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$). Otherwise, there exists a long-run incentive for positive inflation and the optimal long-run inflation, $\overline{\pi}$, is then found to be a positive function of the steady-state efficiency gap, x^*.

In the purely forward-looking *basic neo-Wicksellian model*, the optimal plan implies that inflation evolves according to

$$\pi_t + \varphi_t - \varphi_{t-1} = 0 \tag{51}$$

The increase in output in any period caused by higher inflation in the same period, φ_t, is thus offset by the cost of the reduction in output in the previous period as a result of expected higher inflation, φ_{t-1}. Hence, there is no long-run incentive for positive inflation, the optimal long-run inflation is zero.

The same conclusion holds in the *basic neo-Wicksellian model* with backward-looking price indexation. As in Woodford (2003, Ch. 6, Ch. 7), the conclusion can be reached directly from the result for the Calvo pricesetting. Alternatively, the optimal plan implies that inflation evolves according to

$$(\pi_t - \gamma\pi_{t-1}) - \beta\gamma(\pi_t - \gamma\pi_{t-1}) + \varphi_t - \varphi_{t-1} + \beta\gamma\varphi_t - \beta\gamma\varphi_{t+1} = 0 \tag{52}$$

As in the purely-forward looking *basic neo-Wicksellian model*, the increase in output in any period resulting from higher inflation in the same period, φ_t, is offset by the cost of the reduction in output in the previous period as a result of expected higher inflation, φ_{t-1}. Moreover, the additional increase in output in any period resulting from inflation in the same period, $\beta\gamma\varphi_t$, is also offset by the reduction in output in the subsequent period, $\beta\gamma\varphi_{t+1}$. Once again, there is no long-run incentive for positive inflation, the optimal steady-state inflation is zero.

Fuhrer and Moore's relative contracting model implies an hybrid Phillips curve of the form

$$\pi_t = (1-\varepsilon)E_t\pi_{t+1} + \varepsilon\pi_{t-1} + k_n x_t \tag{53}$$

whereas the hybrid Phillips curve reported in Walsh (2003, 5.65, p. 242) is given by

$$\pi_t = (1-\varepsilon)\beta E_t\pi_{t+1} + \varepsilon\pi_{t-1} + k_n x_t \tag{54}$$

where ε is a measure of the degree of backward-looking behaviour in pricesetting[13]. It is interesting to note that under $\kappa_n = \kappa_1$, $\lambda_n = \lambda_1$, and $\chi_f + \chi_b = 1$ (i.e. $(1-\varepsilon) = \chi_f$ and $\varepsilon = \chi_b$) (40) coincides with (54). $\chi_f + \chi_b = 1$ is then satisfied for $\omega\alpha(1-\beta) = 0$, namely the sum of the coefficients on future expected inflation and lagged inflation in the hybrid Phillips curve implied by backward-looking rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003), is generally greater than one. Given the rigour of mathematics (i.e. $\alpha = 0$ is outside the range for α), the coefficients on future expected inflation and lagged inflation add up to one only in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$) or in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$).

It must be stressed that (54) is simply the NKPC augmented with lagged inflation: the motivation for inflation persistence in (54) is purely empirical. The two Phillips curves differ only for one respect: (54) displays a long-run Phillips-curve trade-off, (53) does not. Assuming the monetary policy objective

$$\sum_{t=0}^{\infty} \beta^t \left[\pi_t^2 + \lambda_n (x_t - x^*)^2\right] \tag{55}$$

the optimal steady-state inflation is then easily seen to be given by[14]

$$\overline{\pi} = \frac{(1-\beta)\varepsilon\lambda_n\kappa_n}{\kappa_n^2 + (1-\varepsilon)(1-\beta)^2\varepsilon\lambda_n} x^* \tag{c}$$

Given $k_n > 0$ and $\lambda_n > 0$, optimal long-run inflation is always positive and collapses to zero in the absence

[13] We use ε in both Phillips curves because the only goal at hand is stressing the importance of a long-run Phillips-curve trade-off.

[14] See Appendix C.

of backward-looking behaviour in pricesetting (i.e. $\varepsilon = 0$), in the absence of steady-state distortions (i.e. $x^* = 0$), and in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$, namely when 53 replaces 54).

3.1 Calibration

Equation (a) contains six structural parameters (α, β, θ, ϖ, σ^{-1}, ω) for which values must be specified[15]. Four parameters are chosen to equal those used by Woodford (2003, p. 431), which stem from the estimation results in Rotemberg and Woodford (1997). These values are given in Table 1.

Structural parameter	β	θ	ϖ	σ^{-1}
Value	0.99	7.88	0.47	0.16

Table 1. Benchmark structural estimates (quarterly)

The steady-state efficiency gap, x^*, is accordingly set equal to 0.2, which is the value implied by $x^* = \Phi_y/(\varpi+\sigma^{-1})$, namely the steady-state distortions are only due to monopolistic competition. Letting α and ω vary over their respective ranges, annualized percentage optimal steady-state inflation is then observed to spike for low values of α, which are empirically unrealistic. Figure 1 thus reports the annualized percentage optimal steady-state inflation for empirically realistic values of the degree of price stickiness (between 2 and 5 quarters, $0.5 \leq \alpha \leq 0.8$). As for ω, Galì and Gertler (1999) report estimates of ω between 0.077 and 0.552, but we extend the range up to $\omega = 0.7$, which implies that the hybrid Phillips curve corresponds closely to the one in Fuhrer and Moore (1995) (i.e. $\varepsilon = 0.5$).

