A note on the indeterminacy and arbitrariness of pena's method of construction of synthetic indicators

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Abstract

In this paper we demonstrate that Pena's method of construction of a synthetic indicator is very sensitive to the order in which the constituent variables (whose linear aggregation yields the synthetic indicator) are arranged. Since m number of constituent variables may be arranged in m-factorial ways, even a moderately large m can give rise to a very large number of synthetic indicators from which one cannot choose the one which best represents the constituent variables. Given that an analyst has too little information as to the order in which a sizeable number of constituent variables must be arranged so as to obtain the best representative synthetic indicator, Pena's method can give only an arbitrary synthetic indicator whose representativeness is indeterminate and uncertain.

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Keywords: Synthetic indicators, Human development, NP-Hard problem

I. Introduction: Synthetic indicators (Z) are the (weighted) linear combination of a large number (say, m) of variables (X) such that Z=Xw, where Z is a column vector with n elements (cases), X is an n x m matrix of constituent variables and w is an m-element column vector of weights. In making a synthetic indicator, the crux of the problem lay in determining the weight vector. Traditionally, the principal component analysis is used to derive the weights, but the critics of this method abound (Munda & Nardo, 2005; Mishra 2010). An alternative method (proposed by Pena, 1977) is currently gaining a remarkable popularity for determining weights and construction of synthetic indicators.

A synthetic indicator constructed by Pena's method (or so-called P2-distance) is claimed to possess almost all desirable properties such as non-negativity, commutativity, triangular Inequality, existence, determination, monotony, uniqueness, transitivity, invariance to change of origin and/or scale of the units in which the variables are defined, invariance to a change in the general conditions and exhaustiveness and reference base. It is also claimed that this indicator solves a large number of problems such as the aggregation of variables expressed in different measures, arbitrary weights and duplicity of information. (Pena, 1977; Zarazosa, 1996; Somarriba & Pena, 2009; Montero et al., 2010; Martína & Fernández, 2011). So claimed, the synthetic indicators constructed by the Principal Component analysis or its alternatives using non-Euclidean norms (Mishra, 2009) are deficient in the one or the other desirable properties in comparison to the Pena's method.

Summarily, Pena's synthetic indicator is Pena's P2-distance (DP2) defined as:

$$DP2_i = \sum_{j=1}^{m}\left[\left(\frac{d_{ij}}{\sigma_j}\right)\left(1-R^2_{j,j-1,\ldots,1}\right)\right]; i=1,2,\ldots,n \qquad \ldots \qquad (1)$$

where: $i=1,2,\ldots,n$ are cases (countries, for example); m is the number of constituent variables, X, such that $x_{ij} \in X; i=1,2,\ldots,n; j=1,2,\ldots,m$; $d_{ij}=\left|x_{ij}-x_{rj}\right|; i=1,2,..,n; j=1,2,\ldots,m$; r is the reference case; σ_j is the standard deviation of variable j; $R^2_{j,j-1,\ldots,1}$; $j>1$ is the coefficient of determination in the regression of x_j over $x_{j-1,}x_{j-2},\ldots,x_1$. Moreover, $R^2_1=0$ (Somarriba & Pena, 2009).

In equation (1), (d_{ij}/σ_j) is merely a change in the origin and the scale if the $\min_i(x_{ij})$ is taken as reference (r). Also, one may use zero as the reference (r) point and [$\max_i(x_{ij})$-$\min_i(x_{ij})$] instead of σ_j as a scaling factor, without any adverse effect on the formula. The real crux, however, lay in the weights, (1-$R^2_{j,j-1,\ldots,1}$). It may be noted that the first variable obtains an absolute weight of unity (1-R^2_1). The subsequent variable $j=2$ obtains a weight (1-$R^2_{2,1}$) and in general, the j^{th} variable obtains a weight of (1-$R^2_{j,j-1,\ldots,1}$). Due to this, it is obvious that the weights assigned to a variable will depend on its position in the order. This is demonstrated by an example as presented in the subsequent sections.

II. A Synthetic Indicator of Human Development with Income Equalities: Using the Human Development Report of UNDP, 2004 data and the additional information on the measures of inequality, Sarker et al. (2007), argued that Human Development Index (HDI) should include income equality measures (EQ) also in addition to the three conventional measures, viz. life expectancy (LE), education (ED) and per capita gross domestic product at the purchasing power parity with the US $ (PCI). Using their data, Mishra (2007; 2010) constructed two synthetic indicators, HDI_1 and HDI_2, by direct maximization of $\sum_{j=1}^{m}\left|r(HDI_1,x_j)\right|$ and $\sum_{j=1}^{m}\left|r(HDI_2,x_j)\right|^2$ respectively. The second indicator (HDI_2) is also obtainable by the principal component analysis. The data and the two synthetic indicators are reproduced here in Table-1. We use these variables (LE, ED, PCI and EQ) to construct Pena's synthetic indicators and also compare them with HDI_1 and HDI_2.

