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# The Arm's Length Principle and Tacit Collusion\*

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## Abstract

The arm's length principle states that the transfer price between two associated enterprises should be the price that would be paid for similar goods in similar circumstances by unrelated parties dealing at arm's length with each other. This paper examines the effect of the arm's length principle on dynamic competition in imperfectly competitive markets. It is shown that the arm's length principle renders tacit collusion more stable. This is true whether firms have exclusive dealings with unrelated parties or compete for the demand from unrelated parties.

**JEL classification numbers:** D43, L13, L41

**Key words:** Transfer price, arm's length principle, tacit collusion, stability of collusion.

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# 1 Introduction

Transfer prices are often used to record intra-firm transactions among various divisions within the firm. On one hand, they generate information for measuring divisional performance and provide the basis for management accounting. On the other hand, they affect the firm's overall tax liability when divisions operate in multiple tax jurisdictions. These twin roles of transfer pricing have been the focus of much research.<sup>1</sup> When some divisions of a firm face competition, research has further shown that transfer prices can also serve strategic purposes (Alles and Datar, 1998; Zhao, 2000; Arya and Mittendorf, 2008). Typical reasoning for the strategic use of transfer pricing relies on the firm's ability to exercise price discrimination: by charging transfer price to its own affiliate that is different from what the firm would charge unrelated buyers, the firm is able to improve the affiliate's competitive posture against the unrelated competitors.

In reality, the firm's ability to exercise such price discrimination may be limited by the so-called arm's length principle (ALP, henceforth). The ALP, set out in Article 9 of the OECD Model Tax Convention, states that the transfer price between two associated enterprises should be the price that would be paid for similar goods in similar circumstances by unrelated parties dealing at arm's length with each other. The ALP is adopted by most countries around the world with only a few exceptions. In applying the ALP, most jurisdictions consider a comparable uncontrolled price to be the most reliable indicator of an arm's length price, and the failure to comply with the ALP may result in a penalty.<sup>2</sup> The main reason for the ALP is given in OECD (2009, p. 27): "[...] to provide broad parity of tax treatment for multinational enterprises and independent enterprises. Because the ALP puts associated and independent enterprises on a more equal footing for tax purposes, it avoids the creation of tax advantages that would otherwise distort the relative competitive positions of either type of entity."

The purpose of this paper is to examine how the ALP changes the nature of dynamic competition in imperfectly competitive markets. Our main point is that the ALP renders tacit collusion between firms more stable than when firms are not subject to the ALP.<sup>3</sup> To understand why, suppose there are two upstream firms each with its own downstream affil-

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<sup>1</sup>Examples of studies on the role of transfer prices for management accounting are Holmström and Tirole (1991) and Anctil and Dutta (1999). Tax implications of transfer pricing are examined, for example, in Gordon and Wilson (1986), Kant (1990), and Goolsbee and Maydew (2000). Studies that consider both roles of transfer pricing include Elitzur and Mintz (1996), Sansing (1999), Smith (2002), Baldenius et al. (2004), Hyde and Choe (2005), and Choe and Hyde (2007).

<sup>2</sup>See Choe and Hyde (2007) for discussions of the penalty and how the firm needs to factor this into account in its transfer pricing policy.

<sup>3</sup>Shor and Chen (2009) show that multi-divisional firms can use transfer prices and decentralized organizational structure to coordinate onto tacit collusion. But they do not consider the effect of the ALP.

iate and some unrelated downstream buyers. Upstream firms supply intermediate goods to downstream buyers and we take the ALP to require that the price charged by an upstream firm to unrelated buyers be equal to the upstream firm's internal transfer price. Thus the ALP ensures a level-playing field for the upstream firms' affiliates and unrelated buyers. In the absence of the ALP, it is optimal for the upstream firms to exercise price discrimination to foreclose all unrelated downstream buyers, which transfers the upstream duopoly to the downstream market. When upstream firms follow the ALP, however, they cannot foreclose unrelated downstream buyers. This makes downstream competition tougher than without the ALP, which in turn makes upstream competition tougher, and hence tacit collusion more attractive. Moreover, total surplus under collusion is lower under the ALP, primarily because the collusive wholesale price is higher under the ALP. Once again this is due to the fact that the ALP compels upstream firms not to foreclose unrelated buyers. Our findings suggest that, while the ALP can ensure the level-playing field for multinationals' affiliates and unrelated enterprises in a static setting, it can render tacit collusion more attractive in a dynamic setting, which can further reduce welfare.

Two recent studies have also argued that adherence to the ALP may lead to distortions that could reduce overall welfare. Devereux and Keuschnigg (2009) consider a model in which multinationals procure intermediate goods offshore and assemble final goods in home country. They show that application of the ALP can distort a multinational's organizational choice between foreign direct investment and outsourcing. Specifically, the ALP can reduce financial strength of a multinational's foreign affiliates, which can lead the multinational to choose outsourcing over foreign direct investment even if the latter can be more valuable. Behrens et al. (2010) consider a model where a firm sells final goods in a foreign market either by establishing its own foreign subsidiary or through contracting with independent distributors at arm's length. They show that the independent arm's length relationship generates inefficiency because of double marginalization and that application of the ALP weakens the firm's incentives to establish its own foreign subsidiary. In sum, these studies show how the ALP can affect the firm's choice of organizational structure, which can have efficiency implications. Our paper complements these studies by highlighting the potentially anti-competitive effect of the ALP in a dynamic setting.