The deviation from full price stability is observed to be minimal. In effect, in developed countries inflation targets vary between 2% and 4% per year whereas slightly higher targets are observed in developing countries. However, low levels of annualized optimal steady-state inflation hinge on the extremely low relative weight on output fluctuations (i.e. $\lambda_1 = 0.003$ under the benchmark structural estimates).

Indeed, (c), which, under the three conditions above, generalises (a), is easily seen to be increasing in λ_n. Equation (c) contains five parameters (β, ε, λ_n, κ_n, x^*) for which values must be specified. Keeping β and x^* set equal to 0.99 and 0.2 respectively, the remaining parameters are chosen to equal those used in Walsh (2003, p. 527, p. 539). These values are given in table 2.

Parameter	ε	λ_n	κ_n
Value	0.5	0.25	0.05

Table 2. Benchmark estimates (quarterly)

[15] Under Steinsson's rule-of-thumb the structural parameters are seven (α, β, δ, θ, ϖ, σ^{-1}, ω). Given the absence of an estimate for δ, we focus on Galì and Gertler's rule-of-thumb. Note that we use the same terminology in Altissimo et al. (2006) thus distinguishing between estimates and structural estimates.

Under benchmark estimates, optimal steady-state inflation is 1.995% per year. Figure 2 reports the annualized percentage optimal steady-state inflation for values of ε up to 0.7 and values of λ_n up to 0.5[16].

4 Conclusion and Discussion

This paper makes two distinct contributions to the literature on structural inflation persistence and optimal monetary policy.

First, we show how backward-looking rule-of-thumb behaviour by price setters, specified either à la Galì and Gertler (1999) or à la Steinsson (2003), breaks the surprising robustness of zero long-run inflation target, namely backward-looking rule-of-thumb behaviour by price setters results in optimal positive long-run inflation.

The result gives a blow to skepticism about the application of existing New Keynesian models to policy advice and to empirical analysis. New Keynesian economics is undoubtedly providing a major input to our understanding of how central banks and governments interact in the macroeconomic policy arena, using their own policy instruments.

Second, comparing theoretical explanations for structural inflation persistence, which share the assumption of backward-looking behaviour, suggests that the features that seem capable of delivering an endogenously optimal positive inflation target are costly disinflation, long-run Phillips-curve trade-off, and steady-state distortions.

On the one hand, this paper highlights the trickiness of microfounding structural inflation persistence; on the other hand, costly disinflation seems capable of bringing the short-run in line with the long-run.

Given a long-run Phillips-curve trade-off, the dichotomy short-run and long-run is, to say the least, weakened. Arguably, the biggest virtue of New Keynesian economics is having tackled the conventional wisdom regarding the steady-state Phillips curve.

Last but not least, this paper reveals that the widespread practice in the New Keynesian literature on optimal monetary policy of restricting the attention to the case of an efficient nonstochastic steady state is far from being innocuous . What we show here is that a policy that is optimal for an economy with an efficient steady state differs from what is optimal in an economy where the empirically unrealistic subsidies that achieve Pareto efficiency are unavailable. Overall, fiscal policy shall not be assumed to fully offset steady-state distortions, fiscal policy can either exacerbate or ameliorate steady-state distortions.

[16] The annualized percentage optimal steady-state inflation is an arithmetic progression in ε.

The result is obviously sensitive to Calvo's (1983) assumption of a constant probability of price adjustment, irrespective of the duration of prices. Sheedy (2007a) drops such assumption and derives a simple and tractable expression for the Phillips curve that exhibits intrinsic inflation persistence. Inflation persistence is intrinsic, rather than structural, in the sense that inflation determination is partially backward-looking even though all agents remain forward-looking. Sheedy (2007b) goes on to analyse optimal monetary policy in response to shocks, but the steady state he considers is Pareto efficient. Extending the analysis to the case of an inefficient steady state is a natural way to confirm the importance of steady-state distortions for the optimality of positive inflation targets.

It should be stressed however that the model studied in this paper is a basic closed economy New Keynesian model. Papers such as Khan et al. (2003) and Altig et al. (2005) also consider other features such as transaction frictions, wage stickiness, capital goods, and investment in addition to price stickiness. In future research, we are particularly interested in extending the analysis to a model that is capable to account fairly well for business-cycle fluctuations.

