The Economics of doping

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2002

The Economics of Doping[1]

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March 2002

Abstract

This paper considers a strategic game in which two players, with unequal prospects of winning the game, decide simultaneously and secretly to use performance-enhancing drugs before they compete. In the mixed strategy equilibrium, the favorite player is more likely to take these drugs than is the underdog, yet, for some parameter values, he is less likely to win the game with doping opportunities than without. The paper then analyzes the anti-doping regulations adopted by the International Olympic Committee, comparing its rules with a ranking-based sanction scheme. Two results emerge from this comparison: First, while IOC regulations cannot satisfy participation and incentive compatibility constraints and implement the no-doping equilibrium in all circumstances, a more effective ranking-based sanction scheme with these properties exists. Second, ranking-based punishment schemes are less costly to implement than are IOC regulations because fewer tests are needed to attain the no-doping equilibrium.

Keywords: Doping, Doping Regulation, Contests, Tournaments.

JEL: C72, D74, D78

1   Introduction

The use of performance-enhancing drugs in sports (doping) has a long history.[2] The first recorded instances were noted by Philostratus and Galerius in the 3rd century B.C. at the ancient Olympic Games. An large number of small statues of Jupiter, found around ancient sports arenas, bear witness to a desire to obtain divine pardon for contravention of the rules and indicates the scale of the phenomenon. In modern times, the first doping fatality occurred in the Bordeaux-Paris bicycle race in 1879, when an English cyclist, Linton, died after being doped with tri-methyl. Around 1910, we find a fashion to use oxygen by Belgian and English football teams. Later, boxers used strychnine and mixtures of brandy and cocaine. More recently, in 1988, the famous 100-m runner, Ben Johnson, lost the gold medal he had won at the Olympics in Seoul after being found guilty of using an anabolic steroid.

In 1963, the European Parliament, responding to growing concern about the use of drugs in sports, proposed the first legal definition of doping as "the administration or use of exogenous substances in an abnormal form or way to healthy persons where the only goal is to achieve an artificial and unfair improvement of the performance of the athlete" (Reiter 1994). Following the legal definition of the European Parliament, enforcement institutions were established at the national and international levels in all major sports (Bird and Wagner 1997). These institutions rely without exception on a regulatory framework that is based on a *negative list* enumerating all banned substances. The negative list is usually enforced by random blood or urine tests of performing athletes. The sanctions imposed are exclusion from the game and suspension from competition for some length of time.[3]

The surge in doping cases in the late 1990's suggests that anti-doping regulations do

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[2] The quote above, *the* folk theorem of sports, has been attributed to Red Sanders of Vanderbuilt University (Bird and Wagner 1997). The historical episodes recounted here are taken from De Merode (1999).

[3] In 1999, the IOC proposed two penalty schemes for first-time offenders: a) If the substance used is ephedrine, phenylpropanolamine, pseudoephedrine, caffeine, strychnine or a related substance the offender will receive either i) a warning; ii) a ban on participation in one or several sports competitions; iii) a fine of up to $100,000; or iv) suspension from any competition for a period of one to six months. b) If the substance used is one other than those referred to in paragraph a) above the offender will receive either i) a ban on participation in one or several sports competitions; ii) a fine of up to $100,000; or iii) suspension from any competition for a period of two years.

not attain their goal of achieving a drug-free environment in sports.[4] This paper, therefore, proposes a *ranking-based* punishment scheme that changes the athletes' incentives towards the no-doping equilibrium. A ranking-based sanction scheme is developed that implements the no-doping equilibrium in all circumstances - a property not attained by the sanction scheme of the International Olympic Committee (IOC). Moreover, it is shown that ranking-based sanctions are more cost effective than IOC regulations because fewer doping tests are needed to attain the no-doping equilibrium.

To analyze the doping problem, the paper considers a strategic game where two athletes simultaneously and secretly decide to use a performance-enhancing drug before competing. In the existing literature, the doping problem is considered to be a prisoner's dilemma (Breivik 1987, Wagner 1993, Bird and Wagner 1997). It is shown that this is only true if the players have equal prospects of winning. With unequally talented players, however, mixed-strategy equilibria exist that exhibit interesting properties. Perhaps surprisingly, for some parameter values, the favored player (the player who wins the game with probability $\rho > \frac{1}{2}$) is more likely to use performance-enhancing drugs than is the underdog, yet he is less likely to win with doping opportunity than without.

The model suggests three justifications for implementing anti-doping measures: First, doping never increases welfare; rather, for a large set of parameter values welfare is lower with doping opportunities than it is without. Second, and even worse, there exists no equilibrium where the expected payoff of any competitor is larger with a doping opportunity than without. Third, the "wrong" player may win the game because doping changes the probabilities of winning.

While the model provides a clear case for anti-doping regulations, the major problem of implementation is that doping tests sometimes provide faulty information. Occasionally, tests indicate that the athlete is not doped when he or she is or that an athlete is doped when he or she is not.[5] In the model this problem is taken into account. Moreover, following

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[4]The most prominent recent offender is Marco Pantani, an Italian cyclist who won the Tour de France in 1998 and was the leader at the Giro d'Italia in 1999 before he was disqualified next to the last day of the race. The most reliable data available at present to assess the use of performance-enhancing drugs are the statistics of the IOC. Currently, 24 IOC-accredited laboratories analyze more than 100,000 samples from athletes, of which more than 40% are out-of-competition samples. The total rate of positive cases is around 1.6%. Anabolic steroids have the highest incidence, at around 65% of the positive cases. Stimulants (20%) and diuretics (4%) follow (Segura 1999).

[5]Testing is most problematical when a substance is also naturally produced by the body. For example, pharmaceutical forms of growth hormone (GH), erythropoietin (EPO), and human chorionic gonadotrophin (hCG) are usually identical to natural forms already present in the body. This means that the tests should differentiate between the two forms, but they cannot (De Merode 1999, Segura 1999).

IOC regulations, I impose the rule that an athlete who tests positive is excluded from the ranking and has to pay a fine. Departing from IOC rules, I allow for a ranking-based punishment.

The paper's topic is related to the theory of contests and tournaments, which studies the effort choice of two or more players who hope to win a prize.[6] In this literature, players' effort choices affect the winning probabilities. Here, use of performance-enhancing drugs affects these probabilities, and effort levels at the contest are given and supposed to be maximal (because athletes prepare for months, or even for years, for important contests, such as the Olympic Games). During the contest athletes attempt to reach peak performance in all circumstances, that is, they choose maximum effort, a behavior that is expressed by the quote "Winning isn't everything, it is the only thing".[7]

The paper is organized as follows: Section 2 presents the model and the equilibrium when there are no doping regulations; section 3 analyzes IOC doping regulations; section 4 derives the ranking-based punishment scheme and compares IOC regulations with ranking-based punishments. Section 5 concludes.

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[6]See Nitzan (1994) for an overview of the literature on rent-seeking contests.

[7]If, instead, one considers training effort and use of performance-enhancing drugs during the precontest-season, these choices could be related.

4

2 The model

Consider a strategic game where two athletes, a and b, choose simultaneously to use performance-enhancing drugs before they compete.[8] The athletes face a discrete choice problem because, through past experience and through their doctors' recommendations, they have learned of substances and quantities that deliver peak-performance during the contest. Then, shortly before they compete, they decide whether to take these quantities and this decision depends on the strength of their opponent, on the enforcement of anti-doping regulations, and on the cost of cheating. The model describes athletes' incentives to take drugs shortly before a contest and so models the choice of so-called *race-day drugs* that have an immediate effect on athletes' performances.[9]

The players have von Neumann Morgenstern preferences over winning and losing the competition. The winning player receives utility $w > 0$ and the losing player receives zero utility. Without loss in generality, assume that if no drugs are taken, player a is at least as likely to win the game as player b, i.e., he wins the competition with probability ρ where $1 > \rho \geq \frac{1}{2}$. When only player a is doped, he wins the game with probability ρ_a where $\rho_a \geq \rho$ and when only player b is doped, he wins the game with probability ρ_b where $\rho_b \geq 1 - \rho$. When both players are doped, the winning probabilities are not affected.

