Default Risk and Risk Averse International Investors

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Abstract

This paper develops a model of debt and default for small open economies that interact with risk averse international investors whose preferences exhibit decreasing absolute risk aversion (DARA). By incorporating risk averse investors who trade with an emerging economy, the present model explains a larger proportion and volatility of the spread between sovereign bonds and riskless assets than the standard model with risk neutral investors. The paper shows that if investors have DARA preferences, then the emerging economy's default risk, capital flows, and bond prices are a function not only of the fundamentals of the economy but also of the level of financial wealth and risk aversion of international investors. In particular, as investors become wealthier or less risk averse, the emerging economy becomes less credit constrained. As a result, the emerging economy's default risk is lower, and its bond prices and capital inflows are higher. Additionally, with risk averse investors, the risk premium in the asset prices of the sovereign countries can be decomposed into two components: a base premium that compensates the investors for the probability of default and an "excess" premium that compensates them for taking the risk of default.

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1 Introduction

In the literature on endogenous sovereign default risk, it is widely recognized that there is a strong relation between the cyclical behavior of the domestic fundamentals of emerging economies and their access to international credit markets.[1] However, despite the growing empirical literature on the subject, it is not as widely acknowledged that the characteristics of the investors should also have an impact on the quality of the emerging economies' access to credit. Examples of investors' characteristics that might affect financial flows towards emerging economies include their attitude towards risk and their wealth level.[2] By taking in account these characteristics, the current paper can account for five stylized facts regarding emerging economies' access to financial markets that have not been explained by the previous literature in endogenous sovereign default risk:

(i) Emerging economies' estimated default probabilities do not account for all of the yield spreads in their sovereign bonds.

(ii) The proportion of sovereign yield spreads explained by emerging economies' own fundamentals is smaller for riskier sovereign bonds than for investment grade bonds.

(iii) Investors' financial performance and their net foreign asset position in emerging economies are positively correlated.

(iv) Emerging economies' credit spreads are positively correlated with spreads of corporate junk bonds from developed countries.

(v) Sovereign bond spreads across emerging economies are highly correlated.

This paper presents a stochastic general equilibrium model of endogenous sovereign default risk in which two types of agents interact in international financial markets: (1) risk averse financial investors whose preferences exhibit decreasing absolute risk aversion (DARA) and (2) the benevolent government of an emerging economy which cannot commit to pay its debts. The interaction between the two parties determines the equilibrium price of the bonds of the emerging economy. The price of the bonds of the economy depends on

[1]See for example Aguiar and Gopinath (2006), Bai and Zhang (2006), Arellano (2008), Hatchondo, Martinez and Sapriza (2008) and Cuadra and Sapriza(2008), and Mendoza and Yue (2008).

[2]The empirical literature on the determination of sovereign spreads establishes an important role for investors' risk appetite and liquidity patterns in the determination of bond spreads. See for example Fitzgerald and Krolzig (2003), Ferruci et al. (2004), Remolona et al.(2007), González and Levy Yeyati (2008), Broner et al.(2010) and Longstaff et al. (2011)

the likelihood of repayment of the debt and on the tolerance towards the risk of default of the investors. Both the investors and the government take as given the price function of the emerging economy's discount bonds, q.

International financial markets are incomplete because the only traded assets are one period no-contingent bonds, and risk free T-Bills. Therefore the investors are not able to insure away the income uncertainty specific to the emerging country.

Households in the economy are identical and risk averse. Each period these households receive a stochastic endowment plus a lump-sum transfer from their government and they choose their consumption subject to their budget constraint. The government of the economy solves a dynamic optimization problem in order to maximize the lifetime utility of the households in the economy. The government uses access to financial markets to smooth the consumption path of these households by trading one-period non-contingent bonds with the investors. If the government defaults on its debts, it is temporarily excluded from international credit markets.

On the other side of the market, it is assumed that all the international investors are identical. The risk averse representative investor solves a dynamic portfolio problem in which she decides the optimal allocation of her portfolio between bonds of the economy and T-Bills. Since the representative investor's preferences exhibit DARA, her tolerance towards risk is highly dependent on her wealth and degree of risk aversion.[3]

Clearly, a model of endogenous default risk with risk neutral international investors cannot explain stylized facts (i) through (v) because in such models spreads are explained only by the default probabilities and these probabilities depend only on the fundamentals of the economy. Instead, in the current model, international investors demand an *excess* risk premium in order to willingly take the risk of default embodied in the emerging economies' sovereign bonds and this risk premium is higher for higher levels of risk. Additionally, since

[3]The assumption of DARA preferences seems to be justified by the characteristics of the investors in emerging financial markets. These investors are both individuals and institutional investors such as banks, mutual funds, hedge funds, pension funds and insurance companies. For individual investors, it is straightforward to assume that these agents are risk averse. For institutional investors risk aversion may follow from two sources: regulations over the composition of their portfolio and the characteristics of the institutions' management. Regarding the first source, banks face capital adequacy ratios; mutual funds face restrictions in their access to leverage against their asset holdings; and pension funds and insurance companies face strict limits on their exposure to risk. Regarding the second source, for each class of institutional investor, managers ultimately make the portfolio allocation decisions. These managers can also be treated as risk averse agents. Additionally, the remuneration—and therefore the wealth—of these agents is closely related to the performance of the portfolio that they manage. These factors suggest that portfolio choices of institutional investors will be consistent with the choices of agents whose preferences exhibit DARA.

investors' preferences exhibit DARA, their tolerance of risk varies with their wealth and their degree of risk aversion. These results imply the following: First, this model can account for fact (i) since the price of the emerging economy's bonds is lower than the world price of T-Bills adjusted by the economy's default probability. Second, this model can account for fact (ii) since the proportion of sovereign yield spreads explained by default probabilities is smaller for riskier sovereign bonds than for less risky bonds. Third, this model can account for fact (iii) because there is a positive correlation between the representative lender's wealth and the lender's investment in the emerging economy. Fourth, this model can account for facts (iv) and (v) because whenever investors' tolerance to risk changes, there must be a change in the spreads of all risky assets. As a consequence, the spreads of the economy sovereign bonds should be correlated with the spreads of other emerging economies and of industrialized economies' junk bonds.

In the quantitative part of the paper, the model is calibrated to the case of the recent default in Argentina and its results are compared to the results of a similar model of endogenous sovereign default risk with risk neutral investors. In general, the model with risk averse investors performs better at explaining the real business statistics in Argentina. The current model explains the spreads of the economy much better than its counterpart because it delivers higher average spreads with a higher volatility which is closer to the observed data. Also the model delivers the observed correlation between investors' wealth (proxied by the $SP500$, as in the empirical literature) and the spreads of the economy. Like the risk-neutral model, the risk-averse model can also account for the negative correlations between output and trade balance, and output and interest rates; and both models account for the positive correlation between interest rates and trade balance. On the downside, neither model delivers sufficiently high average levels of debt.

The paper is organized as follows: section 1 is the introduction; section 2 presents the theoretical model; section 3 characterizes the equilibrium of the model; section 4 discusses the quantitative implications of the model; and section 5 concludes. Two appendixes provide proofs of propositions presented in the main text and the algorithm that solves the model.

2 THE MODEL

2.1 The Emerging Economy

There is a small open economy that is populated by identical risk averse households that maximize their discounted expected lifetime utility from consumption

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{1}$$

where $0 < \beta < 1$ is the discount factor and c is the households' consumption at time t. The households' periodic utility takes the functional form

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In each period, the households receive a stochastic stream of consumption goods, y. This income is non-storable, its realizations are assumed to have a compact support, and the stream of income follows a Markov process drawn from probability space $(y, Y(y))$ with a transition function $f(y' \mid y)$. Households also receive a lump-sum transfer from the government.

The government of the economy is a benevolent government that aims to maximize the lifetime utility of the households in the economy. The government has access to international financial markets in which it trades one-period non-contingent bonds with a representative competitive risk averse international investor. The government uses this access to financial markets to smooth the consumption path of the households in the economy.

In international financial markets the government borrows or saves by buying one period bonds, b', at price $q(b', y, W)$. Both the investors and the government of the economy take as given the price function of the emerging economy's non-contingent discount bonds, $q(b', y, W)$. In each period, the government rebates back to the households all proceeds from its international credit operations in a lump-sum fashion.

Bonds of the emerging economy, b', are risky assets because debt contracts between the government of the emerging economy and the investors are not enforceable. At any time, the government of emerging economy can choose to default on its debt. If the government defaults, all its current debt is erased, and it is temporarily excluded from international financial markets. Defaulting also entails a direct output cost.

Because the investors are risk averse, the bond prices of the emerging economy $q(b', y, W)$ have two components: the price of the expected losses from default $q^{RN}(\delta(b', y, W))$ that corresponds to the price of riskless bonds (hereafter T-Bills), q^f, adjusted by the default probability of the economy $\delta(b', y, W)$,and an "excess" premium or risk premium $\zeta^{RA}(b', y, W)$. This result will be discussed in more detail in the next sub-section.

Obviously when $b' \geq 0$, the probability of default, $\delta(b', y, W)$, is 0, and since the asset is riskless in this case, the risk premium, $\zeta^{RA}(b', y, W)$, is also 0. Therefore the price of the bond of the emerging economy is equal to the price of T-Bills which is $q^f = \frac{1}{1+r}$, where r is the constant international interest rate. Only when $b' \leq 0$ can $\delta(b', y, W)$ and $\zeta^{RA}(b', y, W)$ be different from 0.

When the government chooses to repay its debts, the resource constraint of the emerging economy is given by

$$c = y - q\left(b', y, W\right) b' + b, \tag{2}$$

When the government chooses to default the resource constraint of the emerging economy is given by

$$c = y^{def}, \tag{3}$$

where $y^{def} = h(y)$ and $h(y)$ is an increasing function.

The timing of decisions within each period on the side of the emerging economy is as follows: the government starts with initial assets b, observes the income shock y, and decides whether to repay its debt or to default. If the government decides to repay, then taking as given the bond price schedule $q(b', y, W)$, the government chooses its next period asset position b' subject to the resource constraint. Finally consumption of the emerging economies' households c takes place.

