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Abstract

Empirical research confirms the existence of volatility spillovers across national stock markets. However, the models in use are mostly statistical ones. Much less is known about the actual transmission mechanisms; theoretical literature is scarce, and so is empirical work trying to estimate specific theoretical models. Some economic theory founded tests for such spillovers have been developed for non-overlapping markets; this institutional set up provides a way around the problems of estimating a system of simultaneous equations. However, volatility spillovers across overlapping markets might be as important a phenomenon as across non-overlapping markets. Building on recent advances in econometrics of identifying structural vector autoregressive models, this paper proposes a way to estimate an existing signal-extraction model that explains volatility spillovers across simultaneously open stock markets. Furthermore, a new empirical test for detection of such spillovers is derived. As an empirical application, the theoretical model is fitted to daily data of eurozone stock markets in years 2010--2011. Evidence of volatility spillovers across the countries is found.

JEL Classification: C12, C30, D82, G14, G15

Keywords: Volatility transmission; financial contagion; SVAR identification; hypothesis testing; stock markets; euro debt crisis.

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1   Introduction

Usually, during periods of financial distress, there is a nearly contemporaneous increase in financial market volatilities in a multitude of countries. Literature on volatility spillovers claims that one reason for such global and simultaneous volatility clusters is transmission of volatility from one country to the others. Since the seminal papers by Engle, Ito, and Lin (1990) and by Hamao, Masulis, and Ng (1990), volatility spillovers have been extensively studied and, especially, different GARCH model specifications have been popular.[1]

A quintessential research question has been: Does a rise in the price volatility in one country affect the volatilities in other countries? An alternative hypothesis is that the volatilities are country specific phenomenons and determined by countries' fundamentals. Most of the empirical studies find evidence of the inter-market dependencies in market volatilities. However, in a contrast to the abundance of empirical studies, the theoretical literature on causes of these volatility dependencies is much more limited (Soriano and Climent (2006)). Literature is especially scarce on what it comes to papers trying to estimate theoretical models. It is there where this papers provides its contribution.

In this paper I present a method to estimate the theoretical signal-extraction model of King and Wadhwani (1990). In the model, the transmission of volatility is ultimately a consequence of information asymmetries between rational investors; uninformed investors use observable price changes as signals in their efforts in trying to infer informed investors' private information. King and Wadhwani were unable to identify–and hence to estimate–their model. And, to the best of my knowledge this is the first paper that tries to estimate it.

The estimation is based on the idea of interpreting the King and Wadhwani model as a structural vector autoregressive (SVAR) model. Then, by using a recent identification method by Lanne and Lütkepohl (2010), we are able estimate the model's volatility transmission parameters. However, in order to be able to use this identification method, we need to augment the King and Wadhwani model with an additional assumption concerning the distribution of the error terms. As it is discussed in the paper, this assumption is fully consistent with the assumptions of the King and Wadhwani model.

Then, based on the theoretical model and the applied identification method, a new test for the volatility transmissions is derived. Along the way, an issue with the Lanne and Lütkepohl identification method is discussed. More precisely, it is shown that the identification method, as such, only guarantees a partial estimation of the structural model parameters. This, however, is shown to be enough for the validity of our volatility transmission test. But, the full estimation of the parameters requires additional information. These requirements are also discussed, and a method to for the full estimation is proposed.

This paper most closely relates to the few empirical studies that try to estimate theoretical models explaining the volatility spillovers. Especially, Lin, Engle, and Ito (1994) present and estimate a signal-extraction model that closely resemble that of King and Wadhwani.[2] However, they consider stock markets

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[1]Soriano and Climent (2006) provide an extensive survey on volatility transmission literature. Also, Hong (2001) shorty reviews this literature. Savva (2009) reviews the most popular GARCH model specifications in use.

[2]Actually, Lin, Engle, and Ito (1994) present two models that they call aggregate-shock model and signal-extraction model, where the first one is technically closer to the King and

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(New York and Tokyo) that do not overlap in trading. Once, one is interested in analyzing volatility transmission across overlapping markets with simultaneous trading hours, for example inside the eurozone, the Lin, Engle and Ito estimation method is not applicable as such.

In addition, this paper is related to the fast growing literature on financial contagion. The King and Wadhwani model can be interpreted as a contagion model. This relation is shortly explored later. Also, the paper contributes to the literature on SVAR model identification as it is pinpointed on which assumption the uniqueness of the Lanne and Lütkepohl identification method especially relies on. We also consider an exogenous source of information that can help us to justify a given assumption.

Finally, two prevailing themes in the finance literature concerning the effects of news (or information) on stock markets are (1) transmission on information across national borders and (2) filtering of news in order to extract relevant information. This paper encompasses both of these two themes. In the related literature, Wongswan (2006) shows that information from major economies (the United States and Japan) gets transmitted to smaller economies (Korea and Thailand) and has short-lived effect on the stock market volatilities there. The empirical results of this paper show that information gets transmitted also between small countries stock markets. Concerning the filtering of news, Groß-Klußmann and Hautsch (2011), when analyzing effects of randomly arriving company news, find that only relevant information (news) affects the volatility of company's shares. From the point of view of this paper, especially interesting in their analysis is that the news data they use is filtered in real time by a software that uses linguistic patterns to label news either positive, neutral or negative, and either relevant or not. Their study illustrates the important difference between a piece of news as such and its information content. That difference is an essential theme in the King and Wadhwani model.

The rest of the paper is organized as follows. Section 2 presents the original King and Wadhwani model. Section 3 discusses the additional assumptions that we need to make in order to be able to estimate the model. The section also shows how to test for the volatility transmission effects and how, actually, to estimate the model. As an empirical application, section 4 tests for the volatility transmissions and fits the model to the eurozone stock market data. Finally, section 5 concludes. Appendices provide both formal proofs of the results that are used in section 3 and details on the data that is used in section 4.

2   Model of volatility transmissions

The King and Wadhwani (1990) model of volatility transmissions (henceforth the KW model) is a variant of the Grossman and Stiglitz (1980) model on the impossibility of prices in a competitive equilibrium to fully reveal all information. Before presenting the general version of the KW model, the two country case is considered. The model exposition follows quite closely that of the original article.

Wadhwani model. Basically the difference of their two models comes from assuming news arrival process either possessing conditional heteroskedasticity or not. Their analysis supports there being ARCH-effects in news arrival process.

2.1 The KW model with two countries

Assume two countries with one stock market in each, and risk-neutral investors in both countries with no trading in stocks across the borders[3]. It is assumed that both markets are continuously open around the clock. The model also assumes that news in both countries consists of two components: systematic information u that globally affects the equity values, and idiosyncratic information v that has relevance only to local equity values.

Both of these information components come in two different types, depending on whether the piece of information is observed in country 1 or 2. Hence, if $\eta_t^{(i)}$ denotes the news in country i at time t, we have the following decomposition

$$\eta_t^{(i)} = u_t^{(i)} + v_t^{(i)} \tag{1}$$

for $i = 1, 2$. Superscripts denote in which country, 1 or 2 respectively, the information is observed[4]. All the four information variables are assumed to be uncorrelated of each others and follow white noise processes. The assumption that $u^{(1)}$ and $u^{(2)}$ are uncorrelated implies a restriction that news affecting stock market valuations in both countries are never (correctly) interpreted simultaneously but always only in one of the countries.[5] Hence in the model domestic investors are always the informed ones whereas the foreign investors are uninformed (section 2.1.1 discusses the assumption on information asymmetry).

A change in the stock market indexes between time $t-1$ and t is a function of the news released during that time period. Given that foreign investors never directly observe the domestic systematic information, they need to form expectations. Let E_i denote the expectation operator of country i investors conditional on all information observed in market i at time t. The stock market price indexes will then follow the following equations:

$$\Delta S_t^{(1)} = u_t^{(1)} + \alpha_{12} E_1\left(u_t^{(2)}\right) + v_t^{(1)}, \tag{2}$$

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[3]According to the authors, if we allowed risk neutral investors with possibility of arbitrage between national stock markets, in the equilibrium all information would be revealed. Prohibiting international trade in stocks allows a non-fully-revealing equilibrium (equilibrium with information asymmetries) in a model with risk-neutral investors. This simplifies the model's structure, making the price changes equations linear. Alternatively, one could permit international trade in stocks and obtain the non-fully-revealing equilibrium by assuming risk-averse investors. This would however complicate the model structure with little additional insights.

[4]Decomposition 1 is similar to that of Grossman and Stiglitz (1980) who assume that the return u of a risky asset is decomposable to $u = \theta + \varepsilon$. Both θ and ε are random variables but θ is observable at a cost c and ε is unobservable. In the KW model $u_t^{(i)}$–and hence also $v_t^{(i)}$–is always observable to the investors of country i but never to those of country j.

[5]In the era of modern information technology and international news agencies, it might feel hard to accept an assumption that foreign investors would not be able to observe information as well as domestic investors–surely news are widespread almost instantaneously. King and Wadhwani point out that there is a difference between *news* in the media and *information* as an assessment on consequences of the news to the equity valuations. This type of valuation assessment is not costless and some investors might be better prepared to perform it than others who may find it less costly to try to infer the new valuations from the market price changes. Alternatively, one could argue that some (institutional) investors are specialized to specific regions and, hence, possess better technical and informational capabilities to infer relevant information from regional specific news. In such a case–in a contrast to what is said in the main text, it would probably be large foreign investors who are better equipped to analyze information. This is exactly what Bailey, Mao, and Sirodom (2007) argue based on analysis on Thai and Singaporean stock markets.

$$\Delta S_t^{(2)} = \alpha_{21} E_2 \left(u_t^{(1)} \right) + u_t^{(2)} + v_t^{(2)}, \tag{3}$$

where $\Delta S_t^{(i)}$ denotes the percentage return in country i stock market between time $t-1$ and t measured by the change in the logarithm of the price index. Parameter α_{ij} controls for the importance of systemic information revealed in market j on to the equity prices in market i.

It is assumed that the only information available to the foreign investors about the domestic systematic information is the contemporaneous domestic price change. For example, consider the market 1 investors forming their expectations on market 2 systematic information $u^{(2)}$. Although the unconditional expectation $E(u_t^{(2)})$ is zero,

$$E_1 \left(u_t^{(2)} \right) \neq 0$$

conditional on a nonzero realization of $\Delta S_t^{(2)}$. The price change provides information to market 1 investors about the information observed in market 2. However, because the price change $\Delta S_t^{(2)}$ is a function of both systematic information $u^{(2)}$ and idiosyncratic information $v^{(2)}$, the signal to market 1 is contaminated by the country 2 idiosyncratic information. In addition, the market 1 investors understand that, simultaneously to them forming conditional expectations on $u_t^{(2)}$, the market 2 investors will form conditional expectations on $u_t^{(1)}$. Hence, the market 1 investors need to adjust their expectations accordingly. Symmetric reasoning applies to the market 2 investors.

