Liquidity-adjusted benchmark yield curves: a look at trading concentration and information

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Liquidity-Adjusted Benchmark Yield Curves: A Look at Trading Concentration and Information

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ABSTRACT

Estimation of benchmark yield curve in developing markets is often influenced by liquidity concentration. Based on an affine term structure model, we develop a long run liquidity weighted fitting method to address the trading concentration phenomenon arising from horizon-induced clientele equilibrium as well as information discovery. Specifically we employ arguments from models of liquidity concentration and benchmark security information. After examining time series behavior of price errors against our fitted model, we find results consistent with both the horizon and information hypotheses. Our evidences indicate that trading liquidity carries information effect in the long run, which cannot be fully captured in the short run. Trading liquidity plays a key role in long run term structure fitting. Markets for liquid benchmark government bond issues collectively form a long term equilibrium. Compared with previous studies, our results provide a robust and realistic characterization of the sport rate term structure and related price forecasting over time, which helps portfolio investment of fixed income and long run pricing of financial instruments.

Keywords: Liquidity; Trading Concentration; Information Discovery; Term Structure; Yield Curve.
JEL Classification: D4, D53, D83, E43, G12

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Introduction

Estimation of term structure of interest rate, especially in the emerging markets, is affected by pronounced trading concentration. Implied yields of benchmark on-the-run issues reflect much more than just a point on a hypothetic continuous sport rate curve. The liquidity premium caused by the concentration of trading, without controlling for the related effects, leads to biases in the estimated term structure. Not only ignoring this fact results in distortion of implied interest rates, but factors driving the long-run term structure will be overlooked as well. The theoretic and empirical attention on the liquidity premium in bond markets is well documented in the literature. Theoretically, Duffie, Garleanu and Pedersen (2005) provided a general model for liquidity premium in an OTC market based on search and bargaining mechanism. Vayanos and Wang (2006) further argue that the liquidity premium will be more substantial in markets for specific securities where trading concentrates. Empirically, Amihud and Mendelson (1991) analyze how liquidity affects Treasury bill yields. Warga (1992) suggests that liquidity of the former is priced by the market, where on-the-run issues have lower returns than off-the-run ones. Elton and Green (1998) argue that trading volume as a proxy for liquidity correlates more with bond prices. Evidences in the international markets can be found in Eom, Subramanyam and Uno (2002) for the Japanese market and Diaz, Merrick and Navarro (2006) for the Spanish government bond market. Evidences from the literatures suggest that it is crucial, especially for emerging markets, to control the liquidity concentration effect while estimating yield curve.

The control of liquidity effect can be carried out in several ways. Bolder and Sterilski (1997) use a subset of bonds based on liquidity and estimate the yield curve on the selected issues. It literally estimates a 'liquid yield curve', which may omit issues of very long and short maturities in the emerging market due to low liquidity. The other approach estimates the yield curve jointly with a liquidity function to cover the effect of non-interest rate factors on bond prices, as seen in Elton and Greene (1998) and Alonso, Blanco, del Rio and Sanchis (2004). The regression conducted in that approach could suffer from the nonnegative nature of the liquidity effects on bond prices although it is claimed that the joint estimation of a yield curve and liquidity function is consistent technically. Lastly, the approach of Subramanian (2001) and Dutta, Basu and Vaidyanathan (2005) employ a weighting scheme from one or more liquidity proxies to estimate the yield curve by minimizing the weighted pricing errors. The impact from the more liquid issues are weighted or penalized more in the optimization, so the resulting yield curve estimated is always closer to the observed yields of the liquid issues. However, it has been found that pricing errors generated from the liquidity-weighted yield curve model are systematically related to idiosyncratic rather than systematic factors.

In this study, we develop a method to examine performance of liquidity-adjusted term structure in the long run. Our method is capable of showing specifically how the more liquid issues help the market achieving long term equilibrium. In an emerging bond market where trading concentration is substantial[1], liquidity attached to the most or the more liquid issue is persistently higher than other issues. In the case of Taiwan the liquidity of a 10-year on-the-run issue consistently dominates the rest and therefore correlates with systematic factors. This extreme concentration cannot be explained reasonably alone by the information effect. The fact that the special repos are almost entirely on the same 10-year issue could be a major factor to the concentration in the ourtright market in the sense of Duffie, *et al*. (2005). Furthermore, according to Vayanos and Wang (2006) this concentration is related exogenously to heterogeneous investment horizons among market participants. While the horizon effect can be estimated from the distribution of special repos, the information effect of benchmark issues, according to Yuan (2005), due to systematic variable other than interest rates may be filtered out with a liquidity-weighting yield curve fitting. Applying a model of affine term structure by Duffie and Singleton (1997) and Liu, Longstaff and Mandell (2006), we show how liquidity premium can be consistent with Yuan (2005) and Vayanos and Wang (2006). As Diebold and Li (2006) suggest a time series link between parameters of yield curve for one period and those for the next period, we intend to examine pricing error time series where the information effect is filtered while trading concentration effect remains within the errors. We have modified the weighting of Subramanian (2001) and Dutta *et al.* (2005) to incorporate intraday liquidity measure following Goldreich, Hanke and Nath (2005), which proposed a liquidity measure specifically appropriate for fixed income securities with the concept of expected future liquidity. We further analyzed the time series of forecasting errors provide implications beyond existing methods and previous literatures. Our results address the issue of yield curve forecasting in a long period of time, which is important for portfolio management, derivatives pricing and risk management. Although only the information effect is filtered in our study, the evidences are significantly consistent with Yuan (2005). Our study of the time series pattern of pricing errors compares against Diebold and Li (2006) with a richer theoretical foundation. The cointegration relationship found among pricing errors of benchmark issues further validates the clientele equilibrium as laid out in Vayanos and Wang (2006).

Our study has several contributions to the practice of fixed income securities. First, we provide a consistent reason for applying liquidity adjustment in the estimation of term structure of emerging markets. Other than technical reasons, it helps capturing price premium statically as well as dynamically. Another contribution is that our long term approach[2] is consistent with fundamental characteristics of the government bond market such as the trading facilitation and price discovery. Lastly, the implication our analysis is crucial to fixed income portfolio management for its consistency with long-term market equilibrium. The rest of the paper is organized as follows. In Section 2 we provide a detailed description of our modeling framework with our definition of liquidity weighting. Section 3 reports our data and preliminary estimation results. In section 4, comparisons on results from alternative methods are made and the informational role of liquidity is discussed. Section 5 gives concluding remarks and recommendations.

Horizon and Information Effects in Term Structure

Trading of government bonds especially in the emerging markets have been characterized by limited number of issues on the one hand and liquidity concentration on the other. Darbha (2004) addressed this issue in the Indian government bond market, where trading is concentrated more on medium maturity issues. Alonso, *et al*. (2004) studied the Spanish Treasury bond market and found significant liquid premia for off-the-run issues. The average number of issues in their yield curve estimation is 38. Diaz, *et al*. (2006) also studied the same bond market and indicated specifically that the 10-year on-the-run issue accounted for as much as 23.5% of the total market volume in the period of 1998-2002. The average number of concurrent bonds is only around 30. Although the liquidity concentration phenomenon is not pronounced in the more developed market, Shen and Starr (1998) have studied a liquidity-based term structure model for T-bills. Amihud and Mendleson (1991) have argued, more liquid issues are traded with lower yield. Longstaff (2004) has also demonstrated that the liquidity premium could be as high as 15%. Elton and Green (1998) used volume as a proxy for liquidity and found significant results in explaining US zero-coupon yields. Shen and Starr (1998) used monthly and quarterly bid-ask spreads to analyzing term premium between 6-month and 3-month US T-bills.

Taiwan's government bond market has now reached on the outright transaction a daily trading volume of around 16 billion US dollars on 66 outstanding issues as of November 2006, which is about 87% of the Canadian government bond market volume, 3.2% of the US treasury volume and 5.3% of the Japanese government bond volume. The repo market is also active with a daily volume of around 10.5 billion dollars. Trading primarily takes place in a centralized trade-matching market, the Electronic Bond Trading System (EBTS). Over-the-counter trading is still in practice, where 13 of the 87 dealers are primary dealers, but accounts for about 8 percent of the total volume as of November 2006. The repos are still mainly traded over the counter through dealers. Contract days concentrate from overnight up to 30 days. Special repos are only available on the EBTS for overnight contracts, accounting for about 10 percent of the total repo volume, and hence around 4 percent of the total government bond volume. However, both outright and special repo trading are extremely concentrated in the on-the-run, especially the 10-year, issues, which normally constitutes more than two thirds of the daily trading volume. With this drastic concentration of liquidity, it is reasonable to consider the effect of liquidity in estimating the sport rate term structure for the Taiwanese fixed income market. Studies of on this market have not made formal treatment on this issue in fitting the yield curve although some consideration of liquidity measures has been noted. We will try to establish in this section a theoretical framework for our subsequent analysis. Our focus of trading concentration will be on the information and repo dealing effects of the heavily traded benchmark issues in this market.

