Dynamics in a environmental model with tourism taxation

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Abstract–The purpose of this work is to analyze the dynamics of a model describing the interaction between tourists (T) and environmental resource (E) in the presence or absence of a tourist tax β, used to protect the environmental resource. The model highlights how the introduction of tourist tax complicates the dynamics of the system, thus giving origin a new internal equilibrium that is a saddle point, which the stable manifold separates the basin attraction of the locally attractive internal positive point from the one equilibrium point $(K, 0)$, which is also locally stable. Moreover, starting from a system with $\beta = 0$, which has an unstable internal equilibrium, a suitable combination of tourist tax and defensive expenditures leads to a stabilization the protect system.

Key–Words–tourism economics, tourism taxation, Hopf bifurcation, environmental quality, economic modelling

I. INTRODUCTION

The increasing importance of tourism has triggered an interest in public intervention. For instance, some tourism economies strive for a change of the pattern of specialization from the mass tourism to "quality" tourism. In some cases as well, there is a demand for public intervention to correct environmental externalities generated by the tourism sector. To reach these targets several policy instruments have been used such as tourism taxes (room taxes, entry taxes and exit taxes), quality requirements imposed on the suppliers of tourism services, or the provision of public infrastructures related to the tourism activity. Moreover, an important share of the tourism sector is its interdependency with the environmental quality of the destination. On one hand, tourism, as well as all the economics activities, directly affects the environment. The tourism sector and policy makers are interested in investing on the environmental quality and on a sustainable utilization of the local resources. However, on the other hand, the tourism sector depends on the natural environment; the environmental quality of a tourism destination is therefore an important tool that hotels have to hold the tourism demand. From a tourist point of view, the importance of the environmental quality is out of the question, since tourists are mainly interested in it. In this respect, [11] analyze the impact of the environment on holiday destination choices of prospective UK tourists. These authors found out that tourists are willing to pay more in order to visit a destination with high environmental quality (see also [16] and [5]). From all those studies appears clear that environmental quality is important for tourists and that in a large number of cases, they are willing to pay for quality From a tourist point of view, the importance of the environmental quality is out of the question, since tourists are mainly interested in it. From all those studies appears clear that environmental quality is important for tourists and that in a large number of cases, they are willing to pay for quality. This is another peculiar characteristic of the tourism demand, and one of the components of the model analyzed in this paper. Tourist taxes have become an important source of revenue for many tourist destinations. Taxes on accommodation are upheld by their proponents as a way of shifting the local tax burden on to non-residents, while the travel industry claims that these levies do significant damage to their level of competitiveness. Additionally, we assume the existence of a lump sum tax in the accommodation sector. Taxing became in fact a very common policy instrument, with the aim of controlling the negative impact of tourism on the environment. There are many economic studies about tourism taxation, as for example [8], [1], [2], [10], [3], [17], [7] and [13]. Especially [14] investigates how the introduction of user fees and defensive expenditures change the complex dynamics of a discrete-time model, which represents the interaction between visitors and environmental quality in a Open-Access Protected-Area. In this paper is analyzed a continuous-time model. Further [15] modeling the difference between the revenues from visitors and the sum of expenditures on recreation investments and defensive expenditures

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for ensuring the preservation of natural and cultural heritage by formulating a optimal control problem.

II. THE MODEL

Renewable environmental resources such as fisheries and forests reproduce and grow, but are also subject to both natural mortality and human disturbance. If left undisturbed, renewable resources are typically assumed to reach a maximum level at which birth and growth exactly balance decay and death (see [9]). This point, denoted the natural carrying capacity of the resource, is sustainable (see [4]). Because, however, the carrying capacity is only obtainable for resources left undisturbed by human use, it is generally not a viable option for resources supporting a tourism industry. Although most tourism depends on multi-attribute bundles of environmental resources, we simplify the model by assuming that the condition of all renewable resources in the community may be appropriately measured by one composite index variable, E, which we denote environmental quality. This index consolidates the notions of resource quality and ecosystem productivity for all types of renewable resources into a single index. Also for simplicity, we assume that all resources on which tourism depends are renewable to some degree; nonrenewable resources are not considered. These simplifications allow us to emphasize the fundamental trade off between visitors and environmental quality (see [6]). For a renewable resource, we assume that environmental quality gradually renews itself, or grows, in proportion to the underlying stock of the resource. The growth functionhere specified using the simple function based on an underlying logistic growth function $h(E)$implies that natural renewal or growth of environmental quality is a mathematical function of E. That is, when environmental quality is highly degraded (i.e.,small), the natural improvement in quality, $h(E)$, will be relatively small. When environmental quality is pristine (at its maximum level or carrying capacity, K), there can be no natural improvement; by definition, $h(K) = 0$. Growth will be fastest at some point between zero and K, peaking at a point of maximum sustainable yield (i.e [12]). The dynamic of the environmental quality combines the negative influence of visitors (T), the positive influence of natural growth $h(E)$ and the protection of the natural resource. It is natural to think that the dynamics of the tourists is positively affected by the environmental quality and negatively effected by the tourist tax and crowding effect. Thus we can write the dynamical system of the model as