Doping improves the performance of the players at cost c where $0 < c < w$. Cost c reflects the fact that people do not like to cheat: Each athlete prefers to win without using performance-enhancing drugs, rather than to win with their use, i.e., $w > w - c$. However, each athlete also prefers to win with performance-enhancing drugs rather than to lose without, i.e., $w - c > 0$. The cost may also represent expected health cost and monetary cost of drugs.

Let the set of players be $\{a, b\}$, the pure strategy space of each player be $\{d, nd\}$ for doping and no doping, and denote by α and β the probability distribution over pure strategies of players a and b, respectively. The payoffs for the strategy profile $\{d, d\}$ are $\{\rho w - c, (1 - \rho) w - c\}$, for $\{d, nd\}$ they are $\{\rho_a w - c, (1 - \rho_a) w\}$, for $\{nd, d\}$ they are

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[8]The analysis in a two-player framework is appropriate for many sports (boxing, tennis, martial arts, etc.). It also applies to team sports, such as soccer or football, where the team manager decides whether her team will use performance-enhancing drugs. It may also apply to other sports, such as cycling or downhill skiing, when two clear leaders compete for the first prize and the other competitors have no influence on this decision.

[9]At this stage of the contest athletes are not willing to experiment with unknown quantities or new drugs. Other substances are taken months before the competition to increase muscle mass and strength. While it is clear that so-called race day drugs can be detected during competition, others used in preparation for the games may no longer be present in the body of athletes at the time of the games (Segura 1999).

$\{(1 - \rho_b) w, \rho_b w - c\}$, and for $\{nd, nd\}$ they are $\{\rho w, (1 - \rho) w\}$.

To proceed, define $\delta_a = \rho_a - \rho$ and $\delta_b = \rho_b - (1 - \rho)$ where δ_i specifies the *effectiveness* of performance-enhancing drugs in the sense of measuring the increase of player i's winning probability when only player i dopes. Throughout the paper, it is assumed that performance-enhancing drugs are more effective for the weak player, i.e., that $\delta_b \geq \delta_a$.[10] A special case is $\rho_a = \rho_b = 1$ where a doped player wins with certainty if the other player is not doped. In this case, $\delta_b > \delta_a$ when $\rho > \frac{1}{2}$, because $\delta_a = 1 - \rho$ and $\delta_b = \rho$. Solving the noncooperative game yields the following results:

Lemma 1 *There is a critical value $\widetilde{c} = \frac{c}{w}$ such that the following is true: if $\delta_a \geq \widetilde{c}$, there exists a pure-strategy equilibrium (PSE) that entails $\alpha = 1$ and $\beta = 1$; if $\delta_b \leq \widetilde{c}$, there exists a PSE that entails $\alpha = 0$ and $\beta = 0$; if $\delta_a = \widetilde{c}$ ($\delta_b = \widetilde{c}$), there exists a nongeneric PSE that entails $\alpha = 1$ and $\beta = 0$ ($\alpha = 0$ and $\beta = 1$); if $\delta_b > \widetilde{c} > \delta_a$ there exists a mixed-strategy equilibrium (MSE) that entails $\alpha = \alpha^* \in (0, 1)$ and $\beta = \beta^* \in (0, 1)$, where*

$$\alpha^* = \frac{\delta_b - \widetilde{c}}{\delta_b - \delta_a} \ \ and \ \beta^* = \frac{\widetilde{c} - \delta_a}{\delta_b - \delta_a}.$$

Lemma 1 characterizes the existence regions of the different types of equilibria. The equilibria are unique except for the nongeneric parameter values $\delta_b = \widetilde{c}$, $\delta_a = \widetilde{c}$, and $\delta_a = \delta_b = \widetilde{c}$. Interestingly, a necessary condition for a (generic) mixed-strategy equilibrium is that performance-enhancing drugs affect athletes asymmetrically ($\delta_b > \delta_a$). If $\delta_a = \delta_b$, we have a symmetric solution in the sense that either both agents are doped or they are not doped. Note also that $\alpha^* \geq \beta^*$ if performance-enhancing drugs are sufficiently effective, that is, if $\delta_a + \delta_b \geq 2\widetilde{c}$.

In the mixed-strategy equilibrium, depending on parameter values, either competitor has a better chance to win compared to the game without a doping opportunity. To see this, denote by ρ_m player a's probability of winning in the mixed strategy equilibrium of the game with doping opportunity and note that $\rho_m - \rho = (\delta_b \delta_a - \widetilde{c}^2)(\delta_b - \delta_a)^{-1}$. Thus, if $\sqrt{\delta_b \delta_a} < \widetilde{c}$, the more talented player a is less likely to win with doping opportunities than without. Moreover, if $\delta_a + \delta_b > 2\widetilde{c}$ and if $\sqrt{\delta_b \delta_a} < \widetilde{c}$, the favored player is more likely to use performance-enhancing drugs than the underdog, yet he is less likely to win with doping opportunities than without.

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[10]In the appendix, I present a model that demonstrates how doping affects athletes' performances and how such a change affects the winning probabilities ρ_a and ρ_b, respectively, δ_a and δ_b. I show that under quite plausible assumptions $\delta_b \geq \delta_a$, i.e., performance-enhancing drugs are more effective for the weak player.
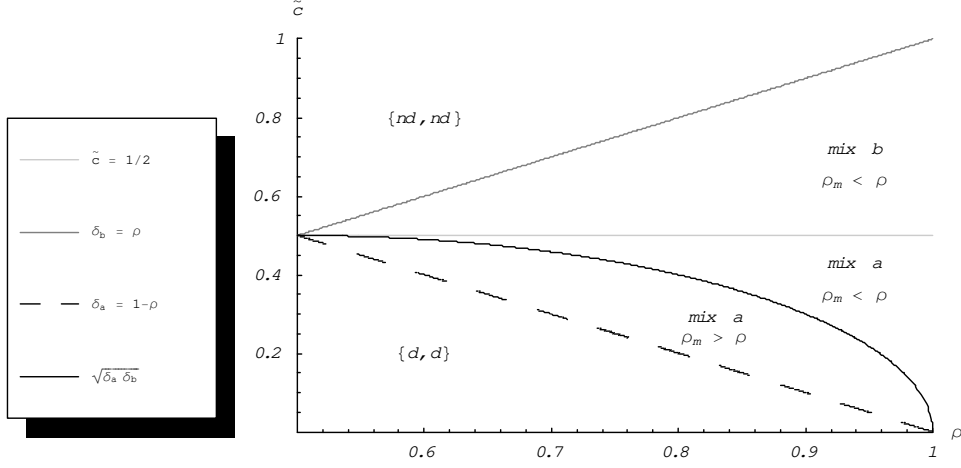
Figure 1: Existence regions when $\delta_a = 1 - \rho$ and $\delta_b = \rho$.

Figure 1 characterizes the existence regions of the different types of equilibria for all values of ρ and $\widetilde{c}$ when $\delta_a = 1 - \rho$ and $\delta_b = \rho$. Pure strategy equilibria are labelled $\{nd, nd\}$ and $\{d, d\}$. Mixed-strategy equilibria exist for parameter values in the regions labelled mix a and mix b. The favorite player is more (less) likely to be doped than is the underdog in the region labelled mix a (mix b). The region where player a's winning probability is larger (smaller) with doping opportunities than without is labelled $\rho_m > \rho$ ($\rho_m < \rho$). One can see that there are parameter values where the favorite player is more likely to use performance-enhancing drugs (mix a), yet he is less likely to win with doping opportunities than without ($\rho_m < \rho$).