Define $V^0(b, y, W)$ as the value function of the government that has the option to default. The government starts the current period with assets b and income y and faces the representative international investor that has wealth W. The government decides wether to default or repay its debts to maximize the households' welfare. Given the option of default, $V^0(b, y, W)$ satisfies

$$V^0(b, y, W) = \max_{\{R,D\}} \left\{V^R(b, y, W), V^D(y, W + b)\right\} \tag{4}$$

where $V^R(b, y, W)$ is the value to the government of repaying its debt and $V^D(y, W + b)$ is the value of defaulting in the current period. If the government repays its debt the investors' wealth is W. Otherwise investors face an asset loss of $-b$ and their actual wealth corresponds to $W + b$.

If the government defaults the value of default is given by

$$V^D(y, W+b) = u(y^{def}) + \beta \int_{y'} [\theta V^0(0, y', W') + (1-\theta)V^D(y', W')] f(y, y')dy'.$$

where θ is the probability that the economy regains access to credit markets.

If the government repays its debts, the value of not defaulting is given by

$$V^R(b, y, W) = \max_{\{b'\}} \left\{ u(y - q(b', y, W)b' + b) + \beta \int_{y'} V^0(b', y', W') f(y, y')dy' \right\}.$$

Let $s = \{b, y, W\}$ be the aggregate state of the model. For the government of the emerging country the decision of default/repayment depends on the comparison between the value of repaying its debt, $V^R(s)$, versus the value of opting for financial autarky, $V^D(y, W + b)$. The repayment/default decision of the government is summarized by the indicator variable d. In the context of this model, when the government pays back its debt this variable takes the value of 1 and when the government does not pay back this variable has the value of 0. The functional form of the default/repayment decision is given by

$$d = \left\{ \begin{array}{ll} 1 & \text{if } V^R(s) > V^D(y, W+b) \\ 0 & \text{otherwise} \end{array} \right\} \tag{5}$$

This repayment/default decision is a period by period decision.

It is also important to notice that the government faces a lower bound on debt $\underline{\underline{B}} < 0$ that prevents Ponzi schemes. This lower bound on debt $\underline{\underline{B}}$ is not binding in equilibrium (i.e. $b' \geq \underline{\underline{B}}$).

Following closely Arellano (2008), and conditional on the representative investor's wealth level W, the emerging economy's default policy can be characterized by repayment and default sets:

Definition 1 *For a given level of wealth, W, the default set $D(b \mid W)$ consists of the equilibrium set of y for which default is optimal when the government's asset holdings are b:*

$$D(b \mid W) = \left\{ \; y \in Y : V^R \; (s) \leq V^D \; (y, W+b) \mid W \right\}. \tag{6}$$

The repayment set $A(b \mid W)$ is the complement of the default set and corresponds to the equilibrium set of y for which repayment is optimal when the government's asset holdings are b:

$$A(b \mid W) = \left\{ \, y \in Y : V^{R} \ (s) \geq V^{D} \ (y, W + b) \mid W \right\}. \tag{7}$$

Equilibrium default sets, $D(b' \mid W'(s))$, are related to equilibrium default probabilities, $\delta(b', s)$, by the equation

$$\delta\left(b', s\right) = 1 - E\left\{ \ d'\left(b', s\right)\right\} = \int\limits_{D\left(b' \mid W'(s)\right)} f\left(y' \mid y\right) dy'. \tag{8}$$

If the default set is empty for b', then for all realizations of the economy's endowment, $d' = 1$ and the equilibrium default probability $\delta(b', s)$ is equal to 0. In this case, it is not optimal for the government to default in the next period for any realization of the economy's endowment. On the other hand, if the default set for b' includes the entire support for the endowment realizations, i.e. $D(b' \mid W'(s)) = Y$, then $d' = 0$ for all realizations of the economy's endowment. As a consequence, the equilibrium default probability $\delta(b', s)$ is equal to 1. Otherwise, when the default set is not empty but does not include the whole support for the endowment realizations, $0 < \delta(b', s) < 1$.

Associated with the default sets we can define two concepts, the *maximum credit constraint* and the *maximum safe level of debt*:

Definition 2 *The maximum credit constraint is the maximum level of assets, $\underline{b}(W)$, that is low enough such that no matter what the realization of the endowment, default is the optimal choice and $D(\underline{b}(W) \mid W) = Y$.*

Definition 3 *The maximum safe level of debt is the minimum level of assets $\overline{b}(W)$ for which repayment is the optimal choice for all realizations of the endowment. In this case, $D(\overline{b}(W) \mid W) = \emptyset$.*

Because the value of repayment is monotonically decreasing in b, it is obvious that $\underline{b}(W) \leq \overline{b}(W) \leq 0$. $\underline{b}(W)$ and $\overline{b}(W)$ are single-valued functions.[4]

[4] The stochastic process for the endowments has a compact support, and conditional on W, the value of the credit contract is monotonically decreasing in b. Monotonicity of the credit contract and compactness of the endowment support are sufficient conditions to guarantee that $\underline{b}(W)$ and $\overline{b}(W)$ are single-valued functions.

Given W', any investment in the emerging economy's bonds in excess of $\underline{b}(W')$ would imply $\delta(b', s) = 1$. Since the default likelihood is one of the components of the prices of the bonds of the economy, these investments will have a $q(b', s) = 0$. On the other hand, all investments in the emerging economy's bond of an amount lower than $\overline{b}(W')$ imply $\delta(b', s) = 0$. Because these investments are riskless it follows that $q(b', s) = q^f$.

In this case, conditional on investors' wealth W, the main results of comparative statics of the model of endogenous sovereign risk with risk neutral international investors follow (see Aguiar and Gopinath (2006) Arellano(2008)). That is, default sets are shrinking in the economies assets (i.e. if $b_1 < b_2$ then $D\left(b_2 \mid W\right) \subseteq D\left(b_1 \mid W\right)$) therefore the probability of default $\delta(b', s)$ is decreasing on b'. Also, the emerging economy only defaults when it is facing capital outflows (i.e. $b - q\left(b'(s), s\right) b'\left(s\right) < 0$). Finally, conditional on the persistence of the endowment process not being too high, the default risk is larger for lower levels of income. Since the economic intuition of these results is identical to the intuition in the model of endogenous sovereign default risk with risk neutral investors, it will not be discussed in detail here.

It is worthwhile mentioning that since $\delta(b', s)$ is decreasing in b', the risk premium $\zeta^{RA}(b', s)$ is also decreasing in b'. Therefore bond prices $q(b', s)$ are, as in the model with risk neutral investors, increasing on b'. This result will be discussed in more detail in the next sub-section.

2.2 International investors

There are a large but finite number of identical competitive Investors who will be represented by a representative investor. The representative investor is a risk averse agent whose preferences over consumption are defined by a constant relative risk aversion (CRRA) periodic utility function with parameter $\gamma^L > 0$. The investor has perfect information regarding the income process of the emerging economy, and in each period the investor is able to observe the realizations of this endowment.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$E_0 \sum_{t=0}^{\infty} \beta_L^t v\left(c_t^L\right) \tag{9}$$

where c^L is the investor's consumption. The periodic utility of this agent is given by

$$v(c^L) = \frac{\left(c^L\right)^{1-\gamma^L}}{1-\gamma^L} \tag{10}$$

The representative investor is endowed with some initial wealth, W_0, at time 0, and in each period, the investor receives an exogenous income X.

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of q^f. The representative investor can also invest in non-contingent bonds of the emerging economy which have an endogenously determined stochastic price of $q(b', s)$. As was mentioned before, this price is taken as given by both the investors and the government of the emerging economy.

The timing of decisions within each period on the side of the investors is as follows: The investors start by observing their wealth, W, which is composed of their asset position in T-Bills, ϑ^{TB}, and their asset position in bonds of the emerging economy ϑ, $W = \vartheta^{TB} + \vartheta$. After the government of the economy decides on defaulting or repaying its debt, the investors realize their gains/losses and their actual wealth is either W if the government has honored its debt or $W - \vartheta = \vartheta^{TB}$ if the government chooses to default. If the government has paid back its debt, the investors choose their next period asset position in the emerging economy, ϑ', and in the T-Bills, $\vartheta^{TB'}$. If the government has defaulted the investors choose their next period asset position only in T-Bills. Finally consumption of international investors, c^L, takes place.

Whenever the government of the emerging economy has payed back its debt the representative investor faces the budget constraint

$$c^{L,\, ndef} = X + W - q^f \vartheta^{TB\prime} - q(b', y, W)\vartheta' \tag{11}$$

It is assumed that investors cannot go short in their investments with emerging economies. Therefore whenever the emerging economy is saving, the representative international investor receives these savings and invests them completely in $\vartheta^{TB'}$. Therefore $\vartheta' = -b'$ if the economy is borrowing, and it is equal to 0 otherwise.[5]

[5]This assumption does not seem to be inconsistent with reality. For example, mutual funds are strictly restricted by The Investment Company Act in their ability to leverage or borrow against the value of securities in their portfolio. On the other hand, hedge funds and other types of investors face no such restrictions. Because of these regulations it seems reasonable to make the simplifying assumption that international

On the other hand, if the government of the economy is in financial autarky because of default in the current period or past default without yet regaining access to credit markets, the investor's budget constraint is

$$c^{L,\ def} = X + \vartheta^{TB} - q^f \vartheta^{TB\prime} \tag{12}$$

The law of motion of the representative investor's wealth is given by

$$W' = \ d'(b', s)\vartheta' + \vartheta^{TB\prime}. \tag{13}$$

where $d'(b', s)$ is defined as in the emerging economy's sub-section.

Define $V_L^0(s)$ as the value function of the representative investor with an asset position of W facing a government with assets b and income y at the start of the period, which might default.

Given the option of default, $V_L^0(s)$ satisfies

$$V_L^0(s) = \left\{ \begin{array}{ll} V_L^R(s) & \text{if } d = 1 \\ V_L^D(y, \vartheta^{TB}) & \text{if } d = 0 \end{array} \right\} \tag{14}$$

where $V_L^R(s)$ is the value to the investors when the government repays its debt, and $V_L^D(y, \vartheta^{TB})$ is the value to the investors when the government defaults in the current period. As said before, if the government repays its debt the investors wealth is W otherwise they face an asset loss of $-b$ and their actual wealth corresponds to ϑ^{TB}.