To address the liquidity concentration phenomenon in the asset market, Vayanos and Wang (2006) introduced a search-based model of asset trading where clientele equilibrium prescribes endogenous concentration of trading liquidity in the market. Basically, participants in the market of heterogeneous investment horizons are faced with separate thresholds of liquidity risks and become endogenously buyers, owners or sellers in respective markets. An asset traded in the market with trading concentration would require the lowest search time. The equilibrium price of the asset is higher than one with identical payoff but traded otherwise in a less liquid market. This horizon effect characterizes well the liquidity premium appearing on off-the-run government bond issues. When there is a repo market using liquid government securities as collateral, the concentration of trading is reinforced by the search-related motive arising from special repos which designate the delivery of a specific issue. In the case of Taiwan, the concentration of liquidity on benchmark issues grew substantially after the repos started trading, and the concentration on the 10-year on-the-run issues became pronounced after trading of special repos started on the EBTS. This phenomenon is consistent with Duffie, *et al*. (2005) in general, and Vayanos and Wang (2006) in particular. While they focus on a one-stage endogenous equilibrium where the horizon distribution is exogenous and may not change within a short period of time. But in the study of Yuan (2005), information about systematic and idiosyncratic risks can be extracted from the trading of benchmark security. In a market of asymmetric information, Yuan (2005) implies that over time the liquidity-related changes will then affect subsequent trading of non-benchmark securities. This intertemporal relation is central to our analysis. Diebold and Li (2006) modeled specifically a parameter process in fitting term structure to reflect this issue. In another word, fitted price errors at a given time help predicting fitted errors in the next period. The information effect provides connection of term structure across time. Instead of considering liquidity-related premium in a single period context, our study that follows will examine fitting results along the evolution of major government bonds. As an issue turns from on-the-run to off-the run, the information effect diminishes. Our work is then in the very spirit of Goldreich *et al.* (2005) and consistent with Alonso, *et al*. (2004).

Both the horizon and the information effects are contained in the liquidity of traded government bonds and on-the-run issues would have contained the most of it. Based on the effects outlined above studies, we propose a model as follows to conduct our empirical analysis.

The model

We adopt in this section a model in the spirit of Duffie and Singleton (1997), Duffee (2002) and Liu, *et al*. (2006) to characterize liquidity premium for the noises from horizon and information. Their affine framework of fixed income is straightforward and maintains generality for the purpose of illustration. We will consider only two classes of default-free bonds in our model, one being liquid and the other illiquid. Assume a highly liquidly traded default-free zero-coupon bond maturing at T has a value at time t as in (1).

$$H(t,T) = E_Q\left[\exp\left(-\int_t^T r_s ds\right)\right], \tag{1}$$

where r_s is the instantaneous spot rate. The expectation E_Q is on a risk-neutral probability measure Q, corresponding to the physical or objective measure P. For the value of a less liquid default-free bond, there is a liquidity premium γ_s and its value at time t can be expressed as

$$L(t,T) = E_Q\left[\exp\left(-\int_t^T [r_s + \gamma_s]ds\right)\right]. \tag{2}$$

The liquidity premium γ_s reflects on the one hand the difference in yields between bonds with different terms at issuance. On the other hand, it can be also considered as characterizing the difference between on-the-run and off-the-run issues of a given maturity at different time t in the sense of Goldreich, *et al.* (2005).

Following the specification of Duffie and Singleton (1997), we consider the two endogenous variables as driven by a set of four state variables given by the vector X, where X= [X_1, X_2, X_3, X_4] is Markovian under the equivalent martingale measure Q and square-root diffusions. The short rate is assumed to be driven by three state variables of common shocks to the economy,

$$r_t = \delta_0 + X_1 + X_2 + X_3, \tag{3}$$

where δ_0 is a constant. The instantaneous liquidity spread at time t in the less liquid bond is assumed to take the form of

$$\gamma_t = \delta_1 + X_4, \tag{4}$$

where δ_1 is also a constant and the X_4 is the state variable on liquidity related uncertainty. The affine term-structure model with general Gaussian processe X is defined as

$$dX = -\beta X dt + \Sigma dB^Q, \tag{5}$$

where β is a diagonal matrix and B^Q is a vector of independent standard Brownian motions under the risk-neutral measure of Q. Σ is a lower diagonal matrix containing covariances among the state variables, and it is assumed also that the covariance matrix $\Sigma\Sigma'$ is of full rank to allow correlations of state variables. Corresponding to this affine structure is the dynamics under the physical measure P,

$$dX = \eta(\theta - X)dt + \Sigma dB^P, \tag{6}$$

where η is also a diagonal matrix and θ is a vector of long-term value of the state variables. The prices of the liquid and illiquid bonds can be solved under the risk-neutral dynamics (5). Their closed-form representations, following Liu, *et al.* (2004), are as follows,

$$H(t,T) = \exp\left[-\delta_0(T-t) + a(t) + b'(t)X\right] \tag{7}$$

$$L(t,T) = \exp\left[-(\delta_0 + \delta_1)(T-t) + c(t) + d'(t)X\right], \tag{8}$$

where

$$a(t) = \frac{1}{2}M'\beta^{-1}\Sigma\Sigma'\beta^{-1}M(T-t) - M'\beta^{-1}\Sigma\Sigma'\beta^{-2}(I - e^{-\beta(T-t)}) + \sum_{i,j}\frac{1 - e^{-(\beta_{ii}+\beta_{jj})(T-t)}}{2\beta_{ii}\beta_{jj}(\beta_{ii}+\beta_{jj})}\Sigma\Sigma' M_i M_j,$$

$$b(t) = \beta^{-1}(e^{-\beta(T-t)} - I)M$$

and I is the identity matrix and $M' = [1,1,1,0]$. Functions c(t) and d(t) are the same as a(t) and b(t) except that M' is defined as [1,1,1,1].

On the physical measure P, the difference in return arising from liquidity related reason can derived from (7) and (8) as

$$\gamma_t + \lambda_3(t)[(\beta - \eta)X_t + \eta\theta], \tag{9}$$

where $\lambda_3(t)$ is a function of parameters. For a given maturity, the first term in (9) compensates holding the less liquid riskless bond at time t. It can be considered as characterized by the horizon component of liquidity spread[3] in the sense of Vayanos and Wang (2006). The second term is a premium compensating for possible future liquidity related price changes, which amounts to about 73 b.p. for a 10-year bond based on US data according to Liu, *et al.* (2004). It can also be treated as the information-induced component and reflects the extent of price discovery of a benchmark bond in the context of Yuan (2005)[4]. So the less liquid bond carries a higher yield due to lack of information content in addition to higher search cost and lower demand from the market as collateral for repos.

Liquidity-adjusted term structure fitting

To determine the information effect contained in a benchmark government bond, we need to compare yields on issues with different market liquidity. As part of the extra yields on the less liquid issues exists to compensate their lack of price informativeness, a term structure fitted without this component would be free of the information effect. Fitting a yield curve without accounting for the role of liquidity would not have captured information contained in the benchmark issue about the correlation between the liquidity state variable X_4 and state variables which lies in the second term of (9). So the ordinary way of term structure fitting, which normally minimizes fitted errors, would have left fitted errors correlated overtime. Based on the model considered above, we will try to fit a liquidity-adjusted term structure and examine the information effect over time. The validity of our analysis would be crucial in understanding the liquidity concentration phenomenon in the fixed income market.

In this study we will follow the works of Subramanian (2001) and Vaidyanathan, Dutta and Basu (2005) to fit the Taiwanese term structure with a liquidity-weighted optimization process. Their liquidity measure are however daily volume and trades due to data limitation. Our study uses both daily volume and intra-day trading liquidity measures to emulate market depth in capturing the liquidity effect in a more realistic sense. Specifically, we have considered the *expected future liquidity* concept proposed by Goldreidch, Hank and Nath (2005) as it is unique for fixed income securities. Average quote spread and effective quote spread are reported as the two most prominent liquidity measures for treasuries. Fleming (2003) has also reported that the intra-day measures such as bid-ask spreads are better in tracking the liquidity of treasury issues than quote and trade size. To account for the need to modify functional form of the fitting objective in a less developed market, we adopt the optimization function of Vaidyanathan, Dutta and Basu (2005) to incorporate the effect of liquidity in the estimation procedure. Two sets of weights are used in all the models to contrast the unweighted fitting model. One contains only weights for liquidity, which are based on intra-day liquidity measures instead of daily trading volume. The other set of weights utilizes both liquidity and duration to examine the validity of liquidity effect.

In order for the term structure to identify pricing errors, using a squared error criterion tends to amplify pricing errors since large error terms from the presence of liquidity premiums contribute more to the objective function than to the errors on liquid securities. The objective functions have two variations, where one is minimizing the mean squared errors while the other minimizes mean absolute deviations. A reciprocal of the average bid-ask spreads in a given day is an ideal liquidity function, in addition the volume of trade and the number of trades are good candidates too. They are characterized by

$$\min[\xi(\theta)] = \min\left[\sum_{i=1}^{n} w_i \left(B_i - \hat{B}_i\right)^2\right] \tag{10}$$

and

$$\min[\xi(\theta)] = \min\left[\sum_{i=1}^{n} w_i \left|B_i - \hat{B}_i\right|\right] \tag{11}$$

where B_i and $\hat{B}_i$ are actual and fitted bond prices respectively. The weights w_i in (10) and (11) are defined by

$$w_i = \frac{\tilde{W}_i}{\sum \tilde{W}_i}$$

and

$$\tilde{W}_i = \tanh(\frac{-v_i}{v_{\max}}) + \tanh(\frac{-s_i}{s_{\max}}) \tag{12}$$

where v_i and s_i are daily trading volume and average spread or the i_{th} security respectively, while v_{max} and s_{max} are the maximum volume of trades and the maximum number of trades among all the securities traded for the day respectively. The adoption of the hyperbolic tangent function is to incorporate asymptotic behaviour in the liquidity function. The relatively liquid securities would have v_i/v_{max} and s_i/s_{max} close to 1 and hence the weights of liquid securities would not be significantly different. However, the weights would fall at a fast rate as liquidity decreases.