How do doping opportunities affect the players' expected utilities? In equilibrium $\{d, d\}$ both agents bear the cost c which makes them strictly worse off relative to the game without a doping opportunity. Interestingly, in equilibrium $\{d, nd\}$, the expected payoff of player a equals his expected payoff in the game without a doping opportunity and player b is strictly worse off. In the equilibrium $\{nd, d\}$ an equivalent result holds where player b's expected payoff equals the expected payoff of the game without a doping opportunity and player a is strictly worse off. To compare the expected payoffs in the mixed-strategy equilibrium, denote by π_a and π_b the expected payoffs for player a and b, respectively. One can show that $\pi_a - \rho w = -\beta^* \left(\widetilde{c} + \delta_b \right) w < 0$ and that $\pi_b - (1 - \rho) w = -\alpha^* \left(\widetilde{c} + \delta_a \right) w < 0$. Thus, in the mixed-strategy equilibrium, the expected payoffs are strictly smaller with doping opportunities than without.

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3 IOC doping regulations

The model provides three justifications for implementing anti-doping measures. First, a doping opportunity never increases welfare; rather, for a large set of parameter values, welfare is lower with doping opportunities. Second, and even worse, there exists no equilibrium where the expected payoff of any competitor is strictly larger with a doping opportunity than without. Third, the "wrong" player could win the game because doping changes the probabilities of winning.

To attain a drug-free sports environment, the IOC defines a list of banned classes of substances (the negative list) that is enforced with random urine or blood tests of performing athletes. If with conditional probability θ_d the doping test indicates that an athlete is doped when he is, the probability that the test reads falsely negative is $1 - \theta_d$ and if with conditional probability θ_{nd}, the test indicates that the athlete is doped when he is not the probability that it reads falsely positive is θ_{nd}. Throughout the paper it is assumed that $\theta_d \geq \theta_{nd}$.

In reality, θ_d and θ_{nd} depend on the drug for which the test is designed. For certain doping substances, particularly substances that are not naturally produced by an athlete's body, doping tests are highly reliable, that is, θ_d is large and θ_{nd} is small. For other substances, however, such as erythropoietin (EPO) and the human growth hormone (hGH), which are naturally produced by the body, it is difficult to distinguish an exogenous administration from a natural endogenous concentration, which means that θ_d and θ_{nd} are relatively close. Moreover, for each test, by choosing a threshold for the substance they test for, regulators have a choice over θ_d and θ_{nd}.[11] The problem, however, is the positive correlation between θ_d and θ_{nd}, so that there is a *trade-off* between the type of mistake (false positive/false negative) anti-doping regulators make more often: When a low threshold for a doping substance is chosen there is a low probability that the test produces a falsely negative result and a high probability that it produces a falsely positive result and vice versa.

Under IOC regulations, an athlete who tests positive is disqualified and must pay a fine

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[11]For example, because the pharmaceutical and natural forms of EPO cannot, as yet, be distinguished, the International Cycling Union measures the haematocrit content of the blood. The current threshold is set at 50 percent for men and 47 percent for women. If the measured value is larger than 50 percent, it is considered to be because of an exogenous administration of EPO. Actually, a cyclist with an abnormally high haematocrit content is declared temporarily unfit for at least fifteen days, which means that he is not allowed to finish the race (see footnote 3 about Marco Pantani). It seems that the tests are so unreliable that the International Cycling Union - to avoid legal dispute - resorts to this health argument when excluding a cyclist from a race.

in terms of utility. Departing from IOC rules, I allow for ranking-based sanctions where S_1 is the punishment for the winner and S_2 is the punishment for the loser. Then, if $s_1 = \frac{S_1}{w}$ and if $s_2 = \frac{S_2}{w}$, the payoffs, which are divided by w, are

$$(1-\theta_d)(1-\theta_d)\rho + (1-\theta_d)\theta_d - \theta_d\left(\rho s_1 + (1-\rho)\,s_2\right) - \widetilde{c} \qquad (a_{11})$$

$$(1-\theta_d)(1-\theta_d)\left(1-\rho\right) + (1-\theta_d)\theta_d - \theta_d\left((1-\rho)\,s_1 + \rho s_2\right) - \widetilde{c} \qquad (b_{11})$$

$$(1-\theta_d)(1-\theta_{nd})\rho_a + (1-\theta_d)\theta_{nd} - \theta_d\left(\rho_a s_1 + (1-\rho_a)\,s_2\right) - \widetilde{c} \qquad (a_{12})$$

$$(1-\theta_{nd})(1-\theta_d)\left(1-\rho_a\right) + (1-\theta_{nd})\theta_d - \theta_{nd}\left((1-\rho_a)\,s_1 + \rho_a s_2\right) \qquad (b_{12})$$

$$(1-\theta_{nd})(1-\theta_d)\left(1-\rho_b\right) + (1-\theta_{nd})\theta_d - \theta_{nd}\left((1-\rho_b)\,s_1 + \rho_b s_2\right) \qquad (a_{21})$$

$$(1-\theta_d)(1-\theta_{nd})\rho_b + (1-\theta_d)\theta_{nd} - \theta_d\left(\rho_b s_1 + (1-\rho_b)\,s_2\right) - \widetilde{c} \qquad (b_{21})$$

$$(1-\theta_{nd})(1-\theta_{nd})\rho + (1-\theta_{nd})\theta_{nd} - \theta_{nd}\left(\rho s_1 + (1-\rho)\,s_2\right) \qquad (a_{22})$$

$$(1-\theta_{nd})(1-\theta_{nd})\left(1-\rho\right) + (1-\theta_{nd})\theta_{nd} - \theta_{nd}\left((1-\rho)\,s_1 + \rho s_2\right) \qquad (b_{22})$$

a_{11} and b_{11} are the expected payoffs of players a and b, respectively, for the strategy profile $\{d, d\}$. For example, consider a_{11}: With probability $(1-\theta_d)(1-\theta_d)$ neither tests positive and agent a wins the game with probability ρ. With probability $(1-\theta_d)\theta_d$ player a is tested negative and player b positive and player a wins the game with certainty. With probability θ_d he is tested positive, disqualified, and he pays the fine s_1 if he is the (disqualified) winner or s_2 if she is the (disqualified) loser. a_{12} and b_{12} are the expected payoffs for the strategy profile $\{d, nd\}$. a_{21} and b_{21} are the expected payoffs for the strategy profile $\{nd, d\}$ and a_{22} and b_{22} are the expected payoffs for the strategy profile $\{nd, nd\}$.

The equilibria under the IOC punishment scheme $s_1 = s_2 = s$ can be characterized in the same way as in Lemma 1 for the game without doping regulations. However, because a full characterization is rather cumbersome, for Lemma 2 parameter values are restricted as follows: $\rho_a = \rho_b = \widetilde{\rho}$ and $\rho > \widehat{\rho} = \frac{(\theta_d - \theta_{nd})^2 + (1-\theta_d)(1-\theta_{nd})}{(1-\theta_d)^2 + (1-\theta_{nd})^2}$.[12] In order to prepare the characterization, consider the four critical values σ_i, $i = 1, .., 4$. The critical value σ_1 is the value of s that solves $b_{11} = b_{12}$:

$$\sigma_1 = \frac{(1-\theta_d)\left[(1-\theta_d)\left(1-\rho\right) - (1-\theta_{nd})\left(1-\widetilde{\rho}\right)\right] - \theta_d\left(\theta_d - \theta_{nd}\right) - \widetilde{c}}{\theta_d - \theta_{nd}} \qquad (1)$$

Thus, σ_1 is the value of punishment s that makes player b indifferent between doping and not doping when player a is doped. Note that if $s > \sigma_1$, $b_{12} > b_{11}$, which implies that player b's best response is not to dope when a dopes. The critical value σ_2 is the value of s that solves $a_{22} = a_{12}$:

$$\sigma_2 = \frac{(1-\theta_{nd})\left[(1-\theta_d)\widetilde{\rho} - (1-\theta_{nd})\,\rho\right] - \theta_{nd}(\theta_d - \theta_{nd}) - \widetilde{c}}{\theta_d - \theta_{nd}} \qquad (2)$$

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[12]Note that in the following section, which compares the IOC anti-doping measures with the ranking-based punishment scheem, parameter values are not restricted.