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6 Figures

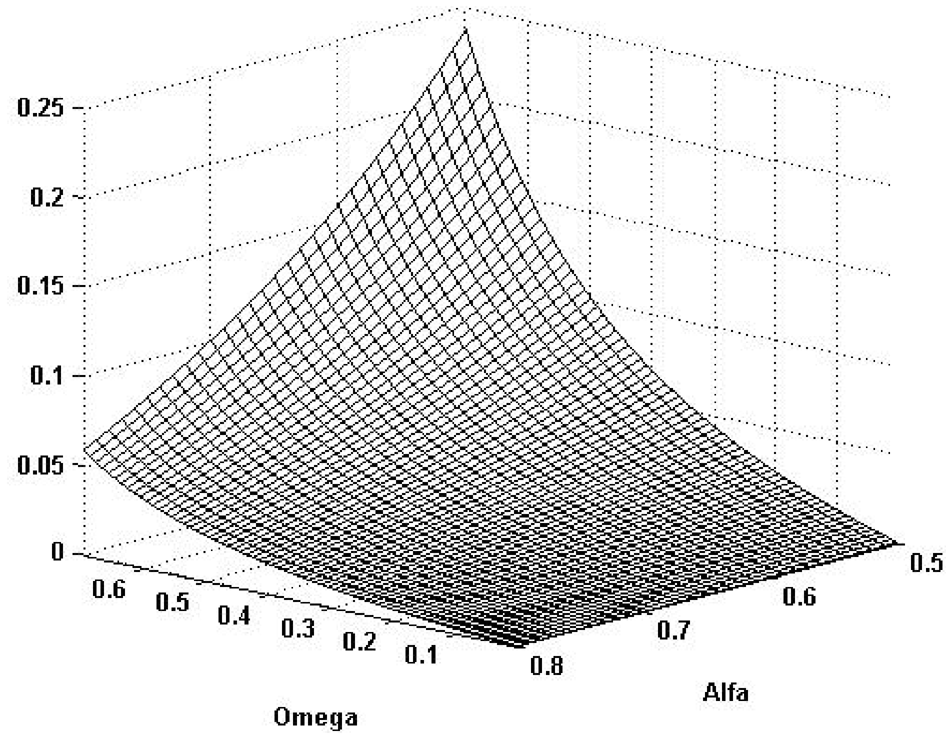

Figure 1: The annualized percentage optimal steady-state inflation as a function of α and ω.

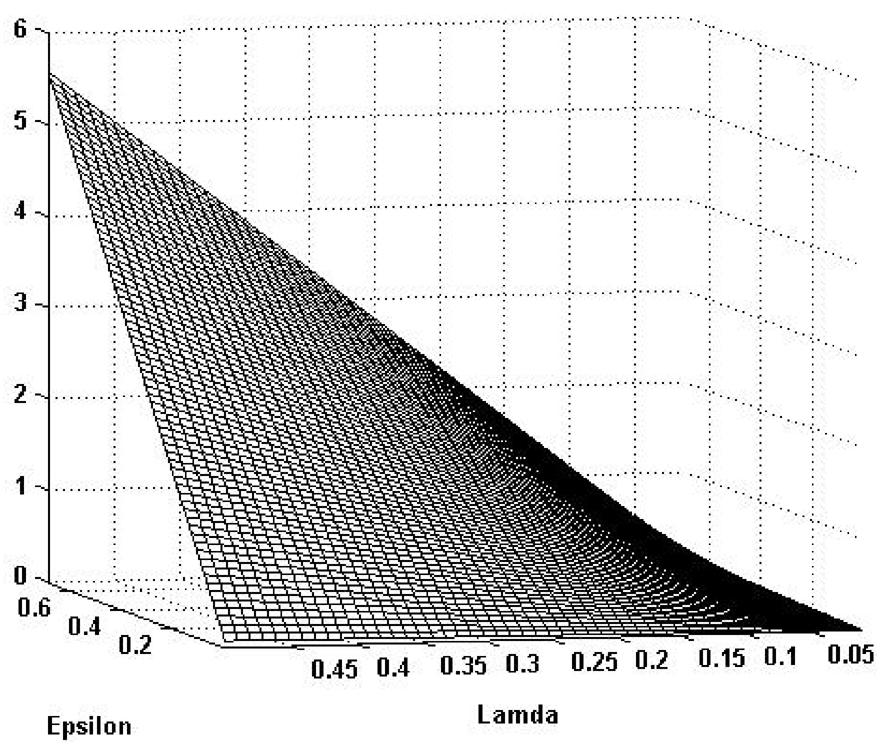

Figure 2: The annualized percentage optimal steady-state inflation as a function of ε and λ_n.