If the government of the emerging economy defaults, the value of default to the investors is given by

$$V_L^D(y, \vartheta^{TB}) = \max_{\{\vartheta^{TB\prime}\}} \left\{ v(X + \vartheta^{TB} - q^f \vartheta^{TB\prime}) + \beta_L \int_{y'} [\theta V_L^0(0, y', \vartheta^{TB\prime}) + (1-\theta) V_L^D(y', \vartheta^{TB\prime})] f(y, y') dy' \right\}.$$

If the government repays its debts the value to the investors is given by

$$V_L^R(s) = \max_{\{\vartheta',\ \vartheta^{TB\prime}\}} \left\{ v(X + W - q^f \vartheta^{TB\prime} - q(b', y, W)\vartheta') + \beta_L \int_{y'} V_L^0(s') f(y, y') dy' \right\}.$$

The representative investor faces a lower bound on her asset holdings $\underline{\underline{W}} < 0$ that

investors are able to leverage the riskless asset, ϑ^{TB}, but must have a non-negative position in the emerging economy's asset.

prevents Ponzi schemes,

$$W' \geq \underline{\underline{W}} \tag{15}$$

$\underline{\underline{\text{W}}}$ corresponds to the "natural" debt limit discussed in Aiyagari (1994). Additionally the investors asset position in bonds of the emerging economy is non-negative, i.e. $\vartheta \geq 0$.

The optimization problem that the representative investor faces can be described as one in which in each period, t, she optimally chooses her portfolio according to her preferences in order to maximize her discounted expected lifetime utility from consumption, subject to her budget constraint, the law of motion of her wealth, the no-ponzi condition, and the condition that $\vartheta \geq 0$.

Because $v(c^L)$ satisfies the standard Inada conditions, and X sufficiently large, $c_L > 0$ always. Because the representative investor is not credit constrained (Equation (15)), when the government does not default in the current period the solution to the stochastic dynamic problem for this investor can be characterized by the following first order conditions:
For $\vartheta^{TB\prime}$

$$q^f v_{c^L}\left(c^L\right) = \beta_L \int_{y'} \left[v_{c^L}\left(c^{L'}\right)\right] f(y,y')dy'. \tag{16}$$

For ϑ'

$$q v_{c^L}\left(c^L\right) = \beta_L \int_{y'} \left[v_{c^L}\left(c^{L'}\right)\ d'(b',s)\right] f(y,y')dy'. \tag{17}$$

On the other hand, when the government of the emerging economy is in financial autarky the solution to the stochastic problem of the investor is characterized by the following order condition
For $\vartheta^{TB\prime}$

$$q^f v_{c^L}\left(c^L\right) = \beta_L \int_{y'} \left[v_{c^L}\left(c^{L'}\right)\right] f(y,y')dy'. \tag{18}$$

Equation (17) highlights the fact that the endogenous risk of default by the emerging economy—i.e. the possibility that $d'(b',s) = 0$ for some states of the world in the next period—reduces the representative investor's expected marginal benefit of investing in the emerging economy. Everything else equal, this result tends to reduce the allocation of resources to the emerging economy relative to the case where the emerging economy could commit to repayment.

It is possible to manipulate equation (17) to get

$$\begin{aligned} q(b',s) &= \beta_L \int_{y'} \frac{\left[v_{c^L}\left(c^{L'}\right)\ d'(b',s)\right]}{v_{c^L}\left(c^L\right)} f(y,y')dy'. \\ &= \beta_L \frac{Cov\left[v_{c^L}\left(c^{L'}\right),\ d'(b',s)\right]}{v_{c^L}\left(c^L\right)} + q^{RN}(b',s). \\ &= \zeta^{RA}(b',s) + q^{RN}(b',s). \end{aligned} \tag{19}$$

where $q^{RN}(b',s) = q^f\left(1-\delta(b',s)\right)$. Equation (19) shows the two components of the bond prices of economies that trade financially with risk averse investors. The first component, $q^{RN}(b',s)$, compensates the investors for the expected loss from default. The second component, $\zeta^{RA}(b',s)$, corresponds to the risk premium that sovereign bonds have to carry in order to induce risk averse investors to hold them. This term is the principal source of the differences between the results of this model and the model of endogenous sovereign risk with risk neutral investors.

The main determinant of the risk premium $\zeta^{RA}(b',s)$ is the covariance term in equation (19). This risk premium is different from 0 only when the covariance term is different from 0. In turn, this covariance term is non-zero only for bonds with face value b' such that the government of the emerging economy finds it optimal to default in its debt next period in some, but not all states of the world, that is for bonds, b', for which $0 < \delta(b',s) < 1$. For these bonds the covariance term is non-positive: $Cov\left[v_{c^L}\left(c^{L'}\right),\ d'(b',s)\right] \leq 0$.[6] Since the covariance term is non-positive the emerging economy's bond prices in this model are lower than the prices that would be observed in a model with risk neutral investors even in the case in which $\delta(b',s)$ is identical in both models.

[6]If for some bond b' the government of the economy does not default next period in any state of the world (i.e. the default set for b' is empty), then $d'(b',s) = 1$ for all states, and $\delta(b',s) = 0$, $Cov\left[v_{c^L}\left(c^{L'}\right),\ d'(b',s)\right] = 0$, and $q(b',s) = q^f$. On the other hand, if for some other b' the government of the economy defaults next period in all states of the world (i.e. the default set for b' includes the complete support of the endowment realizations), then $d'(b',s) = 0$ for all states and therefore $\delta(b',s) = 1$, $Cov\left[v_{c^L}\left(c^{L'}\right), d'(b',s)\right] = 0$, and $q(b',s) = 0$. If $0 < \delta(b',s) < 1$, then for the states of the word next period in which the government of the economy repays $\left[W' \mid_{d'(b',s)=1}\right] = \vartheta' + \vartheta^{TB\prime}$, and for the states in which the government defaults $\left[W' \mid_{d'(b',s)=0}\right] = \vartheta^{TB\prime}$. Because $\left[W' \mid_{d'(b',s)=1}\right] > \left[W' \mid_{d'(b',s)=0}\right]$ then $\left[c^{L'} \mid_{d'(b',s)=1}\right] > \left[c^{L'} \mid_{d'(b',s)=0}\right]$ and by concavity of $v(\cdot)$ $\left[v_{c^L}\left(c^{L'}\right) \mid_{d'(b',s)=1}\right] < \left[v_{c^L}\left(c^{L'}\right) \mid_{d'(b',s)=0}\right]$, as a consequence, for b' with more $d'(b',s) = 1$ $v_{c^L}\left(c^{L'}\right)$ is lower. Clearly for this case $Cov\left[v_{c^L}\left(c^{L'}\right),\ d'(b',s)\right] < 0$.

It is worth examining how $\zeta^{RA}(b', s)$ responds to the variables in the model. First, asset pricing theory implies that the more risky an asset looks in the eyes of the investor the larger should be its risk premium $\zeta^{RA}(b', s)$. Clearly, from an investor's perspective, an asset would seem more risky the less tolerant of risk is this investor. As a consequence, $\zeta^{RA}(b', s)$ is larger for higher levels of γ^L, or for lower levels of W. Second, increasing the investor's exposure to the emerging economy's debt should increase $\zeta^{RA}(b', s)$—even if the intrinsic riskiness of the economy's assets could be kept fixed when the economy's debt level increases. The obvious explanation for this result is that even with a fixed default probability for the economy, a larger exposure to the economy's debt would increase the riskiness of the investor's portfolio, and therefore should command a larger $\zeta^{RA}(b', s)$. Finally, increasing the riskiness of the emerging economy's assets (i.e. $\delta(b', s)$) should increase $\zeta^{RA}(b', s)$.

Leaving aside the behavior of $\zeta^{RA}(b', s)$, it is important to note that for any bond with face value b', the probability of default is higher in the case of a risk averse investor,$\delta\left(s, b'\right)$, than in the case of a risk neutral investor, $\delta^{RN}\left(s, b'\right)$. Therefore, for any bond b', the component of the price that compensates the investor for the expected loss from default is also larger in the case of risk averse investors. In conclusion, for s and b' given, for the emerging economy trading with a risk averse investor, the price of the bonds, $q\left(\delta\left(s, b'\right)\right)$, is always lower or at best equal to price of the same bonds traded with a representative risk neutral investor, $q^{RN}\left(\delta^{RN}\left(s, b'\right)\right)$.

From Equation (19) is clear that if investors are risk averse, $q(b', s)$ depends not only on the emerging economy's fundamentals (i.e. y and b'), but on γ^L and W. In contrast, in models of emerging economies that face risk neutral investors, the price of bonds of the economy depends only on the economy's own fundamentals (i.e, $q(b', y)$).

An implication of having $q(b', s)$ depend on W is that the response of $q(b', s)$ to changes in y is smaller than in an otherwise similar model with risk neutral investors. Therefore, in the risk averse model, small changes in b' generate smaller changes in $q(b', s)$. This muted response of $q(b', s)$ to y implies that the price function $q(b', s)$ is flatter for the risky region of debt. Therefore, as explained in Aguiar and Gopinath (2006), other things given, larger levels of debt can be supported at equilibrium by this model in comparison to the model with risk neutral investors.

2.2.1 Investor's Credit Constraints

Whenever the representative investor faces credit constraints in international credit markets the following Kuhn-Tucker conditions characterize her optimization problem:

For $\theta^{TB\prime}$

$$q^f v_{c^L}\left(c^L\right) - \mu = \beta_L \int_{y'} \left[v_{c^L}\left(c^{L'}\right)\right] f(y, y')dy'.$$

For θ'_j

$$q(b', s, \mu) = q^*(b', s) - \frac{\mu\left(1 - \delta(b', s)\right)}{v_{c^L}\left(c^L\right)}.$$

where, $q^*(b', s)$ corresponds to the bond price consistent with an interior solution for representative investor's optimization problem, and μ corresponds to the multiplier on the representative investor's credit constraint.

Given that credit constraints for the investors increase their opportunity cost of investing in emerging economies, other things equal, these constraints should reduce the equilibrium bond prices of the emerging economy even further in comparison to the default risk adjusted-price (i.e. $q^{RN}(b', s)$).