The KW model assumes that the structure of the model is of common knowledge. Then the following minimum-variance estimators provide then the solution to the investors' signal extraction problem:

$$E_i \left(u_t^{(j)} \right) = \lambda_j \left[\Delta S_t^{(j)} - \alpha_{ji} E_j \left(u_t^{(i)} \right) \right],$$

where

$$\lambda_j = \sigma_{u^{(j)}}^2 / \left(\sigma_{u^{(j)}}^2 + \sigma_{v^{(j)}}^2 \right)$$

for $i, j = 1, 2$ and $i \neq j$, and σ_x^2 denotes the (known) variance of x. Substituting these estimators into the equations (2) and (3) and using the combined news $\eta_t^{(i)}$ notation of equation (1) yields us

$$\Delta S_t^{(1)} = (1 - \alpha_{12} \alpha_{21} \lambda_1 \lambda_2) \eta_t^{(1)} + \alpha_{12} \lambda_2 \Delta S_t^{(2)}, \tag{4}$$

$$\Delta S_t^{(2)} = (1 - \alpha_{12} \alpha_{21} \lambda_1 \lambda_2) \eta_t^{(2)} + \alpha_{21} \lambda_1 \Delta S_t^{(1)}. \tag{5}$$

Because the α and λ parameters are not separately identifiable, let us define

$$\beta_{ij} = \alpha_{ij} \lambda_j$$

for $i, j = 1, 2$ and $i \neq j$. Using this notation and solving the system of equations (4)–(5) with respect to the price changes, we finally get the equilibrium laws of motions for the stock market returns as a function of the combined news:

$$\Delta S_t^{(1)} = \eta_t^{(1)} + \beta_{12} \eta_t^{(2)}, \tag{6}$$

$$\Delta S_t^{(2)} = \beta_{21} \eta_t^{(1)} + \eta_t^{(2)}. \tag{7}$$

Then, for example, the volatility of the market returns in country 1 becomes

$$\text{Var}\left(\Delta S_t^{(1)}\right) = \sigma_{\eta^{(1)}}^2 + (\beta_{12})^2\,\sigma_{\eta^{(2)}}^2. \tag{8}$$

So, as long as $\beta_{12} \neq 0$ volatility in country 2 gets transmitted to country 1.

Note that, due to the information asymmetries, it is the combined (total) news released in market i, $\eta_t^{(i)}$, that affects the price index volatility in market j. Hence, the idiosyncratic shocks $v_t^{(1)}$ and $v_t^{(2)}$, that are country specific volatility factors, get also transmitted across the borders and increase the volatility of price changes in both countries. This is in contrast to a regime where prices fully reveal all information–and hence there are no information asymmetries (for details see the original paper by King and Wadhwani). With full information the "excess" volatility, that idiosyncratic shock $v_t^{(i)}$ creates, is not transmitted across borders.

It is worthwhile to consider what $\beta_{12} = 0$ would imply. Remember that the only reason why idiosyncratic shocks get transmitted across the countries is that we assume there being common shocks $(u^{(i)})$ that affect the valuations in both countries but are not observable by all investors. From the definition of β_{12} we see that, once we assume $\sigma_{u^{(2)}}^2 \neq 0$, $\beta_{12} = 0$ if and only if $\alpha_{12} = 0$. The "systematic" information $u^{(2)}$ has no effect on the equity values in the country 1 and it actually becomes country 2 specific news. Alternatively, $\sigma_{u^{(2)}}^2 = 0$ would also mean that news in country 2 consist only of idiosyncratic information. In both cases, the news in country 2 has no economic value to country 1's stock market valuations.

The authors were not, however, able to estimate the transmission parameters β_{12} and β_{21}. The reason is that the model in equations (6)–(7) is not identifiable[6]; the variances and the covariance of $\Delta S_t^{(1)}$ and $\Delta S_t^{(2)}$ provide us only three equations while there are four parameters $(\beta_{12}, \beta_{21}, \sigma_{\eta^{(1)}}^2, \sigma_{\eta^{(2)}}^2)$ to be estimated. In order to surpass this identification obstacle, in section 3, we will augment the structural model with an assumption concerning the distribution of the total news vector $\boldsymbol{\eta}_t = (\eta_t^{(1)}, \eta_t^{(2)})'$. As discussed more in detail at that occasion, this assumption is consistent with the assumptions of the KW model.

2.1.1 On the assumption of information asymmetry

In the KW model it is assumed that only the domestic investors are able to (correctly) detect the information content of news about their home country. Is there any empirical support for such an assumption of information asymmetries between domestic and foreign investors?

Frankel and Schmukler (1996) analyze differences in the Mexican stock market valuations and the valuations of Mexican closed-end country funds (traded

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[6]Although King and Wadhwani (1990) are unable to estimate their full model, they utilize overlapping opening hours of London and New York stock exchanges to test for implications of their model. Their structural model implies that the opening of NYSE should be visible as a jump in the London price index as the investors (traders) in London try to infer US (New York) specific information from the opening prices of NYSE. They find evidence of such a jump. And, what is still more interesting and fully in-line with the assumptions of the model, it seems that for the traders in London, more important than the main US macro news themselves–these are released an hour before New York opens–is how the New York trades infer these US specific news.

in NYSE) and argue that around the Mexican devaluation in December 1994 local investors were better informed, reacted first to negative local news, than international investors and were the first ones to turn more pessimistic in their expectations. King, Sentana, and Wadhwani (1994) find that changes "observable" factors summarizing information of several macroeconomic variables explain quite poorly time-variation in covariances of national stock markets. Much better explanation is provided by "unobservable" common factors. The authors propose that investor sentiment could be one unobservable common explanatory variable. Then, one can speculate, that some of the variation could be a result of time-variation in the importance the international markets (foreign investors) put on the systematic information of a specific country. This would lead into time-variating β coefficients of the KW model (something that is not allowed here).

More recently, Chen and Choi (2012) analyze the stock market values of 56 Canadian companies listed both in Toronto Stock Exchange (TSX) and New York Stock Exchange (NYSE). They find evidence of local (TSX) investors being better informed than the foreign (NYSE) investors. This information asymmetry, they argue, explains the small share price premiums detected in the NYSE prices over the TSX prices of these companies. Also, Chan, Menkveld, and Yang (2008) find evidence of information asymmetries between Chinese and foreign investors being an explanation of the price differences between the locally owned (A-)shares and the foreign own (B-)shares of Chinese companies.

2.1.2 The KW model as contagion model

In the KW model, because of the asymmetric information, idiosyncratic shocks get transmitted across borders. Also, as the authors show in their paper, the correlation between the market returns is higher in the KW model than in a comparable model with full information. For these reasons, the authors label the model that was presented in section 2.1 as "contagion model". This is in-line with the contagion literature where many authors[7] define contagion as spreading of an idiosyncratic shock–or crisis–to other countries.

Comparable to the KW model, Kodres and Pritsker (2002) analyze contagion in a informed–uniformed investor set up. As in the KW model, in their model contagion is a consequence of uninformed investors trying to infer informed investors' private information from price changes. One interesting insight of their analysis is worth to emphasize: according to their analysis, the magnitude of contagion–or volatility transmission in the KW model's concept–depends on the share of the informed investors over the uninformed ones. Hence, for a given number of the uninformed investors, increasing the amount of the informed investors, in the limit, makes contagion vanish. The intuition is that, as the relative share of the informed investors increases, the assets prices will better reflect their private information and this makes the price system more informative.

Both, King and Wadhwani (1990) and Kodres and Pritsker (2002), consider the share of the informed investors over the uninformed ones as an exogenous

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[7]For example, Kaminsky and Reinhart (2000); Kodres and Pritsker (2002); Corsetti, Pericoli, and Sbracia (2005); Dungey, Fry, Gonzalez-Hermosillo, and Martin (2005); Pesaran and Pick (2007). For surveys on contagion literature, see for example Pericoli and Sbracia (2003); Dungey, Fry, Gonzalez-Hermosillo, and Martin (2005); Dornbusch, Park, and Claessens (2000).

variable. However, at least to some extend, due to costs of acquiring information (examples of such costs are individual time and effort, payments for professional analysts, and so on), some investors might actually choose to stay ignored about macroeconomic details of a specific country. Calvo and Mendoza (2000) analyze investors' incentives to pay for such information. In their model an investor has a choice either to pay a cost to learn country's fundamentals or to remain uninformed. In the latter case she simply tracks a generic global stock market portfolio. Hence, the share of the uninformed investors over the informed ones becomes an endogenous variable.

The authors show that, once there are exogenous information costs or binding institutional or legislative constraints on short-selling, it is possible that the globalization of financial markets induces a rational investor to stay ignored about (macroeconomic) details of any specific country. The intuition is that, for example, the constraints on short-selling limit the opportunities of the informed investors. This decreases the expected value of information which, in its turn, decreases the incentives to pay for it. Meanwhile, however, more global financial markets permit investors to more easily, by mimicking a generic market portfolio, take advantage of the benefits of diversification.

2.2 Generalization of the KW model

The KW model generalizes easily to a multiple country case. All the time following the exposition in King and Wadhwani (1990), assume $n \geq 2$ markets. Then prices in these markets are set by the following equation (comparable to the two market case equations (2) and (3))

$$\mathbf{\Delta S}_t = \boldsymbol{\eta}_t + \mathbf{A}\mathbf{e}_t, \tag{9}$$

where $\mathbf{\Delta S}_t$ is a $n \times 1$ vector of the price changes at time t, $\boldsymbol{\eta}_t$ is a $n \times 1$ vector of the total news at time t with a typical element

$$\eta_t^{(i)} = u_t^{(i)} + v_t^{(i)}$$

depicting news released in country i, $\mathbf{A}$ is a $n \times n$ coefficient matrix with a typical element α_{ij}, $i, j = 1, \ldots, n$, and $\alpha_{ii} = 0$ for all $i = 1, \ldots, n$ (all the main diagonal elements), and finally $\mathbf{e}_t$ is a $n \times 1$ vector of the conditional expectations on the systemic informations $u^{(i)}$, $i = 1, \ldots, n$, held by agents in other markets $j \neq i$ at time t.[8]

The solution to the signal extraction problem is

$$\mathbf{e}_t = \boldsymbol{\Lambda}\left(\mathbf{\Delta S}_t - \mathbf{A}\mathbf{e}_t\right), \tag{10}$$

where $\boldsymbol{\Lambda}$ is a $n \times n$ diagonal matrix with parameter λ_i as the ith element on its main diagonal. Then, by combining equations (9) and (10) and solving for

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[8]Although it is not explicitly stated in King and Wadhwani (1990), and it might be evident, note that equation (9) indicates that, for example, in a three market case we would have $E_1(u_t^{(2)}) = E_3(u_t^{(2)})$. That is, conditional expectation in markets 1 and 3 about the systematic information observed in market 2 are equal. Otherwise $\mathbf{e}$ couldn't be a $n \times 1$ vector. This result is an implication of the assumption that the model structure is of common knowledge and the information components are uncorrelated.