The critical value σ_3 is the value of s that solves $a_{11} = a_{21}$:

$$\sigma_3 = \frac{(1 - \theta_d)\left[(1 - \theta_d)\rho - (1 - \theta_{nd})(1 - \widetilde{\rho})\right] - \theta_d(\theta_d - \theta_{nd}) - \widetilde{c}}{\theta_d - \theta_{nd}} \tag{3}$$

The critical value σ_4 is the value of s that solves $b_{22} = b_{21}$:

$$\sigma_4 = \frac{(1 - \theta_{nd})\left[(1 - \theta_d)\widetilde{\rho} - (1 - \theta_{nd})(1 - \rho)\right] - \theta_{nd}(\theta_d - \theta_{nd}) - \widetilde{c}}{\theta_d - \theta_{nd}} \tag{4}$$

Finally, note that when $\rho > \widehat{\rho}$, $\sigma_1 \leq \sigma_2 \leq \sigma_3 \leq \sigma_4$.

Lemma 2 *Assume $s_1 = s_2 = s$, $\rho_a = \rho_b = \widetilde{\rho}$ and $\rho > \widehat{\rho}$. Then, if $s \leq \sigma_1$, there exists a PSE that entails $\alpha = 1$ and $\beta = 1$; if $\sigma_1 \leq s \leq \sigma_2$, there exists a PSE that entails $\alpha = 1$ and $\beta = 0$; if $\sigma_3 \leq s \leq \sigma_4$, there exists a PSE that entails $\alpha = 0$ and $\beta = 1$; if $\sigma_4 \leq s$, there exists a PSE that entails $\alpha = 0$ and $\beta = 0$; if $\sigma_2 < s < \sigma_3$ or there exists a MSE that entails $\alpha = \alpha_{oc}^* \in (0,1)$ and $\beta = \beta_{oc}^* \in (0,1)$, where*

$$\alpha_{oc}^* = \frac{(1-\theta_{nd})\left[(1-\theta_{nd})(1-\rho) - (1-\theta_d)\widetilde{\rho}\right] + (\theta_d - \theta_{nd})(\theta_{nd} + s) + \widetilde{c}}{(1-\theta_d)(1-\theta_{nd}) - \rho\left[(1-\theta_d)^2 + (1-\theta_{nd})^2\right]}$$

$$\beta_{oc}^* = \frac{(1-\theta_{nd})\left[(1-\theta_{nd})\rho - (1-\theta_d)\widetilde{\rho}\right] + (\theta_d - \theta_{nd})(\theta_{nd} + s) + \widetilde{c}}{(1-\theta_d)(1-\theta_{nd}) - (1-\rho)\left[(1-\theta_d)^2 + (1-\theta_{nd})^2\right]}$$

Lemma 2 characterizes the existence regions for the different types of equilibria. Note that the equilibria are unique. Figure 2 shows the equilibrium strategies of players a and b as a function of punishment s. The solid line is the equilibrium strategy of player a and the doted line is player b's equilibrium strategy. For low values of s, i.e., if $s \leq \sigma_1$, we have a pure doping equilibrium $\{d, d\}$. As we increase the punishment, at $s = \sigma_1$, the equilibrium switches to $\{d, nd\}$. As we increase s even more, at $s = \sigma_2$, we first attain the mixed strategy equilibrium, then, at $s = \sigma_3$, the equilibrium $\{nd, d\}$, and, finally, at $s = \sigma_4$, the no-doping equilibrium $\{nd, nd\}$.
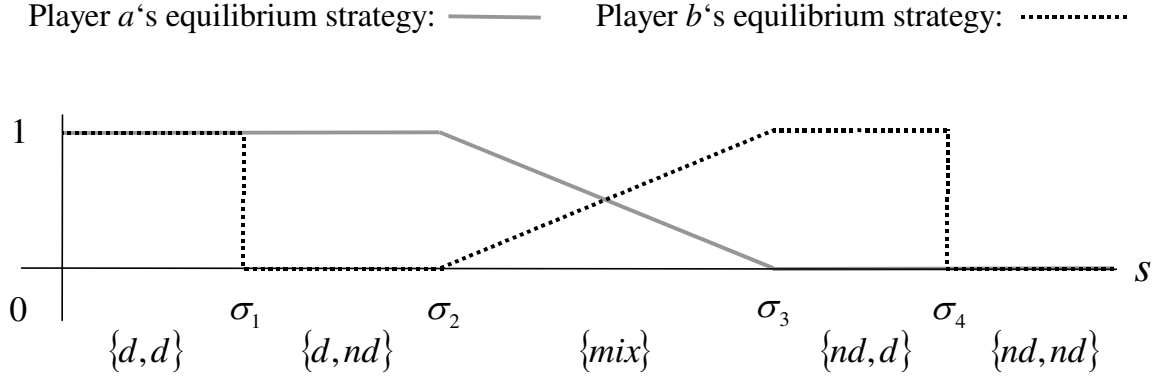
Player a's equilibrium strategy: ——— Player b's equilibrium strategy: ············

Figure 2: Equilibrium strategies of a and b.

Note that the bounds σ_i are decreasing in the relative cost of doping $\widetilde{c}$. Thus, increasing prize w (decreasing $\widetilde{c}$) makes it less likely that the no-doping equilibrium is attained. This result suggests that the ongoing trend towards higher prizes and higher salaries for top-level athletes could be a driving force behind the recent surge in doping cases.[13]

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[13] Heinemann (1999) argues that "Victory and failure are becoming more important due to the increasing

A further reason for the recent increase in doping cases could be improvements of existing and development of new performance-enhancing drugs. The availability of more effective doping substances, i.e., substances with a larger $\widetilde{\rho}$, shifts the bounds σ_i to the right, which makes it more difficult to attain the no-doping equilibrium. In contrast, improvements of test technology, as expressed by an increasing θ_d, counteract this effect because an increase in θ_d shifts the bounds σ_i to the left. Finally, an increase in ρ shifts σ_1 and σ_2 to the left and σ_3 and σ_4 to the right. Accordingly, an increase in ρ makes the existence of a mixed-strategy equilibrium more likely.

Interestingly, while α_{oc}^* is decreasing in s, β_{oc}^* is increasing (see Figure 2). Thus, in the mixed strategy equilibrium, increasing s will increase the probability that the underdog is doped. Moreover, if ρ is sufficiently large, $\beta_{oc}^* > \beta^*$: for certain parameter values the underdog is more likely to be doped under the IOC anti-doping regulations than without regulations.

To explain this effect, assume that player b is very weak and doping costs are very small, i.e., $\rho = 1$ and $\widetilde{c} = 0$, which implies that the underdog has no chance of winning in the game without doping regulations (with these parameter values $\beta^* = 0$ and $\beta_{oc}^* = \frac{(1+s)(\theta_d - \theta_{nd})}{(1-\theta_d)(1-\theta_{nd})}$). With IOC regulations, however, the weak player has a chance of winning if player a is disqualified. The prospect of winning through disqualification of the opponent gives player b an *strategic* incentive to take performance-enhancing drugs, which is not present without doping regulations.