7 Appendix A. Derivations

7.1 The hybrid Phillips curve

(21) can be log-linearised as

$$\widehat{P}_t = (1-\alpha)\widehat{p}_t^* + \alpha\widehat{P}_{t-1} \tag{56}$$

with

$$\widehat{p}_t^* = (1-\omega)\widehat{p}_t^f + \omega\widehat{p}_t^b \tag{57}$$

A log-linearisation to the notional SRAS is given by

$$\log(p_t^f/P_t) = \zeta x_t \tag{58}$$

where ζ is the elasticity of the notional SRAS curve, which, under the assumption of specific labour markets, is given by $\zeta = (\sigma^{-1} + \varpi)(1+\varpi\theta)^{-1} > 0$. Substituting (58) in (19) and quasi-differencing yields

$$\widehat{p}_t^f = (1-\alpha\beta)\zeta x_t + (1-\alpha\beta)\widehat{P}_t + \alpha\beta E_t\widehat{p}_{t+1}^f \tag{59}$$

Log-linearising (20) yields

$$\widehat{p}_t^b = \widehat{p}_{t-1}^* + \pi_{t-1} + \delta x_{t-1} \tag{60}$$

(56) and (57) imply that the aggregate inflation rate, π_t, evolves according to

$$\pi_t = \frac{1-\alpha}{\alpha}\left[(1-\omega)(\widehat{p}_t^f - \widehat{P}_t) + \omega(\widehat{p}_t^b - \widehat{P}_t)\right] \tag{61}$$

Using (56), $\widehat{p}_t^b - \widehat{P}_t$ is given by

$$\widehat{p}_t^b - \widehat{P}_t = \frac{1}{1-\alpha}\pi_{t-1} - \pi_t + \delta x_{t-1} \tag{62}$$

Rewriting (59) in terms of $\widehat{p}_t^f - \widehat{P}_t$ yields

$$\widehat{p}_t^f - \widehat{P}_t = (1-\alpha\beta)\zeta x_t + \alpha\beta E_t(\widehat{p}_{t+1}^f - \widehat{P}_t) \tag{63}$$

Combining (56), (57), and (62), $E_t(\widehat{p}_{t+1}^f - \widehat{P}_t)$ is given by

$$E_t(\widehat{p}_{t+1}^f - \widehat{P}_t) = \frac{1}{(1-\alpha)(1-\omega)}E_t(\pi_{t+1} - \omega\pi_t) - \frac{\omega\delta}{(1-\omega)}x_t \tag{64}$$

Substituting (64) in (63), $\widehat{p}_t^f - \widehat{P}_t$ is given by

$$\widehat{p}_t^f - \widehat{P}_t = (1-\alpha\beta)\zeta x_t + \frac{\alpha\beta}{(1-\alpha)(1-\omega)} E_t(\pi_{t+1} - \omega\pi_t) - \frac{\alpha\beta\omega\delta}{(1-\omega)} x_t \tag{65}$$

Substituting (62) and (65) in (61) yields the hybrid Phillips curve for price inflation

$$\pi_t = \chi_f \beta E_t \pi_{t+1} + \chi_b \pi_{t-1} + \kappa_1 x_t + \kappa_2 x_{t-1} \tag{66}$$

where the parameters are

$$\left\{ \begin{array}{c} \phi = \alpha + \omega - (1-\beta)\omega\alpha; \chi_f = \frac{\alpha}{\phi}; \chi_b = \frac{\omega}{\phi}; \kappa_2 = \frac{(1-\alpha)\omega\delta}{\phi} \\ \kappa_1 = \frac{(1-\omega)\alpha\kappa - (1-\alpha)\alpha\beta\omega\delta}{\phi}; \kappa = \frac{(1-\alpha)(1-\alpha\beta)(\sigma^{-1}+\varpi)}{(1+\varpi\theta)\alpha} \end{array} \right\} \tag{67}$$

7.2 The second-order approximation to the period utility

We pick on the "choice of variables" issue (Woodford (2003), p. 388). The scenario is the one of small steady-state distortions, namely

$$U_c(\overline{Y}, 0) = O\left(\|\Phi_y\|\right) \tag{68}$$

What we show here is that, when (68) holds, a linear approximation to the production function is indeed accurate for the purpose of policy analysis. Considering a first-order and not a second-order approximation to the production function does not alter the approximate welfare measure, still given by (41). The period utility of the representative household, as a function solely of all $y_t(i)$, is given by

$$U_t = \widetilde{u}(Y_t; \widetilde{\xi}_t) - \int_0^1 \widetilde{v}(y_t(i); \widetilde{\xi}_t) di \tag{69}$$

The first term in (69) can be approximated to second order by

$$\widetilde{u}(Y_t; \widetilde{\xi}_t) = \overline{u} + \widetilde{u}_c \widetilde{Y}_t + \widetilde{u}_\xi \widetilde{\xi}_t + \frac{1}{2}\widetilde{u}_{cc}\widetilde{Y}_t^2 + \widetilde{u}_{c\xi}\widetilde{Y}_t\widetilde{\xi}_t + \frac{1}{2}\widetilde{\xi}_t' \widetilde{u}_{\xi\xi}\widetilde{\xi}_t + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right) \tag{70}$$

Substituting $\widetilde{Y}_t = \overline{Y}\widehat{Y}_t$ and dropping the terms that are higher than second order, yields

$$\widetilde{u}(Y_t; \widetilde{\xi}_t) = \overline{u} + \overline{Y}\widetilde{u}_c \widehat{Y}_t + \widetilde{u}_\xi \widetilde{\xi}_t + \frac{1}{2}\overline{Y}^2 \widetilde{u}_{cc}\widehat{Y}_t^2 + \overline{Y}\widetilde{u}_{c\xi}\widetilde{\xi}_t\widehat{Y}_t + \frac{1}{2}\widetilde{\xi}_t' \widetilde{u}_{\xi\xi}\widetilde{\xi}_t + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right) \tag{71}$$