The rest of this paper is organized as follows. Section 2 describes the basic model with two upstream firms where each upstream firm deals with an exclusive set of unrelated downstream buyers. The basic model is analyzed in Section 3, where it is shown that tacit collusion is more stable under the ALP than without it. Section 4 considers the case where upstream firms compete for unrelated downstream buyers and shows that the ALP continues to render more stability to tacit collusion. Section 5 concludes the paper.

## 2 The Model

Consider a vertically related market with two upstream firms indexed  $i = 1, 2$ . Each upstream firm produces an intermediate good at constant marginal cost  $c$ , which it supplies to its downstream affiliate and other unrelated buyers in the downstream market. We assume that one unit of intermediate good is converted to one unit of final good at no further cost,<sup>4</sup> each upstream firm deals with at most  $n - 1$  unrelated downstream buyers, and competition in the downstream market is à la Cournot. These assumptions allow us to address the issue of tacit collusion in the most parsimonious way. We index downstream firms by  $j$  whereby upstream firm 1 supplies to downstream firm  $j = 1$ , its own affiliate, and downstream firms  $j = 2, \dots, n$ , unrelated buyers. Upstream firm 2 supplies to downstream firm  $j = n + 1$ , its own affiliate, and unrelated downstream buyers  $j = n + 2, \dots, 2n$ .<sup>5</sup> We denote the set of downstream firms by  $N = \{1, \dots, 2n\}$ . The downstream market's (inverse) demand is given by  $p = a - Y$  with  $a > c$  where  $Y := \sum_{j \in N} y_j$  and  $y_j$  is the quantity purchased by downstream firm  $j$ . The headquarters of each vertical structure is located upstream and maximizes the total profit from its supply chain while the downstream affiliate maximizes its own profit. That the downstream affiliate maximizes its own profit is standard in the literature on transfer pricing; otherwise, the vertical structure is centralized and the downstream affiliate does not play any meaningful role.

For our discussion of tacit collusion, we consider an infinitely repeated game where  $\delta$  denotes the discount factor between periods. We examine the effect of  $\delta$  on the stability of the collusion between the upstream firms on transfer prices. Along the punishment path, the firms are assumed to use the grim trigger strategy as in Friedman (1971).<sup>6</sup> In each period, the two upstream firms independently and simultaneously set prices for their own affiliates and the unrelated buyers, which is followed by the downstream firms' choice of quantity. Let  $w_j$  be the price downstream firm  $j$  pays the relevant upstream firm. Thus  $w_1$  is upstream firm 1's internal transfer price for its own affiliate and  $w_j$  is the price it charges unrelated buyer  $j = 2, \dots, n$ . Similarly  $w_{n+1}$  is upstream firm 2's internal transfer price and  $w_j$  is the price it charges unrelated buyer  $j = n + 2, \dots, 2n$ .

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<sup>4</sup>Adding additional costs does not make any changes to our main point as long as they are linear in quantity. Also we do not explicitly consider tax. But it is easy to see that our results continue to hold insofar as both upstream firms face the same tax rate.

<sup>5</sup>In our main model, we consider the case where each upstream firm has exclusive dealings with  $n - 1$  additional unrelated buyers. In Section 4, we discuss the case where upstream firms cannot exclude any unrelated buyer.

<sup>6</sup>Although this punishment strategy is not optimal (Abreu, 1988), we use it for simplicity and tractability. Indeed many previous studies have adopted the grim trigger strategy when analyzing stability of agreements. See, among others, Deneckere (1983), Chang (1991), Lambertini et al. (1998), and Maggi (1999).

Given prices  $(w_1, \dots, w_{2n})$ , downstream firm  $j$  chooses  $y_j$  to maximize profit<sup>7</sup>

$$\pi_j := (p - w_j)y_j = \left(a - \sum_{k \in N} y_k - w_j\right)y_j. \quad (1)$$

Given the quantities  $(y_1, \dots, y_{2n})$  determined above, upstream firm 1 chooses  $(w_1, \dots, w_n)$  to maximize its consolidated profit

$$\Pi_1 := (p - c)y_1 + \sum_{k=2}^n (w_k - c)y_k, \quad (2)$$

and upstream firm 2 chooses  $(w_{n+1}, \dots, w_{2n})$  to maximize its consolidated profit

$$\Pi_2 := (p - c)y_{n+1} + \sum_{k=n+2}^{2n} (w_k - c)y_k. \quad (3)$$

Note that the internal transfer prices disappear in the above consolidated profits since they merely track internal transactions between the two affiliated entities and, therefore, cancel out upon consolidation.