III. Pena's Indicators of Human Development: Using the formula given in (1), we construct Pena's synthetic Indicators of Human Development. Since our purpose is to demonstrate the indeterminacy and arbitrariness of Pena's indicators in face of permutation of the constituent variables (LE, ED, PCI and EQ), we construct all possible (24 = 4-factorial) indicators. All permutations of LE, ED, PCI and EQ are listed in Table-3. The order (1,2,3,4) signifies (LE, ED, PCI, EQ), order (2,1,3,4) signifies (ED, LE, PCI, EQ) and so on. Pena's synthetic indicators for those permutations are presented in Tables 4.1 and 4.2.

In Table-5 we have present the absolute weights (w_j=1 for j=1; $1-R^2_{j,j-1,\ldots,1}$ $for\ j>1$), the relative weights ($\omega_j = w_j / \sum_{j=1}^{m} w_j$) and the correlation of the constituent variables with the synthetic Pena Indicator for all possible permutations. We clearly see that the weights (absolute as well as relative) obtained by different constituent variables are different for different permutations or the order in which the constituent variables inter in formula (1). So are the coefficients of correlation of Pena's indicators with the constituent variables.

III. Correlation of Pena's Indicators with Other Synthetic Indicators: How do Pena's indicators (under permutation of the constituent variables – LE, ED, PCI and EQ) correlate with the alternative (and more conventional) synthetic indicators such as HDI_1 and HDI_2 (that are obtained by maximization of $\sum_{j=1}^{m}\left|r(HDI_1, x_j)\right|$ and $\sum_{j=1}^{m}\left|r(HDI_2, x_j)\right|^2$ respectively)? We present the correlation coefficients in Table-6.

It is found that some Pena Indicators have relatively larger coefficients of correlation (although statistically not different among themselves) with HDI_1. In general, HDI_2 has smaller coefficient of correlation with Pena indicators that HDI_1 has. Thus Pena's indicators are farther than the Principal component based synthetic indicator (HDI_2) vis-á-vis HDI_1, which is based on maximization of $\sum_{j=1}^{m}\left|r(HDI_1, x_j)\right|$. It may be noted that HDI_1 is a more inclusive synthetic index than HDI_2 (Mishra, 2010).

IV. Concluding Remarks: It has been demonstrated that Pena's synthetic indicators are sensitive to the order or permutation of the constituent variables in which they enter into the formula (eq. 1). Even for a modestly large number of variables to be aggregated into a synthetic indicator, there will be a very large number of possible Pena's indicators. Say, if the number of variables is 15, the total number of all possible Pena's indicators will be 3628.8 thousand. Thus, obtaining all synthetic indicators (for all permutations) is a real NP-Hard problem. But in practice, there are cases that use 50 or more variables. Due to this limitation, a choice of a particular order of variables to construct Pena's synthetic indicator (as is presently done) must be arbitrary and indeterminate. It cannot be sure that one has chosen the right order of variables. Thus, considering a synthetic indicator constructed by Pena's method better than those constructed by other (rather conventional) methods is a matter of (unfounded) belief.

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Tables:

Table-1. Correlation Maximizing HDI Indicators and their Constituent Variables

Country	LE	ED	PCI	EQ	HDI_1	HDI_2	Country	LE	ED	PCI	EQ	HDI_1	HDI_2
Norway	0.90	0.99	0.99	0.96	0.96	0.96	Turkey	0.76	0.80	0.69	0.66	0.73	0.73
Sweden	0.92	0.99	0.93	0.98	0.95	0.95	Azerbaijan	0.78	0.88	0.58	0.73	0.74	0.74
Canada	0.90	0.98	0.95	0.81	0.91	0.92	Jordan	0.76	0.86	0.62	0.74	0.74	0.74
Netherlands	0.89	0.99	0.95	0.82	0.92	0.92	Tunisia	0.79	0.74	0.70	0.66	0.72	0.73
Australia	0.90	0.99	0.94	0.76	0.90	0.91	China	0.76	0.83	0.64	0.56	0.70	0.71
Belgium	0.90	0.99	0.94	0.98	0.95	0.95	Georgia	0.81	0.89	0.52	0.73	0.73	0.73
United_States	0.87	0.97	0.98	0.64	0.87	0.89	Dominican_Republic	0.70	0.82	0.70	0.50	0.68	0.70
Japan	0.94	0.94	0.93	0.98	0.95	0.94	Sri_Lanka	0.79	0.83	0.60	0.78	0.75	0.74
Luxembourg	0.89	0.91	1.00	0.86	0.92	0.92	Ecuador	0.76	0.85	0.60	0.58	0.70	0.71
Ireland	0.86	0.96	0.98	0.75	0.89	0.90	Iran_Islamic_Rep_of	0.75	0.74	0.70	0.60	0.70	0.71
Switzerland	0.90	0.95	0.95	0.81	0.90	0.91	El_Salvador	0.76	0.75	0.65	0.38	0.64	0.66
Austria	0.89	0.96	0.95	0.87	0.92	0.92	Guyana	0.64	0.89	0.63	0.59	0.69	0.70
United_Kingdom	0.88	0.99	0.93	0.74	0.89	0.90	Uzbekistan	0.74	0.91	0.47	0.94	0.76	0.74
Finland	0.88	0.99	0.93	0.94	0.94	0.93	Algeria	0.74	0.69	0.68	0.76	0.72	0.71
Denmark	0.86	0.98	0.96	0.99	0.95	0.94	Kyrgyzstan	0.72	0.92	0.46	0.89	0.74	0.73
France	0.90	0.96	0.93	0.81	0.90	0.91	Indonesia	0.69	0.80	0.58	0.78	0.71	0.70
New_Zealand	0.89	0.99	0.90	0.74	0.88	0.90	Viet_Nam	0.73	0.82	0.52	0.74	0.70	0.70
Germany	0.89	0.95	0.94	0.91	0.92	0.92	Moldova_Rep_of	0.73	0.87	0.45	0.74	0.69	0.69
Spain	0.90	0.97	0.90	0.82	0.90	0.91	Bolivia	0.64	0.86	0.53	0.56	0.65	0.66
Italy	0.89	0.93	0.93	0.74	0.88	0.89	Honduras	0.73	0.74	0.54	0.34	0.59	0.61
Israel	0.90	0.94	0.88	0.76	0.87	0.88	Tajikistan	0.73	0.90	0.38	0.77	0.69	0.68
Singapore	0.88	0.91	0.92	0.61	0.83	0.86	Nicaragua	0.74	0.73	0.54	0.34	0.59	0.61
Greece	0.89	0.95	0.87	0.76	0.87	0.88	Mongolia	0.64	0.89	0.47	0.57	0.64	0.65
Hong_Kong_China_(SAR)	0.91	0.86	0.93	0.59	0.83	0.85	South_Africa	0.40	0.83	0.77	0.25	0.57	0.60
Portugal	0.85	0.97	0.87	0.69	0.85	0.86	Egypt	0.73	0.62	0.61	0.78	0.68	0.67