Beside the liquidity weights defined in (12), we have also considered the following alternative ones to match the reality of the Taiwanese government bond market. As the 10-year on-the-run issues often trade initially at around 80% to 90% of the market volume, the adjustment of (12) may not be able to restore the liquidity premium accurately. So we also look at an alternative weight definition to restore stability of the curve while adjusting the premium reasonably.

$$\tilde{W}_i = \tanh(\frac{-s_i}{s_{\max}}) + \tanh(\frac{-d_i}{d_{\max}}) \tag{13}$$

where d_i is the duration of the i_{th} security, while d_{max} is the maximum duration among all the securities traded for the day respectively. The adoption of duration measure in the weight construction is intended to capture factors other than liquidity which influence trading concentration, as issues with larger duration measure tend to attract trading interests.

B-splines and variable roughness penalty (VRP) model

Our base fitting method takes the B-Spline approach of Steeley (1991) using the following pricing function

$$B_i = \sum_{j=1}^{k} b_j \left(\sum C_i(t) g_i(t) \right) + \varepsilon_i \tag{14}$$

Where $C_i(t)$ are coupon payments and the B-spline function $g_i(t)$ is defined as suggested in Steeley (1991),

$$g_j^p(t) = \sum_{h=j}^{j+p+1} \left[\left(\prod_{l=j, l\neq h}^{j+p+1} \frac{1}{(t_l - t_h)} \right) \right] \left[\max(t - t_h, 0) \right]^p \quad -\infty < t < \infty \tag{15}$$

where j is the number of control points and p is the order of the spline. Many studies, such as Alonso, *et al.* (2005) and Diebold and Li (2006), adopt an econometric model of Nelson and Siegel (1987) and Svensson (1995) to estimate term structure as it provides economic interpretations for estimated parameters. The Nelson-Siegel-Svensson (NSS) model is however more sensitive to sample size. Due to the limited number of issues available for a given day in the Taiwan market, we choose to use the B-spline model for long-term stability of estimation results.

The VRP or smoothing splines approach used in this study is the one from Waggoner (1997), an extension of the splines approach suggested by McCulloch (1971) and Fisher, Nychka and Zervos (1995). The objective function with VRP has the form

$$\min[\xi(\theta)] = \min\left[\sum_{i=1}^{n} w_i \left(B_i - \hat{B}_i\right)^2 + \int_0^T \lambda_t \left[D''(\theta)\right]^2 dt\right] \tag{16}$$

where λ_t is the smoothing parameter and has the following distribution,

$$\lambda_t = \begin{cases} 0.1 & 0 \le t \le 1 \\ 100 & 1 \le t \le 10 \\ 100{,}000 & 10 \le t\,. \end{cases}$$

and $D(\theta)$ is the yield curve as a function of parameters to be chosen in the fitting process.

The first term in (16) represents the goodness of fit while the second term is the roughness penalty. As cubic splines generate oscillations in the forward rate term structure, an unacceptable behavior, the curvature is penalized accordingly in the optimization under (16). The penalty is smaller in the short end as the forward curve is less oscillating for the shorter maturity. In the long maturity segment, the penalty is the highest. The objective function takes the form of a non-linear regression. Our estimation adopt the estimates from the original B-spline model as the initial values of the VRP model and then obtain parameters with the Gauss-Newton method.

Data and Results

Our data is obtained from the EBTS of Gretai Securites Market in Taipei from January 1, 2003 to June 30, 2005. The trading data contains traded, bid and ask price of all the records submitted through EBTS. We exclude for each issue days without bid and ask records, to avoid non-trading problem. So our data is more practical in the utilization of trading information than using only daily closing price and trading volume. When there is no traded price based on actual trading, the closing price is not backed up by actual market perception. So focusing on intra-day trading activity to some extent improves substantially the reliability of our data. This concern has rarely been explicitly addressed in the literatures on fixed income and particularly about the local market. For the validity and stability of the sample, prices of the 30-year bond and when-issue data are excluded. So our term structure fits only up to 20 years the spot rates of the Taiwanese market. There are altogether 53 issues with valid trading data during this period. We have complied data for 545 days with reasonable number of issues traded each day. The numbers of issues are between 18 to 35 in a given day, with an average of around 22 for the whole period.

Table 1 shows that, according to the weight definition of (12), the weight distribution across issues is quite uneven, while adding duration in the weight construction greatly reduces the unevenness. On the first set of weights, each one of the 10-year issues can account for up to an average of around 80 percent in the first 30 days after issuance and still around 20 percent in an extended trading period. Weights for the 5- and 2-year issues are generally only one-sixth those of the 10-year ones, with weights for 5-year ones slightly larger. On the second weighting scheme, weight for any single 10-year issue accounts for only up to 15 percent in the first 30 days and down to an average of about 7 percent in the long run. The effect of adding in duration in the weight construction is, however, not significant on 5- and 2-year issues. It is obvious that the inclusion of duration has reduced the liquidity adjustment effect substantially.

To reduce possible bias caused by potential fitted price error problems, we have employed fitted yield mean absolute deviation minimization in addition to the least square optimization. Fitted results from the two optimizing scheme differ not much from each other. So we report in the study only results from the least square method. After making the comparison, we would be less concerned with influences from uneven price error distribution. Fitted yield curves under the B-Spline method of a given day is drawn in figure 1. Liquidity-weighted curves, with and without VRP adjustment, are lower than the unweighted curve near the 10-year mar, and higher in the longer and in the shorter terms[5]. It reflects more the market's perception of the spot rate term structure, as compared with the hypothetical curve. After being adjusted with VRP function, the fitted curve runs through all the more liquid issues. To get an idea of how the curves move over time, we plot all the liquidity weighted curves against the unweighted ones in figure 2 for a total of 545 days. It is apparent that the liquidity adjustment has caused swings in the long and in the short end, which reflects the dominant trading weight on the 10-year issues. figure 3 shows the unweighted curves compared with the liquidity and duration weighted ones. The weighting scheme is smoother especially on the longer end as the duration of long bond compensates for its lower liquidity. On the low end of the term, small duration and low liquidity still produce yields higher than the unweighted curve. So the second weighting scheme basically reduces liquidity effect only on the long end.

In a separate analysis, whose results are not reported here, we have compared the performance between the NSS and B-Spline models with and without liquidity or VRP adjustments using root mean squared errors. B-Spline-Liquidity performs the best, while the B-Spline-Liquidity-VRP stands as second and the NSS-Liquidity is the least preferable. Generally for unweighed fitting, NSS is smoother than the B-Spline method as seen in other studies. The performance of unweighted versus liquidity-weighted fitting will be examined in the next section with conceptually more sensible forecast errors. Although the B-Spline-Liquidity model has been identified as the best in performance, it exhibits the most oscillation among the three. The curvature is potentially related to the fact that the on-the-run 10-year issue was trading at a dominating volume of total market. So the weighted yield curve has a dip on the 10-year maturity. The fitted value there is lower than the fitted unweighted 10-year yield by at an average of about 10 b.p., which causes the two humps from optimization under the B-Spline-Liquidity model. The NSS-Liquidity, however, provides a more moderate curve than the other two. In terms of liquidity-weighted estimation, our results are different from that of Dutta, Basu and Vaidyanathan (2005), which identified the NSS-Liquidity model as the most stable one. However, as argued in Bliss (1997), the length of fitting period seems to affect the comparison of performance among models. There he found that the Smoothed Fama-Bliss method performs better in the short run, while the McCulloch Cubic Spline works better in the long run. To the extent that the combination of issues of various long and short term influence the fitting result, our findings exemplifies a working model for the long term.

The forecasting performance of the term structure is based idea of Diebold and Li (2006), which adopted the NSS model and showed that the three coefficients in the yield curve may be interpreted as latent level, slope and curvature factors. In this study we attempt to address the time series properties of our fitted results. The forecasting capability of models we use can be of great interest since the long run properties of the fitting model can be evaluated. We compare results from the two liquidity-weighted models with those from the unweighted one. Each day we apply the B-Spline model to obtain three sets of parameters, derived from traded prices, liquidity and cash flow applicable to that day. Forecasted price for the next day is computed by applying this day's parameter to the cash flow array of the next day. Forecast errors are then computed by subtracted the forecast price from actual traded price each day. This measure is employed in our study in place of the commonly used RMSE measure. From a practical perspective, the forecast price errors tell how a fitting scheme performs in a trading environment. If we can tell how well market participants can infer from the forecast error series, we would be more confident to use the scheme in a practical sense.