Taking all the steps as in Woodford (2003, Appendix E.1) yields

$$\widetilde{u}(Y_t;\widetilde{\xi}_t)=\overline{Y}\widetilde{u}_c\left\{\widehat{Y}_t-\frac{1}{2}\sigma^{-1}\widehat{Y}_t^2+\sigma^{-1}g_t\widehat{Y}_t\right\}+t.i.p+O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{72}$$

The second term in (69), using $\widetilde{Y}_t=\overline{Y}\widehat{Y}_t$, can be approximated to second order by

$$\widetilde{v}(y_t(i);\widetilde{\xi}_t)=\overline{v}+\overline{Y}\widetilde{u}_c(1-\Phi_y)\widehat{y}_t(i)+\widetilde{v}_\xi\widetilde{\xi}_t+\frac{1}{2}\overline{Y}^2\widetilde{v}_{yy}\widehat{y}_t(i)^2+\overline{Y}\widetilde{v}_{y\xi}\widetilde{\xi}_t\widehat{y}_t(i)+\frac{1}{2}\widetilde{\xi}_t'\widetilde{u}_{\xi\xi}\widetilde{\xi}_t+O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{73}$$

(73) delivers

$$\int_0^1\widetilde{v}(y_t(i);\widetilde{\xi}_t)di=\overline{Y}\widetilde{u}_c\left\{\begin{array}{c}(1-\Phi_y)\widehat{Y}_t+\frac{1}{2}\varpi\widehat{Y}_t^2-\varpi q_t\widehat{Y}_t\\+\frac{1}{2}(\theta^{-1}+\varpi)var_i\widehat{y}_t(i)\end{array}\right\}+t.i.p+O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{74}$$

Combining (72) and (74) yields

$$U_t=\overline{Y}\widetilde{u}_c\left\{\begin{array}{c}\Phi_y\widehat{Y}_t-\frac{1}{2}(\sigma^{-1}+\varpi)\widehat{Y}_t^2+(\varpi q_t+\sigma^{-1}g_t)\widehat{Y}_t\\-\frac{1}{2}(\theta^{-1}+\varpi)var_i\widehat{y}_t(i)\end{array}\right\}+t.i.p+O\left(\left\|\Phi_y,\widetilde{\xi},\varrho\right\|^3\right) \tag{75}$$

which then results in (41).

7.3 The second-order approximation to the discounted sum of utility

Under Calvo (1983) staggered pricesetting and backward-looking rule-of-thumb behaviour by price setters, the distribution of prices in any period, $\{p_t(i)\}$, consists of α times the distribution of prices in the previous period, $\{p_{t-1}(i)\}$, an atom of size $(1-\alpha)(1-\omega)$ at the forward-looking reset price, p_t^f, and an atom of size $(1-\alpha)\omega$ at the rule-of-thumb backward-looking reset price, p_t^b

$$\{p_t(i)\}=\alpha\{p_{t-1}(i)\}+(1-\alpha)(1-\omega)p_t^f+(1-\alpha)\omega p_t^b \tag{76}$$

Let $\Delta_t\equiv var_i\log p_t(i)$ denote the degree of price dispersion and $\overline{P}_t\equiv E_i\{\log p_t(i)\}$ denote the average price, hence $\overline{P}_t-\overline{P}_{t-1}=E_i\left[\log\{p_t(i)\}-\overline{P}_{t-1}\right]$. Recalling $\log p_t^*=(1-\omega)\log p_t^f+\omega\log p_t^b$ and using (76), $\overline{P}_t-\overline{P}_{t-1}$ can be rewritten as

$$\begin{aligned}\overline{P}_t-\overline{P}_{t-1}&=\overbrace{\alpha E_i\left[\{\log p_{t-1}(i)\}-\overline{P}_{t-1}\right]}^{0}+(1-\alpha)(1-\omega)(\log p_t^f-\overline{P}_{t-1})+(1-\alpha)\omega(\log p_t^b-\overline{P}_{t-1})\\&=(1-\alpha)(\log p_t^*-\overline{P}_{t-1})\end{aligned} \tag{77}$$

Similarly, Δ_t can be rewritten as

$$\begin{aligned}
\Delta_t &= var_i\left[\log\{p_t(i)\} - \overline{P}_{t-1}\right] = E_i\left\{\left[\log\{p_t(i)\} - \overline{P}_{t-1}\right]^2\right\} - \left[E_i \log\{p_t(i)\} - \overline{P}_{t-1}\right]^2 \\
&= \left[\begin{array}{c} \alpha E_i\left\{\left[\log\{p_{t-1}(i)\} - \overline{P}_{t-1}\right]^2\right\} + (1-\alpha)(1-\omega)(\log p_t^f - \overline{P}_{t-1})^2 \\ +(1-\alpha)\omega(\log p_t^b - \overline{P}_{t-1})^2 - (\overline{P}_t - \overline{P}_{t-1})^2 \end{array}\right]
\end{aligned} \tag{78}$$