Slovenia	0.85	0.96	0.87	0.91	0.90	0.90	Guatemala	0.68	0.65	0.62	0.48	0.61	0.62
Korea_Rep_of	0.84	0.97	0.86	0.84	0.88	0.88	Morocco	0.72	0.53	0.61	0.67	0.63	0.63
Czech_Republic	0.84	0.92	0.84	0.97	0.89	0.88	Namibia	0.34	0.79	0.69	0.00	0.47	0.51
Argentina	0.82	0.96	0.78	0.40	0.75	0.78	India	0.64	0.59	0.55	0.82	0.65	0.63
Estonia	0.78	0.98	0.80	0.72	0.82	0.83	Botswana	0.27	0.76	0.73	0.17	0.50	0.52
Poland	0.81	0.96	0.78	0.84	0.85	0.85	Ghana	0.55	0.65	0.51	0.87	0.64	0.62
Hungary	0.78	0.95	0.82	1.00	0.89	0.87	Cambodia	0.54	0.66	0.50	0.65	0.59	0.58
Slovakia	0.81	0.91	0.81	0.96	0.87	0.86	Papua_New_Guinea	0.54	0.57	0.52	0.43	0.52	0.52
Lithuania	0.79	0.96	0.77	0.83	0.84	0.84	Lao_People's_Dem_Rep	0.49	0.64	0.47	0.72	0.58	0.56
Chile	0.85	0.90	0.77	0.30	0.71	0.75	Swaziland	0.18	0.74	0.64	0.21	0.45	0.47
Uruguay	0.84	0.94	0.73	0.56	0.77	0.79	Bangladesh	0.60	0.45	0.47	0.83	0.58	0.56
Costa_Rica	0.88	0.87	0.75	0.52	0.76	0.78	Nepal	0.58	0.50	0.44	0.73	0.56	0.54
Croatia	0.82	0.90	0.77	0.89	0.84	0.84	Cameroon	0.36	0.64	0.50	0.56	0.52	0.51
Latvia	0.76	0.95	0.75	0.82	0.82	0.82	Pakistan	0.60	0.40	0.49	0.81	0.57	0.55
Mexico	0.81	0.85	0.75	0.35	0.70	0.73	Lesotho	0.19	0.76	0.53	0.17	0.42	0.44
Trinidad_and_Tobago	0.77	0.87	0.76	0.65	0.76	0.77	Uganda	0.34	0.70	0.44	0.60	0.52	0.51
Bulgaria	0.77	0.91	0.71	0.83	0.80	0.80	Zimbabwe	0.15	0.79	0.53	0.30	0.45	0.46
Malaysia	0.80	0.83	0.75	0.46	0.71	0.74	Kenya	0.34	0.74	0.39	0.56	0.51	0.50
Russian_Federation	0.69	0.95	0.74	0.54	0.74	0.75	Yemen	0.58	0.50	0.36	0.80	0.55	0.53
Macedonia_TFYR	0.81	0.87	0.70	0.91	0.82	0.81	Madagascar	0.47	0.60	0.33	0.50	0.47	0.47
Panama	0.83	0.86	0.69	0.31	0.68	0.71	Nigeria	0.44	0.59	0.36	0.43	0.46	0.46
Belarus	0.75	0.95	0.67	0.86	0.81	0.80	Mauritania	0.45	0.42	0.52	0.68	0.52	0.50
Albania	0.81	0.89	0.65	0.91	0.81	0.80	Gambia	0.48	0.40	0.47	0.70	0.51	0.49
Bosnia_and_Herzegovi	0.82	0.84	0.68	0.95	0.82	0.81	Senegal	0.46	0.39	0.46	0.63	0.48	0.47
Venezuela	0.81	0.86	0.67	0.47	0.71	0.73	Guinea	0.40	0.37	0.51	0.65	0.48	0.46
Romania	0.76	0.88	0.70	0.87	0.80	0.79	Tanzania_U_Rep_of	0.31	0.62	0.29	0.70	0.48	0.45
Ukraine	0.74	0.94	0.65	0.89	0.80	0.79	Cote_d_Ivoire	0.27	0.47	0.45	0.55	0.44	0.42
Saint_Lucia	0.79	0.88	0.66	0.60	0.73	0.75	Zambia	0.13	0.68	0.36	0.39	0.39	0.39
Brazil	0.72	0.88	0.73	0.25	0.65	0.69	Malawi	0.21	0.66	0.29	0.44	0.40	0.39
Colombia	0.78	0.84	0.69	0.29	0.66	0.69	Central_African_Rep	0.25	0.43	0.41	0.21	0.33	0.34
Thailand	0.74	0.86	0.71	0.59	0.73	0.74	Ethiopia	0.34	0.39	0.34	0.87	0.48	0.44
Kazakhstan	0.69	0.93	0.68	0.84	0.78	0.78	Mozambique	0.22	0.45	0.39	0.67	0.43	0.41
Jamaica	0.84	0.83	0.61	0.70	0.74	0.75	Guinea-Bissau	0.34	0.39	0.33	0.51	0.39	0.38
Armenia	0.79	0.90	0.57	0.70	0.74	0.74	Burundi	0.26	0.45	0.31	0.80	0.45	0.42
Philippines	0.75	0.89	0.62	0.53	0.70	0.71	Mali	0.39	0.21	0.37	0.44	0.35	0.34
Turkmenistan	0.70	0.93	0.63	0.64	0.73	0.73	Burkina_Faso	0.35	0.16	0.40	0.49	0.35	0.34
Paraguay	0.76	0.85	0.64	0.30	0.64	0.67	Niger	0.35	0.18	0.35	0.44	0.33	0.32
Peru	0.74	0.86	0.65	0.45	0.68	0.70	Note:	From Sarker et al (2006)				Mishra (2007)	

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Table-3. All Permutations of Constituent Variables of Human Development (LE, ED, PCI and EQ)

P	Order					P	Order			
1	1	2	3	4		13	1	3	4	2
2	2	1	3	4		14	3	1	4	2
3	3	1	2	4		15	4	1	3	2
4	1	3	2	4		16	1	4	3	2
5	2	3	1	4		17	3	4	1	2
6	3	2	1	4		18	4	3	1	2
7	4	2	1	3		19	4	3	2	1
8	2	4	1	3		20	3	4	2	1
9	1	4	2	3		21	2	4	3	1
10	4	1	2	3		22	4	2	3	1
11	2	1	4	3		23	3	2	4	1
12	1	2	4	3		24	2	3	4	1