For each issue, from its issuance date we construct the price errors through out our data period in the fashion outlined in Table 1. Table 2 reports the summary information of major issues which will be employed in our regression analyses. As we are concerned with nonconsecutive observations from a practical perspective, we have selected eventually only 14 issues for our purpose. First of all, under all fitting models the 10-year issues tend to be negative across the board, a natural subsequence of fitted higher price (lower yield) than the less liquid 5- and 2-year issues. Forecast price errors generated from the liquidity-weighted model reflect the most upward-adjusted forecast prices. Errors for the 5-year issues are less so in the pattern, while those for the 2-year ones even exhibit positive forecast errors, reflecting suppressed forecast prices computed from parameters meant to downward-adjust fitted prices in matching market reality. It is worth noted that the standard deviation of forecast prices is larger than that of the fitted prices as expected. However, the forecasting scheme preserves the pattern we see in fitted prices. Also, the liquidity-weighted forecast price errors have larger standard deviation, especially on the 10-year issues than the unweighted ones, as the latter have been optimized on squared errors. On the 5- and 2-year issues, the difference is less pronounced. This is again an effect of liquidity, where shorter term issues are less affected. Comparing the liquidity-duration-weighted errors against the unweighted ones, we see a similar pattern. The difference is that here the standard deviation of forecast price errors on the 5- and 2-year issues are even higher than their unweighted counterparts. As liquidity carries smaller weights on the short end of the term, the duration weight factor seems to become the primary source of perturbation to the fitting process for the short term issues. To get a visual understanding of the phenomena discussed above, we have plotted in figure 4, figure 5 and figure 6 forecast errors under the three fitting schemes for the most recent issue in each of the three term categories. They show that the curve of liquidity-weighted price errors are generally smoother and, reflecting what Table 2 reports, stays below the unweighted and the liquidity/duration weighted curves. On the other hand, the liquidity-weighted curves stay above, and are also smoother than, those for the other two fitting schemes,

Regardless of the weighting scheme, forecasted price errors exhibit significantly positive first and second order autocorrelations. Table 3 reports the adjusted *R*-squared measures on the AR(2)[6] regressions of selected issues[7] for all three fitting models. The explanatory power of the liquidity-weighted model is the highest among the three across all issues irrespective of term. 10-year issues also have the higher adjusted *R*-squared's than the other two term groups, indicating the concentration of trading brings better predictive power. For the 2-year issues, the explanatory power of the regression is only half of that for the 10-year ones. The fact that adding duration in the weighting scheme does not bring higher explanatory power in Table 3 indicates, as expected, that duration does not provide any information related function in constructing optimizing weights.

Informational Role of Benchmark Issues

The importance of our liquidity adjustment does not come from its performance on fitting term structure as in Darbha (2004) and Dutta, Basu and Vaidyanathan (2005). Rather, the forecasting errors from our liquidity-weighted estimation help determining how liquidity affects pricing over time, given a spot rate yield curve. Specifically, our methodology extracts the effect that is present from one period to the next, which is information. It also identifies the other market structure related horizon effect in a longer period of time. To explore informational content carried by the liquid benchmark issues, we proceed with further analysis on the influence of liquidity on the persistence of fitted price errors. Two models are analyzed to infer possible relations between their results. The Base Model, whose results are reported in Table 4, is just the AR(2) model considered in Table 3, while the Control Model employs a concurrently active on-the-run 5- or 10-year issues as an control variable added to the Base Model. The only difference between it and the one considered in Table 3 is the number of observations used. In order to pair results from the Base Model against those from the Control Model, data period of the control variable is matched by the variable of interest.

As only the unweighted regression in the Control Model utilizes the same variable as the Base Model, comparing the two tells us that the autocorrelation coefficients tend to go down after a control variable has been added. Significantly positive coefficients for the control variable indicate possibly part of the price error persistence effect is carried by the control variable[8]. This pattern holds also for the liquidity-duration weighted model. However, if we consider the fitted price errors from the liquidity-weighted model, we find uniformly insignificant coefficients for the control variables. The autocorrelation coefficients are even larger. It is natural to argue that during the liquidity-weighting process fitted price errors of the off-the-run issue has incorporated certain information carried by the on-run-issue. So there is little further to be conveyed by the control variable. Larger autocorrelation coefficients suggest a situation the liquidity has actually been incorporated at an extent more than adequate, which is what needs to be examined further. The short term persistence of the fitting errors indicate that the model employed have not captured movements of market yields over time. Part of the traded yield level in a given day reflects not only current market demand and supply, but also certain anticipation of subsequent yield movements. If the fitting is inaccurately modeled, we would have seen negative autocorrelation as a correction. In another word, with both the horizon and information effects affecting the liquid premium, only the latter incorporated in the liquidity-weighting fitting process will influence the potential informational content carried by pricing errors next period. The horizon effect is static and its incorporation in the liquidity-weighting process does not help forecasting in the next period.

To establish a more formal foundation for the information role of trading liquidity, we need to analyze the market system with a long term perspective. The persistence for the fitted price error patterns is strong enough for us to conduct unit root tests to verify its time series stationarity. Evidences suggest a significant non-stationarity is present in the liquidity-weighted price error series, but not in the unweighted and the liquidity-duration-weighted ones. We therefore proceed with a cointegration analysis on a group of 5 time series including the three most recent 10-year issues and the most recent 2- and 5-year issues. In light of the informational effect found in Table 5, it is reasonable to expect certain cointegration mechanism to remove the non-stationarity of some of the liquidity-weghted price error series. Results reported in Table 6 suggest significantly that the 10-year error series, not those from issues of shorter maturities, are the factors restoring the system to long term stationarity. Relative to the most recent 2- and 5-year on-the-run issues, coefficients for all the most recent three 10-year issues are significant within the cointegrating vector. The more recent 10-year on-the-run issues appear to be more significantly the variables in bringing the system to cointegration. So the dominant liquidity behind the 10-year on-the-runs can be considered as one factor that drives the government bond market to certain long run equilibrium. In the context of Vayanos and Wang (2006), liquidity is distributed according to participants' investment horizons in the clientele equilibrium, and the equilibrium liquidity in the most liquid market determines liquidity distribution of other markets sequentially. The exogenous liquidity threshold modeled in Vayanos and Wang (2006) dictates first the number of buyers and sellers in the most liquid market, and subsequently in other markets. The long term stationarity achieved for pricing errors of benchmark issues of from our cointegration analysis validates exactly this prediction.

Conclusions

Concentration of liquidity is in general common in the fixed income securities market and in the government bond market in particular. This phenomenon in the emerging government bond market is especially pronounced, which affects the estimation of term structure. The degree of liquidity concentration and the premium arising from asymmetric pricing has been documented well empirically. However, the cause of the concentration and related implications for the long-term price structure has not been addressed proportionally. This study examines the theoretical foundation of liquidity concentration and verifies empirically what the implications may be.

The importance of our analysis with liquidity adjustment lie not so much in the fitting performance on the term structure, but in the implications brought forward by the behavior of forecasting errors in a long-term context. We find that a liquidity function based on average quoted spread and the total volume models the liquidity fairly well. Estimation using this weighted objective function ensures that liquid bonds in the market are priced consistently with information delivery and long-term market structure. The incorporation of liquidity improves substantially the fitting performance on the one hand. On the other hand, the forecasting performance of the liquidity adjustment provides the justification of its implementation as the outcome is consistent with predictions of underlying theoretical models. We have compared the performance of three different models. The B-Spline model with liquidity weighting adjustment performs the best in terms of efficiency and stability. Liquidity not only affects pricing errors in fitting yield curves cross-sectionally, it also does it along the time series direction. In addition to the cross-sectional analysis in comparing fitting results with and without liquidity-weighted optimization, we take fitted price errors of given issues and observe their time series properties. The results suggest that there are needs to incorporate time-series-wise adjustment as the fitted prices errors are serially correlated. Without the liquidity adjustment, fitted pricing errors of liquid issues carry information about the less liquid ones. However, after adjusted for liquidity, the information effect disappears. This is evidence supporting the information carrying function of benchmark issues as argued in Yuan (2005). To investigate the long-term equilibrium, we further conduct a cointegration analysis to explore a common factor that drives the prices of on-the-run 2- 5- and 10-year issues. Cointegration analysis performed on a group of most recent on-the-run issues indicated that the most active 10-year issues are the ones that restoring the system to stationarity. The most actively traded 10-year issues appear to be the stabilizing factor that removes the long term non-stationarity of the error time series. This is consistent with the clientele equilibrium laid out by Vayanos and Wang (2006) where the most liquid market leads other markets in reaching equilibrium.

Our results contribute to the pricing practice of fixed income securities in several ways. First, we provide a justification for the empirical literatures that apply liquidity adjustment in the estimation of term structure of emerging markets. Liquidity adjustment is necessary not just for technical reasons, but also for capturing price premium arising from static market structure and dynamic information dissemination. Secondly, our long term approach of term structure estimation produces parameters and performance more consistent with fundamental characteristics of the government bond market such as the price discovery function of benchmark issues and the influence of repo market. Thirdly, the implication our analysis is consistent with long-term market equilibrium[9], which is crucial to fixed income portfolio management.

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TABLE 1

Distribution of Average Weights across Issues

Weights from the two weighting schemes are averaged first over all the days where trading prices are available, and then over the first 30 trading days. As trading is more active for an issue while it is on the run, the averages are in general smaller than weights during initial trading days. Weights computed for the liquidity and duration weighting scheme are generally smaller than those weighted only with liquidity due to the fact that the raw weights used are of similar magnitudes to the duration measures. As a result, these weights are substantially smaller.