$\overline{P}_t$ is related to the Constant Elasticity of Substitution Dixit-Stiglitz (1967) price index through the log-linear approximation

$$\overline{P}_t = \log P_t + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{79}$$

the second-order residual follows from the fact that the equilibrium inflation process (as the equilibrium output process) satisfies a bound of second order $O(\left\|\widetilde{\xi}, \varrho\right\|^2)$ together with a second-order bound on the initial (i.e. date -1, policy is implemented at date 0) degree of price dispersion, Δ_{-1}. Note that, as in Woodford (2003), Δ_{-1} is assumed to be of second order (that is why it enters the second-order residual in (79) to the power of 1/2). It then follows that this measure of price dispersion continues to be only of second order in the case of first-order deviations of inflation from zero. Recalling $\log p_t^b = \log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}$ and using (79), $\log p_t^b - \overline{P}_{t-1}$ is given by

$$\begin{aligned}
\log p_t^b - \overline{P}_{t-1} &= \log p_{t-1}^* - \overline{P}_{t-2} - (\overline{P}_{t-1} - \overline{P}_{t-2}) + \pi_{t-1} + \delta x_{t-1} \\
&= \log p_{t-1}^* - \overline{P}_{t-2} + \delta x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right)
\end{aligned} \tag{80}$$

Recalling $\log p_t^* = (1-\omega)\log p_t^f + \omega \log p_t^b$, $\log p_t^b = \log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}$, and using (79), $\log p_t^f - \overline{P}_{t-1}$ is given by

$$\begin{aligned}
\log p_t^f - \overline{P}_{t-1} &= \frac{1}{1-\omega}\log p_t^* - \frac{\omega}{1-\omega}(\log p_{t-1}^* + \pi_{t-1} + \delta x_{t-1}) - \overline{P}_{t-1} \\
&= \left[\begin{array}{c} \frac{1}{1-\omega}(\log p_t^* - \overline{P}_{t-1}) - \frac{\omega}{1-\omega}(\log p_{t-1}^* - \overline{P}_{t-2}) \\ -\frac{\omega\delta}{1-\omega}x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \end{array}\right]
\end{aligned} \tag{81}$$

Using (79), (77) becomes

$$\pi_t = (1-\alpha)(\log p_t^* - \overline{P}_{t-1}) + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{82}$$

Accordingly, (80) and (81) become respectively

$$\log p_t^b - \overline{P}_{t-1} = \frac{1}{1-\alpha}\pi_{t-1} + \delta x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{83}$$

$$\log p_t^f - \overline{P}_{t-1} = \frac{1}{(1-\omega)(1-\alpha)}\pi_t - \frac{\omega}{(1-\omega)(1-\alpha)}\pi_{t-1} - \frac{\omega\delta}{(1-\omega)}x_{t-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^2\right) \tag{84}$$

Substituting (79), (83), and (84) in (78) yields

$$\Delta_t = \alpha\Delta_{t-1} + \frac{\alpha}{(1-\alpha)}\pi_t^2 + \frac{\omega}{(1-\omega)(1-\alpha)}\left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1}\right]^2 + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^3\right)$$

Integrating forward, starting from any small initial degree of price dispersion, Δ_{-1}, the degree of price dispersion in any period $t \geq 0$ is given by

$$\Delta_t = \sum_{s=0}^{\infty}\alpha^{t-s}\begin{bmatrix} \frac{\alpha}{(1-\alpha)}\pi_t^2 + \\ \frac{\omega}{(1-\omega)(1-\alpha)}\left[\pi_t - \pi_{t-1} - (1-\alpha)\delta x_{t-1}\right]^2 \end{bmatrix} + \alpha^{t-1}\Delta_{-1} + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^3\right) \tag{85}$$

The term $\alpha^{t-1}\Delta_{-1}$ is independent of monetary policy. Taking the discounted value of (85) over all periods $t \geq 0$ gives

$$\sum_{t=0}^{\infty}\beta^t\Delta_t = \frac{1}{1-\alpha\beta}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} \frac{\alpha}{(1-\alpha)}\pi_t^2 + \\ \frac{\omega}{(1-\omega)(1-\alpha)}\begin{bmatrix} \pi_t - \pi_{t-1} \\ -(1-\alpha)\delta x_{t-1} \end{bmatrix}^2 \end{bmatrix} + t.i.p + O\left(\left\|\Delta_{-1}^{1/2}, \widetilde{\xi}, \varrho\right\|^3\right) \tag{86}$$

Taking the discounted value of (41) over all periods $t \geq 0$ yields

$$\sum_{t=0}^{\infty}\beta^t U_t = -\frac{\overline{Y}\widetilde{u}_c}{2}\left[(\sigma^{-1}+\varpi)\sum_{t=0}^{\infty}\beta^t(x_t - x^*)^2 + (1+\varpi\theta)\theta\sum_{t=0}^{\infty}\beta^t\Delta_t\right] + t.i.p + O\left(\left\|\Phi_y, \widetilde{\xi}, \varrho\right\|^3\right) \tag{87}$$