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Table-4.1. Pena's Synthetic Indicators of Human Development (First 12 Permutations of Constituent Variables)												
Country	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8)	P(9)	P(10)	P(11)	P(12)
Norway	1.329	1.316	1.381	1.338	1.323	1.379	1.330	1.324	1.330	1.343	1.316	1.329
Sweden	1.336	1.317	1.357	1.342	1.320	1.355	1.332	1.325	1.337	1.350	1.317	1.336
Canada	1.260	1.247	1.296	1.266	1.252	1.294	1.243	1.243	1.259	1.253	1.247	1.260
Netherlands	1.261	1.254	1.300	1.267	1.259	1.299	1.251	1.251	1.261	1.257	1.253	1.261
Australia	1.241	1.231	1.273	1.246	1.236	1.271	1.221	1.224	1.240	1.227	1.231	1.241
Belgium	1.327	1.314	1.359	1.333	1.319	1.357	1.330	1.323	1.328	1.344	1.314	1.327
United_States	1.180	1.176	1.238	1.187	1.184	1.237	1.153	1.160	1.178	1.153	1.175	1.180
Japan	1.333	1.294	1.351	1.341	1.296	1.346	1.311	1.302	1.334	1.345	1.294	1.333
Luxembourg	1.265	1.234	1.328	1.277	1.242	1.324	1.240	1.235	1.265	1.266	1.234	1.265
Ireland	1.215	1.209	1.277	1.224	1.218	1.276	1.201	1.203	1.215	1.205	1.209	1.215
Switzerland	1.252	1.230	1.289	1.259	1.235	1.287	1.228	1.227	1.251	1.245	1.230	1.251
Austria	1.272	1.256	1.313	1.280	1.261	1.311	1.261	1.257	1.273	1.275	1.256	1.272
United_Kingdom	1.220	1.217	1.254	1.225	1.222	1.254	1.205	1.209	1.219	1.206	1.217	1.220
Finland	1.299	1.292	1.333	1.304	1.297	1.332	1.305	1.300	1.299	1.312	1.292	1.299
Denmark	1.311	1.307	1.363	1.318	1.314	1.362	1.326	1.319	1.312	1.334	1.307	1.311
France	1.250	1.232	1.280	1.257	1.235	1.277	1.229	1.228	1.250	1.243	1.232	1.250
New_Zealand	1.219	1.214	1.239	1.222	1.216	1.238	1.202	1.205	1.219	1.204	1.214	1.219
Germany	1.283	1.263	1.320	1.291	1.268	1.318	1.274	1.268	1.284	1.292	1.263	1.284
Spain	1.251	1.235	1.268	1.255	1.237	1.266	1.233	1.233	1.251	1.245	1.235	1.251
Italy	1.209	1.186	1.243	1.217	1.191	1.241	1.177	1.179	1.209	1.194	1.186	1.209
Israel	1.215	1.192	1.226	1.220	1.193	1.223	1.185	1.186	1.215	1.201	1.192	1.215
Singapore	1.145	1.122	1.180	1.153	1.126	1.177	1.099	1.105	1.144	1.113	1.122	1.145
Greece	1.210	1.193	1.220	1.214	1.194	1.218	1.186	1.187	1.210	1.198	1.193	1.210
Hong_Kong_China_(SAR)	1.142	1.096	1.174	1.152	1.099	1.169	1.072	1.077	1.141	1.104	1.096	1.141
Portugal	1.167	1.169	1.188	1.169	1.172	1.188	1.154	1.159	1.166	1.150	1.168	1.167
Slovenia	1.251	1.246	1.272	1.254	1.249	1.271	1.258	1.253	1.251	1.264	1.246	1.251
Korea_Rep_of	1.218	1.221	1.239	1.221	1.224	1.239	1.224	1.223	1.219	1.223	1.221	1.218
Czech_Republic	1.252	1.238	1.267	1.256	1.240	1.265	1.260	1.251	1.253	1.275	1.238	1.252
Argentina	1.016	1.030	1.012	1.013	1.029	1.014	0.982	1.000	1.013	0.963	1.029	1.015
Estonia	1.130	1.155	1.145	1.128	1.158	1.148	1.147	1.152	1.129	1.126	1.155	1.130
Poland	1.184	1.193	1.182	1.182	1.193	1.183	1.199	1.197	1.184	1.192	1.193	1.184
Hungary	1.236	1.248	1.260	1.237	1.252	1.261	1.275	1.266	1.237	1.270	1.248	1.236
Slovakia	1.223	1.216	1.236	1.226	1.218	1.235	1.239	1.230	1.225	1.249	1.216	1.223
Lithuania	1.167	1.182	1.168	1.165	1.183	1.170	1.188	1.187	1.167	1.177	1.182	1.167
Chile	0.974	0.964	0.961	0.973	0.961	0.960	0.906	0.926	0.971	0.904	0.963	0.974
Uruguay	1.074	1.074	1.045	1.070	1.069	1.046	1.046	1.056	1.072	1.040	1.074	1.074
Costa_Rica	1.065	1.033	1.035	1.065	1.028	1.032	1.002	1.011	1.063	1.021	1.033	1.064
Croatia	1.190	1.179	1.185	1.192	1.178	1.184	1.193	1.187	1.191	1.205	1.179	1.190
Latvia	1.140	1.162	1.143	1.137	1.163	1.146	1.168	1.168	1.140	1.153	1.162	1.140
Mexico	0.955	0.942	0.949	0.955	0.940	0.947	0.895	0.910	0.952	0.896	0.941	0.954
Trinidad_and_Tobago	1.059	1.058	1.067	1.060	1.059	1.067	1.048	1.051	1.058	1.047	1.058	1.059
Bulgaria	1.130	1.138	1.117	1.128	1.136	1.118	1.148	1.145	1.131	1.143	1.138	1.131
Malaysia	0.987	0.969	0.985	0.989	0.968	0.983	0.937	0.947	0.985	0.945	0.969	0.987
Russian_Federation	0.990	1.038	1.012	0.985	1.042	1.017	1.014	1.026	0.989	0.973	1.037	0.990
Macedonia_TFYR	1.171	1.154	1.143	1.170	1.149	1.141	1.173	1.165	1.172	1.189	1.154	1.171
Panama	0.940	0.925	0.904	0.938	0.919	0.903	0.872	0.890	0.937	0.874	0.925	0.940
Belarus	1.134	1.159	1.109	1.128	1.156	1.112	1.171	1.169	1.135	1.154	1.159	1.135
Albania	1.166	1.155	1.117	1.162	1.148	1.116	1.174	1.167	1.167	1.185	1.155	1.166
Bosnia_and_Herzegovi	1.180	1.151	1.142	1.180	1.144	1.139	1.176	1.165	1.181	1.203	1.151	1.180
Venezuela	0.989	0.977	0.950	0.985	0.971	0.949	0.944	0.955	0.987	0.947	0.977	0.988
Romania	1.131	1.132	1.118	1.129	1.130	1.118	1.149	1.143	1.132	1.150	1.132	1.131
Ukraine	1.134	1.159	1.105	1.127	1.155	1.108	1.176	1.171	1.135	1.159	1.159	1.134