$\mathbf{\Delta S}_t$, one gets the laws of motion of the price changes in n market setup as a function of the total news:

$$\mathbf{\Delta S}_t = (\mathbf{I}_n + \mathbf{B})\,\boldsymbol{\eta}_t \tag{11}$$

where $\mathbf{B} = \mathbf{A}\boldsymbol{\Lambda}$ is $n \times n$ matrix, and its ijth element β_{ij} is the response of the market i prices to the price changes in market j. The matrix $\mathbf{I}_n$ is $n \times n$ identity matrix. As the matrix $\mathbf{B}$ consists of the volatility transmission coefficients, a simple test for the existence of such a transmission effect, for example, from market j to market j is to test whether the element β_{ij} equals zero or not. Note that by construction the main diagonal elements β_{ii} for all $i = 1, \ldots, n$ are zero.

3 Estimation of the volatility transmission model

Consider the n country KW model. Then, the stock market prices follow equation (11). By defining $\tilde{\boldsymbol{\eta}}_t = (\mathbf{I}_n + \mathbf{B})\,\boldsymbol{\eta}_t$, we get the following simple identity

$$\mathbf{\Delta S}_t = \tilde{\boldsymbol{\eta}}_t. \tag{12}$$

This equation can be interpreted as a zero order reduced form vector autoregressive (VAR) model. Vector $\tilde{\boldsymbol{\eta}}_t$ consists of reduced form errors (here equal to price changes). Also, define $\tilde{\mathbf{B}} = (\mathbf{I}_n + \mathbf{B})$, to rewrite equation (11) equally well as

$$\mathbf{\Delta S}_t = \tilde{\mathbf{B}}\boldsymbol{\eta}_t. \tag{13}$$

This, in its turn, can be interpreted as a n variable, zero order SVAR model. Hence, the $n \times 1$ random vector $\boldsymbol{\eta}_t$ (the KW model total news vector) represents structural shocks of the underlying structural model[9].

Equations (12) and (13) give us the following equality between reduced form errors (changes in stock prices) and structural shocks (total news):

$$\tilde{\boldsymbol{\eta}}_t = \tilde{\mathbf{B}}\boldsymbol{\eta}_t. \tag{14}$$

This equality is consistent with the so-called B-model framework of SVAR models (see, e.g., Lütkepohl (2005), p.362–64) where the n-dimensional reduced form error term $(\tilde{\boldsymbol{\eta}}_t)$ depends on the n structural shocks $(\boldsymbol{\eta}_t)$ via the $n \times n$ coefficient matrix $(\tilde{\mathbf{B}})$. The fundamental question of SVAR models is how to estimate the coefficient matrix and, hence, identify the structural shocks. If we mark the covariance matrix of the reduced from errors as $\boldsymbol{\Sigma}_{\tilde{\eta}}$ and that of the structural shocks as $\boldsymbol{\Sigma}_{\eta}$ (which is by assumption diagonal), we get from equation (14)

$$\boldsymbol{\Sigma}_{\tilde{\eta}} = \tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\eta}\tilde{\mathbf{B}}'.$$

Typically, a SVAR model is normalized by assumption $\boldsymbol{\Sigma}_{\eta} = \mathbf{I}_n$ which gives us the following system of n equations:

$$\boldsymbol{\Sigma}_{\tilde{\eta}} = \tilde{\mathbf{B}}\tilde{\mathbf{B}}' \tag{15}$$

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[9]In the KW model, the news $\eta_t^{(i)}$ and $\eta_t^{(j)}$ of all countries $i, j = 1, \ldots, n$ with $i \neq j$ were assumed to be uncorrelated with each other which is consistent with the usual assumption of structural shocks also being (at least) uncorrelated.

where $\Sigma_{\tilde{\eta}}$ can be estimated consistently with standard estimation methods.

However, as the matrix $\tilde{\mathbf{B}}$ consists of $n \times n$ unknown parameters and equation (15) provides only $n(n+1)/2$ equations, extra information is needed to be able to estimate the matrix $\tilde{\mathbf{B}}$. One standard method is to use economic theory or institutional knowledge to directly restrict (to zero) sufficiently many elements of $\tilde{\mathbf{B}}$. Other methods include, for example, restricting the signs of the impulse responses of the system, or restricting long-run effects of the structural shocks on the observed variables.[10] However, most of the standard identification methods are not suitable for the SVAR model at hand. Now, the non-diagonal elements of $\tilde{\mathbf{B}}$ are the volatility transmission coefficients. Our very goal is to test whether or not some (or all) of these elements are equal to zero.

3.1 Testing the volatility transmission effects

In equation (13) the KW model is interpreted as a SVAR model, and equation (12) shows the corresponding reduced form VAR representation. Now, assume the reduced form errors $\tilde{\boldsymbol{\eta}}_t$, that is the price changes, follow a mixed-normal distribution:

$$\tilde{\boldsymbol{\eta}}_t = \begin{cases} \tilde{\boldsymbol{\eta}}_{1t} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma_1}\right) & \text{with probability} & \gamma, \\ \tilde{\boldsymbol{\eta}}_{2t} \sim N\left(\mathbf{0}, \boldsymbol{\Sigma_2}\right) & \text{with probability} & 1-\gamma. \end{cases} \tag{16}$$

Here $N\left(\mathbf{0}, \boldsymbol{\Sigma}\right)$ denotes a multivariate normal distribution with zero mean and covariance matrix $\boldsymbol{\Sigma}$. The $n \times n$ covariance matrices $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ are assumed to be distinct and $\gamma \in (0,1)$ is the mixture probability. In order to be able to identify the parameter γ one needs to assume that $\boldsymbol{\Sigma}_1 \neq \boldsymbol{\Sigma}_2$. Parts of $\boldsymbol{\Sigma}_1$ and $\boldsymbol{\Sigma}_2$ may still be identical.

The random vector $\tilde{\boldsymbol{\eta}}_t$ has zero mean and covariance matrix $\gamma \boldsymbol{\Sigma}_1 + (1-\gamma)\boldsymbol{\Sigma}_2$. This distributional assumption is fully consistent with the assumptions of the KW model in sections 2.1 and 2.2. There, it was assumed that the elements of $\boldsymbol{\eta}_t$ (total news) are non-correlated. However, the elements of $\tilde{\boldsymbol{\eta}}_t$ (stock market price changes) might well be correlated depending on whether the matrix $\tilde{\mathbf{B}}$ is diagonal or not. Or in other words, whether there is volatility transmissions across the markets or not. Also, as the distribution (16) is non-normal and given that non-normality is a general feature of financial time series, the assumption seems reasonable in this respect.

Lanne and Lütkepohl (2010) show that, given the distributional assumption (16), there exist a diagonal matrix[11] $\boldsymbol{\Psi} = \text{diag}\left(\psi_1, \ldots, \psi_n\right)$ with $\psi_i > 0$ for all $i = 1, \ldots, n$, and a nonsingular $n \times n$ matrix $\mathbf{W}$ such that $\boldsymbol{\Sigma_1} = \mathbf{WW}'$ and $\boldsymbol{\Sigma_2} = \mathbf{W}\boldsymbol{\Psi}\mathbf{W}'$. The model in equations (12) and (16) can be estimated by the method of maximum likelihood (ML). The distribution of $\boldsymbol{\Delta}\mathbf{S}_t$ can be written as (for details about deriving a conditional density for a VAR model with lagged values of dependent variable, see Lanne and Lütkepohl (2010))

$$\begin{aligned} f\left(\boldsymbol{\Delta}\mathbf{S}_t\right) &= \gamma \det\left(\mathbf{W}\right)^{-1} \exp\left\{-\tfrac{1}{2}\boldsymbol{\Delta}\mathbf{S}_t'\left(\mathbf{WW}'\right)^{-1}\boldsymbol{\Delta}\mathbf{S}_t\right\} \\ &\quad + (1-\gamma)\det\left(\boldsymbol{\Psi}\right)^{-1/2}\det\left(\mathbf{W}\right)^{-1}\exp\left\{-\tfrac{1}{2}\boldsymbol{\Delta}\mathbf{S}_t'\left(\mathbf{W}\boldsymbol{\Psi}\mathbf{W}'\right)^{-1}\boldsymbol{\Delta}\mathbf{S}_t\right\}. \end{aligned}$$

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[10]Kilian (2011) provides a good survey of the different SVAR model identification methods.

[11]Notation $X = \text{diag}\left(x_1, \ldots, x_n\right)$ means that matrix X is a $n \times n$ diagonal matrix with the first main diagonal element being x_1, the second x_2, and so on.

Collecting all the parameters into vector Θ, the log-likelihood function can be written as

$$l_T\left(\boldsymbol{\Theta}\right) = \sum_{t=1}^{T} \log f\left(\boldsymbol{\Delta}\mathbf{S}_t\right).$$

This can be maximized with standard nonlinear optimization algorithms.

As long as all the elements $\psi_i > 0$ are distinct, the matrix $\mathbf{W}$ is unique apart from changing all signs in a column. The covariance matrix of the reduced form error vector $\tilde{\boldsymbol{\eta}}_t$ can then be written as

$$\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \gamma\boldsymbol{\Sigma}_1 + (1-\gamma)\boldsymbol{\Sigma}_2 = \gamma\mathbf{W}\mathbf{W}' + (1-\gamma)\mathbf{W}\boldsymbol{\Psi}\mathbf{W}'$$

which becomes

$$\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \mathbf{W}\left(\gamma\mathbf{I}_n + (1-\gamma)\boldsymbol{\Psi}\right)\mathbf{W}'. \tag{17}$$

Comparing this with equation (15) let us choose

$$\tilde{\mathbf{B}} = \mathbf{W}\left(\gamma\mathbf{I}_n + (1-\gamma)\boldsymbol{\Psi}\right)^{1/2}, \tag{18}$$

where, as already mentioned, $\mathbf{W}$ is a nonsingular $n \times n$ matrix. Clearly, the $n \times n$ matrix $(\gamma\mathbf{I}_n + (1-\gamma)\boldsymbol{\Psi})$ is diagonal. Hence, as long as all the n elements $\psi_i > 0$ are distinct, the matrix $\tilde{\mathbf{B}}$ is (locally) unique and the structural shocks $\boldsymbol{\eta}_t$ are identified.