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dependence of top-level sports on the resources of the economy, the mass media, spectators and the state... The specific economic value of sport is, that it produces its "product", victory, exciting competitions, exemplary and superlative heroes. But it is increasingly impossible to fulfil theses requirements without doping."

4   Ranking-based vs IOC punishments

This section develops a ranking-based punishment scheme that is incentive compatible and individually rational and that implements the no-doping equilibrium for all parameter values of the game. A mechanism with these properties is called a *perfect* mechanism. The section also compares ranking-based punishments with IOC anti-doping regulations. Two results emerge from this comparison: First, IOC anti-doping regulations are not a perfect mechanism, i.e., they cannot attain the no-doping equilibrium for all parameter values of the game. Second, if IOC regulations attain the no-doping equilibrium, there is always a cheaper ranking-based punishment scheme that attains the no-doping equilibrium too. Ranking-based punishments are cheaper because fewer doping tests are required to attain the no-doping equilibrium.

4.1   Perfect mechanism

A perfect mechanism satisfies the following conditions:

$$a_{22} \geq 0 \text{ and } b_{22} \geq 0 \tag{5}$$

$$a_{22} \geq b_{22} \tag{6}$$

$$a_{22} \geq a_{12}, \, a_{21} \geq a_{11} \text{ and } b_{22} \geq b_{21} \quad \text{or} \tag{7}$$

$$b_{22} \geq b_{21}, \, b_{12} \geq b_{11} \text{ and } a_{22} \geq a_{12}$$

Inequalities (5) and (6) are the participation and incentive-compatibility constraints, respectively. If (5) holds, it is rational for player a and b to participate in the game. If (6) holds, player a is better off being the more talented player. If the first (last) three inequalities of condition (7) hold, player a (player b) has a weakly dominant strategy not to use performance-enhancing drugs, and the other player's best response is not to dope. This requirement is akin to the concept of a "dominant-strategy mechanism" (Fudenberg and Tirole 1991).

Proposition 1 *Assume $s_1 = s_2 = s$. Then there is no sanction s that satisfies (5)-(7) for all parameter values.*

Proposition 1 implies that IOC regulations, which set $s_1 = s_2$, are *not* a perfect mechanism. The problem is the participation constraint of the weak player. This constraint limits the sanction that can be imposed to punish player b if he tests positive. This constraint is:

$$s \leq \overline{s} = \frac{(1-\rho)(1-\theta_{nd})^2 + (1-\theta_{nd})\theta_{nd}}{\theta_{nd}}. \tag{8}$$

13

According to (8), sanction $s = \bar{s}$ is the largest sanction that can be imposed without violating the participation constraint of player b. Bound $\bar{s}$ is strictly decreasing in θ_{nd} and equal to zero at $\theta_{nd} = 1$. It imposes no limit on s at $\theta_{nd} = 0$. Thus, a necessary condition for failure of the IOC anti-doping regulations is $\theta_{nd} > 0$. Bound $\bar{s}$ is also decreasing in ρ and equal to $1 - \theta_{nd}$ at $\rho = 1$. Thus, the weaker player b is, the smaller is the largest sanction that can be imposed without violating her participation constraint.

For certain parameter values, sanction $s = \bar{s}$ is too low to be consistent with a no-doping equilibrium because the weak player, despite this sanction, will choose to dope. To see this, note that the weak player's no-deviation condition $b_{22} \geq b_{21}$ requires that

$$s \geq \underline{s} = \frac{(1 - \theta_{nd})\left[(1 - \theta_d)\rho_b - (1 - \theta_{nd})(1 - \rho)\right] - \widetilde{c} - \theta_{nd}(\theta_d - \theta_{nd})}{\theta_d - \theta_{nd}}. \tag{9}$$

The constraint (9) gives a lower bound on the sanction s. If $s \geq \underline{s}$, the weak player has no incentive to deviate from the no-doping strategy profile $\{nd, nd\}$. If the precision of the test, $\theta_d - \theta_{nd}$, converges to zero and the nominator of (9) is positive, $\underline{s}$ approaches infinity. Thus, for certain parameter values, $\underline{s} > \bar{s}$, and, therefore, IOC anti-doping regulations cannot implement the no-doping equilibrium for all parameter values.

The failure of the IOC sanction scheme to satisfy the criteria for a perfect mechanism provides a rationale to search for an alternative sanction scheme. Proposition 2 presents such a mechanism.

Proposition 2 *The ranking-based punishment scheme $s_1 = \frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d}$ and $s_2 = 0$ is a perfect mechanism.*

This establishes the existence of a perfect mechanism if one allows for different punishments for winner and loser. The first thing to note is that the incentive compatibility constraint (6) provides an upper limit on the difference $s_1 - s_2$, which equals

$$s_1 - s_2 \leq \frac{(1 - \theta_{nd})^2}{\theta_{nd}}. \tag{10}$$

Thus, any perfect mechanism must satisfy (10). Moreover, the participation constraint of the weak player and (10) imply that the sanction for the winner must satisfy

$$s_1 \leq \frac{(1 - \theta_{nd})^2}{\theta_{nd}} + 1 - \theta_{nd}. \tag{11}$$

Similarly, the participation constraint of the strong player and (10) imply that the sanction of the loser must satisfy

$$s_2 \leq 1 - \theta_{nd}. \tag{12}$$

14

As with the IOC regulation, the problem of any sanction scheme is the incentive of the weak player to deviate from the no-doping strategy profile $\{nd, nd\}$. Because the weak player is less likely to win, a first guess, therefore, is to choose $s_2 = 0$ and, then, to search for a s_1 that satisfies (10), (11), and the weak player's no-deviation condition $b_{22} \geq b_{21}$.[14]

To see why a ranking-based punishment scheme does not fail to satisfy the criteria for a perfect mechanism, even if the precision of the test, $\theta_d - \theta_{nd}$, is very low, it is instructive to evaluate the weak player's no-deviation constraint at $\theta_d = \theta_{nd}$:

$$s_1 - s_2 \geq \frac{(1 - \theta_{nd})^2 - \widetilde{c}/(\rho_b - (1 - \rho))}{\theta_{nd}} \tag{13}$$

In contrast to inequality (9) the right-hand side of (13) is bounded at $\theta_d = \theta_{nd}$. Thus, even if the precision of the test, $\theta_d - \theta_{nd}$, is very low, we do not need drastic punishment to satisfy the no-deviation condition $b_{22} \geq b_{12}$. What we need instead is a sufficiently large difference in sanctions for winner and loser.

4.2   Cost effectiveness

This section compares the cost of ranking-based punishments and IOC anti-doping regulations. For this purpose, assume that testing is costly and that regulators can save cost by reducing the frequency of doping tests that they carry out after competitions. Up to this point, the paper has assumed that player a and b are tested with certainty. In the following, the assumption is that they are tested with probability τ, so that $\tau\theta_d$ ($\tau\theta_{nd}$) is the probability that a doped (not doped) athlete is tested positive. Regulators' goal is choose the smallest value of τ that is consistent with the no-doping equilibrium.