As usual, we solve the game backwards. First, given  $(w_1, \dots, w_{2n})$ , we find Cournot equilibrium quantities in the downstream market denoted by  $(y_1^*, \dots, y_{2n}^*)$ , and the equilibrium price denoted by  $p^*$ . Let  $A$  be the set of downstream firms that are active in equilibrium. That is,  $A := \{j \in N \mid y_j^* > 0 \text{ given } (w_1, \dots, w_{2n})\}$ . Let  $n_A$  be the cardinality of  $A$ . Then it is easy to see that, for all  $j \in A$ , we have

$$y_j^* = \frac{a + \sum_{k \in A} w_k - (n_A + 1)w_j}{n_A + 1}, \quad p^* = \frac{a + \sum_{k \in A} w_k}{n_A + 1}. \quad (4)$$

Next we solve for each upstream firm's optimal prices given the above Cournot equilibrium. However, the details of its pricing decision depend on whether or not it complies with the ALP. If an upstream firm complies with the ALP, then its optimization problem is subject to the constraint that it cannot charge different prices to unrelated buyers and its own affiliate. Of course, compliance with the ALP is the firm's choice in that violation of the principle can result in a penalty that depends on the magnitude of non-compliance (Choe and Hyde, 2007). Since our main focus is on the effect of the ALP on tacit collusion, however, we do not model this choice explicitly but focus on the two polar cases of compliance vis-à-vis

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<sup>7</sup>As is typical in the transfer pricing literature (Arya and Mittendorf, 2008), we assume that downstream firms observe all internal transfer prices before choosing their quantities.

non-compliance. Adding the penalty for non-compliance will complicate analysis at no additional insight because the firm's decision to collude or deviate will depend sensitively on the way the penalty is modelled.

### 3 Tacit Collusion and the ALP

In this section we analyze the stability of tacit collusion with or without the ALP. Since firms are assumed to use the grim trigger strategy, three payoffs are relevant in each period: the profit under tacit collusion superscripted by 'C', the profit from deviation superscripted by 'D', and the profit during the competitive phase superscripted by 'E'.

#### 3.1 Outcomes under the ALP

In applying the ALP, most jurisdictions consider a comparable uncontrolled price to be the most reliable indicator of an arm's length price. Since the comparable uncontrolled price in our setting is the price the upstream firm charges unrelated buyers, we have, under the ALP,  $w_1 = w_j$  for all  $j = 2, \dots, n$  and  $w_{n+1} = w_k$  for all  $k = n+2, \dots, 2n$ . That all unrelated buyers for each upstream firm face the same price is due to symmetry. Then we have  $A = N$  in Cournot equilibrium and, from (??),  $y_1^* = y_j^* = [a + nw_{n+1} - (n+1)w_1]/(2n+1)$  for all  $j = 2, \dots, n$ ,  $y_{n+1}^* = y_k^* = [a + nw_1 - (n+1)w_{n+1}]/(2n+1)$  for all  $k = n+2, \dots, 2n$ , and  $p^* = [a + n(w_1 + w_{n+1})]/(2n+1)$ . Using the above and (??), we can derive upstream firm 1's reaction function:

$$w_1 = \frac{(2n^2 - n - 2)a + n(n+1)(2n+1)c}{2(n+1)(2n^2 - 1)} + \frac{n(2n^2 - n - 2)w_{n+1}}{2(n+1)(2n^2 - 1)}. \quad (5)$$

As shown in (??) and noting that  $n \geq 2$ , prices are strategic complements in this case since firm cannot exclude unrelated buyers when they comply with the ALP. Because firms are symmetric, we focus, without loss of generality, on upstream firm 1's problem and derive its consolidated profit under collusion denoted by  $\Pi_1^C$ , its consolidated profit from deviation denoted by  $\Pi_1^D$ , and its consolidated profit from competition denoted by  $\Pi_1^E$ .

First, the collusive price  $w^C := w_1 = w_{n+1}$  maximizes joint profit  $\Pi_1 + \Pi_2$ , leading to

$$w^C = \frac{(2n^2 + n - 2)a + n(2n+1)c}{2(2n^2 + n - 1)}.$$

The resulting profit from collusion, the profit of each unrelated firm, and the consumer surplus are

$$\Pi_1^C = \frac{n^2(a-c)^2}{4(n+1)(2n-1)}, \quad \pi_j = \frac{n^2(a-c)^2}{4(n+1)^2(2n-1)^2}, \quad CS^C = \frac{n^4(a-c)^2}{2(n+1)^2(2n-1)^2}. \quad (6)$$

Second, we find upstream firm 1's optimal deviation. Given  $w_1$  and  $w_{n+1} = w^C$ , the quantity purchased by each downstream firm that deals with upstream firm 2 is

$$y_{n+1}^* = \dots = y_{2n}^* = \frac{2n(2n-1)w_1 - n(2n-3)a - n(2n+1)c}{2(2n-1)(2n+1)}.$$

We divide analysis into two cases. First, if upstream firm 1's choice of  $w_1$  is such that  $y_j^* > 0$  for all  $j \geq n+1$ , then we have  $A = N$ . The required condition in this case is  $w_1 > [(2n-3)a + (2n+1)c]/(4n-2)$ . Then upstream firm 1's optimal deviation can be found from (??):

$$w_1^D = \frac{(2n^2 - n - 2)(2n^3 + 5n^2 - 2)a + n(2n+1)(6n^3 + 5n^2 - 2n - 2)c}{4(n+1)^2(2n^2 - 1)(2n-1)}.$$