There is one severe limitation in a straightforward application of the identification method of Lanne and Lütkepohl: as it is formally shown in the appendix A, when equation (18) holds, we can equally well choose as our matrix $\tilde{\mathbf{B}}$ the following matrix $\hat{\mathbf{B}}$:

$$\begin{aligned}
\hat{\mathbf{B}} &= [\mathbf{W}\mathbf{P}'][\mathbf{P}(\gamma\mathbf{I}_n + (1-\gamma)\boldsymbol{\Psi})\mathbf{P}']^{1/2} \\
&= (\mathbf{W}\mathbf{P}')(\gamma\mathbf{P}\mathbf{P}' + (1-\gamma)\mathbf{P}\boldsymbol{\Psi}\mathbf{P}')^{1/2} \\
&= (\mathbf{W}\mathbf{P}')(\gamma\mathbf{I}_n + (1-\gamma)\mathbf{P}\boldsymbol{\Psi}\mathbf{P}')^{1/2} \\
&= \hat{\mathbf{W}}\hat{\boldsymbol{\Psi}}^{1/2}
\end{aligned}$$

where $\mathbf{P}$ is an arbitrary $n \times n$ permutation matrix, $\hat{\mathbf{W}} = \mathbf{W}\mathbf{P}'$, $\hat{\boldsymbol{\Psi}} = \gamma\mathbf{I}_n + (1-\gamma)\mathbf{P}\boldsymbol{\Psi}\mathbf{P}'$, and we have used fact that $\mathbf{P}\mathbf{P}' = \mathbf{I}_n$. Matrix $\mathbf{P}\boldsymbol{\Psi}\mathbf{P}'$ is diagonal with a different permutation of the elements $\{\psi_1, \ldots, \psi_n\}$ on its main diagonal than their permutation on the main diagonal of the matrix $\boldsymbol{\Psi}$. Also, in matrix $\hat{\mathbf{W}}$ columns of $\mathbf{W}$ have changed their place according to the permutation. However– and this is important, the row indexes have remained the same.

Clearly $\tilde{\mathbf{B}}$ in equation (18) is not equivalent to the matrix $\hat{\mathbf{B}}$ unless $\mathbf{P} = \mathbf{I}_n$. The permutation matrix $\mathbf{P}$ was arbitrary. All in all, there are $n!$ possible matrices $\mathbf{P}$. Hence, there are equally many possible matrices $\hat{\mathbf{B}}$ (the matrix $\tilde{\mathbf{B}}$ in equation (18) included, corresponding to $\mathbf{P} = \mathbf{I}_n$). An implication of this is that the matrix $\hat{\mathbf{B}}$ will be unique once the permutation of the elements $\{\psi_1, \ldots, \psi_n\}$ is given[12].

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[12]This can be seen in the following way: assume we have some given permutation of the main diagonal elements $\{\psi_1, \ldots, \psi_n\}$ (not necessarily in order $\mathrm{diag}\,(\psi_1, \ldots, \psi_n)$) that is a result of matrix multiplication $\mathbf{P}\boldsymbol{\Psi}\mathbf{P}'$, where $\boldsymbol{\Psi} = \mathrm{diag}\,(\psi_1, \ldots, \psi_n)$ and $\mathbf{P}$ can be identity matrix or any other permutation matrix. Then, because each resulting permutation of the elements on the main diagonal of $\mathbf{P}\boldsymbol{\Psi}\mathbf{P}'$ corresponds to some unique permutation matrix $\mathbf{P}$, if we fix the resulting order of the elements $\{\psi_1, \ldots, \psi_n\}$, we necessarily fix also the matrix $\mathbf{P}$. Then it follows that matrix $\mathbf{W}\mathbf{P}'$–and subsequently matrix $\hat{\mathbf{B}}$ equally well–is given.

So, one can conclude, the identification following the method of Lanne and Lütkepohl is unique up to the given permutation of the elements $\{\psi_1, \ldots, \psi_n\}$. In practice, this means that once we have estimated the model as described above, and especially once we have our (initial) estimate of the matrix $\tilde{\mathbf{B}}$ from equation (18), the actual (full) estimation of the structural KW model parameters requires us to identify the "correct" permutation of the elements $\{\psi_1, \ldots, \psi_n\}$, or in other words the correct matrix $\hat{\boldsymbol{\Psi}}$.

However, as it is now explained, the partial estimation–referring to situation when one does not (yet) know which of the all possible matrices $\hat{\boldsymbol{\Psi}}$ is the "correct" one–of the KW model is sufficient for our first objective of testing whether there is any volatility transmissions *to* market i *from* all the other $n-1$ markets (combined). And also, whether there is volatility transmission *from* market i *to* all the other $n-1$ markets. The first test would correspond to a null-hypothesis of $\beta_{ik} = 0$, and the latter to $\beta_{ki} = 0$, for all $k \in \{1, \ldots, n\}/\{i\}$.

In what it follows, for notational simplicity, let us denote $\hat{\mathbf{B}} = \tilde{\mathbf{B}}$. This should not create any confusion as, whenever $\tilde{\tilde{\mathbf{B}}}$ refers only to the matrix $\tilde{\mathbf{B}}$ in equation (18), this reference is made clear. Also, for convenience, let us denote $\hat{\mathbf{W}} = \mathbf{W}$ and $\mathbf{P}\boldsymbol{\Psi}\mathbf{P}' = \boldsymbol{\Psi}$. This way, unless other is stated, when we refer to any of these matrices we refer–somewhat loosely–to the partially estimated matrices, or in other words, to the case where the "correct" permutation has not yet been identified. Again. this should not cause any confusion.

In our volatility transmission testing everything is based on assuming the theoretical model in section 2 holds: daily stock market returns in our n countries are function of total news (our structural shocks) $\eta^{(i)}$, $i = 1, \ldots, n$. Then, the $n!$ possible permutations of our matrix $\tilde{\mathbf{B}}$ correspond to all $n!$ different ways to shuffle the ordering of total news $\eta^{(i)}$, $i = 1, \ldots, n$, in a $n \times 1$ vector. Only one of these permutations will coincide with the correct order of $\boldsymbol{\eta}_t = (\eta^{(1)}, \eta^{(2)}, \ldots, \eta^{(n)})$ where the country 1 total news is situated first, country 2 second, and so on. Hence, only one permutation corresponds to the (correct) matrix $\tilde{\mathbf{B}}$.

Then, the fact that all the $n!$ possible matrices $\mathbf{W}$ differ only by the order of their columns while the row indexes remain the same enables us to to conduct our testing. The idea of the test is easiest to demonstrate with a two country example. When $n = 2$ there are only two possible matrices $\boldsymbol{\Psi}$: $\boldsymbol{\Psi}^{(1)} = \text{diag}(\psi_1, \psi_2)$, and $\boldsymbol{\Psi}^{(2)} = \text{diag}(\psi_2, \psi_1)$. Let's then redefine

$$\tilde{\boldsymbol{\Psi}}^{(i)} = \left(\gamma\mathbf{I}_n + (1-\gamma)\boldsymbol{\Psi}^{(i)}\right)^{1/2},$$

for $i = 1, 2$. The matrices

$$\tilde{\boldsymbol{\Psi}}^{(1)} = \text{diag}(\tilde{\psi}_1, \tilde{\psi}_2) \text{ and } \tilde{\boldsymbol{\Psi}}^{(2)} = \text{diag}(\tilde{\psi}_2, \tilde{\psi}_1).$$

correspond to the matrices $\boldsymbol{\Psi}^{(1)}$ and $\boldsymbol{\Psi}^{(2)}$, respectively, and differ only by the order of their main diagonal elements $\{\tilde{\psi}_1, \tilde{\psi}_2\}$. As it is shown in the appendix A, then there are two alternative $\tilde{\mathbf{B}}$ matrices:

$$\tilde{\mathbf{B}}^{(1)} = \begin{bmatrix} \tilde{\psi}_1^{\frac{1}{2}} w_{11} & \tilde{\psi}_2^{\frac{1}{2}} w_{12} \\ \tilde{\psi}_1^{\frac{1}{2}} w_{21} & \tilde{\psi}_2^{\frac{1}{2}} w_{22} \end{bmatrix} \text{ and } \tilde{\mathbf{B}}^{(2)} = \begin{bmatrix} \tilde{\psi}_2^{\frac{1}{2}} w_{12} & \tilde{\psi}_1^{\frac{1}{2}} w_{11} \\ \tilde{\psi}_2^{\frac{1}{2}} w_{22} & \tilde{\psi}_1^{\frac{1}{2}} w_{21} \end{bmatrix}.$$

Now, assuming our theoretical model in section 2 holds, one of these two matrices $\tilde{\mathbf{B}}^{(1)}$ or $\tilde{\mathbf{B}}^{(2)}$ must correspond to the correctly identified permutation

of the elements $\{\psi_1, \psi_2\}$. Only the correct permutation sets the (estimated) total news into their correct ordering of $\boldsymbol{\eta}_t = (\eta^{(1)}, \eta^{(2)})$ where $\eta^{(1)}$ refers to country 1 total news, and $\eta^{(2)}$ to country 2. Now, assume, for example, the matrix $\tilde{\mathbf{B}}^{(1)}$ corresponds to the correct permutation. Then, the only difference that using $\tilde{\mathbf{B}}^{(2)}$ instead of $\tilde{\mathbf{B}}^{(1)}$ would do, is that it would set the total news into their reverse order; namely to $\{\eta^{(2)}, \eta^{(1)}\}$ instead of $\{\eta^{(1)}, \eta^{(2)}\}$. And, we would then mistakenly interpret $\eta^{(2)}$ as country 1 total news and $\eta^{(1)}$ as country 2.

Of course, in reality we do not know which one the matrices, $\tilde{\mathbf{B}}^{(1)}$ or $\tilde{\mathbf{B}}^{(2)}$, corresponds to the correctly identified model. However, from the equations of these two matrices we see that the coefficient of volatility transmission, for example, from country 2 to country 1 is either $\tilde{\psi}_2^{0.5} w_{12}$ or $\tilde{\psi}_1^{0.5} w_{11}$ (and the other one will then be the country 1's own total news effect). On the other hand, according to the KW model the country 1's own total news effect needs to be non-zero. Hence, either $\tilde{\psi}_2^{0.5} w_{12}$ or $\tilde{\psi}_1^{0.5} w_{11}$ must be statistically different from zero. So, in case we find that both of them are statistically different from zero, we know that there must exist volatility transmission from country 2 to country 1. In contrast, if we find that only one of them is statistically significant, we will know that there is not volatility transmission from from country 2 to country 1.