$$\begin{cases} \dot{E} = r(1-\frac{E}{K})E - \alpha T^2 + \beta\rho T \\ \dot{T} = T(-\beta - aT + \sigma E) \end{cases} \tag{1}$$

where r measures the rate of growth of the environmental quality, α measures the environmental impact associated with a unit of visitors , ρ is the technology parameter that measures the effectiveness of protection the natural resource policy and β represents the tourist tax. Further the parameter a and σ represent the crowding effect and tourist preference respectively. All parameters are strictly greater than zero except β, which may take the value zero.
In fact now analyze the case where you do not include any tax stay $(\beta = 0)$ and then if it is asked by police makers a tourist tax $(\beta > 0)$.

III. DYNAMICS WITH NO TOURIST TAXATION $(\beta = 0)$

The model (1) becomes

$$\begin{cases} \dot{E} = r(1-\frac{E}{K})E - \alpha T^2 \\ \dot{T} = T(-aT + \sigma E) \end{cases} \tag{2}$$

Proposition 1. *For all parameters values,* (2) *has three fixed points:*

a) $O(0,0)$

b) $P(K,0)$

c) $S(E_\infty, T_\infty)$

where $E_\infty = \dfrac{ra^2K}{ra^2+\sigma^2\alpha K}$, $T_\infty = \dfrac{ra\sigma K}{ra^2+\alpha\sigma^2 K}$.

Note that the phase portrait of the system (2), is constituted from an ellipse with center $C_e = (\frac{K}{2}, 0)$ and from a straight(r_1) with equation $E = \frac{a}{\sigma}T$.

Let E^* and T^* the values of the fixed point O, P, S, then the characteristic equation of dynamic system (2) is

$$\lambda^2 + [aT^* + r(1-\frac{2E^*}{K})]\lambda + \\ T^*[2\sigma\alpha T^* - ra(1-\frac{2E^*}{K})] = 0 \tag{3}$$

Therefore, we can state the following propositions:

Proposition 2. *The fixed point $O(0,0)$, for all values of the parameters is a non-hyperbolic point.*

Proof. From (3) the eigenvalues are $\lambda_1 = 0$, $\lambda_2 = r$.

Proposition 3. *The fixed point $P(K,0)$, for all values of the parameters is a saddle point.*

Proof. From (3), the eigenvalues are $\lambda_1 = -r$, $\lambda_2 = \sigma K$.

Proposition 4. *The fixed point $S(E_\infty, T_\infty)$, is an attractor point if and only if $\alpha\sigma^2 K - a^2(r+\sigma K) < 0$, else it is a repellor point.*

Proof. We consider the following equations obtained from the coefficients of the characteristic polynomial (3)

$$\begin{cases} aT^* - r(1 - \dfrac{2E^*}{K}) = 0 \\ 2\sigma\alpha T^* - ra(1 - \dfrac{2E^*}{K}) = 0 \end{cases} \tag{4}$$

these are two straights (respective r_3 and r_2). Substituting the fixed point $S(E_\infty, T\infty)$ in (4) and by straightforward computations we obtain that the straight r_3 passes for the points $(C_e, \frac{T^*}{2})$, while the straight r_2 passes for the points $(C_e, \frac{r\sigma K}{2ar + a\sigma K})$. By Routh-Hurwitz criterion if the right-hand sides of (4) are strictly positive then the fixed point S is an attractor, being always $T_3 < T^*$ then this happens if and only if $T_2 < T^*$, namely when $\alpha\sigma^2 K - a^2(r+\sigma K) < 0$ is hold. If $T_2 > T^*$ then the right-hand side of the first equation of (4) is strictly negative, so the eigenvalues are either real and strictly positive or with negative real part (see Figure 1).