However, one can check that, given the above  $w_1^D$ , we have  $y_j^* \leq 0$  for all  $j \geq n+1$  except when  $n = 2$ . Thus upstream firm 1's optimal deviation that retains the upstream duopoly, hence  $A = N$ , is possible only when  $n = 2$ . In this case, we have  $w_1^D = (34a + 155c)/189$ . Second, if upstream firm 1's choice of  $w_1$  is such that  $y_j^* \leq 0$  for all  $j \geq n+1$ , then we have  $A = \{1, \dots, n\}$ . Thus upstream firm 1's deviation in this case results in all the downstream firms that deal with upstream firm 2 shutting down. The condition required for this case is  $w_1 \leq [(2n-3)a + (2n+1)c]/(4n-2)$ . Then we have  $y_1^* = \dots = y_n^* = (a - w_1)/(n+1)$ . Plugging these into upstream firm 1's consolidated profit (??), one can show that the profit is strictly increasing in  $w_1$  for all  $w_1 \leq [(2n-3)a + (2n+1)c]/(4n-2)$ . Therefore upstream firm 1's optimal deviation in this case is obtained at the corner:  $w_1^D = [(2n-3)a + (2n+1)c]/(4n-2)$ . Summarizing the above, upstream firm 1's optimal deviation retains the upstream duopoly when  $n = 2$  but it monopolizes the upstream market when  $n \geq 3$ . Putting these together, upstream firm 1's optimal deviation is given by

$$w_1^D = \begin{cases} \frac{34a + 155c}{189}, & n = 2, \\ \frac{(2n-3)a + (2n+1)c}{4n-2}, & n \geq 3. \end{cases}$$

The resulting profit from deviation is

$$\Pi_1^D = \begin{cases} \frac{289(a-c)^2}{1701}, & n = 2, \\ \frac{(2n+1)(2n^3 - n^2 - n + 4)(a-c)^2}{4(n+1)^2(2n-1)^2}, & n \geq 3. \end{cases} \quad (7)$$



Third, when firms return to the competitive phase, Nash equilibrium price can be calculated from (??):

$$w^E := \frac{(2n^2 - n - 2)a + n(n+1)(2n+1)c}{2n^3 + 5n^2 - 2}.$$

This leads to the profit from competition:

$$\Pi_1^E = \frac{n^2(n+1)(2n^2-1)(a-c)^2}{(2n^3+5n^2-2)^2}. \quad (8)$$

### 3.2 Outcomes without the ALP

When upstream firms are not subject to the ALP, it is optimal for them to use price discrimination to exclude unrelated buyers and supply only to their affiliates, hence  $A = \{1, n+1\}$ . When unrelated buyers are excluded, downstream firm  $j$ 's Cournot quantity is  $y_j^* = (a + w_i - 2w_j)/3$ ,  $i \neq j$  and upstream firm 1's consolidated profit is  $\Pi_1 = (a - y_1^* - y_{n+1}^* - c)y_1^*$ . From this, upstream firm 1's reaction function is given by

$$w_1 = \frac{6c - a - w_{n+1}}{4}. \quad (9)$$

Prices in this case are strategic substitutes as shown in (??). When unrelated buyers are excluded, downstream Cournot competition is translated into upstream price competition. This is the strategic effect of transfer price discussed, for example, in Zhao (2000), and Arya and Mittendorf (2008). As before, we focus on upstream firm 1's problem and derive its consolidated profit under collusion denoted by  $\hat{\Pi}_1^C$ , its consolidated profit from deviation denoted by  $\hat{\Pi}_1^D$ , and its consolidated profit from competition denoted by  $\hat{\Pi}_1^E$ .

First, the collusive price  $\hat{w}^C := w_1 = w_{n+1}$  maximizes joint profit  $\Pi_1 + \Pi_2$ , which leads to  $\hat{w}^C = (a + 3c)/4$ . The resulting profit and the consumer surplus are

$$\hat{\Pi}_1^C = \frac{(a-c)^2}{8}, \quad \hat{CS}^C = \frac{(a-c)^2}{8}. \quad (10)$$

Second, given  $w_{n+1} = \hat{w}^C = (a + 3c)/4$ , upstream firm 1's optimal deviation can be derived from (??),  $\hat{w}_1^D := (21c - 5a)/16$ , resulting in

$$\hat{\Pi}_1^D = \frac{25(a-c)^2}{128}. \quad (11)$$

Third, when upstream firms return to the competitive phase, we can again use (??) to

solve for equilibrium transfer prices,  $\hat{w}^E := w_1 = w_{n+1} = (6c - a)/5$ . This leads to

$$\hat{\Pi}_1^E = \frac{2(a - c)^2}{25}. \quad (12)$$

### 3.3 Stability of tacit collusion

Following the literature on tacit collusion, we measure the stability of collusion by the minimum discount factor that sustains collusion. Since firms are assumed to use the grim trigger strategy, any deviation leads the game to the competitive phase perpetually thereafter. Thus tacit collusion is sustainable by the grim trigger strategy if and only if the discounted payoff from perpetual collusion is not smaller than the sum of one-off payoff from deviation and the discounted payoff from competition thereafter. Under the ALP, this condition is

$$\frac{\Pi_1^C}{1 - \delta} \geq \Pi_1^D + \frac{\delta \Pi_1^E}{1 - \delta}. \quad (13)$$