2.3 Recursive Equilibrium

The recursive equilibrium for this model is defined as a set of policy functions for (i) the emerging economy's consumption $c(s)$, (ii) the government's asset holdings $b'(s)$, (iii) the government's default decisions $d(s)$ and the associated default sets $D\left(b|W\right)$, (iv) the representative investor's consumption $c^L(s)$, (v) the representative investor's holdings of emerging economy's bonds $\vartheta'(s)$, (vi) the representative investor's holdings of T-Bills $\vartheta^{TB'}(s)$, and (vii) the emerging economy's bond price function $q(b', s)$ such that:

(i) Taking as given the representative investor's policies and the bond price function $q(s, b')$, the emerging economy's consumption $c(s)$ satisfies the economy's resource constraint. Additionally, the policy functions $b'(s)$, $d(s)$ and default sets $D\left(b|W\right)$ satisfy the optimization problem of the emerging economy.

(ii) Taking as given the government's policies, and the bond price function $q(b', s)$, the representative investor's consumption $c^L(s)$ satisfies the investor's budget constraint. Also, the representative investor's policy functions $\vartheta'(s)$ and $\vartheta^{TB'}(s)$ satisfy the optimization problem of the representative investor, and the law of motion of the investor's wealth.

(iii) Bond prices reflect the government's probability of default and the risk premium demanded by the representative international investor. These prices clear the market for the emerging economy's bonds:

$$b\,'(s) = -\vartheta\,'(s) \qquad \text{if } \; b\,'(s) < 0 \tag{20a}$$

$$0 = -\vartheta\,'(s) \qquad \text{if } \; b\,'(s) \geq 0. \tag{20b}$$

This condition implies that the representative investor and the representative agent of the emerging economy agree on a financial contract (b', q) that is optimal for both agents.

3 Default Risk and Investor's Characteristics

3.1 Default Sets and Risk Aversion of International Investors

Proposition 1 *For any state of the world, s, as the risk aversion of the international investor increases, the probability that the government defaults increases.*

Proof. See Appendix. ■

In this model, the more risk averse are international investors, the higher is the default risk of the emerging economy and the tighter is the emerging economy's endogenous credit. The economic intuition behind this result is straightforward: To induce a very risk averse investor to hold sovereign bonds the government has to accept a very low price for this bonds. However, other things equal, with lower bond prices incentives to default are stronger. Therefore for any given state of the world, $\delta(b', s)$ is increasing in γ^L.

As $\delta(b', s)$ changes so too will the capital flows to the economy: For $\gamma_L^1 < \gamma_L^2$, Proposition 1 implies that $D\left(b \mid W; \gamma_L^1\right) \subseteq D\left(b \mid W; \gamma_L^2\right)$. Therefore

$$\begin{aligned} \underline{b}\left(W; {\gamma^2}_L\right) &\geq \underline{b}\left(W; {\gamma^1}_L\right). \\ \overline{b}\left(W; {\gamma^2}_L\right) &\geq \overline{b}\left(W; {\gamma^1}_L\right). \end{aligned}$$

The maximum credit constraints $\underline{b}\,(W)$ for the government are tighter when international investors are more risk averse—some contracts that are feasible with less risk averse investors are not feasible with more risk averse investors.

3.2 Default Sets and Investor's Wealth

Proposition 2 *Default sets are shrinking in assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b in some states y, given W_2, then default will be optimal for b for the same states y, given W_1. Therefore $D(b \mid W_2) \subseteq D(b \mid W_1)$*

Proof. See Appendix. ■

The investor's wealth also affects the emerging economy's performance. The intuition for this result is simple: given some $\delta(b', s)$, it is less costly in terms of current utility for the investor to invest in the emerging economy when she is wealthy than when she is poor. So keeping constant the degree of risk that the investor faces, any investment that she is willing to undertake when she is poor she will also be willing to undertake when she is rich. Intuitively, financial contracts available to the government of the emerging economy when the investor is relatively rich have to be at least as good as the feasible contracts to which the government has access when the investor is relatively poor. Additionally, the previous effect implies that the government of the economy faces stronger incentives to default when the wealth of the investor is relatively low. Therefore $\delta(b', s)$ is decreasing in the wealth of the investor. These two effects amplify and reinforce each other. That $\delta(b', s)$ is decreasing in W implies that the economy's bond prices $q(b', s)$ are increasing in W.

Proposition 2 also implies that for $W_1 < W_2$

$$\begin{aligned} \underline{b}(W_1) &\geq \underline{b}(W_2) \\ \overline{b}(W_1) &\geq \overline{b}(W_2) \end{aligned}$$

In other words, the maximum credit limit that the government faces is tighter for lower levels of wealth of the investor ($\underline{b}(W_1) \geq \underline{b}(W_2)$)—some portfolio investments that are feasible when the investor is wealthy cannot be an equilibrium outcome when the investor is poor.

3.3 Default as an equilibrium outcome of the model and Investors characteristics

In the current model, in order to observe default it is necessary to have some $b' < \overline{b}(W'(s))$ for which by increasing its borrowing beyond $\overline{b}(W'(s))$ the government is able to increase its current capital inflows $-q(b', s,)b'$. In what follows we limit the analysis to the case in which the incentives to default of the government are stronger the lower the endowment.[7]

Definition 4 *The conditional default boundary function, $y^*(b|W)$, corresponds to the endowment level, y^*, for a given level of debt, $b \in (\underline{b}(W), \overline{b}(W))$, conditional on the representative investor's assets W such that the value of repayment and the value of default are equal for the emerging economy: $V^R(b, y^*, W) = V^D(y^*, W + b)$.*

Conditional on the investor's wealth, W, $y^*(b|W)$ divides the space $\{y, b\}$ into the default and repayment regions. From the previous discussion of the model, it is possible to establish that $y^*(b|W)$ is decreasing in the government's assets,b, and the investor's assets, W, and increasing in the investor's risk aversion γ^L. Using $y^*(b|W)$, the equilibrium bond price function, $q(b', s)$, can be written as:

$$\begin{aligned} q(s, b' \mid W'(s)) &= q^f[1 - F(y^*(b'|W'(s)))] + \beta_L \frac{Cov\left[v_{c^L}(c^{L'}(s)), d'(s, b' \mid W'(s))\right]}{v_{c^L}(c^L(s))} \\ &= \int_{y^*(b'|W'(s))}^{Y} \beta_L \frac{v_{c^L}\left(c^{L'}(s)\right)}{v_{c^L}\left(c^L(s)\right)} f\left(y' \mid y\right) dy'. \end{aligned}$$

where F is the cumulative probability distribution of shocks.

Clearly since $y^*(b'|W'(s))$ is decreasing in b', and $\zeta(b', s)$ is increasing in b', as debt, b', increases, $q(s, b' \mid W'(s))$ goes to zero. We define the *endogenous borrowing limit* $b^*(s \mid W')$ as follows:

Definition 5 *The endogenous borrowing limit $b^*(s \mid W')$ is the level of debt for which $\pi \equiv -q(s, b^*(s \mid W'))b^*(s \mid W')$ satisfies*

$$\pi = \max_{b'} \left[-\left(q^f[1 - F(y^*(b'|W'))] + \beta_L \frac{Cov[v_{c^L}(c^{L'}(s)),\ d'(s, b' \mid W')]}{v_{c^L}(c^L(s))} \right) b' \right].$$

For any state s, $b^*(s \mid W')$ is the endogenous borrowing limit because conditional on W', for any $b' < b^*(s \mid W')$ then $V^R(s, b' \mid W') < V^R(s, b^*(s \mid W'))$, and $b' < b^*(s \mid W')$ cannot be optimal.

[7] The incentives to default for the government are stronger when endowments are low as long as the persistence of the endowment shocks is not too high.

Definition 6 *For any state s the relevant risky region of the model is limited to contracts with* $b' \in \left[b^*\left(s \mid W'\right), \overline{b}(W')\right)$.

The relevant risky region of the model is not empty and default is a possible outcome of the time series of the model only if there exists some $b^*\left(s \mid W'\right)$ such that $b^*\left(s \mid W'\right) < \overline{b}(W')$. For such $b^*\left(s \mid W'\right)$ to exist it is necessary that $q(b', s)$ does not decrease "too fast" when b' decreases.

Investors' characteristics contribute to a non-empty risky region in two ways which have opposite effects: First, from proposition 1, proposition 2, and equation (21) the speed at which $q(b', s)$ decreases when b' decreases is increasing in γ^L and decreasing W. This effect implies that default is less likely to be observed at equilibrium for economies trading with international investors that are more risk averse or less wealthy.

Second, for smaller $\overline{b}\left(W'\left(s\right)\right)$, there is a higher chance that there exists $b^*\left(s\right) < \overline{b}\left(W'\left(s\right)\right)$. Intuitively, because investors must be compensated in order to induce them to take some default risk, this risk imposes an additional cost of borrowing for the government of the economy. For the borrower, the cost of borrowing beyond $\overline{b}\left(W'\left(s\right)\right)$ must be paid over the total amount of resources borrowed, and not only over the marginal amount of borrowing. Therefore, the larger is the base over which this additional cost of borrowing has to be paid—i.e. the larger is $\overline{b}\left(W'\left(s\right)\right)$—the higher is the cost of default risk and the lower is the likelihood that the government would ever choose to borrow beyond $\overline{b}\left(W'\left(s\right)\right)$. As stated before, because of proposition 1 and proposition 2 $\overline{b}\left(W'\left(s\right)\right)$ is decreasing in γ^L and increasing in W. This effect implies that default is more likely to de observed at equilibrium for economies trading with international investors that are more risk averse or less wealthy.

Because of the two opposing effects, it is not possible to establish analytically how the equilibrium default probability responds to changes in γ^L or W. However the numerical simulations of the model performed here suggest no impact of W on the probability of default, and some impact of γ^L. Specifically, the relation between γ^L and $\delta(b', s)$ is non-linear: for high and low levels of γ^L, $\delta(b', s)$ is larger than for intermediate values of γ^L.

3.4 Connection of the Model with Empirical Evidence

This section connects the main results of the model with the results of the existing empirical literature on the subject of the determination of the spreads of sovereign economies.