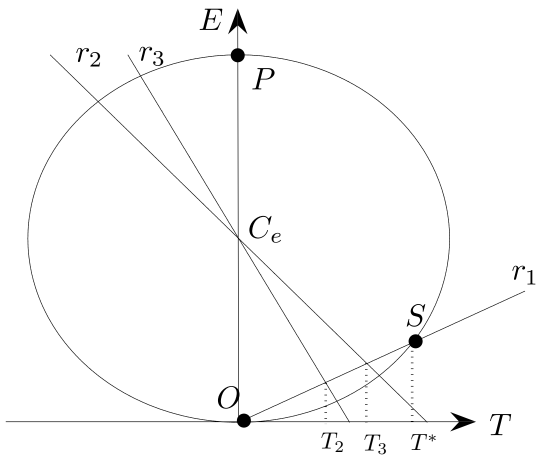


Figure 1: The graphs of isoclines system (2) (ellipse $\dot{E} = 0$ and straight (r_1) $\frac{\dot{T}}{T} = 0$) , and straights r_2 and r_3 used in the proof of the Proposition 4

A. Hopf bifurcation and limit cycles

In order to analyze the Hopf bifurcation and the existence of limit cycles, we choose as bifurcation parameters before σ and after a.
Remember that, the parameter σ represents the attractiveness associated with high environmental quality, while the parameter a may be thought as the crowding coefficient.

Proposition 5. *For any choice of the parameter K, r, and α the equation*

$$\alpha\sigma^2 K - a^2(r + \sigma K) = 0 \tag{5}$$

represents a bifurcation curve, moreover

a) for any value of σ then a limit cycle arise if $a := a_H = \sigma\sqrt{\dfrac{\alpha K}{r+\sigma K}}$ is hold.

b) for any value of a then a limit cycle arise if $\sigma := \sigma_H = \dfrac{a}{2\alpha}(a + \sqrt{a^2 + 4\dfrac{\alpha r}{K}})$ is hold.

Proof. The first and crucial condition for a Hopf bifurcation concerns the existence of a pair purely imaginary eigenvalues. This in turn requires according to Proposition 4, a solution of the equation (5).
Now we prove the second condition of the Hopf bifurcation theorem, namely that the imaginary axis is crossed at non-zero velocity with respect before to the bifurcation parameter a and after σ.
Differentiating the real part of the eigenvalues of (2) with respect to a yields

$$\frac{dRe(\lambda(a))}{da} = -(\frac{2}{\sigma K} + 1)\frac{2\alpha\sigma^3 a K^2}{(ra^2 + \alpha\sigma^2 K)^2} \neq 0 \tag{6}$$

for all a and in particular for $a = a_H$.
Differentiating the real part of the eigenvalues of (2) with respect to σ yields

$$\frac{dRe(\lambda(\sigma))}{d\sigma} = \frac{a}{2\alpha}(a + \sqrt{a^2 + \frac{4\alpha r}{K}}) \neq 0 \tag{7}$$

for all σ and in particular for $\sigma = \sigma_H$.

Figure 2 show the Hopf bifurcation curve (for a generic values of the parameters K, r, α). This curve divides the plane in two regions, a region is characterized by fixed points attractors, other by repellors.

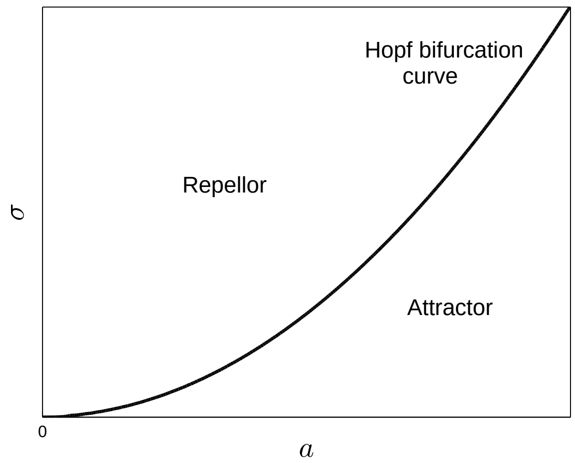


Figure 2: Hopf bifurcation curve of the system (2), in the parameter (a, σ)-plane

Fixed the value of attractiveness associated with high environmental quality, σ, only if the congestion parameter a is greater of a_H is catches up the stable fixed point. In fact, if tourists are not very sensitive crowding, then the system is not in equilibrium, the tourists increase and the environmental resource is damaged, increasing their sensitivity, the tourists desert the site tourism bringing the system in the stable equilibrium.

B. Comparative statics

Consider la $T^*(a)$ e $E^*(a)$, Figure 3 shows the the trend in the number of tourists and the stock of good environmental at the equilibrium in relation to the parameter values for a increasing the parameter σ (ceteris paribus). The function $T^*(a)$ has a maximum at $a_M = \sigma\sqrt{\frac{\alpha K}{r}}$. For values of $a > a_M$the number of tourists (always equilibrium) decreases namely because it increases sensitivity tourists to the over-crowding. One can easily calculate that if $a_M < a_H$ then the maximum value of the tourists equilibrium is always an attractive fixed point. The trend of the function $E^*(a)$ increases with crowding, to at end the bearing capacity of the K stock of environmental resource.