But, by assumption, both ψ_1 and ψ_2–and hence also $\tilde{\psi}_1$ and $\tilde{\psi}_2$–are distinct from zero. Hence, the only way $\tilde{\psi}_2^{0.5} w_{12}$ and $\tilde{\psi}_1^{0.5} w_{11}$ can be zero is by the elements w_{12} and w_{11} being zero, respectively. So, the testing of statistical significance of the elements β_{ij}, $i, j = 1, 2$, boils down to testing whether the elements w_{12} and w_{11} are statistically significant or not. Repeating our argument, we can conclude that in case *both* w_{12} and w_{11} are found to be non-zero, there must be volatility transmission from country 2 to country 1. Contrary, in case only one of these elements is found statistically significant, the evidence speaks against there being volatility transmission from country 2 to country 1. Symmetric reasoning applies to testing volatility transmission from country 1 to country 2.

The test generalizes to all cases of $n \geq 2$. So for example, in case all of the elements w_{ik}, where $k = 1, \ldots, n$, on the ith row of the matrix $\mathbf{W}$ are found to be non-zero, then there is evidence of volatility transmission from *all* of the other $n - 1$ countries to country i. On the contrary, if one finds that $m + 1$ row i elements $(0 \leq m \leq n - 1)$ are non-zero, then there's evidence of volatility transmission from m countries to country i. And this irrespective of which of the possible permutations of matrix $\mathbf{W}$ we are currently working with.

Especially, we can always test whether matrix $\mathbf{W}$ is diagonal or not. If it happens to be diagonal, then also matrix $\tilde{\mathbf{B}}$ is diagonal. In such a case, we can conclude that there is no volatility transmissions across the markets. In addition, then, we have also fully estimated the structural model. This is because, as already explained, according to our theoretical model in section 2, for each country, at least the country's own total news must explain changes in its stock market prices. Hence, if we have found matrix $\tilde{\mathbf{B}}$ being diagonal, in which case for each country there is one and unique statistically significant structural shock (estimated total news), then necessarily, the estimated restricted model corresponds to the structural model.

3.2 Estimation of the KW model parameters

Lanne, Lütkepohl, and Maciejowska (2010) note the sensitivity of the estimated matrix $\tilde{\mathbf{B}}$ to different permutations of the main diagonal elements of $\boldsymbol{\Psi}$. They propose to use either the order from the smallest to the largest or from the largest to the smallest. However, nothing guarantees that either of these two permutations would identify the correct order of the total news (structural shocks). Here, an alternative identification method is proposed.

Recall the identity in equation (14) between the reduced form error vector $\tilde{\boldsymbol{\eta}}_t$ (denoting stock market price changes) and structural shocks vector $\boldsymbol{\eta}_t$ (denoting total news). Given any permutation of the elements $\{\psi_1, \ldots, \psi_n\}$, the Lanne and Lütkepohl (2010) identification method guarantees a locally unique matrix $\tilde{\mathbf{B}}$. Assume this matrix is also invertible. Then, by premultiplying the equation (14) with $\tilde{\mathbf{B}}^{-1}$, one gets

$$\boldsymbol{\eta}_t = \tilde{\mathbf{B}}^{-1} \tilde{\boldsymbol{\eta}}_t.$$

We can now calculate the covariance matrix of the total news as a function of (the estimated) matrix $\tilde{\mathbf{B}}$ and the covariance matrix of market volatilities $\boldsymbol{\Sigma}_{\tilde{\eta}}$:

$$\boldsymbol{\Sigma}_\eta = \tilde{\mathbf{B}}^{-1} \boldsymbol{\Sigma}_{\tilde{\eta}} (\tilde{\mathbf{B}}')^{-1}. \tag{19}$$

In the KW model the total news covariance matrix $\boldsymbol{\Sigma}_\eta$ is a diagonal but not an identity matrix, hence, this equation is not a trivial identity. It gives us estimates of the variances of the total news $(\sigma^2_{\eta^{(i)}})$ for each country $(i = 1, \ldots, n)$. In particular, the estimated order of magnitude of these variances $\{\sigma^2_{\eta^{(1)}}, \ldots, \sigma^2_{\eta^{(n)}}\}$ depends on the specific matrix $\boldsymbol{\Psi}$.

Hence, if we could find–from some other sources–some proximate variables for the each country's total news, we would be able to get an alternative estimate for their variances. Especially, we are interested in the order of magnitude of these proximate news' variances. If the order of the proximate total news variances is unambiguous in such a way that no two or more countries share same ranking, we can use this alternative ranking and equation (19) to identify the correct matrix $\tilde{\mathbf{B}}$. We simply select among our $n!$ possibilities the model that produces the same ordering of the countries' total news variances on the main diagonal of $\boldsymbol{\Sigma}_\eta$ as does our alternative news' ranking. In the next section data from the Google trends is used as a proximate total news data.

As a last note, assume we have identified the correct $\boldsymbol{\Psi}$, using equation (18) to estimate matrix $\tilde{\mathbf{B}}$ does not guarantee the resulting matrix would have diagonal elements equal to one–as there should be based on the assumptions of the KW model. The Lanne and Lütkepohl identification method is based on normalizing the covariance matrix of structural shocks $\boldsymbol{\Sigma}_\eta$ to an identity matrix. The elements of $\tilde{\mathbf{B}}$ are let to vary freely. But, the KW model should be interpreted as a SVAR model where, in contrast, the diagonal elements of $\boldsymbol{\Sigma}_\eta$ are allowed to vary freely but the diagonal elements of $\tilde{\mathbf{B}}$ are normalized to one. These are simply two alternative ways to normalize a SVAR model. As shown in appendix B, one can easily swap the first normalization to the latter one in the following way: on each column $k = 1, \ldots, n$ of matrix $\tilde{\mathbf{B}}$ provided by equation (18), divide all the elements on that column $[\tilde{\mathbf{B}}]_{ik}$, $i = 1, \ldots, n$, by the main diagonal element $[\tilde{\mathbf{B}}]_{kk}$ of the same column. We then get the estimate of matrix $\mathbf{B}$ simply by $\mathbf{B} = \tilde{\mathbf{B}} - \mathbf{I}_n$.

Table 1: Summary statistics of volatilities

	Mean	SD	Skewness	Kurtosis
Italy (ITA)	-0.00	0.02	-0.06	5.94
Spain (ESP)	-0.00	0.02	0.57	9.55
Ireland (IRE)	-0.00	0.01	-0.14	5.23
Greece (GRE)	-0.00	0.03	0.71	6.52

Source: Yahoo! Finance, own calculations.

4 Application of the volatility transmission test and estimation of the KW model

As an empirical example of how to test for volatility transmission and how to estimate the KW model, let us consider the European stock markets. Since early 2010, or late 2009, the eurozone has been in the middle of a debt crisis. Some of the countries in the spotlight have been Italy, Spain, Ireland, and–especially–Greece. Figure 1 (at the end of this paper) depicts how the equity prices in these countries have changed during 2010–2011 (indexes have been rescaled; for details about the data, see appendix C). In Greece the prices have decreased by almost 80 percent, in Italy and Spain by around 30 percent, and by less than ten percent in Ireland. Our empirical analysis will focus on these markets. Of course, considering only four countries might be too restricted, but this simplifies our model identification task considerably: for example the number of possible models when $n = 4$ is only 24 compared to 120 if $n = 5$ was chosen.[13]

4.1 Data

We will consider daily close-to-close price changes. Hence, consistent with what was said in section 2.1, stock market returns are calculated from the daily closing values of each stock market price index by first taking the logarithmic transformation of the price and then taking first differences:

$$\Delta S_t^i = \log P_{C,t}^i - \log P_{C,t-1}^i,$$

where $P_{C,t}^i$ denotes the closing value of the price index in country i at date t, and $i \in \{ITA, ESP, IRE, GRE\}$ (for shortenings, see table 1).

There are 517 closing values for each country which gives 516 observations of returns. However, as indicated in appendix C, for reasons of national banking holidays every country has some missing observations. I have substituted these missing values with the closing value of the previous (open) trading day. Table 1 summarizes the volatility data; the statistics indicate that the empirical distributions of the returns differ from normal distributions. This lends support to using a non-normal error distribution in our VAR model (see equation 16). Time series of the returns are depicted in figure 2.

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[13]Especially, dropping off Germany from the analysis could be questioned. However, the correlation coefficient between Italian and German volatilities is 0.86, so perhaps it is plausible to assume Italy also acts as proxy for Germany.

Table 2: Estimation results of unrestricted model (estimated standard errors in parentheses)

| | Elements of each vector | | | |
	1	2	3	4
$\mathbf{W}[\mathbf{1}, \cdot] \times 100$	1.10*** (0.09)	0.01 (0.43)	−0.16 (0.22)	0.04 (0.21)
$\mathbf{W}[\mathbf{2}, \cdot] \times 100$	0.95*** (0.2)	0.52 (0.37)	−0.22 (0.27)	0.15 (0.26)
$\mathbf{W}[\mathbf{3}, \cdot] \times 100$	0.74*** (0.14)	0.22 (0.34)	0.54*** (0.2)	−0.18 (0.40)
$\mathbf{W}[\mathbf{4}, \cdot] \times 100$	0.78*** (0.22)	0.11 (0.48)	0.65 (1.13)	1.74*** (0.47)
$\mathbf{\Psi}$	6.33*** (1.07)	4.37*** (0.68)	3.07*** (0.51)	2.56*** (0.49)
γ	0.66*** (0.05)			

NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.
$\mathbf{W}[i, \cdot]$ indicates ith row of matrix $\mathbf{W}$.
(**) / (***) indicates statistical significance at 5 % / 1 % significance level.
Results for $\mathbf{W}$ are reported for estimates multiplied by 100.

4.2 Testing volatility transmissions

Let us first test for volatility transmissions employing the testing procedure detailed in section 3.1. The test consists of testing with the likelihood ratio (LR) test the statistical significance of the elements of matrix $\mathbf{W}$. In order to be able to estimate the model we need, first, to determine some order for the main diagonal elements of matrix $\mathbf{\Psi}$. Following Lanne, Lütkepohl, and Maciejowska (2010), let us first select the descending order. Then, as discussed in section 3.1, even if this permutation would not yet fully estimate the structural model parameters, the partial estimation is enough for testing the number of volatility links between the countries. The actual full estimation of the structural parameters is discussed below, in section 4.3.