Let  $\delta^*$  be the value of  $\delta$  that satisfies (13) with equality. That is,  $\delta^* = (\Pi_1^D - \Pi_1^C)/(\Pi_1^D - \Pi_1^E)$ . Then tacit collusion is sustainable under the ALP for all  $\delta \geq \delta^*$ . From (13), (14), and (15), we have

$$\delta^* = \begin{cases} \frac{289}{478}, & n = 2, \\ \frac{(2n^3 + 5n^2 - 2)(2n^4 - n^3 - 2n^2 + n + 1)}{16n^{10} + 48n^9 + 24n^8 - 80n^7 - 59n^6 + 81n^5 + 65n^4 - 32n^3 - 28n^2 + 4n + 4}, & n \geq 3. \end{cases} \quad (14)$$

Similarly, let  $\hat{\delta}^* = (\hat{\Pi}_1^D - \hat{\Pi}_1^C)/(\hat{\Pi}_1^D - \hat{\Pi}_1^E)$  be the minimum discount factor that sustains tacit collusion without the ALP. From (13), (14), and (15), we have

$$\hat{\delta}^* = \frac{25}{41}. \quad (15)$$

Based on (13) and (14), one can show that  $\hat{\delta}^* > \delta^*$  if and only if  $n \neq 3$ . Moreover when  $n \geq 3$ , it can be shown that  $\delta^*$  decreases monotonically as  $n$  increases. We summarize the main result from this section below.

*Proposition 1: Suppose each upstream firm supplies to its own downstream affiliate and  $n - 1$  additional unrelated downstream buyers that have exclusive dealings with each upstream firm. Then tacit collusion is more stable with the ALP than without it in all cases except when  $n = 3$ . Moreover tacit collusion under the ALP becomes more stable as the*

*number of unrelated downstream buyers increases.*

We offer some intuition behind the above proposition. When upstream firms do not comply with the ALP, they can foreclose all unrelated downstream buyers and enjoy duopoly profits. This is independent of the number of unrelated downstream buyers. Compliance with the ALP changes this, however. When upstream firms adhere to the ALP, they cannot exclude unrelated downstream buyers. As the number of unrelated buyers increases, downstream competition intensifies, which is translated into tougher upstream competition. It is easy to verify that the wholesale price from competition,  $w^E$ , converges to  $c$  as  $n$  increases. Thus as the number of unrelated downstream buyers increases, equilibrium profit decreases, which makes collusion more attractive. Indeed it is routine to check from (??) and (??) that  $\Pi_1^C$  increases in  $n$  and  $\Pi_1^E$  decreases in  $n$ . Consequently, the critical discount factor  $\delta^* = (\Pi_1^D - \Pi_1^C)/(\Pi_1^D - \Pi_1^E)$  decreases in  $n$ . The situation is illustrated in Figure 1.

— **Figure 1 goes about here.** —

Before we close this section, we briefly discuss the effect of the ALP on welfare when collusion is sustained. We measure welfare as total surplus, which is the sum of all firms' profits and consumer surplus. Under the ALP, it can be calculated from (??) and, without the ALP, from (??). It is easy to see that both consumer surplus and total surplus are larger without the ALP. The main reason for this is that the collusive wholesale price is higher under the ALP, as can be verified by comparing  $w^C$  and  $\hat{w}^C$ . When upstream firms collude under the ALP, they continue to supply to unrelated buyers and, therefore, they do not want to lower the collusive wholesale price too much as it also benefits unrelated buyers. Without the ALP, they do not face such a problem since they can foreclose all unrelated buyers. Figure 2 shows total surplus and consumer surplus under collusion in the two cases.

— **Figure 2 goes about here.** —

## 4 When Unrelated Buyers Cannot Be Excluded

So far we have assumed that the relationship between each upstream firm and its unrelated downstream buyers was exclusive. That is, unrelated downstream firms that purchase from one upstream firm do not purchase from the other upstream firm. We now consider the case where upstream firms cannot exclude unrelated downstream buyers, who purchase from the upstream firm that charges lower price. Intuitively this makes upstream competition

tougher and collusion more attractive than the previous case. We show that the ALP continues to make collusion more stable.

We modify our basic model slightly. There are two upstream firms, each with one affiliate downstream firm, and  $n - 2$  additional unrelated downstream firms. For  $i = 1, 2$ , upstream firm  $i$ 's downstream affiliate is downstream firm  $i$ , and unrelated downstream firms are indexed by  $j = 3, \dots, n$ . Unrelated downstream firms choose the upstream firm that charges lower price. But each downstream affiliate purchases from its own upstream affiliate regardless of any price difference. If upstream firms charge the same price, we assume that they divide unrelated buyers' demand equally. All other elements of the model are the same as before. Since the outcomes for the case without the ALP are the same as before, we consider below only the case with the ALP.