Issues	**No. of Days Traded**	**Liquidity Weighted**		**Liquidity and Duration Weighted**	
		Entire Period	*First 30 Trading Days*	*Entire Period*	*First 30 Trading Days*
2-year					
A92101	169	0.0037	0.0021	0.0091	0.0129
A92109	97	0.0134	0.0320	0.0131	0.0145
A93101	227	0.0334	0.0404	0.0132	0.0183
A93105	225	0.0559	0.0763	0.0139	0.0205
A94101	50	0.0770	0.0582	0.0144	0.0155
5-year					
A92102	417	0.0077	0.0051	0.0232	0.0298
A92106	192	0.0096	0.0131	0.0254	0.0265
A92108	199	0.0242	0.0893	0.0311	0.0375
A93102	244	0.0478	0.1485	0.0310	0.0433
A93107	167	0.0774	0.1025	0.0319	0.0409
A94102	39	0.1432	0.1195	0.0311	0.0321
10-year					
A92104	395	0.2171	0.6541	0.0666	0.0988
A92107	239	0.1994	0.8863	0.0698	0.1535
A92110	277	0.1466	0.8334	0.0634	0.1447
A93103	283	0.0700	0.3091	0.0818	0.1077
A93108	130	0.6967	0.7825	0.1064	0.1293

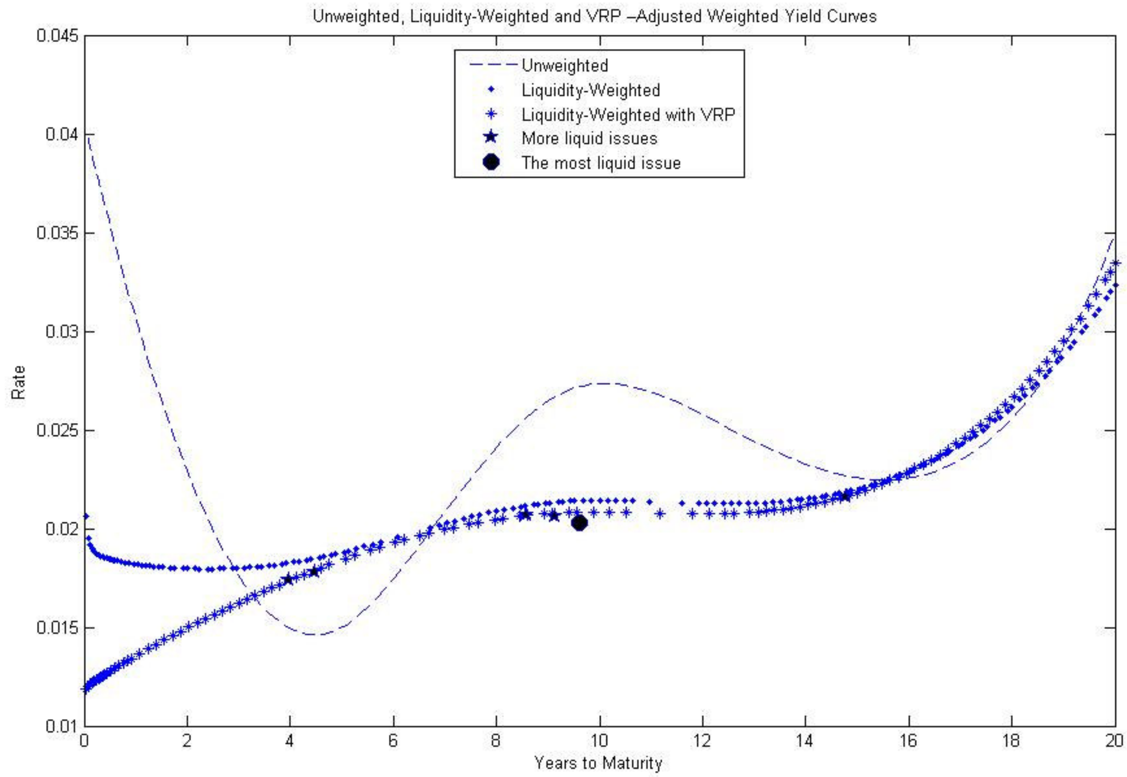

Figure 1 Typical fitted yield curves with different methods

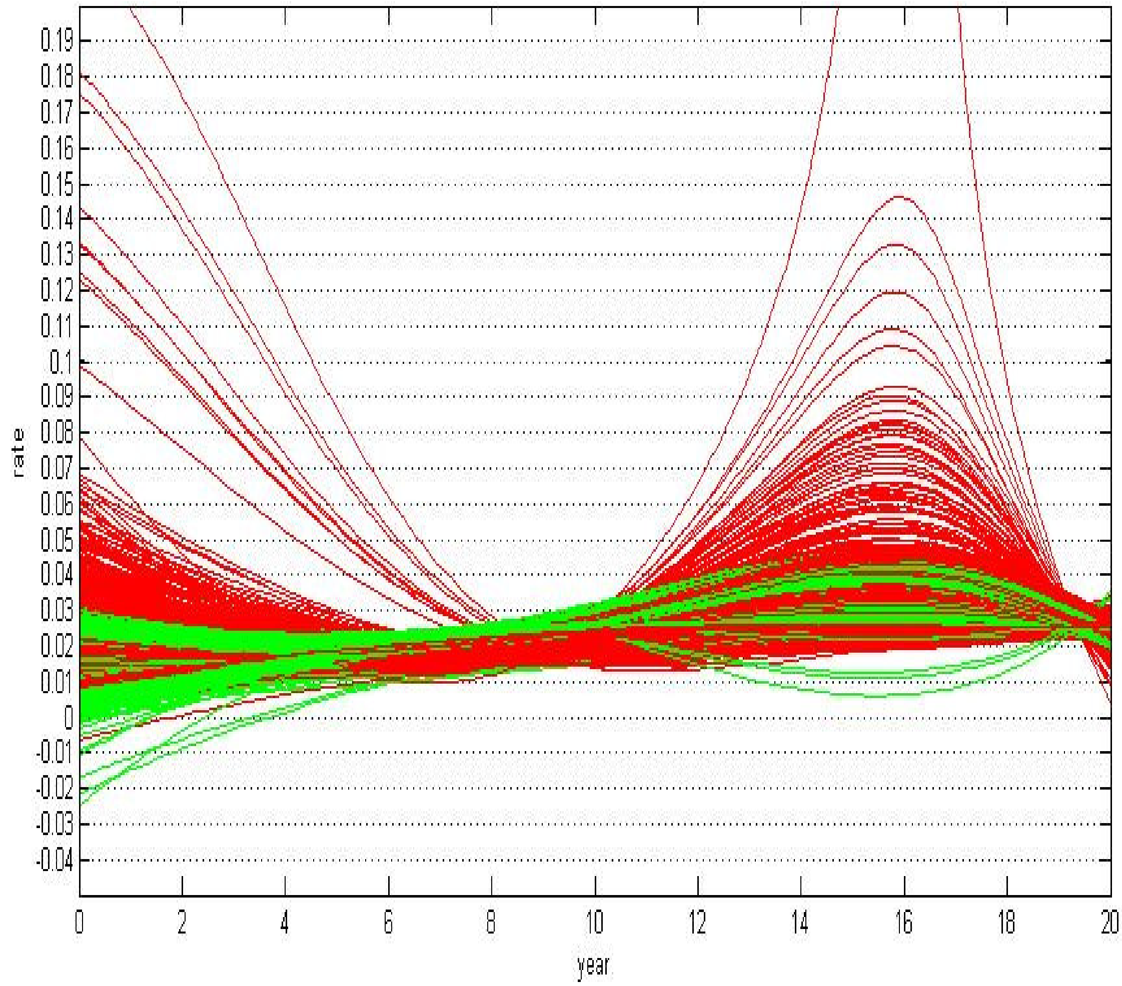

Figure 2 B-spline fitted yield curves, unweighted (light-colored), and liquidity weighted models (dark-colored)