IV. DYNAMICS WITH TOURIST TAXATION ($\beta > 0$)

Analyze the dynamics with $\beta > 0$, with reference to research fixed points of the system (1).
Let $P = [r, K, \alpha, \beta, \alpha, \rho, a, \sigma]$ the array of parameter of system (1) then apply the following proposition

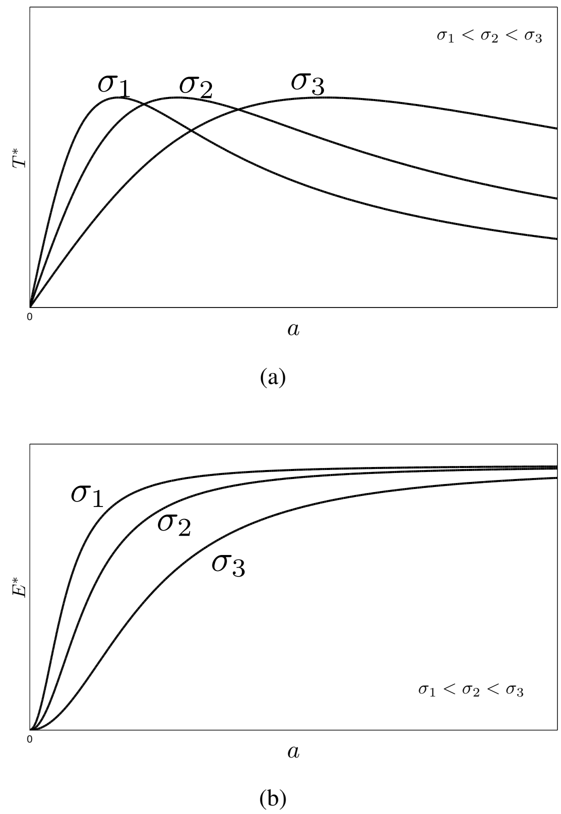


Figure 3: Comparative static on the parameter σ

Proposition 6. *For any choice of the parameters of the array P, the system* (1) *has always two fixed points:*

a) $O(0, 0)$

b) $P(K, 0)$

Moreover

i.) if $\beta < \sigma K$ then the system (1)*, has a unique fixed point S_1 strictly positive;*

ii.) if $K > \frac{ra}{\rho\sigma^2}$ and $\sigma K < \beta < \beta_0$ then the system (1)*, has two fixed points S_1 and S_2 (with $E_1^* > E_2^*$ and $T_1^* > T_2^*$) strictly positive*

where β_0 is the solution of the equation

$$(\rho^2 - \frac{4r}{\sigma^2 K}(a\rho + \alpha))\beta^2 + \frac{2r}{\sigma}(a\rho + 2\alpha)\beta + \frac{r^2a^2}{\sigma^2} = 0 \qquad (8)$$

Proof. The fixed points of the system (1), are given by solution

$$F(E, T) = r(1 - \frac{E}{K})E - \alpha T^2 + \beta\rho T = 0 \qquad (9)$$

$$G(E, T) = T(-\beta - aT + \sigma E) = 0 \qquad (10)$$

Define:

$$A := (\frac{ra^2}{\sigma^2 K} + \alpha)$$
$$B := \frac{r\beta a}{\sigma K} + \frac{ra}{\sigma}(\frac{\beta}{\sigma K} - 1) - \rho\beta$$
$$C := \frac{r\beta}{\sigma}(\frac{\beta}{\sigma K} - 1)$$

Then the fixed points S_1 and S_2 are

$$E^\star_{1,2} = \frac{a}{\sigma}T^\star_{1,2} + \frac{\beta}{\sigma} \qquad T^\star_{1,2} = -\frac{B}{2} \pm \frac{1}{2}\sqrt{B^2 - 4AC}$$

The condition $C < 0$, that is $\beta < \sigma K$, guarantees the existence of only a fixed point.
Necessary and sufficient conditions for existence of two fixed points are $C > 0$, $B < 0$ and $\Delta = B^2 - 4AC > 0$ From easy calculation, the first and second conditions state that $\beta > \sigma K$ and $K > \frac{ra}{\rho\sigma^2}$, while the third condition is verified if $\beta < \beta_0$, with β_0 solution of the equation (8) of the Proposition 6

Figure 4 shows the complete classification of possible cases.