Let  $w_i$  be the wholesale prices set by upstream firm  $i$ ,  $i = 1, 2$ . Under the ALP,  $w_1$  and  $w_2$  are offered to all unrelated downstream buyers, who choose the upstream firm with lower price. Then we can derive upstream firm 1's consolidated profit function:

$$\Pi_1 = \begin{cases} \left( \frac{a+(n-1)w_2+w_1}{n+1} - c \right) \left( \frac{a+(n-1)w_2-nw_1}{n+1} \right) & \text{if } w_1 > w_2, \\ \left( \frac{a+nw_1}{n+1} - c \right) \left( \frac{a-w_1}{n+1} \right) + \frac{(n-2)(w_1-c)(a+w_2-2w_1)}{2(n+1)} & \text{if } w_1 = w_2, \\ \left( \frac{a+(n-1)w_1+w_2}{n+1} - c \right) \left( \frac{a+w_2-2w_1}{n+1} \right) + \frac{(n-2)(w_1-c)(a+w_2-2w_1)}{n+1} & \text{if } w_1 < w_2. \end{cases} \quad (16)$$

Upstream firm 2's consolidated profit function can be written in a symmetric way.

As before, we solve for upstream firm 1's consolidated profit from collusion, deviation and competition. First, the collusive price  $w^C := w_1 = w_2$  maximizes joint profit  $\Pi_1 + \Pi_2$ ,

$$\Pi_1 + \Pi_2 = \left( \frac{a + nw_1}{n + 1} - c \right) \frac{2(a - w_1)}{n + 1} + \frac{(n - 2)(w_1 - c)(a - w_1)}{n + 1}$$

which leads to

$$w^C = \frac{(n^2 + n - 4)a + n(n + 1)c}{2(n - 1)(n + 2)}.$$

The resulting profit is

$$\Pi_1^C = \frac{n^2(a - c)^2}{8(n - 1)(n + 2)}. \quad (17)$$

Second, given  $w_2 = w^C$ , upstream firm 1's optimal deviation should necessarily be undercutting upstream firm 2's price. Thus it can be found from the third line in (??):

$$w_1^D = \frac{(n^2 - 5)(3n^2 + 3n - 8)a + (n + 1)(5n^3 - 17n + 8)c}{8(n - 1)(n + 2)(n^2 - 3)}.$$

However, given the above  $w_1^D$  and  $w^C$ , one can show that upstream firm 2's downstream affiliate, i.e., downstream firm 2, purchases a positive quantity only when  $n \leq 5$ . When  $n \geq 6$ , upstream firm 1's deviation leads to  $y_2^* = 0$ . The condition required for this case is  $w_1 \leq [(n^3 - n^2 - 6n + 4)a + n^2(n + 1)c]/[2(n + 2)(n - 1)^2]$ . In this case, all the remaining downstream firms' Cournot quantity is  $y_1^* = y_3^* = \dots = y_n^* = (a - w_1)/n$ . Plugging these into upstream firm 1's consolidated profit, one can show that the profit is strictly increasing in  $w_1$  for all  $w_1 \leq [(n^3 - n^2 - 6n + 4)a + n^2(n + 1)c]/[2(n + 2)(n - 1)^2]$ . Therefore upstream firm 1's optimal deviation in this case is obtained at the corner:  $w_1^D = [(n^3 - n^2 - 6n + 4)a + n^2(n + 1)c]/[2(n + 2)(n - 1)^2]$ . Summarizing the above, firm 1's optimal deviation retains the upstream duopoly when  $n \leq 5$  but it monopolizes the upstream market when  $n \geq 6$ . Putting these together, upstream firm 1's optimal deviation is given by

$$w_1^D = \begin{cases} \frac{(n^2 - 5)(3n^2 + 3n - 8)a + (n + 1)(5n^3 - 17n + 8)c}{8(n - 1)(n + 2)(n^2 - 3)}, & n \leq 5, \\ \frac{(n^3 - n^2 - 6n + 4)a + n^2(n + 1)c}{2(n + 2)(n - 1)^2}, & n \geq 6. \end{cases}$$

The resulting deviation profit is

$$\Pi_1^D = \begin{cases} \frac{(3n^2 + 3n - 8)^2(a - c)^2}{32(n + 2)^2(n^2 - 3)}, & n \leq 5, \\ \frac{n(n + 1)(n^4 - 2n^3 - 4n^2 + 11n - 4)(a - c)^2}{4(n + 2)^2(n - 1)^4}, & n \geq 6. \end{cases} \quad (18)$$