There are now in total 21 parameters to be estimated. Table 2 reports the estimation results for the model with the aforementioned descending order.[14] First four rows of the table corresponds to the four rows of matrix $\mathbf{W}$ (multiplied by 100). The fifth row in the table shows the estimates of the matrix $\mathbf{\Psi}$ main diagonal vector which are by assumption for the moment in a descending order. The estimate of matrix $\mathbf{\Psi}$ do not have any particular interpretation. The last table row reports the estimated mixture probability γ; with a probability of around 66.0 percent the VAR model errors are from the multi-normal distribution with smaller variances.

To briefly recap what was said in section 3.1: always making sure that at least one element on each row and column of matrix $\mathbf{W}$ remains non-zero, one can test the existence of volatility transmission, for example, from other countries to the first country by restricting to zero three elements on the first row of matrix $\mathbf{W}$. The results in table 2 suggest that at least some of the elements of matrix

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[14]All calculations were done with programs in the GAUSS CMLMT library.

Table 3: Estimation results of restricted model (estimated standard errors in parentheses)

	Elements of each vector			
	1	2	3	4
$\mathbf{W}[\mathbf{1}, \cdot] \times 100$	1.12*** (0.08)	..	−0.13*** (0.03)	..
$\mathbf{W}[\mathbf{2}, \cdot] \times 100$	1.00*** (0.08)	0.52*** (0.03)	..	..
$\mathbf{W}[\mathbf{3}, \cdot] \times 100$	0.73*** (0.06)	..	0.41*** (0.12)	−0.53*** (0.12)
$\mathbf{W}[\mathbf{4}, \cdot] \times 100$	0.89*** (0.1)	..	1.38*** (0.27)	1.08*** (0.38)
$\boldsymbol{\Psi}$	6.05*** (0.99)	4.30*** (0.68)	3.09*** (0.52)	2.80*** (0.52)
γ	0.66*** (0.05)			

NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.
$\mathbf{W}[i, \cdot]$ indicates ith row of matrix $\mathbf{W}$.
(**) / (***) indicates statistical significance at 5 % / 1 % significance level.
Sign $\cdot\cdot$ signifies the parameter is statistically insignificant.
Results for $\mathbf{W}$ are reported for estimates multiplied by 100.

$\mathbf{W}$ are statistically insignificant.

As a way forward, let us first restrict to zero all elements with the greatest p-values and then test with the LR test if the restriction(s) are supported by the data. Table 3 reports the estimation results of a restricted model that is achieved by restricting to zero matrix $\mathbf{W}$ elements up to a point where no more restrictions are supported. The LR test statistic for the joint restrictions in the table is 1.99 which is less than 12.6, the critical value of χ^2 distribution at 5 % significance level and six degrees of freedom. Because in every possible permutation of the elements $\{\psi_1, \ldots, \psi_4\}$ the row indexes of the matrix $\mathbf{W}$ stay the same, the first row of matrix $\mathbf{W}$ refers to Italy, the second to Spain, the third to Ireland, and the fourth to Greece. Hence, according to our test results, there is evidence of volatility transmissions: from one of the other three countries to Italy, same to Spain, and to Ireland and Greece from two other countries.[15] (Remember, the own total news effect needs to be significant for every country by assumption.) Also, as a consequence of the results in table 3, a test hypothesis of matrix $\mathbf{W}$ being diagonal–and there not being any volatility transmissions–is clearly rejected.

4.3 Estimation of the KW model

Previous section concluded that there is evidence of volatility transmissions between the four countries. In order to identify the source countries of spillovers for each country, we need to fully estimate the KW model. To do this, it was

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[15] As discussed in section 3.1 this interpretation relies on assuming the KW model of section 2.2 holds here, and the specific way in which different permutations of the main diagonal elements of matrix $\boldsymbol{\Psi}$ shuffle the ordering of structural shocks.

Table 4: Variances of the weekly percentage changes in search volume index, and descending order rank of the countries according to the variances

Country	Variance	Rank
Italy	3831.0	1
Spain	1184.0	3
Ireland	367.0	4
Greece	1842.0	2

suggested in section 3.2 that one could use some proximate variables that mimic total news variances $\sigma^2_{\eta^{(i)}}$, $i = 1, \ldots, 4$. This would provide an alternative estimate for the ordering of these countries' total news variances (structural shock variances). Because each of the possible $4! = 24$ matrices $\tilde{\mathbf{B}}$ corresponds to a specific matrix $\boldsymbol{\Psi}$, one could possibly be able to identify the correct matrix $\boldsymbol{\Psi}$. Here, it is proposed that search volume data from Google Trends is used to calculate proximate variables for the total news variances.

Figure 3 shows the rescaled index of global Google search volumes about the economic conditions of the four countries considered here. The data covers weekly observation for years 2010 and 2011 and is rescaled so that the first week in 2010 equals 100.[16] (More details on the data, and the actual search keywords, are are provided in appendix C.) Hence, for example, when according to the figure the global search traffic on the economic conditions of Greece peaked at around 1600 during the spring 2010, it means that during that week the average search volume on Greece–relative to the average of all search traffic in Google that week–was 1500 percent larger than during the first week of the year. It would then make sense to assume that such a heavy increase in search traffic on the Greek economy somehow reflects new information, or news released, about the country's condition at that time.[17]

Figure 4 in its turn shows the weekly percentage changes in the search volume index. This is now the data that I consider as a proxy for the arriving news, and hence use the variances of these time series as a proximate variable for each country's total news variance[18]. Table 4 reports the variances and the country ranking when the ranking is based on descending order of the variances. Italy has the largest variance, followed by Greece, Spain, and lastly by Ireland. There is only one matrix $\boldsymbol{\Psi}$ that creates this same ordering of the variances of total news. The parameter estimates of this model are reported in table 5 (to save some place I report only the estimates of the restricted model comparable to the restricted model in table 3).

The structural model becomes identified and, hence, we can calculate the estimates of the volatility transmission parameters of the KW model (for details

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[16]In the data week is considered to start on Sunday, so week 1 in year 2010 began on Sunday January 3rd, 2010.

[17]Of course, the peak coincides with the onset of the euro debt crisis and the first Greek bailout package, but even so, this doesn't contradict with what is said in the text.

[18]The results are, naturally, qualitatively the same if I, instead of percentage changes, use either first differences or first differences of logarithmic transformations.

Table 5: Estimation results of restricted KW model (estimated standard errors in parentheses)

	Elements of each vector			
	1	2	3	4
$100 \times \mathbf{W}[\mathbf{1}, \cdot]$	1.10*** (0.08)	..	..	0.13*** (0.03)
$100 \times \mathbf{W}[\mathbf{2}, \cdot]$	0.98*** (0.07)	0.51*** (0.03)	..	..
$100 \times \mathbf{W}[\mathbf{3}, \cdot]$	0.72*** (0.06)	..	−0.53*** (0.12)	−0.42*** (0.12)
$100 \times \mathbf{W}[\mathbf{4}, \cdot]$	0.87*** (0.10)	..	1.09*** (0.38)	−1.37*** (0.28)
$\mathbf{\Psi}$	6.28*** (1.04)	4.37*** (0.70)	2.70*** (0.50)	3.05*** (0.51)
γ	0.65*** (0.05)			

NOTE:
Standard errors obtained from the inverse Hessian of the log-likelihood function.
$\mathbf{W}[i, \cdot]$ indicates ith row of matrix $\mathbf{W}$.
(**) / (***) indicates statistical significance at 5 % / 1 % significance level.
Sign .. signifies the parameter is statistically insignificant.
Results for $\mathbf{W}$ are reported for estimates multiplied by 100.

see section 3.2). That is we can estimate the structural equation (13)[19]:

$$
\begin{bmatrix} \Delta S_t^{ITA} \\ \Delta S_t^{ESP} \\ \Delta S_t^{IRE} \\ \Delta S_t^{GRE} \end{bmatrix} = \begin{bmatrix} 1.00 & .. & .. & -0.10 \\ 0.89 & 1.00 & .. & .. \\ 0.65 & .. & 1.00 & 0.30 \\ 0.79 & .. & -2.07 & 1.00 \end{bmatrix} \begin{bmatrix} \hat{\eta}_t^{ITA} \\ \hat{\eta}_t^{ESP} \\ \hat{\eta}_t^{IRE} \\ \hat{\eta}_t^{GRE} \end{bmatrix}. \tag{20}
$$

In equation (20), for example, the last coefficient on the first row, −0.10, means that one percent increase in stock market prices in Greece decreases the stock market prices in Italy by 0.1 percent. But the effect on the volatility in Italy is the squared value of the coefficient (see equation (8)). Here it would be $(-0.10)^2 = 0.01$. Table 6 reports the spillover effects on volatilities across all countries. According to the estimates, volatility from Italy gets always transmitted to the other countries, unlike that of Spain which doesn't have any effect on the others. Ireland and Greece affect each other, and Greece is the only country whose volatility has had an effect to volatility in Italian stock markets. Interestingly, the coefficient that the Irish volatility has on the Greece on is 4.28, by far the greatest coefficient by its magnitude.

As it was shortly discussed in section 2.1, see below equation (8), the finding that a particular volatility transmission coefficient is found insignificant, is perhaps best understood as meaning that the markets consider news released in–or on–the particular source country consisting (mostly) of idiosyncratic information. Hence for example, our estimation results suggest that, given our group of countries, news in Spain are considered in markets as irrelevant infor-

---

[19]The sign .. signifies that the parameter is not statistically significant.

Table 6: Estimated effects across countries' volatilities

To	From			
	Italy	Spain	Ireland	Greece
Italy	1.00	..	..	0.01
Spain	0.79	1.00	..	..
Ireland	0.42	..	1.00	0.09
Greece	0.62	..	4.28	1.00

(··) indicates no effect

mation for the company valuations in the other countries. On the other hand, given that Italy has significant coefficients against all the others, markets seem to consider news in Italy having, at least some, effect on the valuations in other countries. From the perspective of the contagion literature, there is evidence of contagion (1) from Italy to other countries, (2) from Ireland to Greece, and (3) from Greece to Italy and Ireland.

5   Conclusions and discussion

In this paper, first, an existing theoretical model, that provides an explanation for volatility spillovers across overlapping stock markets, was presented. The key insight of the theoretical model is that volatility transmission could be a result of uninformed investors' efforts to try to infer private information of informed investors. The realized price changes work as signals of this private information. But because this signal is contaminated by idiosyncratic shocks, also "excess" volatility gets transmitted from one country to the others.

Second, by interpreting the theoretical model as a structural VAR model and by augmenting it with an additional distributional assumption, recently introduced SVAR identification methodology was exploited to develop a new test for volatility transmissions. However, as it was shown in the paper, the distributional assumption only guarantees a partial estimation of the structural model parameters. This turn out to be enough for the validity of our test but not enough for one being able to fully estimate the structural model parameters.