Substituting (86) in (87) and normalizing on inflation, the discounted sum of utility of the representative household can be approximated to second-order by

$$\sum_{t=0}^{\infty}\beta^t U_t = -\frac{\overline{Y}\widetilde{u}_c(\sigma^{-1}+\varpi)\theta}{2\kappa}\sum_{t=0}^{\infty}\beta^t\begin{bmatrix} \pi_t^2 + \frac{\kappa}{\theta}(x_t - x^*)^2 \\ +\frac{\omega}{(1-\omega)\alpha}\begin{bmatrix} \pi_t - \pi_{t-1} \\ -(1-\alpha)\delta x_{t-1} \end{bmatrix}^2 \end{bmatrix} + t.i.p + O(\left\|\Phi_y, \widetilde{\xi}, \varrho, \Delta_{-1}^{1/2}\right\|^3) \tag{88}$$

8 Appendix B. Proof of Proposition 1

Proof. Condition (45) has a solution with inflation constant over time only if the Lagrange multiplier is also constant over time. Substituting a constant value for the Lagrange multiplier in (45) and (46), the two conditions can be simultaneously satisfied only if

$$\overline{\pi} = \left\{ \begin{array}{c} \frac{\{(\kappa_1+\beta\kappa_2)[(1-\beta)(1-\alpha)\delta\lambda_2]\}+\left\{(1-\chi_f-\beta\chi_b)\left[\lambda_1-\beta\lambda_2(1-\alpha)^2\delta^2\right]\right\}}{(\kappa_2+\beta\kappa_3)}\overline{x} \\ +\frac{(1-\chi_f-\beta\chi_b)\lambda_1}{(\kappa_2+\beta\kappa_3)} \end{array} \right\} \tag{89}$$

The hybrid Phillips curve (38) implies an upward-sloping relation

$$\overline{x} = \frac{(1-\beta\chi_f-\chi_b)}{(\kappa_2+\kappa_3)}\overline{\pi} \tag{90}$$

between long-run inflation and long-run output gap. Combining (89) and (90) yields the optimal steady-state inflation

$$\overline{\pi} = \frac{(1-\chi_f-\beta\chi_b)(\kappa_2+\kappa_3)\lambda_1}{(\kappa_2+\beta\kappa_3)(\kappa_2+\kappa_3)+(1-\chi_f\beta-\chi_b)\left\{ \begin{array}{c} \{(1-\chi_f-\beta\chi_b)\left[\lambda_1-\beta\lambda_2(1-\alpha)^2\delta^2\right]\} \\ -\{(\kappa_2+\beta\kappa_3)\left[(1-\beta)(1-\alpha)\delta\lambda_2\right]\} \end{array} \right\}}x^* \tag{91}$$

The sign of the relationship is more easily determined by substituting for all the parameters in (91) in terms of structural parameters (keeping κ implicit)

$$\overline{\pi} = \frac{(1-\alpha)(1-\beta)\kappa\theta^{-1}\omega\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right]}{\left\{ \begin{array}{c} (1-\omega)(1-\alpha)(\theta^{-1}-\delta)(1-\beta)^2\alpha\omega\kappa \\ \left[(1-\omega)\alpha\kappa+(1-\alpha)^2\beta\omega\delta\right]\left[(1-\omega)\alpha\kappa+(1-\alpha)(1-\alpha\beta)\omega\delta\right] \end{array} \right\}}x^* \tag{92}$$

which is (b). Given $k > 0$ and the rigour of mathematics (i.e. $\alpha = 1$ is outside the range for α as it would imply dividing by zero in deriving (38)), optimal long-run inflation is always positive and collapses to zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$), in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$). We now turn to the case of backward-looking rule-of-thumb behaviour by price setters a là Galì and Gertler (1999). What constitutes optimal long-run inflation is implied by setting $\delta = 0$ in (b). Here we prefer to derive it. A central bank acting under commitment faces the problem of choosing bounded deterministic paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise (43) subject to the constraint that the sequences must satisfy (40) each period. We form the following Lagrangian

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \frac{1}{2} \left[\pi_t^2 + \lambda_1 (x_t - x^*)^2 + \lambda_2 (\Delta \pi_t)^2 \right] + \varphi_t \left[\pi_t - \chi_f \beta \pi_{t+1} - \chi_b \pi_{t-1} - \kappa_1 x_t \right] \right\} \tag{93}$$

Differentiating with respect to π_t and x_t, we get the two first-order conditions

$$\pi_t + \lambda_2(\pi_t - \pi_{t-1}) - \beta \lambda_2 (\pi_{t+1} - \pi_t) + \varphi_t - \chi_f \varphi_{t-1} - \beta \chi_b \varphi_{t+1} = 0 \tag{94}$$

$$\lambda_1 (x_t - x^*) - \kappa_1 \varphi_t = 0 \tag{95}$$

for each $t \geq 0$. Condition (94) has a solution with inflation constant over time only if the Lagrange multiplier is also constant over time. Substituting a constant value for the Lagrange multiplier in (94) and (95), the two conditions can be simultaneously satisfied only if

$$\overline{\pi} = -\frac{(1 - \chi_f - \beta \chi_b)\lambda_1}{\kappa_1} (\overline{x} - x^*) \tag{96}$$