Third, when firms return to the competitive phase, we have Bertrand competition since each upstream firm vies for unrelated downstream buyers (see  $\Pi_1$  in (??) which discontinuously changes around the neighbourhood of  $w_1 = w_2$ ). Insofar as its price is above marginal cost, each upstream firm has incentives to undercut each other. Only when  $w_1 = w_2 = c$ , no upstream firm has an incentive to change its price. Thus equilibrium price in the competitive phase is

$$w^E := c,$$

and the resulting profit is

$$\Pi_1^E = \frac{(a - c)^2}{(n + 1)^2}. \quad (19)$$

As before, let  $\delta^* = (\Pi_1^D - \Pi_1^C)/(\Pi_1^D - \Pi_1^E)$ . Then tacit collusion is more stable with the ALP than without it if and only if  $\delta^* < \hat{\delta}^* = 25/41$ . From (??), (??) and (??), one can calculate  $\delta^*$  and verify that  $\delta^* < \hat{\delta}^* = 25/41$  for all  $n$ . Unlike the previous case, however,  $\delta^*$  monotonically increases to  $1/2$  as  $n$  increases. This is because, when upstream

firms compete for unrelated downstream buyers, deviation is more profitable than when the upstream-downstream relationship is exclusive. Nonetheless, tacit collusion is more stable under the ALP than without it for any number of unrelated downstream buyers. We summarize the main result from this section below.

*Proposition 2: Suppose each upstream firm supplies to its own downstream affiliate and competes for additional unrelated downstream buyers. Then tacit collusion is more stable with the ALP than without it for any number of unrelated downstream buyers.*

Figure 3 compares various profits and the critical discount factors in the two cases under the ALP. To compare them with equal number of downstream firms, we have chosen the number of downstream firms in Section 4 to be  $2n$ . Then profit from collusion is the same in both cases. Not surprisingly, equilibrium profit is everywhere smaller and deviation profit larger when upstream firms cannot exclude unrelated downstream buyers. Because upstream competition is tougher when upstream firms compete for unrelated downstream buyers, collusion is more attractive and the critical discount factor for collusion is smaller. Thus the ALP continues to render more stability to tacit collusion.

— **Figure 3 goes about here.** —

## 5 Concluding Remarks

This paper has examined how the ALP changes the nature of dynamic competition in imperfectly competitive markets. Our main finding is that the ALP makes tacit collusion between firms more stable than when firms are not subject to the ALP. The primary reason for this is that the ALP compels the firm not to price-discriminate between its own affiliates and unrelated buyers. On the other hand, the firm can exclude unrelated buyers if it is not subject to the ALP. Therefore the ALP makes competition tougher in the market where the firm's affiliates operate. This makes collusion more attractive. We have also shown that total surplus under collusion is lower when firms comply with the ALP, mainly because the collusive price is higher.

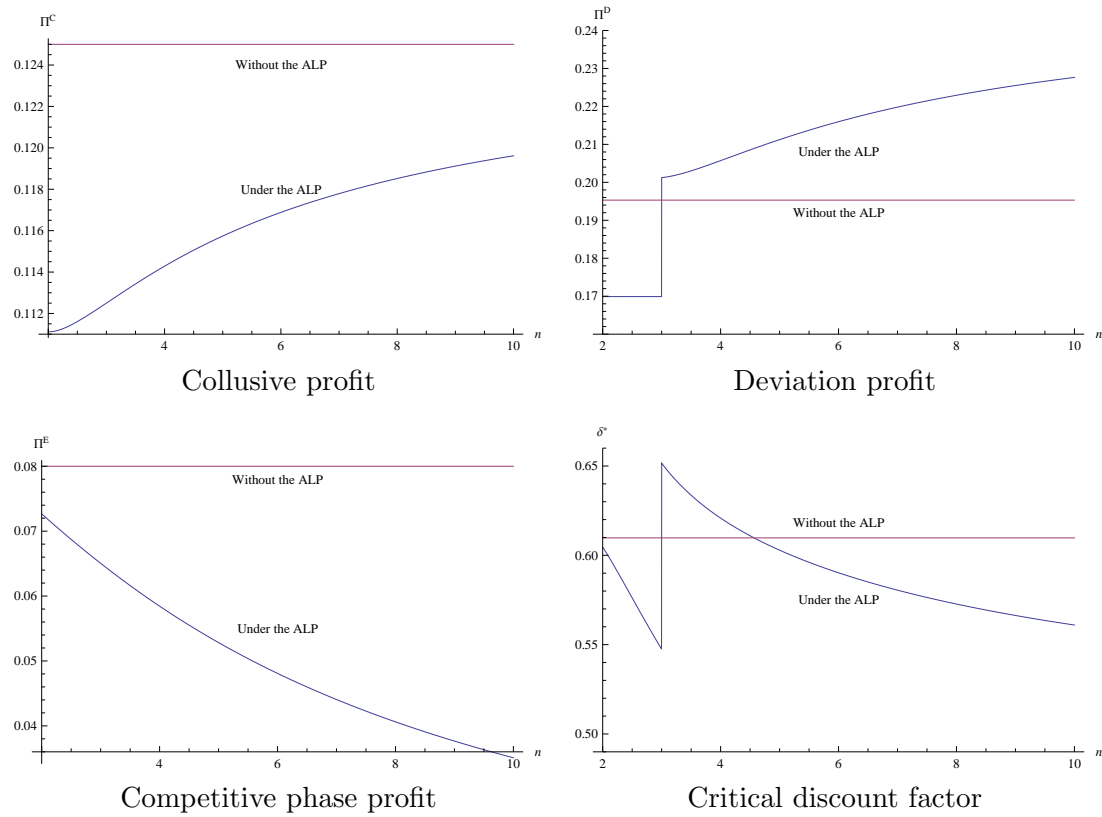
Our findings stand counter to the original rationale of the arm's length principle. Although the principle can ensure the level-playing field for multinationals' affiliates and unrelated enterprises in a static setting, it can lead to anti-competitive outcomes in a dynamic setting, which can further reduce welfare.