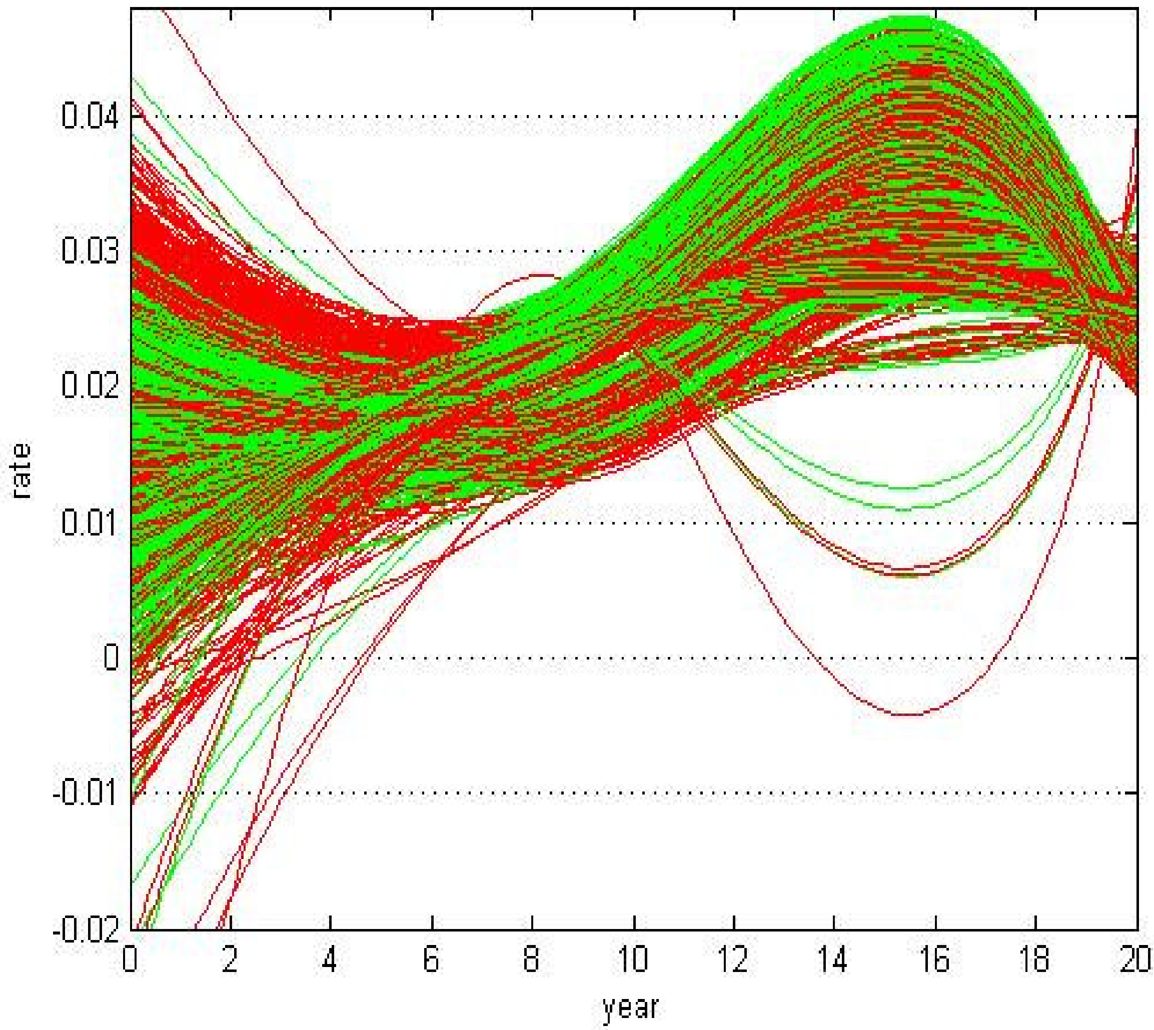

Figure 3 B-spline fitted yield curves, unweighted (light-colored), and liquidity/duration weighted (dark-colored)Models

TABLE 2

Summary Statistics of Forecasted Price Errors on Government Bond Issues

Fitted price errors are computed by subtracting forecasted prices from the actual traded prices. The forecasted prices are derived by applying parameters estimated with the B-Spline method to the cash flow data on the forecasted day. Liquidity weighted resulted are those based on parameters estimated with a liquidity-weighted objective function, whereas liquidity and duration weighted results are from weights comprised of both liquidity and duration.

Issues	Data Period*	Unweighted				Liquidity Weighted				Liquidity and Duration Weighted			
		Median	*Std. Dev.*	*Max.*	*Min.*	*Median*	*Std. Dev.*	*Max.*	*Min.*	*Median*	*Std. Dev.*	*Max.*	*Min.*
2-year													
A92109	03.11.05~04.01.05	0.4426	0.5901	1.0874	-1.0009	2.1620	1.1608	6.3221	-0.7324	0.8383	1.1286	2.0348	-1.6872
A93101	03.12.29~08.19.04	0.3074	0.7712	1.7802	-2.9009	0.8839	0.9484	5.5409	0.0497	0.7572	1.1766	2.5810	-5.7234
A93105	04.04.05~05.02.24	0.8975	0.7912	2.0629	-1.5918	0.8842	0.4149	2.3203	-0.3781	1.3102	1.1828	2.9670	-3.1111
A94101	04.12.28~05.03.16	0.1339	0.7911	1.5998	-1.2905	0.2501	0.3585	1.0931	-0.3836	0.2315	1.5363	2.7552	-2.7355
5-year													
A92106	03.07.03~03.09.01	0.1511	1.1013	2.3339	-2.3678	1.0459	1.0699	3.3415	-1.8024	0.3133	1.3742	2.4396	-3.0493
A92108	03.10.20~04.04.06	-0.6785	0.6228	0.3081	-2.7645	-0.5282	0.7829	0.2283	-4.4778	-0.6919	1.0522	1.1130	-3.8111
A93102	00.01.14~04.08.04	-0.5890	0.9515	1.3098	-3.9662	-1.0528	0.7442	0.2303	-3.3779	-0.5929	1.2600	2.0694	-5.5585
A93107	04.07.12~05.03.16	-0.3028	0.6433	1.2401	-2.0249	-0.7040	0.4218	0.2745	-1.6283	0.1687	1.1647	2.1157	-3.5876
A94102	05.01.12~05.03.16	-0.2484	0.6338	0.9376	-1.2924	0.0589	0.3141	0.8577	-0.4001	-0.2594	1.2896	1.5287	-2.9675
10-year													
A92104	03.11.05~04.01.29	1.4866	2.4151	5.8690	-3.5955	-1.4253	2.7873	3.1587	-7.6352	0.6503	2.0776	3.7349	-3.8663
A92107	03.09.08~04.02.05	-1.3794	0.8940	1.3171	-2.8652	-3.0966	1.2041	0.1010	-6.1199	-1.6447	0.9320	1.4637	-3.5284
A92110	03.11.25~04.09.06	-1.2546	0.9944	1.5444	-4.3761	-2.8938	1.7149	-0.1301	-7.5928	-1.8388	1.0452	1.0213	-4.5723
A93103	04.01.29~05.03.16	-1.7444	2.9575	6.9702	-9.5557	0.4828	4.3143	11.1500	-23.6870	-1.3581	2.6457	6.5382	-8.6387
A93108	04.09.03~05.03.16	1.1269	1.0391	4.4436	-1.2908	0.6596	1.2748	4.4659	-1.0828	0.7572	1.0116	3.9136	-1.9517

* Data period characterizes when the issue has active trading, where there are traded prices for consecutive days.

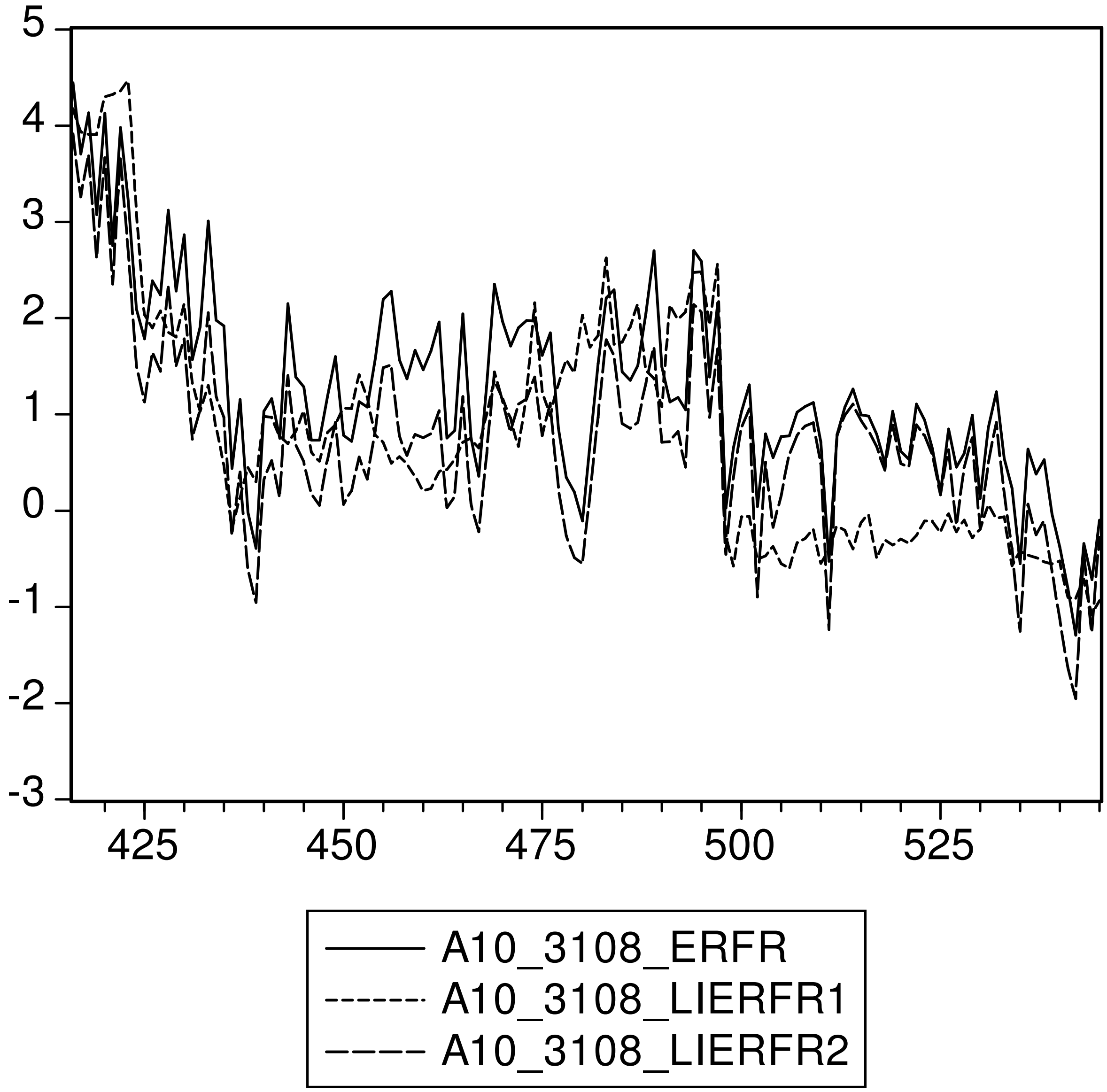

Figure 4 10-year issue forecasted price errors, unweighted (solid line) and two weighted schemes (liquidity weighted: short dashed line; liquidity and duration weighted: long dashed line)