A. Stability analysis

Now we analyze the stability of fixed points of the system (1). We obtain the following propositions.

Proposition 7. *The fixed point $O(0,0)$ is a saddle point, while $P(K,0)$ is a saddle point if $\beta < \sigma K$, else it is a attractor point.*
Moreover

If S_1 exist it is a attractor or repellor point, while if S_2 exist it is a saddle point

Proof. The Jacobian matrix $J(P^\star)$, evaluated at a stationary state $P^\star = (E^\star, T^\star)$ can be expressed as follows:

$$J(P^\star) = \begin{pmatrix} r(1 - 2\frac{E^\star}{K}) & -2\alpha T^\star + \beta\rho \\ \sigma T^\star & -\beta - 2aT^\star + \sigma E^\star \end{pmatrix}$$

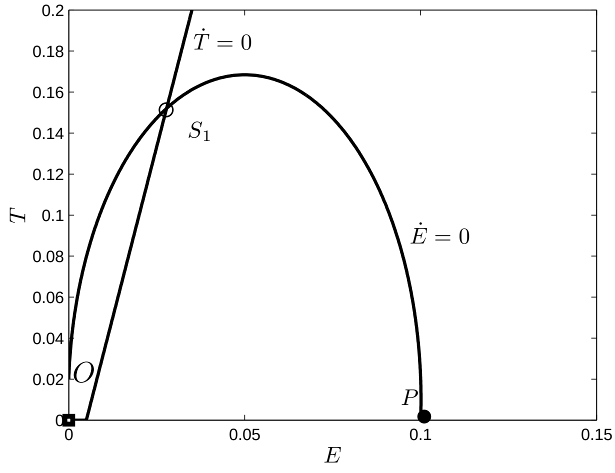
The eigenvalues of $J(P^\star)$ are the roots of the following characteristic polynomial:

$$P_1(\lambda) = \lambda^2 - tr(J)\lambda + det(J) \tag{11}$$

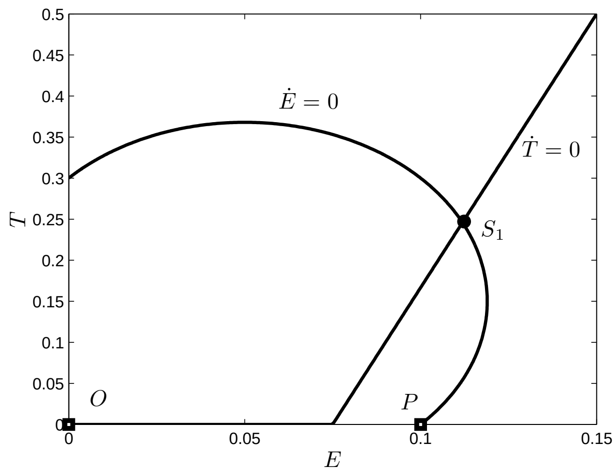
where

$$tr(J) = r(1 - 2\frac{E^\star}{K}) - \beta - 2aT^\star + \sigma E^\star \tag{12}$$
$$det(J) = r(1 - 2\frac{E^\star}{K})(-\beta - 2aT^\star + \sigma E^\star) - \sigma T^\star(-2\alpha T^\star + \beta\rho) \tag{13}$$

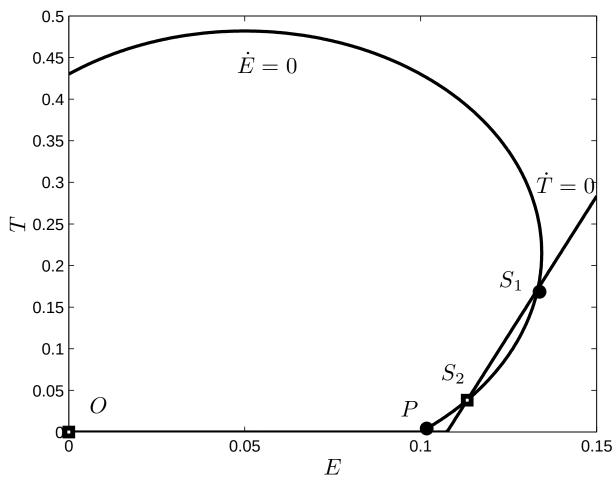
Therefore



(a) $\beta = 0.001$



(b) $\beta = 0.015$



(c) $\beta = 0.0215$

Figure 4: Number of fixed points for different values of β taxation and all the other values as in the previous figures.

i. if $(E^\star, T^\star) = (0,0)$, then the eigenvalues are $\lambda_1 = -r$ and $\lambda_2 = \beta$;

ii. if $(E^\star, T^\star) = (K,0)$, then the eigenvalues are $\lambda_1 = -r$ and $\lambda_2 = -(\beta - \sigma K)$.