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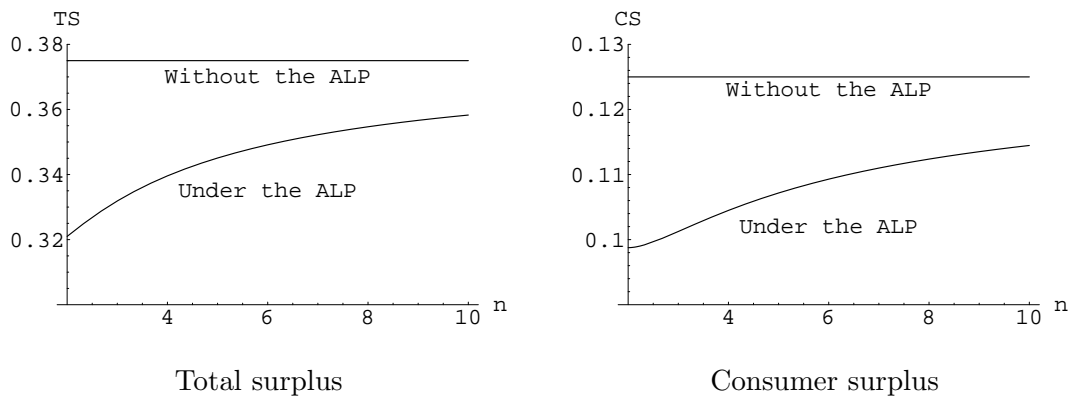
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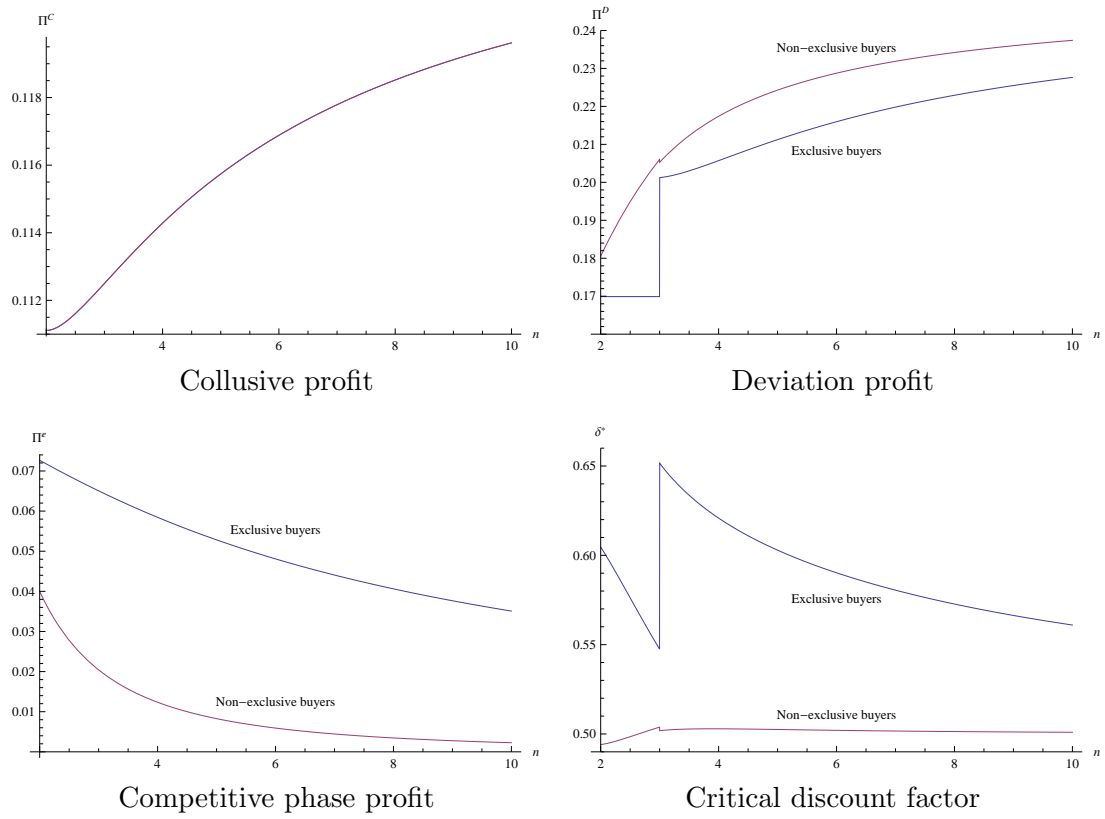




**Figure 1: Profits and Critical Discount Factors for Collusion**



**Figure 2: Total Surplus and Consumer Surplus under Collusion**



**Figure 3: Profits and Critical Discount Factors in the Two Cases under the ALP.**