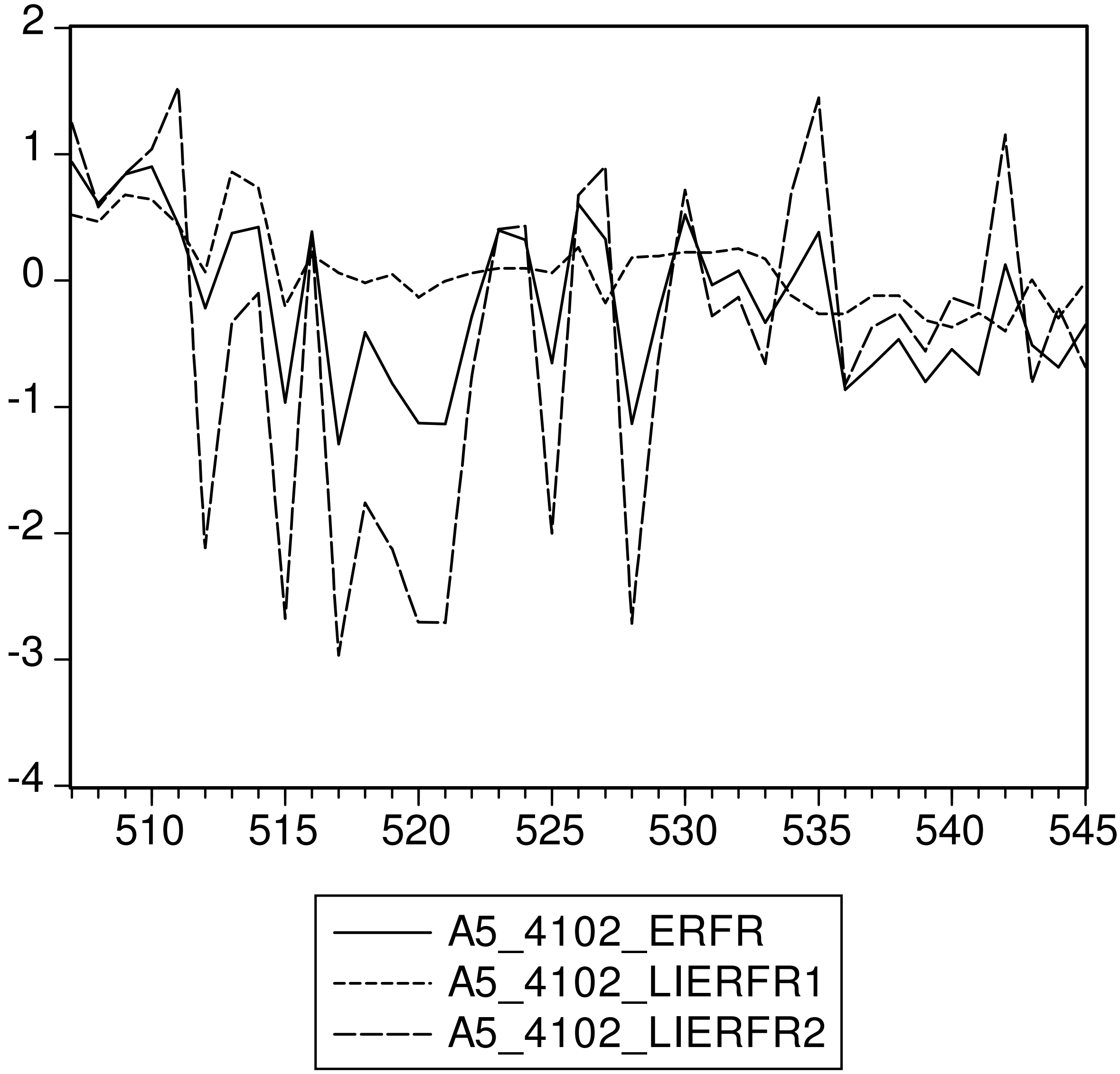

Figure 5 5-year issue forecasted price errors, unweighted (solid line) and two weighted schemes (liquidity weighted: short dashed line; liquidity and duration weighted: long dashed line)

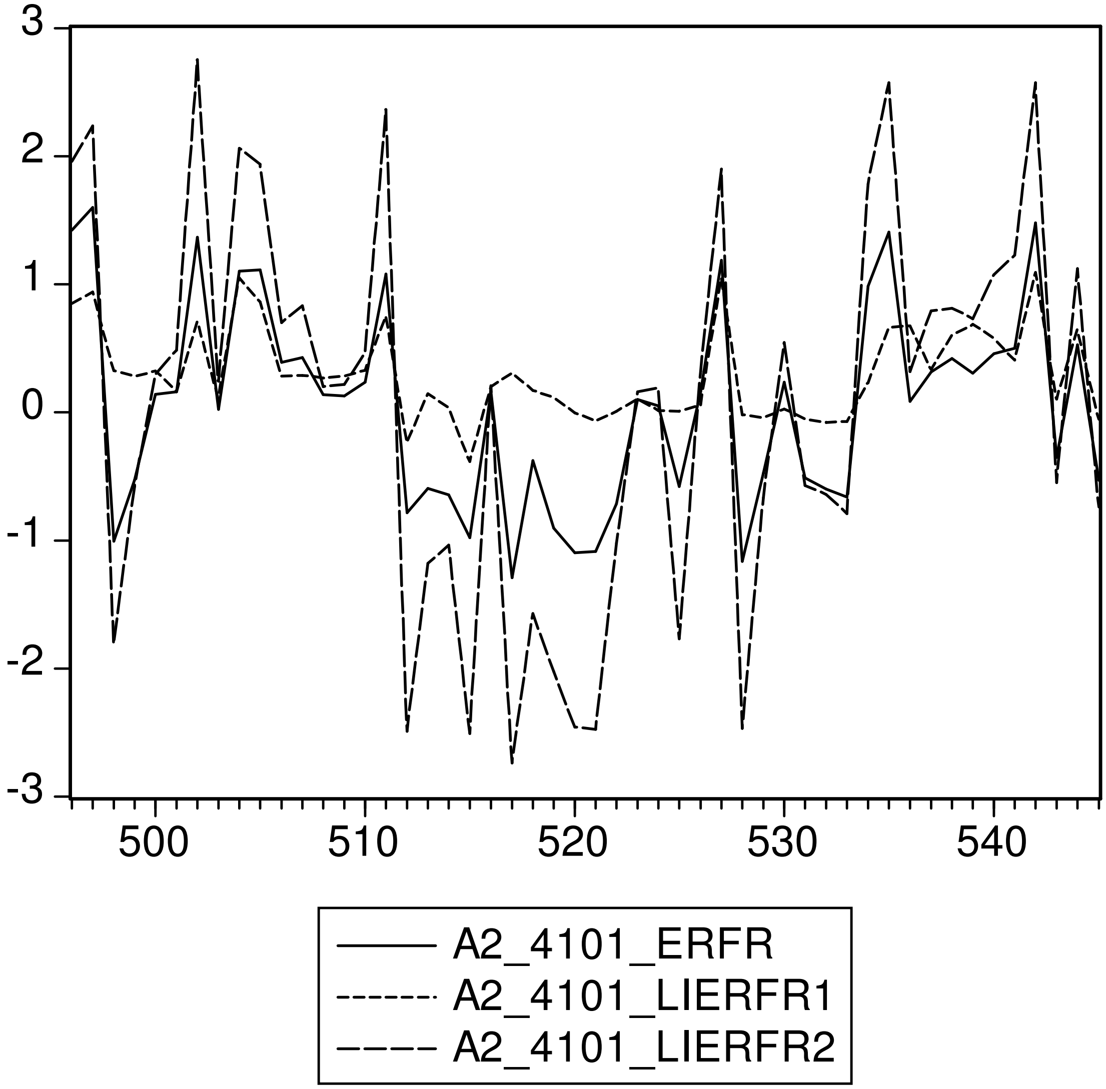

Figure 6 2-year issue forecasted price errors, unweighted (solid line) and two weighted schemes (liquidity weighted: short dashed line; liquidity and duration weighted: long dashed line)

TABLE 3

Comparisons of Explanatory Power of Forecast Price Errors,
Unweighted, Liquidity-Weighted and Liquidity-Duration-Weighted

All the forecasted price errors from the three models are filtered through an AR(2) model. The model is selected its overall performance across all issues and models according to the Akaike information criteria and Durbin-Watson statistics. The adjusted *R*-squared's of the filter are compared across forecasted errors from the three fitting models. Issues with limited number of consecutive trading days are excluded for the reliability of comparisons.