Saint_Lucia	1.033	1.030	0.995	1.028	1.024	0.995	1.013	1.019	1.032	1.011	1.030	1.032
Brazil	0.871	0.895	0.884	0.868	0.897	0.887	0.839	0.860	0.868	0.810	0.895	0.871
Colombia	0.900	0.895	0.881	0.898	0.891	0.880	0.843	0.861	0.897	0.837	0.894	0.900
Thailand	1.006	1.013	1.005	1.005	1.013	1.006	0.998	1.004	1.005	0.990	1.013	1.006
Kazakhstan	1.091	1.128	1.090	1.085	1.129	1.094	1.142	1.139	1.091	1.115	1.128	1.091
Jamaica	1.075	1.042	1.005	1.071	1.031	1.002	1.038	1.038	1.075	1.061	1.043	1.075
Armenia	1.059	1.062	0.985	1.050	1.051	0.986	1.056	1.058	1.059	1.051	1.062	1.059
Philippines	0.978	0.992	0.937	0.971	0.986	0.939	0.968	0.978	0.977	0.952	0.992	0.978
Turkmenistan	1.007	1.046	0.984	0.999	1.043	0.988	1.035	1.042	1.007	1.003	1.045	1.007
Paraguay	0.886	0.890	0.853	0.881	0.884	0.853	0.840	0.858	0.883	0.827	0.889	0.886
Peru	0.939	0.949	0.915	0.934	0.945	0.916	0.917	0.929	0.937	0.903	0.948	0.939
Turkey	1.024	1.007	1.012	1.025	1.005	1.010	1.003	1.003	1.024	1.015	1.007	1.024
Azerbaijan	1.062	1.061	0.996	1.054	1.052	0.997	1.061	1.061	1.062	1.060	1.061	1.062
Jordan	1.058	1.057	1.015	1.053	1.050	1.015	1.059	1.059	1.058	1.060	1.057	1.058
Tunisia	1.026	0.983	1.012	1.031	0.979	1.007	0.980	0.978	1.026	1.013	0.983	1.026
China	0.983	0.976	0.950	0.980	0.971	0.950	0.959	0.965	0.982	0.960	0.976	0.983
Georgia	1.069	1.063	0.970	1.058	1.048	0.970	1.060	1.061	1.069	1.063	1.063	1.069
Dominican_Republic	0.936	0.945	0.947	0.936	0.947	0.948	0.924	0.932	0.935	0.913	0.945	0.936
Sri_Lanka	1.078	1.059	1.019	1.073	1.049	1.017	1.066	1.063	1.078	1.081	1.059	1.078
Ecuador	0.988	0.987	0.939	0.982	0.980	0.939	0.971	0.977	0.987	0.968	0.987	0.988
Iran_Islamic_Rep_of	0.981	0.951	0.979	0.985	0.949	0.976	0.943	0.943	0.981	0.965	0.950	0.981
El_Salvador	0.892	0.866	0.868	0.893	0.862	0.865	0.831	0.842	0.891	0.845	0.866	0.892
Guyana	0.945	0.990	0.942	0.937	0.991	0.947	0.979	0.986	0.944	0.941	0.990	0.945
Uzbekistan	1.109	1.126	1.011	1.095	1.112	1.014	1.152	1.144	1.111	1.141	1.127	1.110
Algeria	1.021	0.977	1.017	1.027	0.975	1.012	0.992	0.983	1.022	1.029	0.977	1.021
Kyrgyzstan	1.080	1.106	0.983	1.064	1.093	0.987	1.126	1.122	1.081	1.107	1.107	1.080
Indonesia	1.012	1.014	0.978	1.007	1.009	0.979	1.028	1.024	1.013	1.028	1.014	1.012
Viet_Nam	1.011	1.008	0.940	1.003	0.998	0.941	1.015	1.013	1.011	1.017	1.008	1.011
Moldova_Rep_of	1.010	1.023	0.910	0.996	1.008	0.912	1.027	1.027	1.011	1.016	1.023	1.011
Bolivia	0.905	0.943	0.865	0.894	0.938	0.869	0.930	0.938	0.904	0.897	0.943	0.905
Honduras	0.836	0.817	0.778	0.831	0.808	0.777	0.780	0.792	0.834	0.787	0.817	0.835
Tajikistan	1.016	1.037	0.887	0.997	1.019	0.890	1.043	1.043	1.017	1.026	1.037	1.017
Nicaragua	0.838	0.814	0.778	0.834	0.804	0.776	0.777	0.789	0.836	0.788	0.814	0.838
Mongolia	0.905	0.952	0.840	0.889	0.944	0.845	0.939	0.947	0.904	0.898	0.952	0.905
South_Africa	0.694	0.798	0.826	0.694	0.820	0.836	0.760	0.780	0.690	0.674	0.798	0.693
Egypt	0.990	0.930	0.966	0.997	0.924	0.959	0.951	0.940	0.992	1.002	0.930	0.991
Guatemala	0.856	0.823	0.850	0.860	0.821	0.846	0.810	0.812	0.855	0.832	0.823	0.855
Morocco	0.917	0.836	0.901	0.928	0.831	0.892	0.850	0.839	0.919	0.916	0.836	0.917
Namibia	0.536	0.652	0.658	0.533	0.673	0.670	0.590	0.619	0.531	0.490	0.651	0.535
India	0.938	0.895	0.919	0.942	0.891	0.914	0.928	0.913	0.940	0.967	0.895	0.938
Botswana	0.565	0.690	0.726	0.566	0.717	0.739	0.653	0.674	0.562	0.551	0.690	0.565
Ghana	0.917	0.918	0.908	0.916	0.917	0.908	0.958	0.944	0.920	0.964	0.918	0.918
Cambodia	0.826	0.836	0.816	0.823	0.835	0.817	0.850	0.846	0.827	0.844	0.836	0.826
Papua_New_Guinea	0.719	0.707	0.722	0.721	0.707	0.721	0.699	0.701	0.718	0.707	0.707	0.719
Lao_People's_Dem_Rep	0.815	0.834	0.810	0.812	0.834	0.812	0.859	0.852	0.817	0.849	0.834	0.815
Swaziland	0.509	0.655	0.664	0.506	0.682	0.679	0.629	0.647	0.506	0.512	0.655	0.509
Bangladesh	0.866	0.796	0.836	0.873	0.790	0.828	0.840	0.820	0.869	0.902	0.797	0.866
Nepal	0.823	0.776	0.786	0.826	0.769	0.780	0.806	0.792	0.825	0.848	0.776	0.824
Cameroon	0.688	0.748	0.736	0.685	0.757	0.743	0.761	0.760	0.689	0.717	0.748	0.688
Pakistan	0.848	0.765	0.829	0.859	0.760	0.820	0.809	0.788	0.852	0.882	0.765	0.849
Lesotho	0.482	0.633	0.590	0.472	0.653	0.606	0.600	0.622	0.479	0.479	0.632	0.482
Uganda	0.698	0.780	0.725	0.689	0.786	0.734	0.796	0.796	0.698	0.734	0.780	0.698
Zimbabwe	0.520	0.689	0.638	0.509	0.711	0.656	0.672	0.689	0.518	0.539	0.688	0.520
Kenya	0.683	0.777	0.688	0.669	0.781	0.699	0.786	0.790	0.683	0.714	0.778	0.683
Yemen	0.835	0.787	0.766	0.834	0.775	0.761	0.826	0.809	0.838	0.869	0.787	0.835
Madagascar	0.679	0.697	0.628	0.670	0.690	0.630	0.700	0.701	0.679	0.685	0.697	0.679