Hence, it was also discussed, what sort of additional information one would need to make in order to identify the correct model permutation that permits us to estimate the underlying structural model. In the paper it was proposed that one such source of information could be the data on weekly web searches on economic conditions of given countries. Changes in web traffic would then proximate news released in (or about) the country.

The empirical part of the paper demonstrates how to apply the test and, also, to fully identify the structural model. The data is stock market data for four eurozone countries: Italy, Spain, Ireland, and Greece. Statistical testing finds evidence of volatility transmissions between several countries. The application of the full estimation method, in its turn, shows that the volatility spillovers across the stock markets of these countries seem quite asymmetric; Italy affects all the countries, whereas Spain seems to have least effect on others. The purpose of the empirical exercise was to demonstrate how to actually estimate the KW

model. More detailed empirical analysis with more elaborated conclusions is a task left for future research.

Appendices

A   Note on Lanne and Lütkepohl identification method

This appendix shows why the identification method of Lanne and Lütkepohl (2010) provides only a partial estimation of the coefficient matrix $\tilde{\mathbf{B}}$. Assume the following $n \times n$ matrices: $\boldsymbol{\Psi} = \mathrm{diag}\left(\psi_1, \ldots, \psi_n\right)$ with $\psi_i > 0$, for all $i = 1, \ldots, n$; and $\mathbf{W}$. Assume also a mixture probability γ such that $0 < \gamma < 1$. Also, assume $n \times n$ reduced form error vector's covariance matrix $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}}$ can be written as

$$\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}} = \mathbf{W}\left(\gamma \mathbf{I}_n + (1-\gamma)\,\boldsymbol{\Psi}\right)\mathbf{W}' = \mathbf{W}\bar{\boldsymbol{\Psi}}\mathbf{W}', \tag{21}$$

where $\mathbf{I}_n$ is $n \times n$ identity matrix, and $\bar{\boldsymbol{\Psi}} = \gamma \mathbf{I}_n + (1-\gamma)\,\boldsymbol{\Psi}$. Clearly $\bar{\boldsymbol{\Psi}}$ is also diagonal matrix with its ith main diagonal element being $\bar{\psi}_i = \gamma + (1-\gamma)\psi_i$. Hence, there is bijective mapping between $\bar{\boldsymbol{\Psi}}$ and $\boldsymbol{\Psi}$. So, we can concentrate on different permutations of $\bar{\boldsymbol{\Psi}}$.

Now, take an arbitrary $n \times n$ permutation matrix $\mathbf{P}$ such that $\mathbf{P} \neq \mathbf{I}_n$. Because permutation matrices are orthogonal matrices, it holds that $\mathbf{P}'\mathbf{P} = \mathbf{I}_n$, where $\mathbf{P}'$ denotes the transpose of $\mathbf{P}$. Hence, we can write

$$\begin{aligned}
\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\eta}}} &= \left(\mathbf{W}\mathbf{P}'\right)\left(\mathbf{P}\bar{\boldsymbol{\Psi}}\mathbf{P}'\right)\left(\mathbf{W}\mathbf{P}'\right)' \\
&= \hat{\mathbf{W}}\hat{\boldsymbol{\Psi}}\hat{\mathbf{W}}',
\end{aligned} \tag{22}$$

where we have redefined $\hat{\mathbf{W}} = \mathbf{W}\mathbf{P}'$ and $\hat{\boldsymbol{\Psi}} = \mathbf{P}\bar{\boldsymbol{\Psi}}\mathbf{P}'$.

It is straightforward to see that the matrix $\hat{\boldsymbol{\Psi}}$ is diagonal with a different permutation of elements $\{\psi_1, \ldots, \psi_n\}$ on its main diagonal than the matrix $\bar{\boldsymbol{\Psi}}$. First, write

$$\hat{\boldsymbol{\Psi}} = \mathbf{P}(\bar{\boldsymbol{\Psi}}^{1/2}\bar{\boldsymbol{\Psi}}^{1/2})\mathbf{P}' = (\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2})(\mathbf{P}\bar{\boldsymbol{\Psi}}'^{1/2})' = (\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2})(\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2})',$$

which is possible because $\bar{\psi}_i > 0$ for all i. Mark as $\mathbf{e}_k$ a $1 \times n$ vector whose kth element equals one and all the other elements equal zero. Then the jth column of $\bar{\boldsymbol{\Psi}}^{1/2}$ can be written as $\mathbf{e}_j'\bar{\psi}_j^{1/2}$. Now, consider the permutation

$$\Pi : \{1, \ldots, n\} \to \{1, \ldots, n\}, \; \Pi(k) = \pi_k \; \forall k \in \{1, \ldots, n\},$$

that corresponds to the permutation matrix $\mathbf{P}$. Then the ith row of $\mathbf{P}$ is $\mathbf{e}_{\pi_i}$. And so, the ijth element of matrix $\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2}$ is

$$\left[\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2}\right]_{ij} = \mathbf{e}_{\pi_i}\mathbf{e}_j'\bar{\psi}_j^{1/2}$$

that equals zero whenever $\pi_i \neq j$ and $\bar{\psi}_{\pi_i}^{1/2}$ when $\pi_i = j$. The ijth element of

matrix $(\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2})'$ equals to the the jith element of matrix $\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2}$. Hence,

$$
\begin{aligned}
\left[\hat{\boldsymbol{\Psi}}\right]_{ij} &= \left[(\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2})(\mathbf{P}\bar{\boldsymbol{\Psi}}^{1/2})'\right]_{ij} \\
&= \sum_{k=1}^{n}\left(\mathbf{e}_{\pi_i}\mathbf{e}_k'\bar{\psi}_k^{1/2}\right)\left(\mathbf{e}_{\pi_j}\mathbf{e}_k'\bar{\psi}_k^{1/2}\right) \\
&= \sum_{k=1}^{n}\bar{\psi}_k\mathbf{e}_{\pi_i}\mathbf{e}_k'\mathbf{e}_{\pi_j}\mathbf{e}_k'
\end{aligned}
$$

which equals zero whenever $i \neq j$ and $\bar{\psi}_{\pi_i}$ when $i = j$. Hence, the matrix $\hat{\boldsymbol{\Psi}}$ is diagonal and the order of its main diagonal elements corresponds to the permutation Π.

So, based on equations (21) and (22) we have now two equally possible choices for matrix $\tilde{\mathbf{B}}$ (see equation (18)):

$$
\tilde{\mathbf{B}}_{(1)} = \mathbf{W}\bar{\boldsymbol{\Psi}}^{1/2} \text{ or } \tilde{\mathbf{B}}_{(2)} = \hat{\mathbf{W}}\hat{\boldsymbol{\Psi}}^{1/2}.
$$

These two alternatives of matrix $\tilde{\mathbf{B}}$ are not the same as long as $\mathbf{P} \neq \mathbf{I}_n$. Because there are $n!$ different matrices $\mathbf{P}$, there will also be $n!$ alternative matrices $\tilde{\mathbf{B}}$. Hence, unless we know which permutation $\mathbf{P}$ to use, the structural model parameters will not be (fully) estimated. Concerning our test of volatility transmission effects in section 3.1, note that multiplying matrix $\mathbf{W}$ from right with permutation matrix $\mathbf{P}'$ will only permute the columns of $\mathbf{W}$. Hence, in $\hat{\mathbf{W}} = \mathbf{W}\mathbf{P}'$ only the order of the columns of $\mathbf{W}$ has changed. Row indexes remain the same.

For demonstration, let's consider the case of $n = 2$. Then we have only two alternative permutation matrices:

$$
\mathbf{P}_{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \mathbf{P}_{(2)} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}.
$$

So there are two alternatives for matrix $\tilde{\mathbf{B}}$:

$$
\begin{aligned}
\tilde{\mathbf{B}}_{(1)} &= \left(\mathbf{W}\mathbf{P}'_{(1)}\right)\left(\mathbf{P}_{(1)}\bar{\boldsymbol{\Psi}}\mathbf{P}'_{(1)}\right)^{1/2} \\
&= \begin{bmatrix} \tilde{\psi}_1^{\frac{1}{2}}w_{11} & \tilde{\psi}_2^{\frac{1}{2}}w_{12} \\ \tilde{\psi}_1^{\frac{1}{2}}w_{21} & \tilde{\psi}_2^{\frac{1}{2}}w_{22} \end{bmatrix},
\end{aligned}
$$

where we have used the fact that $\mathbf{P}^{(1)} = \mathbf{I}_2$, and

$$
\begin{aligned}
\tilde{\mathbf{B}}_{(2)} &= \left(\mathbf{W}\mathbf{P}'_{(2)}\right)\left(\mathbf{P}_{(2)}\bar{\boldsymbol{\Psi}}\mathbf{P}'_{(2)}\right)^{1/2} \\
&= \begin{bmatrix} w_{12} & w_{11} \\ w_{22} & w_{21} \end{bmatrix}\begin{bmatrix} \tilde{\psi}_2 & 0 \\ 0 & \tilde{\psi}_1 \end{bmatrix}^{1/2} \\
&= \begin{bmatrix} \tilde{\psi}_2^{\frac{1}{2}}w_{12} & \tilde{\psi}_1^{\frac{1}{2}}w_{11} \\ \tilde{\psi}_2^{\frac{1}{2}}w_{22} & \tilde{\psi}_1^{\frac{1}{2}}w_{21} \end{bmatrix}.
\end{aligned}
$$

Clearly, $\tilde{\mathbf{B}}_{(1)} \neq \tilde{\mathbf{B}}_{(2)}$.

B Swapping between alternative SVAR normalizations

The notation in this appendix is independent of the one used in the main text. Assume, $n \times 1$ reduced from error vector $\mathbf{u}_t$ of a VAR model and $n \times 1$ vector of structural shocks $\boldsymbol{\varepsilon}_t$ of a SVAR model. The reduced from error vector is assumed to be linear transformation of the structural shocks vector; $\mathbf{u}_t = \mathbf{B}\boldsymbol{\varepsilon}_t$. The structural shocks are distributed as $\boldsymbol{\varepsilon}_t \sim (\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon)$ where $\boldsymbol{\Sigma}_\varepsilon$ is a diagonal matrix. The reduced form errors are distributed as $\mathbf{u}_t \sim (\mathbf{0}, \boldsymbol{\Sigma}_u)$ where $\boldsymbol{\Sigma}_u$ is a general $n \times n$ matrix. We have the two following possible SVAR model normalizations:

(1) Assume $\boldsymbol{\Sigma}_\varepsilon = \mathbf{I}_n$ but let the elements of $\mathbf{B}$ vary freely, or

(2) Assume $\text{diag}(\mathbf{B}) = \mathbf{1}_{n \times 1}$, but let the (diagonal) elements of $\boldsymbol{\Sigma}_\varepsilon$ vary freely,

where $\mathbf{1}_{n \times 1}$ refers to $n \times 1$ vector with all elements equal to one.