Issue Code	Adjusted *R*-squared		
	Unweighted	*Liquidity Weighted*	*Liquidity and Duration Weighted*
2-year			
A93101	0.3601	0.5702	0.1310
A93105	0.5396	0.7202	0.4034
5-year			
A93102	0.5460	0.7968	0.3167
A93107	0.1340	0.8558	0.1668
A94102	0.0507	0.3795	0.0245
10-year			
A92104	0.8363	0.9456	0.8382
A92107	0.3904	0.6096	0.4611
A92110	0.5849	0.8281	0.6944
A93103	0.7201	0.8625	0.7064
A93108	0.5600	0.8805	0.5175

TABLE 4

Autocorrelation of Fitted Price Errors (Base Model)

Price errors fitted from the unweighted scheme are filtered through an AR(2) mode, as the Base Model. The Base Model is specified as

$$e_t^i = \alpha^i + \beta_1^i e_{t-1}^i + \beta_2^i e_{t-2}^i + \varepsilon_t^i$$

with i denoting the ith bond issue.

Issue Code	Lag 1	Lag2	Adj. R-squared
2-year			
A92109	0.4132 (0.1516)**	0.2960 (0.1468)**	0.4294
A93101	0.3140 (0.0870)**	0.3274 (0.0872)**	0.2774
A93105	0.4762 (0.0736)**	0.4107 (0.0737)*	0.6672
5-year			
A92108	0.6240 (0.1983)**	0.2246 (0.1908)**	0.3702
A93102	0.6053 (0.0819)**	0.3489 (0.0820)**	0.6542
A93107	0.4880 (0.1595)**	0.3601 (0.1551)*	0.0529
10-year			
A92110	0.7125 (0.0780)**	0.2780 (0.0781)	0.6769

* Significant at the 5% level.
** Significant at the 1% level.

TABLE 5

Informational Role of Liquidity in Fitted Price Errors, Control Model with Near-by On-the-run Issues

Price errors fitted from the unweighted scheme are filtered through an AR(2) mode, as the Basic Model. Fitted price errors from the unweighted and two weighted models are put in the Control Model with the control variable as the 10-year on-the-run issue when the issue in interest is issued. When a subsequent 5- or 10-year on-the-run issue is out, the control variable is changed to the new on-the-run issue in a different regression. The Control Model is specified as

$$e_t^i = \alpha^i + \beta_1^i e_{t-1}^i + \beta_2^i e_{t-2}^i + \gamma^i e_{10,t}^i + \varepsilon_t^i ,$$

where $e_{10,t}^i$ is the price error concurrent on-the-run 10-year issues.

Issue Code	Model	Control Variable[a]	Lag 1	Lag2
2-year				
A92109[b]	*Unweighted*	0.1259 (0.0697)	0.4145 (0.1475)**	0.2630 (0.1440)
	Liq. Weighted	-0.3691 (0.1248)**	0.4097 (0.1648)*	0.2970 (0.1418)**
	Liq./Dur. Weighted	0.2916 (0.0912)**	0.3154 (0.1256)*	0.2854 (0.1228)*
A93101[c]	*Unweighted*	0.0669 (0.0287)*	0.2916 (0.0860)**	0.3055 (0.0861)**
	Liq. Weighted	-0.0204 (0.0236)	0.4789 (0.0806)**	0.4331 (0.0798)**
	Liq./Dur. Weighted	0.2090 (0.0484)**	0.2295 (0.0830)**	0.2406 (0.0831)**
A93105[d]	*Unweighted*	0.1098 (0.0280)**	0.4746 (0.0704)**	0.4272 (0.0706)**
	Liq. Weighted	0.0103 (0.0087)	0.5333 (0.0727)**	0.4548 (0.0736)**
	Liq./Dur. Weighted	0.3584 (0.0387)**	0.4069 (0.0591)**	0.3912 (0.0586)**
5-year				
A92108[e]	*Unweighted*	0.1844 (0.1245)	0.5671 (0.1971)**	0.1366 (0.1953)
	Liq. Weighted	0.0462 (0.0297)	0.5489 (0.1188)**	0.2991 (0.1162)**
	Liq./Dur. Weighted	0.2515 (0.1696)	0.1866 (0.0934)	0.2346 (0.1984)
A92108[f]	*Unweighted*	0.1416 (0.0566)*	0.3166 (0.1384)*	0.4627 (0.1333)**
	Liq. Weighted	0.0175 (0. 0119)	0.7415 (0.1144)**	0.1996 (0.1141)
	Liq./Dur. Weighted	0.2586 (0.0717)**	0.1635 (0.1220)	0.3170(0.1152)**
A93102[g]	*Unweighted*	0.0439 (0.0197)*	0.5445(0.0852)**	0.2712(0.0880)**
	Liq. Weighted	-0.0100 (0.0066)	0.7179 (0.0823)**	0.2922 (0.0832)**
	Liq./Dur. Weighted	0.0768 (0.0287)**	0.0412 (0.0863)**	0.2476 (0.0872)**
A93107[g]	*Unweighted*	0.1001 (0.0443)*	0.3470 (0.1620)*	0.2028 (0.1605)
	Liq. Weighted	0.0515 (0.0518)	0.9227 (0.1733)**	-0.0278 (0.1668)
	Liq./Dur. Weighted	0.0846 (0.0441)	0.0831 (0.1712)	0.0043 (0.1723)
10-year				
A92110[g]	*Unweighted*	0.0664 (0.0205)**	0.6470 (0.0783)**	0.2334 (0.0770)**
	Liq. Weighted	0.0126 (0.0097)	0.9362 (0.0818)**	0.0548 (0.0816)
	Liq./Dur. Weighted	0.0655 (0.0222)**	0.6340 (0.0777**	0.2808 (0.0768)**

* Significant at the 5% level.
** Significant at the 1% level.
[a] The Major Index for each issue is the most recent on-the-run 5- or 10-year issue.
[b] A92108, issued on 03.10.30 is the *Major Index* for these issues.
[c] A93102, issued on 04.01.30 is the *Major Index* for these issues.
[d] A93107, issued on 04.07.22 is the *Major Index* for these issues.
[e] A92107, issued on 03.12.05 is the *Major Index* for these issues.
[f] A92110, issued on 03.10.19 is the *Major Index* for these issues.
[g] A93103, issued on 04.02.10 is the *Major Index* for these issues.

TABLE 6

Vector Error Correction Cointegration Analysis, 2-, 5- and 10-year on-the run issues and 10-year off-the-run issues

Price errors from Liquidity-weighted B-Spline models are used in the cointegration analysis as unit roots are detected. Coefficients for cointegration equations from Vector Error Correction models are reported with each of the three on-the-run issues as the reference variable with an coefficient of 1. The model is conducted with 2 lags and with an intercept in the estimation of the cointegration equation.

Cointegrating Variable	10-year on-the run	5-year on-the run	2-year on-the run
2-year			
A93105	-0.0048 (0.1639)	0.1988 (6.8275)	
5-year			
A93107	-0.0241 (0.3546)		5.0291(84.2043)
10-year			
A92110	-0.3497 (0.1350)**	14.5122 (6.1796)**	72.9836 (34.3293)**
A93103	0.5823 (0.1555)**	-24.1616 (5.5232)**	-121.5114 (32.4746)**
A93108		-41.4947 (8.3164)**	-208.6811 (47.4212)**

* Significant at the 5% level.
** Significant at the 1% level.

Footnotes

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[1] See Darbha (2004) and Diaz, *et al.* (2006), among others.

[2] By long term approach, we mean one studying daily term structure over a certain period of time, as what is adopted in Diebold and Li (2006). With the analysis on the dynamic patterns of fitting performance, we are able to identify how a certain fitting method can be consistent with long term equilibrium econometrically without resulting unpredictable instability.

[3] Liu, *et al.* (2004) included repo yield in their estimation to reflect the possibility that liquid bonds may trade special in the repo market. So the liquidity process γ_t can be viewed as a direct measure of the specialness of Treasury bonds relative to the repo rate.

[4] As there are few riskless fixed income securities in Taiwan, off-the-run treasury bonds would not contain the so-called 'flight-to-liquidity' premium as in Longstaff (2004).

[5] As noted earlier, B-Spline cannot fit the short end well especially when there are few short term issues. So in Figure 1 the liquidity-weighted curve is supposed to be above the real curve. This can be seen by the fact that all the liquid short yields are below the liquidity-weighted, with and without VRP, curves.

[6] We have also examined autoregression of other orders with the Akaike Information Criterion (AIC). Although an AR(3) process is also supported with slightly smaller AIC, we choose an AR(2) for better comparisons, considering especially issues with small number of observations.

[7] The magnitudes of the autocorrelation coefficients based on part of each time series can be referred to Table 4, where the truncation is done to match observations of a regressor used in Table 5. The coefficients for the entire series of each issue are not reported in Table 3 as the focus of our discussion is not on the magnitudes of them. We have only included issues with meaningful results in the discussion here for the clarity of our analysis.

[8] We have conducted alternative analysis with fitted price errors of an on-the-run issue as dependent variable, while an off-the-run variable is modeled as the control variable. We are not able to obtain consistently significant coefficients.

[9] The long term equilibrium is in an econometric sense within the system of the three types of issues included in our analysis.