IOC regulations cannot implement the no-doping equilibrium if $\overline{s} < \underline{s}$ where $\underline{s}$ and $\overline{s}$ are defined in (8) and (9), respectively. Thus, if $\overline{s} < \underline{s}$, ranking-based punishments are clearly better than IOC regulations. To exclude this case, assume that $0 < \underline{s} < \overline{s}$ where inequality $0 < \underline{s}$ excludes that the no-doping equilibrium is attained without punishment. To take into account that $\tau \leq 1$, inequalities (8) and (9) need to be redefined. Under IOC regulations they are:

$$s \leq \overline{s}(\tau) = \frac{(1 - \rho)(1 - \tau\theta_{nd})^2 + (1 - \tau\theta_{nd})\tau\theta_{nd}}{\tau\theta_{nd}} \tag{14}$$

$$s \geq \underline{s}(\tau) = \frac{(1 - \tau\theta_{nd})[(1 - \tau\theta_d)\rho_b - (1 - \tau\theta_{nd})(1 - \rho)] - \widetilde{c} - \tau^2\theta_{nd}(\theta_d - \theta_{nd})}{\tau(\theta_d - \theta_{nd})} \tag{15}$$

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[14]The proof, which is in the appendix, also establishes that the scheme (s_1, s_2) satisfies $a_{22} \geq a_{12}$ and $b_{12} \geq b_{11}$, which is also needed for a perfect mechanism.

The first thing to note is that the bounds $\underline{s}(\tau)$ and $\overline{s}(\tau)$ are decreasing in τ. Thus, for any sanction s with $\underline{s}(1) \leq s \leq \overline{s}(1)$, the smallest testing probability satisfying $\underline{s}(\tau) \leq s \leq \overline{s}(\tau)$ is the value of τ that satisfies $s = \underline{s}(\tau)$. Denote this value by τ_{oc} (index oc for Olympic Committee) and note that the participation constraint is satisfied because $s \leq \overline{s}(1) \leq \overline{s}(\tau_{oc})$.

Next, consider the ranking-based punishments $\left(s_1 = s + \frac{\rho(1-\tau\theta_{nd})^2}{\tau\theta_{nd}}, s_2 = s - \frac{(1-\rho)(1-\tau\theta_{nd})^2}{\tau\theta_{nd}}\right)$ where s is again any sanction that satisfies $\underline{s}(1) \leq s \leq \overline{s}(1)$ under IOC regulations. The participation constraint and the no-deviation condition for the weak player, respectively, are

$$s \quad \leq \quad \overline{s}(\tau) \tag{16}$$

$$s \quad \geq \quad \underline{s}(\tau) - g(\tau) \tag{17}$$

where $\overline{s}(\tau)$ and $\underline{s}(\tau)$ are defined in (14) and (15), respectively, and

$$g(\tau) = \frac{\theta_d(1-\tau\theta_{nd})^2}{\theta_{nd}} \frac{[\rho_b\rho - (1-\rho_b)(1-\rho)]}{\tau(\theta_d - \theta_{nd})}.$$

At $\tau = \tau_{oc}$ the participation constraint is satisfied because $s \leq \overline{s}(1) \leq \overline{s}(\tau_{oc})$. Next, note that at $\tau = \tau_{oc}$ the left-hand side of (17) is larger than the right-hand side of (17) because $s = \underline{s}(\tau_{oc})$. Thus, for any s there is a $\tau \leq \tau_{oc}$ that satisfies the weak player's no-deviation constraint, i.e., $s = \underline{s}(\tau_{oc}) \geq \underline{s}(\tau) - g(\tau)$. Denote such a value by τ_{rb} (index rb for ranking based) and note that the weak player's participation constraint is also satisfied, i.e., $s \leq \overline{s}(\tau_{oc}) \leq \overline{s}(\tau_{rb})$. Finally, note that by construction the punishments s_1 and s_2 are a perfect mechanism. Thus, for any IOC sanction scheme (s, τ_{oc}) there is a ranking-based mechanism (s_1, s_2, τ_{rb}) with $\tau_{oc} \geq \tau_{rb}$.

The previous result relies on the ability of the regulators to increase punishment for the winner and to decrease it for the loser, that is, $s_1 \geq s \geq s_2$. Increasing punishment is not always feasible because a natural upper bound or legal restrictions may put a limit on the largest feasible punishment.[15] In the following, I compare the cost of ranking-based punishments and the IOC regulations when there is an upper bound on feasible sanctions. Denote this bound by s_l (index l for limit) and assume this punishment is sufficiently large to implement the no-doping equilibrium under IOC regulations, that is $\underline{s}(1) \leq s_l \leq \overline{s}(1)$. Again, the smallest (optimal) testing probability is the value of τ that solves (15) at equality, i.e., $s_l = \underline{s}(\tau_{oc})$.

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[15] For example, a natural upper bound is a lifelong ban from all further competitions. Under the IOC regulations, an offender receives this punishment if he or she has been caught twice.

Next, consider the ranking-based punishments $(s_1 = s_l, s_2 = 0)$. The no-deviation constraint of the weak player requires that:

$$s_l \geq \frac{(1 - \tau\theta_{nd})\left[(1 - \tau\theta_d)\rho_b - (1 - \tau\theta_{nd})(1 - \rho)\right] - \widetilde{c} - \tau\theta_{nd}(\tau\theta_d - \tau\theta_{nd})}{\tau\left[\rho_b\theta_d - \theta_{nd}(1 - \rho)\right]} \quad (18)$$

The right-hand side of (15) is larger than the right-hand side of (18) if $\frac{\theta_{nd}}{\theta_d} \geq \frac{1 - \rho_b}{\rho}$. Thus, if $\frac{\theta_{nd}}{\theta_d} \geq \frac{1 - \rho_b}{\rho}$, $\tau_{oc} \geq \tau_{rb}$, where τ_{rb} is the value of τ that solves (18) at equality. Accordingly, if $\frac{\theta_{nd}}{\theta_d} \geq \frac{1 - \rho_b}{\rho}$, the ranking-based punishment scheme $(s_1 = s_l, s_2 = 0, \tau_{rb})$ requires less doping tests to attain the no-doping equilibrium than the corresponding IOC regulations $(s = s_l, \tau_{oc})$ and this is attained by just setting the punishment for the loser to zero.

5   Conclusion

The paper has analyzed the incentives provided by performance-enhancing drugs and anti-doping regulations in sport contests. When there are no anti-doping regulations, the model shows some interesting effects of doping. For example, in the mixed-strategy equilibrium, the favored player is more likely to use performance-enhancing drugs than is the underdog and, yet, for some parameter values, he is less likely to win the game with a doping opportunity than without. These effects may be also relevant in other economic circumstances, such as rent-seeking or promotion tournaments, when there is scope for cheating to advance one's interests.

The paper then characterizes the equilibrium under IOC regulations and shows that IOC regulations are not individual rational, incentive compatible, and implement the no-doping equilibrium in all circumstances, i.e., they are not a perfect mechanism. For some parameter values, these regulations cannot attain the no-doping equilibrium because the punishment needed to prevent the weak player's deviation from the no-doping strategy profile violates her participation constraint.

The paper then develops a ranking based sanction scheme that is a perfect mechanism and compares the cost of this scheme with the cost of the anti-doping regulations adopted by the IOC. Two results emerge from this comparison: First, ranking-based punishment schemes are less costly than IOC regulations because less tests are needed to attain the no-doping equilibrium. Second, this last result relies on regulators' ability to increase punishments for the winner above the punishment set by the IOC, which might not always be feasible. However, the model shows that even without increasing the punishment for the winner (by just setting punishment for the loser to zero) there are parameter values where such a ranking-based punishment scheme requires less test to attain the no-doping equilibrium than the corresponding IOC regulations.

A question deserving future research is whether these results are robust in a model with $n \geq 2$ players and $m = n$ prizes (positions 1 to n in a contest). This research could follow along the lines of Clark and Riis (1998) who consider a rent-seeking model where more than one prize is at stake or of Rosen (1986) who studies the incentive properties of prices in sequential elimination tournaments.