Bond Prices In the model, bond prices of emerging economies have two components: a part that gives a price to the default risk, and a part that corresponds to a risk premium for the investors. The empirical literature on the determination of sovereign spreads of emerging economies is consistent with this result. For example, using data on credit default swap (CDS) spreads and default histories of rated bonds for 26 emerging economies, Remolona et al.(2007) estimate expected losses from default and risk premia. The authors find that the expected loss plays a small role in determining the spread and that the risk premium plays a bigger role. Similarly, looking at 11 emerging economies for the period $1990-2009$, Broner et al.(2010) find that there is a positive risk premium which is possible only if investors are risk averse. This result corresponds to stylized fact 1.[8]

Risk Premium The study of risk premium in the model establishes that for higher riskiness of the economy's bonds this term becomes more important. The result is consistent with the empirical evidence documented by Cantor and Pecker (1996), Kamin and von Kleist (1999), Cunningham et al.(2001), and Remolona et al.(2007). These authors note the fact that the proportion of sovereign yield spreads explained by emerging economies' own fundamentals is smaller for riskier sovereign bonds than for investment grade bonds. Broner et al. (2010) find that the term premium increases with Maturity. The 5 more volatile countries in their sample have a higher term premium which shows that investors require higher returns to compensate for higher riskiness. This result is stylized fact 2.

Risk Appetite Proposition 1 establishes that capital flows to emerging economies are smaller, and the spreads of the bonds of the economies are larger, when international investors are more risk averse. Much empirical evidence supports this proposition: using the spread between the yield of three month T-bills and the US federal funds rate as a proxy for market turbulence, Arora and Cerisola (2001) find that heightened macroeconomic uncertainty in the US has a positive significant effect on sovereign credit spreads for emerging markets. Using high-low yield spreads on US corporate bonds as a proxy for risk aversion of US investors, FitzGerald and Krolzig (2003), Ferruci et al.(2004), Gonzlez and Levy (2006), and Longstaff et al. (2008) find that sovereign bond spreads increase when the risk aversion of US investors increases. Mody and Taylor (2004), Ferruci et al.(2004), and FitzGerald

[8]Using annual data of 24 emerging economies for the period $1970-2000$, Klingen et al. find an almost zero ex-post spread for emerging debt. This result is not inconsistent with the notion of ex-ante risk premium in sovereign debt; the result might be reflecting the short length of the period under study given the low frequency of the data and the possibility of over-representation of default in the sample during the $1970-1989$ period.

and Krolzig (2003) find that risk aversion of US investors is an important determinant of capital flows to emerging economies: a higher US high-low yield spread—interpreted as a reduction in investor risk appetite—results in a reduced supply of capital to emerging economies. Finally, for a broader class of assets Broner et al. (2006) find that changes in the risk appetite of the investors affect portfolio decisions and stock prices.

Investors' Wealth Proposition 2 determines that the capital flows to the emerging economy are smaller and sovereign spreads are larger when the investors' wealth is lower. Also, much empirical evidence supports Proposition 2: For the period 1984 to 1993, Warther (1995) finds that an inflow to corporate bond funds of around 1% of the mutual fund's assets results in a permanent increase of 2.1% in those bond prices. Using world and U.S. equity indexes respectively as proxies for the business climate (an increase in these indexes is associated with a better business climate), Ferruci et al. (2004), Gonzlez and Levy (2006), and Longstaff et al.(2008) find a negative relation between economic expansion in the investors' countries and sovereign yield spreads of emerging economies. FitzGerald and Krolzig (2003) find a positive and significant relationship between US output and capital inflows to emerging economies. Finally, Mody and Taylor (2003) find that a higher growth in industrial production in the US has a positive effect on the supply of capital to emerging economies. This result is stylized fact 3.

Asset Returns Correlations Proposition 2 also implies than when the wealth of the investors falls their demand for all risky assets in their portfolio should fall. If we think of a world in which investors hold not only one emerging economy's bonds but other risky assets, and if the demand of the investors for these risky assets has an important impact on the price of those assets, then other risky assets spreads should increase whenever the emerging economy's spread increase as a result of the change in the investors' wealth. This implication of the model is consistent with the findings of FitzGerald and Krolzig (2003), Ferruci et al.(2004), and Mody and Taylor (2004) that establishes that emerging economies' credit spreads are positively correlated with spreads of corporate junk bonds from developed countries. This result is stylized fact 4. Furthermore, this implication of the model is consistent with the findings of Valdes (1996), Baig and Goldfajn (1998), Forbes and Rigobon (1999), Baig and Goldfajn (2000), and Longstaff et al.(2008). These authors establish that sovereign bond spreads across emerging economies are highly correlated. This

Table 1: Business Cycle for Argentina and International Investors

	std(x)	corr(x,output)	corr(x,sp 500)	corr(x,spread)
Interest Rate Spread	5.42	-0.60	-0.39	
Trade Balance	1.83	-0.59	-0.07	0.38
Consumption	1.94	0.93	0.35	-0.78
Output	1.91		0.15	
Consumption USA	0.31	-0.08	0.40	-0.08
Output USA	0.40		0.50	-0.10
SP500	14.79	0.15		
Dow	11.22	0.11	0.94	-0.30

result is stylized fact 5.[9]

4 Quantitative Analysis

The model in this paper is used to study the case of Argentina and its default at the end of 2001. The idea is to compare the quantitative performance of the model of endogenous sovereign risk with risk averse investors to the performance of the model of endogenous sovereign risk with risk neutral investors. The model is solved numerically at a quarterly frequency and its parameters are chosen to replicate important features of the Argentinean economy and the international investors in emerging economies for the period 1983:Q1-2001:Q4. In order to draw clear implications of what considering risk aversion can add to the existing literature on the dynamics of emerging economies, some parameters are not calibrated to match specific targets in the data, but instead are taken from the previous literature on the subject of endogenous sovereign risk that looks at the Argentinean default.

Table 1 describes the relevant business cycle features for the period under study. For the Argentinean output, consumption, and trade balance, and for the U.S. output and consumption, the source of the data is the IFS; for the yield of 3-months U.S. Treasury Bills the source is the Federal Reserve Board; for the SP500 index and the Dow-Jones Industrial Average index the source is Bloomberg; finally, for the interest rate of Argentina the source is Neumeyer and Perri (2005). The data for the business cycle statistics includes the period 1983:Q1-2001:Q4 for all series except for Argentina's private consumption. For

[9]Regarding contagion across emerging economies sovereign markets, there is a large body of empirical literature that presents evidence that financial links play a significant role in explaining simultaneous financial crises and correlated spreads across emerging economies. See, for example, Kaminsky and Reinhart (1998), Kaminsky and Reinhart (2000), Van Rijckeghem and Weder (2001), Kaminsky et al.(2001), Hernández and Valdes (2001), Broner et al.(2006) and Hau and Rey (2008).

Table 2: Parameter Values

Parameter	Value
Emerging Economy's Mean Income $E[y]$	1
Std. Dev. Emerging Economy's Income $std[y]$	0.025
Autocorr. Emerging Economy's Income Process	0.945
Emerging Economy's Mean Income $E[y]$	1
Emerging Economy's Discount Factor β	0.953
Emerging Economy's Risk Aversion γ	2
Probability of re-entry τ	0.282
Critical level of output for asymmetrical output cost	$\hat{y} = 0.969E(y)$
Representative investor's Income X	0.01
Representative Investor's Discount Factor β^L	0.98
Representative investor's Risk Aversion γ^L	5
Risk Free Interest Rate $r^f = \frac{1}{q^f}$	0.017

Argentina's private consumption data is only available from 1993:Q1 on. Therefore, for this variable the business cycle statistics corresponds to the period from the initial moment in which it is available to first quarter of 2008. Output and consumption for Argentina and the U.S., and the SP500 and the Dow-Jones indexes are seasonally adjusted, in logs, and filtered with the H-P filter. [10] The Argentinean trade balance is reported as a percentage of the output. The interest spread is defined as the difference between the Argentinean interest rate and the yield of a 3 month U.S. T-Bill.

4.1 Calibration

Table 2 gives the parameters which are considered in the numerical analysis of the model. To make the comparison straightforward between the results of this model and the model with risk neutral investors, the parameters for the emerging economy are taken from the calibration in Arellano (2008). The mean income of the emerging economy is normalized to 1; the coefficient of risk aversion of the economy is 2, a standard value considered in the business cycle literature; the free interest rate is set to 1.7%, to match the period under study with the quarterly US interest rate of a bond with a maturity of 5 years; GDP is assumed to follow a log-normal AR(1) process $log(y_t) = \rho log(y_{t-1}) + \varepsilon^y$ with $E[\varepsilon^y] = 0$ and $E[\varepsilon^{y2}] = \sigma_y^2$. The values estimated by Arellano(2008) for the Argentinean economy are $\rho = 0.94$ and $\sigma_y = 0.025$, and the shock is discretized into a 21 state Markov chain.

[10] The Dow-Jones index is a price-weighted index of 30 blue-chip stocks from U.S. firms that are generally leaders in the industry. The SP500 index is a capitalization-weighted index of 500 stocks that represent all industries that is designed to measure performance of the broad economy.

Following a default there is an asymmetrical function for the output loss that follows:

$$\phi(y) = \left\{ \begin{array}{ll} \widehat{y} & \text{if } y > \widehat{y} \\ y & \text{if } y \leq \widehat{y} \end{array} \right\} \tag{21}$$

with $\widehat{y} = 0.969E(y)$, which in the model with risk neutral investors targets a value of 5.53% for the average debt service to GDP ratio. The probability of re-entry to credit markets after defaulting is set at 0.282, the value chosen in Arellano (2008).[11] The model with risk neutral investors targets a volatility of 1.75 for the trade balance. The discount factor is set at 0.953 which in the model with risk neutral investors targets an annual default probability of 3%.

The parameters for the international investors are set as follows: the representative investor's discount factor is set to 0.98. In order to have a well defined distribution for the investor's assets, it is necessary to have a value of the discount factor such that $\frac{\beta^L}{q^f} < 1$. The value of $\beta^L = 0.98$ is one in the range commonly used in business cycle studies of industrialized countries such that the asset distribution of the investors is well defined given an international interest rate of 1.7%. However the results in the paper are not very sensitive to the choice of β^L as long as this choice is in the range of commonly used values in the real business literature.