Nigeria	0.638	0.664	0.609	0.631	0.660	0.612	0.660	0.663	0.638	0.639	0.664	0.638
Mauritania	0.728	0.697	0.767	0.738	0.702	0.763	0.731	0.717	0.730	0.763	0.697	0.728
Gambia	0.737	0.691	0.748	0.745	0.691	0.742	0.728	0.711	0.739	0.770	0.691	0.737
Senegal	0.694	0.652	0.708	0.702	0.653	0.703	0.682	0.668	0.696	0.720	0.652	0.694
Guinea	0.674	0.644	0.727	0.685	0.651	0.723	0.680	0.665	0.676	0.711	0.644	0.674
Tanzania_U_Rep_of	0.669	0.737	0.651	0.657	0.737	0.659	0.771	0.764	0.671	0.724	0.738	0.669
Cote_d_Ivoire	0.580	0.619	0.645	0.582	0.630	0.649	0.644	0.637	0.581	0.619	0.619	0.580
Zambia	0.480	0.624	0.544	0.467	0.638	0.560	0.626	0.635	0.480	0.515	0.624	0.481
Malawi	0.524	0.638	0.536	0.508	0.643	0.548	0.643	0.649	0.523	0.555	0.638	0.524
Central_African_Rep	0.416	0.456	0.475	0.417	0.466	0.479	0.444	0.451	0.415	0.412	0.456	0.416
Ethiopia	0.699	0.690	0.704	0.702	0.691	0.702	0.756	0.732	0.703	0.775	0.691	0.700
Mozambique	0.583	0.629	0.641	0.583	0.639	0.646	0.673	0.660	0.585	0.645	0.630	0.583
Guinea-Bissau	0.556	0.553	0.557	0.556	0.553	0.556	0.575	0.567	0.557	0.581	0.553	0.556
Burundi	0.639	0.672	0.654	0.637	0.676	0.657	0.730	0.712	0.642	0.715	0.673	0.640
Mali	0.514	0.447	0.525	0.526	0.446	0.517	0.467	0.455	0.516	0.524	0.447	0.514
Burkina_Faso	0.505	0.434	0.542	0.520	0.438	0.534	0.465	0.449	0.507	0.527	0.434	0.505
Niger	0.480	0.417	0.497	0.492	0.417	0.490	0.441	0.428	0.482	0.496	0.417	0.481