The paper has analyzed sanction schemes based on the negative list of banned substances. Bird and Wagner (1997) and Wagner (1993) have fundamentally criticized the use of a negative list. They argue that this list creates strong incentives to develop new performance-enhancing drugs. As an alternative, the authors propose anti-doping regulations based on a drug diary. No substances are forbidden, but each athlete is required

to record all drugs taken in a diary. The athletes are randomly tested for substances not mentioned in the diary. If a substance not mentioned is found, the athlete is sanctioned. This proposal has its virtues because it encourages honesty, transparency, and equal access to doping information. Because their proposal requires doping tests and sanctions as well, my guess is that a ranking-based sanction scheme in the drug diary approach would have the same benefits as in the negative list approach.

Appendix

Derivation of δ_b and δ_a Suppose performance of athlete i during the contest is an independent normally distributed random variables γ_i where $\gamma_i \sim N(\mu_i, \sigma^2)$ if the athlete is not doped and $\gamma_i \sim N(\mu_i + e, \sigma^2)$ if the athlete is doped. For example, μ_i could be the expected distance that a 100-m runner runs in ten seconds, or, it could be the expected number of aces and winners that a tennis player scores during a match if the player is not doped, and $\mu_i + e$ is the expected performance if the player is doped. Note that doping increases the performance of both athletes equally. Nevertheless, if the players' talents are unequal, the incentives to take performance-enhancing drugs are unequal, in the sense that performance-enhancing drugs affects the winning probabilities asymmetrically.

To see this assume that player a is at least as talented as player b, i.e., $\mu_a \geq \mu_b$, and define $\gamma = \gamma_a - \gamma_b$ and $\mu = \mu_a - \mu_b$. Use of doping affects the distribution of γ in the following way: If both athletes are doped or if they are not, $\gamma \sim N(\mu, 2\sigma^2)$, if only player a dopes, $\gamma \sim N(\mu + e, 2\sigma^2)$, and if only players b dopes, $\gamma \sim N(\mu - e, 2\sigma^2)$. Player a wins the competition if $\gamma \geq 0$. Accordingly, the winning probabilities are $\rho = P[\gamma \geq 0]$ where $\gamma \sim N(\mu, 2\sigma^2)$, $\rho_a = P[\gamma \geq 0]$ where $\gamma \sim N(\mu + e, 2\sigma^2)$, and $\rho_b = P[\gamma \leq 0]$ where $\gamma \sim N(\mu - e, 2\sigma^2)$.

To proceed define $\delta_a = \rho_a - \rho$ and $\delta_b = \rho_b - (1 - \rho)$ where δ_i specifies the *effectiveness* of performance-enhancing drugs in the sense that it measures the increase of player i's winning probability when only player i dopes. If $\mu_a > \mu_b$, $\delta_b > \delta_a$ because the probability density function of γ is increasing at $\gamma = 0$. For the normal distribution, $\delta_a = \left(Erf\left(\frac{\mu + e}{2\sqrt{2}\sigma}\right) - Erf\left(\frac{\mu}{2\sqrt{2}\sigma}\right)\right)2^{-1}$ and $\delta_b = \left(Erf\left(\frac{e - \mu}{2\sqrt{2}\sigma}\right) + Erf\left(\frac{\mu}{2\sqrt{2}\sigma}\right)\right)2^{-1}$, where $Erf(.)$ is the error function. Thus, performance-enhancing drugs affect the winning probabilities asymmetrically: They are more effective for weak players.

Proof of Lemma 1 Consider, first, the PSE $\{\alpha = 1, \beta = 1\}$. It exists if $\rho w - c \geq (1 - \rho_b)w$ and if $(1 - \rho)w - c \geq (1 - \rho_a)w$, i.e., if $\delta_b \geq \delta_a \geq \widetilde{c}$. The PSE $\{\alpha = 1, \beta = 0\}$ exists if $\rho_a w - c \geq \rho w$ and if $(1 - \rho_a)w \geq (1 - \rho)w - c$, i.e., $\delta_a = \widetilde{c}$. The PSE $\{\alpha = 0, \beta = 1\}$ exists if $(1 - \rho_b)w \geq \rho w - c$ and if $\rho_b w - c \geq (1 - \rho)w$, i.e., $\delta_b = \widetilde{c}$. The PSE $\{\alpha = 0, \beta = 0\}$ exists if $\rho w \geq \rho_a w - c$ and if $(1 - \rho)w \geq \rho_b w - c$, i.e., $\delta_a \leq \delta_b \leq \widetilde{c}$. In the MSE the doping probabilities are given by $\alpha^* = \frac{\delta_b - \widetilde{c}}{\delta_b - \delta_a}$ and $\beta^* = \frac{\widetilde{c} - \delta_a}{\delta_b - \delta_a}$ where $\alpha^*, \beta^* \in (0, 1)$ if $\delta_a \leq \widetilde{c} \leq \delta_b$. ∎

Proof of Lemma 2 Note, first, that $\widehat{\rho}$ is the value of ρ that solves $\sigma_2 = \sigma_3$:

$$\widehat{\rho} = \frac{(\theta_d - \theta_{nd})^2 + (1 - \theta_d)(1 - \theta_{nd})}{(1 - \theta_d)^2 + (1 - \theta_{nd})^2} \tag{19}$$

20

Manipulation of (19) reveals that $1 \geq \widehat{\rho} \geq 1/2$. Manipulations of equations (1)-(4) reveal that if $\rho \geq \widehat{\rho}$, $\sigma_1 \leq \sigma_2 \leq \sigma_3 \leq \sigma_4$.

Consider, first, the PSE $\{\alpha = 1, \beta = 1\}$. It exists if $a_{11} \geq a_{21}$ and $b_{11} \geq b_{12}$, i.e., $s \leq \sigma_1 \leq \sigma_3$. The PSE $\{\alpha = 1, \beta = 0\}$ exists if $a_{12} \geq a_{22}$ and $b_{12} \geq b_{11}$, i.e., $\sigma_1 \leq s \leq \sigma_2$. The PSE $\{\alpha = 0, \beta = 1\}$ exists if $a_{21} \geq a_{11}$, $b_{21} \geq b_{22}$, i.e., $\sigma_3 \leq \rho \leq \sigma_4$ and the PSE $\{\alpha = 0, \beta = 0\}$ exists if $a_{22} \geq a_{12}$ and $b_{22} \geq b_{21}$, i.e., $s \geq \sigma_4 \geq \sigma_2$.

In the mixed-strategy equilibrium, the doping probabilities are given by

$$\alpha = \frac{b_{22} - b_{21}}{b_{11} - b_{12} + b_{22} - b_{21}} \text{ and } \beta = \frac{a_{22} - a_{12}}{a_{11} - a_{21} + a_{22} - a_{12}}.$$

The first thing to note is that $b_{11} - b_{12} - b_{21} + b_{22} \leq 0$. Thus, $\alpha \in (0, 1)$ if $b_{22} < b_{21}$ and if $b_{11} < b_{12}$, i.e., $\sigma_1 < s < \sigma_4$. $\beta \in (0, 1)$ if either $a_{22} > a_{12}$ and $a_{11} > a_{21}$, i.e., $\sigma_2 < s < \sigma_3$ or if $a_{22} < a_{12}$ and $a_{11} < a_{21}$, i.e., $\sigma_3 < s < \sigma_2$. This case is excluded by the assumption $\rho \geq \widehat{\rho}$. Thus, a MSE exists if $\sigma_2 < s < \sigma_3$.[16] ∎

Proof of Proposition 1 The proof is a direct consequence of the discussion in the text. ∎

Proof of Proposition 2 It is first shown that for all parameter values and punishments $s_2 = 0$ and $s_1 = \frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d}$ the strategy profile $\{nd, nd\}$ is a Nash equilibrium that is incentive compatible and individual rational. For this purpose assume $s_2 = 0$. Then, condition (5) is satisfied if $s_1 \leq \frac{(1 - \theta_{nd})^2}{\theta_{nd}} + \frac{(1 - \theta_{nd})}{\rho}$ and if $s_1 \leq \frac{(1 - \theta_{nd})^2}{\theta_{nd}} + \frac{(1 - \theta_{nd})}{(1 - \rho)}$. Condition (6) is satisfied if $s_1 \leq \frac{(1 - \theta_{nd})^2}{\theta_{nd}}$. Thus, for any value of ρ, (6) is binding.