The representative investor's coefficient of risk aversion is set at 5; this value was chosen in order to generate a mean spread for model that is as close as possible to the mean spread in Argentina for the period of study, which corresponds to 12.67%. For values of the risk aversion parameter of the investors that are commonly used in the real business literature, the mean of the simulated sovereign spreads is increasing in the investors' risk aversion.

The representative investor receives a deterministic income of $X = 1\%$ of the emerging economy's mean income in each period. This parameter is included to preclude the investors from not investing in the emerging economy in order to avoid a negative consumption level in the case of default by the government of the economy. Potentially this parameter could have important effects on the results of the model because it determines the borrowing limit that international investors face in international credit markets.[12, 13] Therefore, the

[11]The value of this parameter is consistent with the empirical results in Gelos et al. (2002) regarding the length of the period exclusion from credit markets of defaulting countries, however Benjamin and Wright(2008) find much longer periods of exclusion.

[12]The natural debt limit faced by the investors defined in equation (15) is given by $\underline{\underline{W}} = \frac{-X(1+r^f)}{r^f}$.

[13]When X is large the portfolio choice of the investors is not very sensitive to W and γ^L, and the results of the model tend to the results of the model with risk neutral investors. By affecting the leverage ability

Table 3: **Business Cycle Statistic: The Model and the Data.**

Statistics	Data	RN	RA	Data-4	RN-4	RA-4	Data-pD	RN-pD	RA-pD
mean $(r - r^f)$ %	12.67	4.65	7.06	22.26	7.13	16.54	29.93	11.45	29.66
std $(r - r^f)$ %	5.42	3.70	8.78	13.59	5.77	12.97			
mean $(-(b/y))$ %	53.30	5.84	4.95		3.21	3.40		2.49	2.94
std (c)/ std(y)	1.02	1.25	1.20	1.24	1.21	1.20			
std (tb/y) %	1.83	1.43	1.10	2.11	1.31	1.19			
corr (y, c)	0.93	0.94	0.96	0.98	0.62	0.61			
corr $(y, r - r^f)$	-0.60	-0.34	-0.32	-0.90	-0.42	-0.54			
corr $(tb/y, y)$	-0.59	-0.43	-0.47	-0.85	-0.18	-0.11			
std (W) %	14.79	N.A	5.16	3.45	N.A	5.52			
corr (W, c)	0.35	N.A	0.03	0.21	N.A	0.19			
corr $(W, r - r^f)$	-0.39	N.A	-0.60	-0.05	N.A	-0.06			
Default Prob. %	0.74	1.36	1.39						

strategy for choosing X is to give it as little importance as possible by choosing a value that is close to 0 and that still allows for interior solutions regarding the investors' investments in the emerging economy's bonds. Overall, the numerical analysis of the model shows that as long as X is not too large (i.e. $X < 100\%$ of the emerging economy's average income) the results of the model are not very sensitive to the value of X.

4.2 Simulations

The model is simulated for $20,000$ periods. From these $20,000$ periods, sub-samples that have the economy staying in the credit market for 74 periods before going into a default are taken to compute the economy's business cycles statistics. This process is repeated $5,000$ times, and the cycle statistics are the average of the statistics derived from each of these repetitions.

The results of the simulations are shown in Table 3. The label RA (RN) refers to the results of the simulations with risk averse investors (risk neutral investors);$RA-4$ ($RN-4$) refers to the results of the simulations for the four periods previous to the default episode; $RA - D$ ($RN - D$) refers to the results of the simulations for the period previous to the default episode.

Other things equal, the model presented here is able to account for a larger proportion of credit spreads than the model with risk neutral investors. In the data, the mean interest rate spread is 12.67%. According to the model, for the whole period, the mean interest rate

and the availability of resources on the side of the investors, X plays a fundamental role in determining the willingness to take risk by these agents.

spread is 7.06%. This value is 2.41% larger than the spread of the model with risk neutral investors. For the year previous to default, the mean interest rate spread in the data is 22.26%. According to the model, the mean interest rate spread is 16.54%. This value is 9.86% larger than the spread of the model with risk neutral investors. Also, for the period before default, in the data the spread rises to 29.93%. The model predicts a spread for the period before default of 29.66%, while the model with risk neutral investors predicts a spread of 11.45%.[14]

The model with risk averse investors replicates better the timing of the spreads of the economy than the model with risk neutral investors: in the risk averse model the interest rate of the economy is high not only in the period immediately before default but also increases substantially in periods of high economic volatility that are not followed by a default (i.e. the year before default).

The model is also more successful than its risk neutral counterpart at explaining the volatility of spreads: The volatility of the spread in the data is 5.42%; the risk averse model over-predicts this volatility with a a value of 8.78%; while the risk neutral model under-predicts the volatility with a value of 3.70%. For the year previous to default, the risk averse model performs much better: the volatility of the spread in the data is 13.59%, the risk averse model predicts a volatility of 12.97%, and the risk neutral model predicts a volatility of 5.77%.

The model introduced here also reproduces the counter-cyclical behavior of domestic interest rates. The value of the correlation predicted by both models is lower than the observed value for the data of -0.60 for the whole period and -0.90 for the year before default. The numerical solution of the model shows that the correlation between domestic interest rates and output is around -0.32 for the whole period and -0.54 for the year before the default episode. The alternative model (i.e. the model with risk neutral investors) predicts values of -0.34 and -0.42 respectively. The model here is consistent with a higher co-movement of the series during periods of economic distress, a result that is not observed for the model with risk neutral investors.

The model also reproduces the counter-cyclical behavior of the trade-balance. In the

[14]In simulating the risk neutral solution, the model in this paper finds a higher default probability and a lower $r - r^f$ for the period before the default episode than those found for the same calibration in Arellano (2008). This difference is likely the result of using a different solution method and a different dimension for the economy's asset position. As discussed in Hatchondo and Martinez (2006), models of endogenous sovereign risk are somewhat sensitive to the solution method employed and how sparse is the grid for the asset position of the economy.

data, the value of the correlation is -0.59 on average and becomes -0.85 for the year previous to default. The performance of both models is similar: The results of the model shows that the correlation between trade-balance and output is around -0.47 for the whole period and -0.11 for the year before the default episode. The alternative model (i.e, the model with risk neutral investors) predicts values of -0.43 and -0.18 respectively.

The model is also consistent with a few statistics that the previous literature in endogenous sovereign risk cannot account for: First, the model is able to match the negative correlation between a measure of the investors' performance, the SP500, and Argentina's interest spread. In the data, the correlation between these measures is -0.39 for the whole period, and falls to -0.05 for the period before default. The model generates this pattern giving a value of -0.60 for the whole sample, and -0.06 for the four periods before default.

Second, in the data, Argentina's consumption and the SP500 are positively correlated at 0.35 for the whole period, and 0.21 for the year previous to default; in the model the correlation between investors wealth and consumption is 0.03 for the whole period, but the value rises to 0.19 for the year previous to the default episode.

There are two important issues in which the performance of the model with risk averse investors does not improve upon the performance of the model with risk neutral investors. (The risk averse model does not do worse either.) First, the introduction of risk aversion does not solve the issue of under-prediction of the debt-to-output ratio observed in the model with risk neutral investors. Despite the flatter price schedule of this model in comparison to the one in the model with risk neutral investors, the problem of low debt-to-output ratios cannot be overcome quantitatively by a different specification of the preferences of the investors; instead this issue might be overcome by considering long term debt, domestic firms' borrowing needs, and domestic holdings of sovereign debt. [15] Nonetheless, it is noteworthy the fact that for the periods of high volatility in the economy (i.e. year before default and previous period before default) the trade-off between fitting the spreads or the debt-to-output ratio of the emerging economy is less acute for the model with risk averse investors than for the model with risk neutral investors.

Second, the model predicts lower overall levels of volatility for the trade balance than the model with risk neutral investors. However, on the plus side, the model delivers the correct timing for this volatility: while the model with risk neutral investors has the volatility falling for the year previous to default, the model with risk averse investors has the volatility

[15] Models of endogenous sovereign risk that look at these subjects include, for example, Arellano and Ramanarayan (2008), Hatchondo and Martinez (2009), Mendoza and Yue (2011), and Sosa-Padilla (2011).

Table 4: **Business Cycle Statistic: Sensitivity of the results to γ^L.**

Statistics	$\gamma^L = 0$	$\gamma^L = 0.5$	$\gamma^L = 1$	$\gamma^L = 2$	$\gamma^L = 5$
mean $(r - r^f)$ %	4.65	5.25	5.25	6.29	7.06
mean $(r - r^f)$-4 %	7.13	7.75	7.76	7.99	16.54
mean $(r - r^f)$-D %	11.45	12.03	12.05	12.28	29.66
std $(r - r^f)$ %	3.70	4.88	4.88	5.78	8.78
std $(r - r^f)$-4 %	5.77	7.01	7.02	7.03	12.97
mean $(-(b/y))$ %	5.92	3.63	3.63	3.97	4.95
mean $(-(b/y))$-4 %	3.26	4.16	4.16	5.50	3.40
mean $(-(b/y))$-D %	2.55	4.11	4.11	5.53	2.94
corr (W, c)	N.A	-0.04	-0.01	0.01	0.03
corr (W, c)-4	N.A	-0.16	-0.02	0.05	0.19
corr $(W, r - r^f)$	N.A	0.15	0.06	-0.08	-0.60
corr $(W, r - r^f)$-4	N.A	0.02	0.01	-0.08	-0.06
Default Prob. %	1.36	0.63	0.63	0.70	1.39

increasing for the year previous to default.

4.2.1 Sensitivity of the results

Table 4 presents the results of the model of endogenous default risk when γ^L varies.[16].

According to the previous table, the spreads of the economy and the volatility of these spreads are increasing in γ^L. It is also interesting to observe that the correlations between W and c and W and $r - r^f$ are very sensitive to γ^L. Low values of γ^L generate correlations with the wrong sign. Finally, the table shows that the probability of default of the economy is not a linear function of γ^L. This default probability is large for very low levels of risk aversion ($\gamma^L = 0$) and for relatively large levels of risk aversion ($\gamma^L = 5$). This last result points to the two previously discussed opposite effects that γ^L has on the speed at which $q(b', s)$ decreases when b' falls.