In the case of fixed points strictly positive, since $\dot{T} = -\beta - aT + \sigma E = 0$, the trace and the determinant of the Jacobian matrix becomes

$$tr(J) = r(1 - 2\frac{E^\star}{K}) - aT^\star \tag{14}$$

$$det(J) = T^\star\big(-ar(1 - 2\frac{E^\star}{K}) - \sigma(-2\alpha T^\star + \beta\rho)\big) \tag{15}$$

a) *Fixed point S_1*

It 's easy to see that for $E^\star > \frac{K}{2}$, the trace is negative, so the fixed point is attractive
In case $E^\star < \frac{K}{2}$, we define the intersection, between the ellipse $\dot{E} = 0$, the straight lines r_1 and r_2 of equation $T = -\frac{2r}{aK}E + \frac{r}{a}$, and $T = -\frac{ar}{\alpha\sigma K}E + \frac{ar + \sigma\rho\beta}{2\alpha\sigma}$ respectively . Below the straight line r_1 we have $tr(J) > 0$, while below the straight line r_2 we have $det(J) < 0$. If we choose parameters such that for $\beta = 0$ the fixed points is a repellor (see figure 1), then it is easy to see that the fixed point is between the two straight lines, therefore increasing β the fixed point becomes the intersection between the ellipse and the straight line r_1, leading to $tr(J) = 0$ e $det(J) > 0$, emerging a limit cycles arising from a Hopf bifurcation of coordinates

$$(E^\star_H = \frac{K(r + \beta_H)}{\sigma K + 2r}, \frac{\sigma}{a}E^\star_H - \frac{\beta_H}{a})$$

with β_H solution of the equation

$$\begin{aligned}&(4r(\alpha + \rho a) + Ka(a + 2\rho\sigma))\beta^2 + \\ &K(K\sigma(a + \rho\sigma) + \rho a\sigma^2 + 2\rho ar\sigma)\beta + \\ &K(Ka^2r\sigma + a^2 + r^2 - \alpha r\sigma^2 K) = 0\end{aligned}$$

Increasing the value of β the fixed point becomes attractive.

b) *Fixed point S_2*

In this case the determinant of Jacobian matrix is negative, thus S_1 is a saddle fixed point.

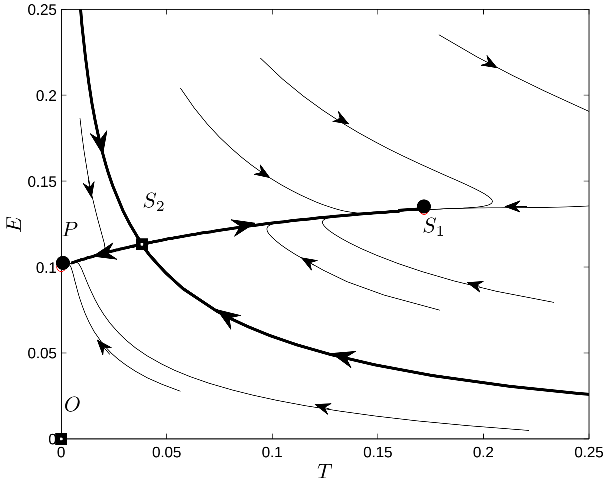


Figure 5: The dynamics in the phase space of (1), with $K = .1$, $\alpha = 0.01$, $r = .01$, $a = .03$, $\sigma = 0.2$, $\beta = 0.0215$, $\rho = 0.2$

We know that case there exist all equilibrium points (see Figure 5 and Figure 4(c)). The one of coordinate $O = (0,0)$ and S_2 (marked with the square) are saddle points, while the other, P and S_2 are locally stable (marked with the bullet). The stable manifold of the saddle point S_2 separates the basin of attraction of the locally attractive internal positive point from the ones of the equilibrium points S_1 and $P = (K,0)$, which is also locally stable.
From the proof of the previous proposition, if the internal fixed point the system without tourist tax is attractive, the introduction of fees and defensive expenditures keeps the system stable. The question that we want answered is: Can suitable values of β and ρ stabilize the unstable system?
Figure 6 to answer to this question. It show a Hopf bifurcation curve(Hbc) varying the parameters β and ρ. Further, are also shown four points,one in the repulsive an three in attractive area. Starting from a repulsive fixed point $A = (\beta_A, \rho_A)$ (we can also think about starting with $\beta = \rho = 0$, that is from origin in Figure 6), may be that (see Figure 6)

i. increases β, leaving unchanged ρ, then the point A moves towards the attractive point $D = (\beta_D, \rho_A)$ with $E^\star = 0.029, T^\star = 0.1567$

ii. increases ρ, leaving unchanged β, then the point A moves towards the attractive point $B = (\beta_A, \rho_B)$ with $E^\star = 0.028, T^\star = 0.1625$

iii. both ρ and β are increased, then the point A moves towards the attractive point $C = (\beta_C, \rho_C)$ with $E^\star = 0.032, T^\star = 0.17$