A no-doping equilibrium exists if $a_{22} \geq a_{12}$ and if $b_{22} \geq b_{21}$. Inequality $b_{22} \geq b_{21}$ is satisfied if

$$s_1 \geq \frac{(1 - \theta_{nd})\left[(1 - \theta_d)\rho_b - (1 - \theta_{nd})(1 - \rho)\right] - \widetilde{c} - \theta_{nd}(\theta_d - \theta_{nd})}{\theta_d \rho_b - \theta_{nd}(1 - \rho)} \tag{20}$$

The right-hand side of (20) is increasing in ρ and in ρ_b. Thus, player $b's$ incentive to deviate from the no-doping equilibrium is largest if he is very weak ($\rho = 1$) and if $\rho_b = 1$. Thus, to prevent b's deviating from the no-doping strategy profile $\{nd, nd\}$ for all ρ and ρ_b we need

$$s_1 \geq \frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d}. \tag{21}$$

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[16]To be complete, if $\rho \leq \widehat{\rho}$, the order of the bounds is $\sigma_1 \leq \sigma_3 \leq \sigma_2 \leq \sigma_4$. For low values of s we again have a pure doping equilibrium $\{d, d\}$. As we increase the punishment the equilibrium switches to $\{d, nd\}$ at $s = \sigma_1$. As we increase s even more, at $s = \sigma_3$, we attain a region with multiple equilibria where the set of equilibria is $\{\{d, nd\}, mix, \{nd, d\}\}$. Then, at $s = \sigma_2$ we attain equilibrium $\{nd, d\}$, and, finally, at $s = \sigma_4$ the no-doping equilibrium $\{nd, nd\}$ is attained.

Inequality $a_{22} \geq a_{12}$ is satisfied if

$$s_1 \geq \frac{(1 - \theta_{nd}) \left[(1 - \theta_d) \rho_a - (1 - \theta_{nd}) \rho \right] - \widetilde{c} - \theta_{nd} (\theta_d - \theta_{nd})}{\theta_d \rho_a - \theta_{nd} \rho} \tag{22}$$

The right-hand side of (22) is decreasing in ρ and decreasing in ρ_a if $[\theta_d - \theta_{nd}] (1 - \theta_{nd}) \rho < \theta_d [\widetilde{c} + \theta_{nd} (\theta_d - \theta_{nd})]$ (case 1) and increasing in ρ_a if $[\theta_d - \theta_{nd}] (1 - \theta_{nd}) \rho > \theta_d [\widetilde{c} + \theta_{nd} (\theta_d - \theta_{nd})]$ (case 2). Consider first case 1: In this case player $a's$ incentive to deviate from the no-doping strategy profile $\{nd, nd\}$ is largest if $\rho = 0.5$ and if $\rho_a = 0.5$. To prevent a's deviation for all ρ and ρ_a we need

$$s_1 \geq \frac{(1 + \theta_{nd}) [\theta_{nd} - \theta_d] - 2\widetilde{c}}{\theta_d - \theta_{nd}} < 0 \tag{23}$$

Consider next case 2: In this case player $a's$ incentive to deviate from the no-doping strategy profile $\{nd, nd\}$ is largest if $\rho = 0.5$ and if $\rho_a = 1$. Thus, we need

$$s_1 \geq \frac{1 - 2\theta_d + \theta_{nd}^2 - 2\widetilde{c}}{2\theta_d - \theta_{nd}} \tag{24}$$

One can show that inequality (21) is binding. Thus, any value of s_1 such that $\frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d} \leq s_1 \leq \frac{(1 - \theta_{nd})^2}{\theta_{nd}}$ satisfies (5), (6), $a_{22} \geq a_{12}$, and $b_{22} \geq b_{21}$. Such a value exists because $\frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d} < \frac{(1 - \theta_{nd})^2}{\theta_{nd}}$. This establishes the existence of a no-doping equilibrium for all values of the game that is incentive compatible and individual rational. The punishment $s_1 = \frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d}$ is the smallest sanction that prevents the weak player to deviate from the no-doping equilibrium.

It remains to show that the punishments $s_2 = 0$ and $s_1 = \frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d}$ also satisfy either $b_{12} \geq b_{11}$ or $a_{21} \geq a_{11}$. Inequality $b_{12} \geq b_{11}$ holds if

$$s_1 \geq \frac{(1 - \theta_d) \left[(1 - \theta_d) (1 - \rho) - (1 - \theta_{nd}) (1 - \rho_a) \right] + (\theta_{nd} - \theta_d)\theta_d - \widetilde{c}}{\theta_d (1 - \rho) - \theta_{nd} (1 - \rho_a)} \tag{25}$$

The right-hand side of (25) is decreasing in ρ and increasing in ρ_a. Thus, the right-hand side is largest if $\rho = 0.5$ and if $\rho_a = 1$ and we need

$$s_1 \geq \frac{(1 - \theta_d)(1 - \theta_d) - 2(\theta_d - \theta_{nd})\theta_d - 2\widetilde{c}}{\theta_d} \tag{26}$$

One can show that the right-hand side of (21) is strictly larger than the right-hand side of (26). This establishes that the sanctions $s_2 = 0$ and $s_1 = \frac{1 - \theta_d - \theta_{nd} + \theta_{nd}^2 - \widetilde{c}}{\theta_d}$ satisfy (5), (6) and (7).∎

Literature

Bird, E., Wagner, G., 1997. *Sport as a common property resource: A solution to the dilemmas of doping.* Journal of Conflict Resolution 41, 749-766.

Breivik, G., 1987. *The doping dilemma: Some game theoretical and philosophical considerations.* Sportwissenschaft 17, 83-94.

Chatlin, D., 1999. *Overview and drug monograph.* International Olympic Committee (ed.), Doping: An IOC White Paper, Lausanne.

Clark, D., Riis, C., 1998. *Influence and the discretionary allocation of several prices.* European Journal of Political Economy 14, 605-625.

De Merode, A., 1999. *Introduction and background.* International Olympic Committee (ed.), Doping: An IOC White Paper, Lausanne.

Dixit, A., 1987. *Strategic behavior in contests*, American Economic Review 77, 891-898.

Fudenberg, D., Tirole, J., 1991. *Game Theory.* MIT-Press, Cambridge Mass./London.

Heinemann, K., 1999: *The economic colonialisation of sport: The loss of autonomy and doping.* Universität Hamburg, Institut für Soziologie, Germany. Available at http://blues.uab.es/olympic.studies/doping/heinemann.htm.

Nitzan, S., 1994. *Modelling rent-seeking contests*, European Journal of Political Economy 10, 41-60.

Reiter, H., et al., 1994. *Bericht der Unabhängigen Dopingkommission des Deutschen Sportbundes (DSB) und des Nationalen Olympischen Komitees (NOK).* In Bette, K.H. (ed.), Doping im Leistungssport - Sozialwissenschaftlich beobachtet, Stuttgart: Nagelschmid, 191-231.

Rosen, S., 1986. *Prizes and incentives in elimination tournaments*, American Economic Review 76, 701-15.

Segura, J., 1999. Sports drug testing: Medical and regulatory aspects. Available at http://blues.uab.es/olympic.studies/doping-segura.htm.

Wagner, G., 1993. *The triple doping dilemma: An economic analysis of anti-doping regulations.* In Gebauer, G. (ed.), Die Aktualität der Sportphilosophie, St. Augustin, 1143-158.