Table 5 helps to understand the effects of other parameters on the equilibrium of the model. This table shows the results of the model with γ^L fixed to 2 and varying β^L and X, and allowing for stochastic shocks to these two previous parameters ($\beta^L(S)$,$X(S)$) to r^f ($r^f(S)$), and and to the preferences of the investors $v(\cdot)(S)$.

The effects of β^L seem to be as follows: lower levels of β^L generate a higher average spread and a lower default probability, implying a higher average risk premium. However the spreads and their volatility for the more volatile periods (year before default and period

[16]Some of the business cycles statistics of the economy are omitted because they are not very sensitive to changes in γ^L.

Table 5: **Business Cycle Statistic: Sensitivity of the Results to other parameters.**

Statistics	$\beta^L = 0.98$ $X = 0.01$	$\beta^L = 0.953$	$\beta^L(S)$	$X = 0.1$	$X(S)$	$r^f(S)$	$v(\cdot)(S)$
mean $(r - r^f)$ %	6.29	7.29	5.42	5.25	5.43	5.16	5.21
mean $(r - r^f)$-4 %	7.99	6.29	8.89	7.76	8.94	6.37	9.94
mean $(r - r^f)$-D %	12.28	7.32	17.30	12.05	17.42	7.81	24.48
std $(r - r^f)$ %	5.78	5.78	5.31	4.88	5.31	4.37	5.65
std $(r - r^f)$-4 %	7.03	4.58	9.26	7.02	9.29	4.45	11.54
mean $(-(b/y))$ %	3.97	3.76	3.65	3.63	3.63	4.24	3.62
mean $(-(b/y))$-4 %	5.50	4.59	4.20	4.16	4.17	7.65	3.54
mean $(-(b/y))$-D %	5.53	4.55	4.15	4.11	4.12	7.80	3.48
std (W) %	6.92	2.51	0.33	8.00	15.90	0.77	0.36
std (W)-4 %	7.54	2.77	1.29	8.43	14.13	3.16	1.40
corr (W, c)	0.01	0.00	0.08	0.00	0.02	0.06	0.03
corr (W, c)-4	0.05	0.02	0.30	0.02	0.01	0.16	0.27
corr $(W, r - r^f)$	-0.08	-0.03	-0.03	-0.04	-0.03	-0.04	-0.06
corr $(W, r - r^f)$-4	-0.08	-0.01	-0.08	-0.01	-0.02	-0.07	-0.06
Default Prob. %	0.70	0.56	0.63	0.63	0.63	1.32	0.82

previous to default) decrease when β^L decreases. Also the level of debt that can be supported at equilibrium is lower with lower levels of β^L. Finally, the correlation between W and the variables of the economy is smaller when β^L is lower.

If β^L is stochastic then the spreads and their volatility in the more turbulent periods increase; also the correlation between W and the variables of the economy increases for these turbulent periods.[17]

The effects of X on the equilibrium are as follow: higher X reduce the average spreads, the volatility of spreads, and the debt-to-output ratio in equilibrium, and reduce the correlation between W and the variables of the economy.

Again, if X is stochastic then the spreads and their volatility in the more turbulent periods increase. Also the model can replicate better the volatility of W, however the correlation between W and the variables of the economy does not change much (in relation to the model with $X = 0.1$).[18]

[17] The process for β^L is simulated by a Markov perfect chain with 5 states. The mean value of β^L is 0.965, the variance corresponds to 0.2775%, and the autocorrelation is set to 0.8501. The calibration for β^L corresponds to the calibration of the process for the cycle of the US discount interest rate for the period 1983-2001, using that variable as a proxy for β^L considering the existence at equilibrium of a tight link between international interest rates and β^L.

[18] The process for X is simulated by a Markov perfect chain with 5 states. The mean value of X is 0.425, the variance corresponds to 11.44%, and the autocorrelation is set to 0.5725. The calibration for X corresponds to the calibration of the process of the cycle of the Dow index for the period 1983-2001. The Dow index is used as a proxy for X considering that X represents other income to the investors not derived

When r^f is stochastic, shocks to r^f are common shocks to the economy and the investors. In this version of the model, the spreads of the economy and their volatility are lower than in the model with a deterministic r^f. On the other hand, the debt-to-output ratio increases despite the fact that the probability of default increases. The correlation between W and the variables of the emerging economy is somewhat higher than when r^f is deterministic.[19]

Finally, when multiplicative shocks to the investors utility function are considered, the effects on the spreads and their volatility are similar to the effects of shocks to X. However in this case, the correlation between W and the variables of the economy is relatively larger than in the model without these shocks.[20]

In general, the results of the model appear relatively robust to changes in the parameters of the utility function of the investors or their budget constraint.

5 Conclusion

This paper presents a stochastic dynamic general equilibrium model of default risk that endogenizes the role of external factors in the determination of small open economies' incentives to default, sovereign bond prices, capital flows and default episodes.

The empirical literature on international finance presents evidence that points to a very relevant role for investors' characteristics—risk aversion and wealth—in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model with endogenous default risk that can account for these empirical findings. By relaxing the assumption of risk neutrality on the side of international investors and assuming that the preferences of these agents exhibit DARA, this model generates a link between international investors' characteristics and emerging economies' sovereign credit markets.

Therefore, the contribution of the paper is twofold. First, the paper qualitatively and

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from T-Bills or bonds of the emerging economy.

[19]The process for r^f is simulated by a Markov perfect chain with 5 states. The mean value of r^f is 1.7%, the variance corresponds to 0.165% and the autocorrelation is set to 0.9125. The calibration for r^f corresponds to the calibration of the process for the cycle of 3-month T-Bills rates for the period 1983-2001.

[20]The process for the multiplicative shocks to $v(\cdot)$ is simulated by a Markov perfect chain with 5 states. The mean value for the shock is 1, the variance corresponds to 8.92% and the autocorrelation is set to 0.6221. The calibration for these shocks corresponds to the calibration of the process of the cycle of the VIX (volatility index) for the period 1990-2001, using that variable as a proxy for the changes in the tolerance to risk of the investors and considering that the shocks to the utility function of the investors will modify their ability to take risk.

quantitatively characterizes the role of investors' characteristics in the determination of small open economies' optimal plans when international credit contracts cannot be enforced. Second, the paper presents a theoretical framework that is extended in a companion paper (Lizarazo (2010)) to a multi-country setup to study endogenous financial links across countries with common investors. This extension can explain endogenously the occurrence of contagion in sovereign debt markets of emerging economies.

Quantitatively, the model developed here outperforms previous models of endogenous default risk in several ways. Compared to risk-neutral models using the same parameterizations, the current model performs better at jointly explaining sovereign spreads levels and equilibrium debt levels. In comparison to those models with risk neutral investors, the present model supports a combination of higher levels of debt at equilibrium (at least for the more volatile periods of the economy) and higher and more volatile spreads. The model is also able to replicate the counter-cyclical behavior of domestic interest rates and the trade balance. The model is also consistent with the observed negative correlation between measures of investors performance and interest rate spreads: this model exhibits the expected negative correlation between investors' wealth and sovereign spreads.

While the model improves on explaining the behavior of prices and quantities with respect to models of the same type that do not consider investors' characteristics, the model is not without shortcomings. For example, the maximum level of debt supported at equilibrium is only around 4.95% of the output, which is much lower than the 53.3% average reached by Argentina at the verge of default reported in Reinhart et al.(2003). Also, from a computational perspective, the inclusion of an additional state variable (the level of wealth of the investors) makes solving this problem much more intensive than the simpler model.

Nonetheless the model presented here opens the door to an important economic issue—that the creditworthiness of a country can be partially explained by factors other than the country's own fundamentals. Specifically, the consideration of risk averse investors explains a large part of sovereign bond spreads and the behavior of borrowers and investors in emerging markets.

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Appendix 1: Proofs.

The proofs assume permanent exclusion of credit markets after a default, therefore the value function of default is independent of γ^L, and W. The quantitative analysis of the model generalizes the results to the case of temporary exclusion. We focus on the case in which the investor is investing in bonds of the economy so $\theta' = -b' > 0$. More borrowing implies a more negative b'.

Proposition 1 *For any state of the world, s, as the risk aversion of the international investor increases, the probability that the government defaults increases.*

Proof. Considering the case in which the government has not defaulted and assuming an interior solution for the allocation to the emerging economy's asset the first order condition of the investor's problem is

$$\phi\left(\vartheta'\right) = E\left\{-qv_c\left(c_L\left(\vartheta'\right)\right) + \beta v_c\left(c'_L\left(\vartheta'\right)\right)d'\right\} = 0.$$

Because $v(\cdot)$ is of the CRRA type and $\gamma_1^L < \gamma_2^L$, then there exists a concave function $\psi(\cdot)$ such that $v_2\left(c;\gamma_2^L\right) = \psi\left(v_1\left(c;\gamma_2^L\right)\right)$. If ϑ'_1 is the optimal allocation when $\gamma^L = \gamma_1^L$, and ϑ'_2 is the optimal allocation when $\gamma^L = \gamma_2^L$ then it holds that

$$\begin{aligned}
\phi_1\left(\vartheta'_1\right) &= E\left\{-qv_{1,c}\left(c_L\left(\vartheta'_1\right)\right) + \beta v_{1,c}\left(c'_L\left(\vartheta'_1\right)\right)d'\right\} = 0. \\
\phi_2\left(\vartheta'_2\right) &= E\left\{-qv_{2,c}\left(c_L\left(\vartheta'_2\right)\right) + \beta v_{2,c}\left(c'_L\left(\vartheta'_2\right)\right)d'\right\} = 0.
\end{aligned}$$

Using $v_2\left(c;\gamma_2^L\right) = \psi\left(v_1\left(c;\gamma_2^L\right)\right)$ it is possible to define

$$\phi_2\left(\vartheta'_1\right) = E\psi'\left[v_1\left(c_L(\vartheta'_1)\right)\right]\left\{-qv_{1,c}\left(c_L\left(\vartheta'_1\right)\right) + \beta\Upsilon(\vartheta'_1)v_{1,c}\left(c'_L\left(\vartheta'_1\right)\right)d'\right\} < 0.$$

where

$$\Upsilon(\vartheta') = \frac{\psi'\left[v\left(c'_L(\vartheta')\right)\right]}{\psi'\left[v\left(c_L(\vartheta')\right)\right]}, \quad \Upsilon(\vartheta') > 0 \text{ and } \Upsilon'(\vartheta') < 0.^{21}$$

[21] The derivative of $\Upsilon(\vartheta')$ is given by

$$\Upsilon'(\vartheta') = \frac{\psi''[v(c'_L(\vartheta'))]v_c(c'_L(\vartheta'))\frac{\partial c'_L(\vartheta')}{\partial\vartheta'} - \psi''[v(c_L(\vartheta'))]v_c(c_L(\vartheta'))\frac{\partial c_L(\vartheta')}{\partial\vartheta'}\Upsilon(\vartheta')}{\psi'\left[v\left(c_L(\vartheta')\right)\right]} < 0.$$

The last inequality comes from the fact that both $\Upsilon(\vartheta')$ and $\psi'(\vartheta')$ are positive and decreasing. The inclusion of these functions in the previous equation implies that $\phi_2(\vartheta_1')$ is lower than $\phi_2(\vartheta_2')$ because $\Upsilon'(\vartheta')$ and $\psi'(\vartheta')$ give little weight to the realizations of $d'=1$, and high weight to the realizations of $d'=0$.[22] Therefore $\phi_2(\vartheta_2') > \phi_2(\vartheta_1')$.