In other words, the increase of β $(i.)$ or ρ $(ii.)$ can stabilize the unstable system, but the increase of both $(iii.)$ bringing the system to higher values at equilibrium. The limit to the increase of the parameters is given by a rapid decrease of $T^\star$ for high values of beta (see Figure 7) or any costs incurred for the protection of the environment[1].

B. Comparative statics

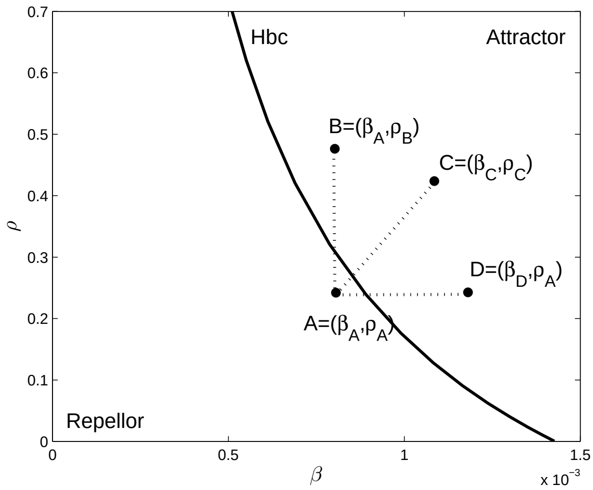


Figure 6: Hopf bifurcation curve of the system (2), in the parameter (β, σ)-plane. The parameters are $K = .1$, $\alpha = 0.01$, $r = .01$, $a = .03$, $\sigma = 0.2$

We should do the usual analysis of comparative statics on the parameters ρ, β, σ and K If there exist only two fixed points with E,$K > 0$, then the one which interests us is the one which is potentially attractive. In particular, we are interested to see how E and T vary when these parameters are varied.
The following propositions investigate the impact of a change in the parameters (we will focus our analysis in particular on ρ, β,σ K) on the values of $E^\star$, $K^\star$. By the symbol $x \uparrow$ and $x \downarrow$ we shall indicate an increase and a decrease in the parameter or variable x respectively.
Differentiating equation (9) and (10) with respect to the parameter $y = \beta, \rho, \sigma, K$, we obtain

$$F(E, K; y) = 0$$
$$G(E, K; y) = 0$$

such that:

$$\frac{\partial F}{\partial E}dE + \frac{\partial F}{\partial T}dK + \frac{\partial F}{\partial y}dy = 0$$
$$\frac{\partial G}{\partial E}dE + \frac{\partial G}{\partial T}dK + \frac{\partial G}{\partial y}dy = 0$$

or

$$J(S_1)\begin{pmatrix} \dfrac{\partial E}{\partial y} \\ \dfrac{\partial T}{\partial y} \end{pmatrix} = -\begin{pmatrix} \dfrac{\partial F}{\partial y} \\ \dfrac{\partial G}{\partial y} \end{pmatrix} \qquad (16)$$

where $J(S_1)$ is the Jacobian matrix evaluated in the potential attractive fixed point S_1.
Thus, we can state the following proposition

Proposition 8. *$\rho \uparrow$ (remember that ρ represent the technology adopt in defensive expenditures) implies $E^\star \uparrow$ and $T^\star \uparrow$..*

Proof. Posing $y = \rho$, after some simple mathematical manipulations,the solution of the system (16) becomes

$$\frac{\partial E}{\partial \rho} = \frac{a\beta T^{\star 2}}{det(J(S_1))} > 0 \quad \frac{\partial T}{\partial \rho} = \frac{\sigma\beta T^{\star 2}}{det(J(S_1))} > 0$$

which gives $\dfrac{\partial E}{\partial \rho} = \dfrac{a}{\sigma}\dfrac{\partial T}{\partial \rho}$
Remembering that $det(J(S_1)) > 0$ the above proposition is proof.