The hybrid Phillips curve (40) implies an upward-sloping relation

$$\overline{x} = \frac{(1 - \beta \chi_f - \chi_b)}{\kappa_1} \overline{\pi} \tag{97}$$

between long-run inflation and long-run output gap. Combining (96) and (97) yields the optimal long-run inflation target

$$\overline{\pi} = \frac{(1 - \chi_f - \beta \chi_b)\lambda_1 \kappa_1}{\kappa_1^2 + (1 - \chi_f - \beta \chi_b)(1 - \beta \chi_f - \chi_b)\lambda_1} x^* \tag{98}$$

The sign of the relationship is more easily determined by substituting for all the parameters in (98) in terms of structural parameters (keeping κ implicit). Here, rather than simply substituting, we can double-check the result obtained. Combining (95) and (94), optimal paths for inflation and output gap satisfy

$$\left[\begin{array}{c} \pi_t + \frac{\omega}{\alpha(1-\omega)}(\pi_t - \pi_{t-1}) \\ -\frac{\beta\omega}{\alpha(1-\omega)}(\pi_{t+1} - \pi_t) \end{array} \right] = \frac{1}{(1-\omega)\alpha\theta} \left[\begin{array}{c} \alpha(x_{t-1} - x^*) + \omega\beta(x_{t+1} - x^*) \\ -\phi(x_t - x^*) \end{array} \right] \tag{99}$$

Solving analytically for the optimal paths for inflation and output gap would require combining (99) with (40) and solve the resulting difference equation. Here we are content with deriving the optimal long-run inflation. The hybrid Phillips Curve (40) can be rewritten in terms of structural parameters as

$$x_t = \frac{1}{\kappa}(\pi_t - \beta\pi_{t+1}) - \frac{\omega\beta}{(1-\omega)\kappa}(\pi_{t+1} - \pi_t) + \frac{\omega}{(1-\omega)\alpha\kappa}(\pi_t - \pi_{t-1}) \tag{100}$$

where the equivalence $\pi_{t+1} \equiv \omega\pi_{t+1} - (1-\omega)\pi_{t+1}$ is used to obtain a term in the rate of inflation acceleration at date $t+1$. Combining (99) and (100) optimal long-run inflation is given by

$$\overline{\pi} = \frac{\omega(1-\alpha)(1-\beta)\kappa}{(1-\omega)\alpha\theta\kappa + \omega(1-\alpha)(1-\beta)^2}x^* \tag{101}$$

which is (a) (i.e. (98) in terms of structural parameters, (b) under $\delta = 0$). Given $k > 0$ and the rigour of mathematics (i.e. $\alpha = 1$ is outside the range for α as it would imply dividing by zero in deriving (40)), optimal long-run inflation is always positive and collapses to zero in the absence of backward-looking rule-of-thumb behaviour (i.e. $\omega = 0$), in the absence of long-run Phillips curve trade off (i.e. $\beta = 1$), and in the absence of steady-state distortions (i.e. $x^* = 0$). ■

9 Appendix C

A central bank acting under commitment faces the problem of choosing bounded deterministic paths for inflation and the output gap, $\{\pi_t, x_t\}_{t=0}^{\infty}$, to minimise (55) subject to the constraint that the sequences must satisfy (54) each period. We form the following Lagrangian.

$$L = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{c} \frac{1}{2}\left[\pi_t^2 + \lambda_n(x_t - x^*)^2\right] \\ +\varphi_t\left[\pi_t - (1-\varepsilon)\beta\pi_{t+1} - \varepsilon\pi_{t-1} - k_n x_t\right] \end{array} \right\} \tag{102}$$

Differentiating with respect to π_t and x_t, we get the two first-order conditions

$$\pi_t + \varphi_t - (1-\varepsilon)\varphi_{t-1} - \beta\varepsilon\varphi_{t+1} = 0 \tag{103}$$

$$\lambda_n(x_t - x^*) - \kappa_n\varphi_t = 0 \tag{104}$$

for each $t \geq 0$. Condition (103) has a solution with inflation constant over time only if the Lagrange multiplier is also constant over time. Substituting a constant value for the Lagrange multiplier in (103) and (104), the two conditions can be simultaneously satisfied only if

$$\overline{\pi} = -\frac{(1-\beta)\varepsilon\lambda_n}{\kappa_n}(\overline{x} - x^*) \tag{105}$$

The hybrid Phillips curve (54) implies an upward-sloping relation

$$\overline{x} = \frac{(1-\beta)(1-\varepsilon)}{\kappa_n}\overline{\pi} \tag{106}$$

between long-run inflation and long-run output gap. Combining (105) and (106) yields the optimal long-run inflation target

$$\overline{\pi} = \frac{(1-\beta)\varepsilon\lambda_n\kappa_n}{\kappa_n^2 + (1-\varepsilon)(1-\beta)^2\varepsilon\lambda_n}x^* \tag{107}$$

which is (c).