The concavity of $V^L(\cdot)$ implies that given q and the risk of default (represented by the expected realizations of d', corresponding to the default probability δ) $\phi(\vartheta')$ is a decreasing function, and as consequence $\vartheta_2' < \vartheta_1'$ which in equilibrium implies $b_2' > b_1'$.

Then for any s and taking as given q and δ, a higher γ^L would result in this agent allocating a lower proportion of her portfolio to the economy's sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to the emerging economy that are not available when the investor is more risk averse. Consequently, given q and δ then $V_1^C(s;\gamma_1^L) \geq V_2^C(s;\gamma_2^L)$.

Because the utility of autarky for the emerging economy does not depend on γ^L, it is clear that if for some s, default is optimal when $\gamma^L = \gamma_1^L$, then for the same s default would be optimal when $\gamma^L = \gamma_2^L$. Additionally, because incentives to default would be higher whenever $\gamma^L = \gamma_2^L$, than when $\gamma^L = \gamma_1^L$ at equilibrium $\delta(s,b';\gamma_2^L) > \delta(s,b';\gamma_1^L)$, and therefore $q(s,b';\gamma_2^L) < q(s,b';\gamma_1^L)$. ■

Proposition 2 *Default sets are shrinking in assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b in some states y, given W_2, then default will be optimal for b for the same states y, given W_1. Therefore $D(b \mid W_2) \subseteq D(b \mid W_1)$*

Proof. Because $v(\cdot)$ exhibits DARA $v(W_1,\vartheta')$ is a concave transformation of $v(W_2,\vartheta')$ so if ϑ_1' is the optimal allocation when $W = W_1$, and ϑ_2' is the optimal allocation when $W = W_2$, it is possible to define $v_1(\vartheta_1') = v(W_1,\vartheta_1')$ and $v_2(\vartheta_2') = v(W_2,\vartheta_2')$, where $v_1(\vartheta') = \psi(v_2(\vartheta'))$. The first order conditions of the investor are

$$
\begin{aligned}
\phi_1(\vartheta_1') &= E\left\{-qv_{1,c}(c_L(\vartheta_1')) + \beta v_{1,c}(c_L'(\vartheta_1'))\, d'\right\} = 0,\\
\phi_2(\vartheta_2') &= E\left\{-qv_{2,c}(c_L(\vartheta_2')) + \beta v_{2,c}(c_L'(\vartheta_2'))\, d'\right\} = 0,
\end{aligned}
$$

[22] From the problem when $\gamma^L = \gamma_1^L$ we know that $E\{-qv_{1,c}(c_L(\vartheta_1')) + \beta v_{1,c}(c_L'(\vartheta_1'))\, d'\} = 0$. But since $\psi'(\vartheta')$ is positive and decreasing then it weights the realizations of $d' = 0$ more than the realizations of $d' = 1$ and $E\psi'[v_1(c_L(\vartheta_1'))]\{-qv_{1,c}(c_L(\vartheta_1')) + \beta v_{1,c}(c_L'(\vartheta_1'))\, d'\} < 0$. Also since $\Upsilon'(\vartheta')$ is positive and decreasing this function also weights the realizations of $d' = 0$ more than the realizations of $d' = 1$, and $E\{-qv_{1,c}(c_L(\vartheta_1')) + \beta\Upsilon(\vartheta_1')v_{1,c}(c_L'(\vartheta_1'))\, d'\} < 0$. Combining these results we have $\phi_2(\vartheta_1') < 0$.

and therefore

$$\phi_1\left(\vartheta_2'\right)=E\psi'\left[v_2\left(\vartheta_2'\right)\right]\left\{-qv_{2,c}\left(c_L\left(\vartheta_2'\right)\right)+\beta\Upsilon(\vartheta_2')v_{2,c}\left(c_L'\left(\vartheta_2'\right)\right)d'\right\}<0.$$

$\Upsilon(\vartheta')$ is defined as before, and as before the inequality comes from the fact that $\Upsilon(\vartheta')$ and $\psi'(\vartheta')$ are both positive and decreasing. Therefore $\phi_1\left(\vartheta_2'\right)<\phi_1\left(\vartheta_1'\right)$.

Again the concavity of $V^L(\cdot)$ implies that given q and δ, $\phi(\vartheta')$ is a decreasing function, and as consequence $\vartheta_2'>\vartheta_1'$ which in equilibrium implies $b_2'<b_1'$.

Then for any s and taking as given q and δ, a lower level of W would result in this agent allocating a lower proportion of her portfolio to the economy's sovereign bonds. Therefore, when the investor is more wealthy there are financial contracts that are available to the emerging economy that are not available when the investor is less wealthy. Consequently, given q and δ then $V_1^C(y,b,W_2)\geq V_2^C(y,b,W_1)$.

Because the utility of autarky for the emerging economy does not depend on W, it is clear that if for some b in some states y, default is optimal when $W=W_2$, then for the same states y default would be optimal when $W=W_1$. Additionally, because incentives to default would be higher whenever $W=W_1$, than when $W=W_2$ at equilibrium $\delta(s,b';W_1')>\delta(s,b';W_2')$, and therefore $q(s,b';W_1')<q(s,b';W_2')$. ■

Appendix 2: Solution Method.

The state space of the model is discretized for the state variables of the model, b,y,W. y is approximated with a discrete Markov chain with 21 possible realizations. b takes 300 possible discrete values. W, takes 10 possible discrete values. Interpolating over the grid points on W, we allow a continuous range for W. The solution algorithm has the following steps:

(i) Make an initial guess for the government's value function, $V^0(s)$, next period asset position, $b^{\prime,0}(s)$, default/repayment decision $d^0(s)$, and *equilibrium* price function $q^{APC,(0)}(s)$.

(ii) Taking $b^{\prime *,(-i)}(s)$, $d^{*,(-i)}(s)$ and $q^{APC(-i)}(s)$ as given, and assuming equilibrium in emerging credit markets given by

$$\theta^{\prime *,(i)}(s)=\begin{cases} b^{\prime *,(-i)}(s) & \text{if } \ b^{\prime *,(-i)}(s)<0 \\ 0 & \text{if } \ b^{\prime *,(-i)}(s)\geq 0 \end{cases},$$

solve the problem of the investor to find her value function $V^{L(i)}(s)$ and her optimal policy functions $W'^{*,(i)}(s)$.

(iii) Solve the problem of the government to find its value function $V^{(i)}(s)$, its optimal policy functions $b'^{*,(i)}(s)$, and $d^{*,(i)}(s)$ and the new equilibrium price function $q^{EE(i)}\left(s; b'^{(i)}(s)\right)$. This maximization involves the following sub-steps:

(a) Take $q^{APC,(-i)}(s)$ and $W'^{*,(i)}(s)$ as given to compute $c_L^{(i)}(s; b')$.

(b) Given $c_L^{(i)}(s; b')$ and $W'^{*,(i)}(s)$, compute

$$A^{(i)}\left(s, b'\right) = \beta_L \int_{y^*(b'|W'^{*(i)})} \left(c^{L'}\right)^{-\gamma^L} f\left(y' \mid y, W'^{*,(i)}(s)\right) dy'$$

(c) For any s, b' solve for $q^{(i)}(s, b')$ by solving the non-linear equation on $q^{(i)}(s, b')$ that is derived from (17):

$$q\left(s, b'\right)^{-\gamma^L} - b' A^{(i)}\left(s, b'\right) q\left(s, b'\right) - \widetilde{c}_L^{(i)}\left(s; b'\right) A^{(i)}\left(s, b'\right) = 0$$

where $\widetilde{c}_L^{(i)}(s; b') = X + W - W'^{*,(i)} q^f - b' q^f$.

(d) For any s, b' given $W'^{*,(i)}(s)$ compute

$$\beta \int V^{C(i)}\left(s; b'\right) f\left(y' \mid y, W'^{,(i)}(s)\right) dy'.$$

(e) Maximize

$$u\left(y + b - b' q\left(s, b'\right)\right) + \beta \int V^{C(i)}\left(\ s; b'\right) f\left(y' \mid y, W'^{,(i)}(s)\right) dy'$$

with respect to b' to find $V^{C(i)}(s)$ and the associated $b'^{*,(i)}(s)$ and $q^{(i)}\left(s, b'^{(i)}(s)\right)$.

(f) Determine $d^{*,(i)}(s)$ by comparing $V^{C(i)}(s)$ to V^D.

(g) Determine the equilibrium price of bonds by setting

$$q^{EE(i)}\left(s; b'^{(i)}(s)\right) = \begin{cases} q^{(i)}\left(s, b'^{(i)}(s)\right) & \text{if} \quad d^{(i)}(s) = 1 \\ 0 & \text{otherwise} \end{cases}.$$

(iv) If $\left|q^{EE(i)}\left(s; b'^{(i)}(s)\right) - q^{APC,(-i)}\left(s; b'^{(-i)}(s)\right)\right| < \varepsilon$ stop. Otherwise, set $q^{APC,(i)}\left(s; b'^{(i)}(s)\right) = q^{EE(i)}\left(s; b'^{(i)}(s)\right)$, and repeat steps 2 to 4.