Proposition 9. *$K \uparrow$ (remember that K represent the carrying capacity of the environmental) implies $E^\star \uparrow$ and $T^\star \uparrow$.*

Proof. Posing $y = K$, after some simple mathematical manipulations,the solution of the system (16) becomes

$$\frac{\partial E}{\partial K} = \frac{arT^\star E^\star}{K^2 det(J(S_1))} > 0 \quad \frac{\partial T}{\partial K} = \frac{\sigma r T^\star E^\star}{K^2 det(J(S_1))} > 0$$

which gives $\dfrac{\partial E}{\partial K} = \dfrac{a}{\sigma}\dfrac{\partial T}{\partial K}$
Remembering that $det(J(S_1)) > 0$ the above proposition is proof.

Proposition 10. *$\sigma \uparrow$ (remember that σ represents the attractiveness associated with high environmental quality) implies $E^\star \uparrow$ and $T^\star \uparrow$.*

[1]A further study will analyze the dynamics of the problem of optimal control where, for example, the utility function is

$$U(T, \rho) = pT - \frac{1}{2}q\rho^2$$

with p average price paid by tourists

Proof. Posing $y = \sigma$, after some simple mathematical manipulations,the solution of the system (16) becomes

$$\frac{\partial E}{\partial \sigma} = \frac{aT^{\star 2}E^{\star}}{det(J(S_1))} > 0 \quad \frac{\partial T}{\partial \sigma} = \frac{\sigma T^{\star 2}E^{\star}}{det(J(S_1))} > 0$$

which gives $\frac{\partial E}{\partial K} = \frac{a}{\sigma}\frac{\partial T}{\partial K}$
Remembering that $det(J(S_1)) > 0$ the above proposition is proof.

Proposition 11. *$\beta \uparrow$ (remember that σ represents the taxation) implies $E^{\star} \uparrow\downarrow$ and $T^{\star} \uparrow\downarrow$.*

Proof. Posing $y = \beta$, after some simple mathematical manipulations,the solution of the system (16) becomes

$$\frac{\partial E}{\partial \beta} = \frac{a\rho T^{\star 2} - T^{\star}(-2\alpha T^{\star} + \rho\beta)}{det(J(S_1))} \lesseqgtr 0$$
$$\frac{\partial T}{\partial \beta} = \frac{\sigma\rho T^{\star 2} + rT^{\star}(1 - \frac{2}{K}E^{\star})}{det(J(S_1))} \lesseqgtr 0$$

In order to the last proposition we can not uniquely determine the impact of a change of β on the values of $E^{\star}$ and $T^{\star}$.
Figure 7, shows how there exist a value of β such that $T^{\star}$ decrease. A further increase of β, involves a rapid decrease of $T^{\star}$, then the defensive expenditures $\rho\beta T^{\star}$ are such as to bring a decrease of environmental quality $E^{\star}$.

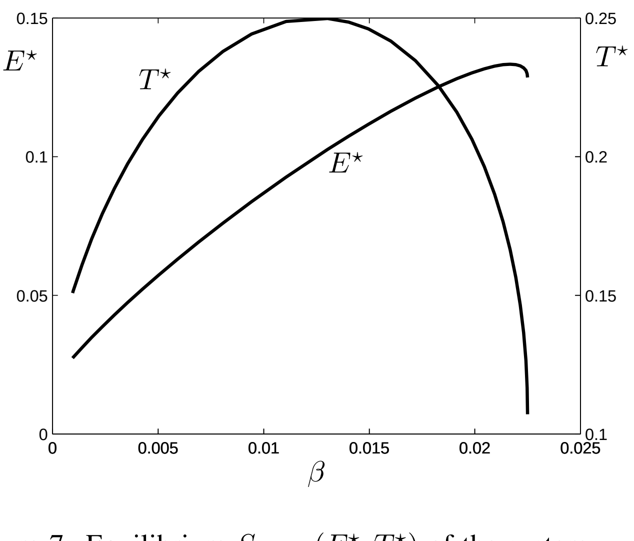


Figure 7: Equilibrium $S_1 = (E^{\star}, T^{\star})$ of the system (1), varying β with $\rho = 0.2$. The other parameters are $K = .1$, $\alpha = 0.01$, $r = .01$, $a = .03$, $\sigma = 0.2$

V. CONCLUSION

The purpose of this work is to analyze the dynamics of a model describing the interaction between tourists (T) and environmental resource (E) in the presence or absence of a tourist tax β. The model highlights how the introduction of tourist tax complicates the dynamics of the system, thus giving origin a new internal equilibrium that is a saddle point, which the stable manifold separates the basin attraction of the locally attractive internal positive point from the one equilibrium point $(K, 0)$, which is also locally stable. It turns out that, for reasonable parameter values (σ and β), a Hopf bifurcation exist. In addition, we have seen how a change of the parameters β and ρ can stabilize or destabilize the system.

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