The first normalization is used in the paper by Lanne and Lütkepohl (2010) and the second in the KW model. From this papers point of view, the relevant question is: How to switch from the normalization (1) to (2)?

First, assume normalization 1. For notational simplicity, let's suppress time indexation. Then, we have $n \times 1$ random vectors $\boldsymbol{\varepsilon}$ and $\mathbf{u}$, and $n \times n$ matrices $\boldsymbol{\Sigma}_\varepsilon$ and $\mathbf{B}$, where $\boldsymbol{\Sigma}_\varepsilon = \mathbf{I}_n$. Following the identity $\mathbf{u} = \mathbf{B}\boldsymbol{\varepsilon}$, we have

$$\boldsymbol{\Sigma}_u = \text{E}(\mathbf{u}\mathbf{u}') = \mathbf{B}\text{E}(\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}')\mathbf{B}' = \mathbf{B}\boldsymbol{\Sigma}_\varepsilon\mathbf{B}' = \mathbf{B}\mathbf{B}', \tag{23}$$

where E() is expectations operator.

Second, assume normalization 2. Then, we have $n \times 1$ random vectors $\tilde{\boldsymbol{\varepsilon}}$ and $\mathbf{u}$, where $\tilde{\boldsymbol{\varepsilon}}$ is not necessarily equal to $\boldsymbol{\varepsilon}$ in the previous paragraph. Also, we have $n \times n$ matrices $\boldsymbol{\Sigma}_{\tilde{\varepsilon}}$ and $\tilde{\mathbf{B}}$, where $\text{diag}(\tilde{\mathbf{B}}) = \mathbf{1}_{n \times 1}$. Again, following the identity $\mathbf{u} = \tilde{\mathbf{B}}\tilde{\boldsymbol{\varepsilon}}$, we also have

$$\boldsymbol{\Sigma}_u = \text{E}(\mathbf{u}\mathbf{u}') = \tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}\tilde{\mathbf{B}}' = \left(\tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}\right)\left(\tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}\right)' \tag{24}$$

Form equations (23) and (24) we get an identity

$$\mathbf{B} = \tilde{\mathbf{B}}\boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}}. \tag{25}$$

For simplicity, let's limit our discussion to the two variable case ($n = 2$). Thus, we have

$$\mathbf{B} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \ \tilde{\mathbf{B}} = \begin{bmatrix} 1 & \tilde{b}_{12} \\ \tilde{b}_{21} & 1 \end{bmatrix}, \text{ and } \boldsymbol{\Sigma}_{\tilde{\varepsilon}}^{\frac{1}{2}} = \begin{bmatrix} \tilde{\sigma}_1 & 0 \\ 0 & \tilde{\sigma}_2 \end{bmatrix}.$$

Equation (25) becomes

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} \tilde{\sigma}_1 & \tilde{\sigma}_2\tilde{b}_{12} \\ \tilde{\sigma}_1\tilde{b}_{21} & \tilde{\sigma}_2 \end{bmatrix}.$$

This gives us following four equations:

$$\begin{cases} b_{11} = \tilde{\sigma}_1 \\ b_{21} = \tilde{\sigma}_1\tilde{b}_{21} \\ b_{12} = \tilde{\sigma}_2\tilde{b}_{12} \\ b_{22} = \tilde{\sigma}_2 \end{cases}.$$

By solving for $\tilde{b}_{12}$ and $\tilde{b}_{21}$, we get

$$\begin{cases} \tilde{b}_{21} = b_{21}/b_{11} \\ \tilde{b}_{12} = b_{12}/b_{22} \end{cases}.$$

Hence, we have derived that

$$\tilde{\mathbf{B}} = \begin{bmatrix} 1 & \frac{b_{12}}{b_{22}} \\ \frac{b_{21}}{b_{11}} & 1 \end{bmatrix}.$$

So, once we have an estimate of $\mathbf{B}$ that is based on normalization (1), we can swap to normalization (2)–and get an estimate of $\tilde{\mathbf{B}}$–by dividing every column of $\mathbf{B}$ by the corresponding main diagonal element. Clearly, the result generalizes to all $n \geq 2$.

C   Data details

Stock market data

The upper part of table 7 provides the details of the stock market price indexes that are used in this paper. All the stock market data is downloaded from Yahoo! Finance. In total the period under consideration covers 517 trading days. Due to banking holidays, none of the individual stock exchanges were open at every possible trading day. When there was a missing value for a trading day, I took the closing value from the previous (open) trading day.

Google trends data

Google Trends provides data on how different topics (search terms) have been searched (in English) over time and provides weekly observations of Google's search volume index. The search index reports the average amount of traffic (Google searches) on the chosen topic relative to worldwide search traffic (in Google) during a week. Given that the data is only available for searches in English, the generality of our results in section 4.3 could, of course, be questioned. However, because we are actually interested in changes of the search volume data, as long as the data for the English tracks well searches in other languages, this shortcoming should not affect too much our results. It seems hard to imagine that trends in particular searches done in English would considerably differ (on average) from searches done in other languages.

   The raw data that Google provides is scaled relative to the first observation of each time series (this is the fixed scaling option that Google provides). However, I have rescaled the time series so that for each series the first week in 2010 equals to 100. So, the data tells the average global traffic for the topics relative to their own global average traffic in week 1, 2010. The lower part of table 7 reports the details of both the search topics I was interested to find data on and the actual Google Trends keywords I used to find the time series. The bar sign "|" between the keywords means that I wanted to find search data for searches including at least one of the keywords. This labeling corresponds to the Google Trends convention.

Table 7: Data details: Stock market indexes, and Google Trend search volume index

Stock market price indexes, daily closing values for time period Jan 4,2010–Dec 30, 2011

Country: Index [Yahoo! Finance ticker]	Includes	# trading days	# of missing obs.
Italy: FTSE MIB [FTSEMIB.MI]	40 most traded stocks	512	5
Spain: IBEX 35 [^IBEX]	35 most traded stocks	513	4
Ireland: ISEQ Overall Index [^ISEQ]	All stocks	514	3
Greece: FTSE/ASE 20 [FTASE.AT]	20 most traded stocks	503	14

Topics and the specific keywords that were used in Google Trends

Search topic	Actual keyword in Google Trends
Italian economy OR debt OR stock market	(italy gdp) | (italy debt) | (italy stock)
Spanish economy OR debt OR stock market	(spain gdp) | (spain debt) | (spain stock)
Irish economy OR debt OR stock market	(ireland gdp) | (ireland debt) | (ireland stock)
Greece economy OR debt OR stock market	(greece gdp) | (greece debt) | (greece stock)

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Figure 1: Stock market price indexes, daily closing values (January 4, 2010–December 30, 2011)
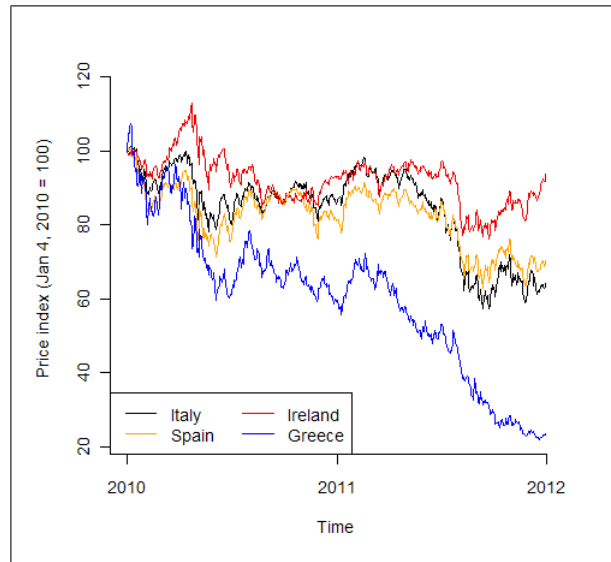

Figure 2: Daily stock market returns (January 5, 2010–December 30, 2011)
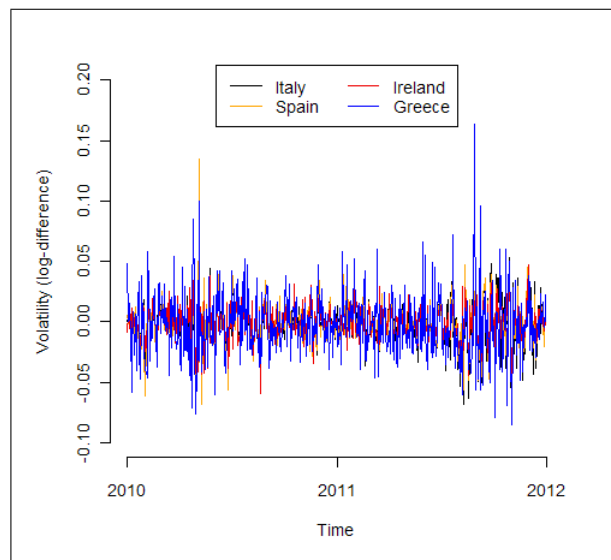
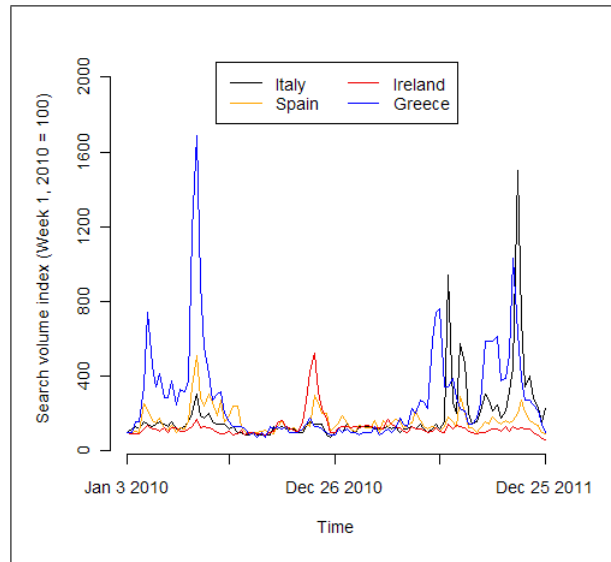Figure 3: Rescaled Google search volume index of searches on the economic conditions of the countries, weekly data (w1 2010=100)

Figure 4: Percentage changes in weekly Google search volume index