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Table-4.2. Pena's Synthetic Indicators of Human Development (Last 12 Permutations of Constituent Variables)												
Country	P(13)	P(14)	P(15)	P(16)	P(17)	P(18)	P(19)	P(20)	P(21)	P(22)	P(23)	P(24)
Norway	1.338	1.381	1.351	1.338	1.386	1.384	1.382	1.384	1.330	1.336	1.383	1.330
Sweden	1.342	1.358	1.355	1.342	1.364	1.367	1.364	1.362	1.328	1.334	1.361	1.327
Canada	1.266	1.296	1.259	1.265	1.291	1.284	1.282	1.290	1.247	1.247	1.290	1.248
Netherlands	1.267	1.300	1.262	1.266	1.296	1.289	1.288	1.295	1.256	1.256	1.295	1.257
Australia	1.246	1.272	1.232	1.245	1.265	1.255	1.254	1.263	1.228	1.225	1.264	1.229
Belgium	1.334	1.359	1.350	1.334	1.366	1.368	1.366	1.364	1.327	1.334	1.364	1.326
United_States	1.187	1.238	1.160	1.186	1.223	1.205	1.204	1.222	1.168	1.161	1.222	1.171
Japan	1.342	1.351	1.353	1.342	1.357	1.360	1.355	1.353	1.304	1.313	1.352	1.303
Luxembourg	1.278	1.328	1.278	1.277	1.327	1.319	1.315	1.323	1.243	1.247	1.323	1.243
Ireland	1.223	1.277	1.213	1.223	1.269	1.257	1.256	1.268	1.211	1.209	1.269	1.213
Switzerland	1.259	1.289	1.252	1.259	1.285	1.278	1.275	1.282	1.231	1.232	1.282	1.232
Austria	1.280	1.313	1.282	1.280	1.313	1.309	1.307	1.311	1.262	1.265	1.311	1.263
United_Kingdom	1.224	1.254	1.210	1.224	1.246	1.236	1.236	1.246	1.214	1.210	1.246	1.215
Finland	1.304	1.333	1.317	1.304	1.338	1.339	1.338	1.337	1.304	1.309	1.337	1.303
Denmark	1.319	1.364	1.341	1.319	1.373	1.375	1.373	1.372	1.325	1.332	1.371	1.324
France	1.257	1.280	1.249	1.256	1.276	1.269	1.267	1.273	1.232	1.233	1.273	1.233
New_Zealand	1.222	1.238	1.207	1.222	1.230	1.222	1.222	1.230	1.208	1.204	1.230	1.209
Germany	1.291	1.321	1.299	1.291	1.324	1.322	1.319	1.321	1.273	1.278	1.321	1.272
Spain	1.255	1.268	1.249	1.255	1.265	1.261	1.259	1.263	1.235	1.235	1.263	1.235
Italy	1.217	1.243	1.201	1.216	1.235	1.225	1.223	1.233	1.183	1.182	1.233	1.184
Israel	1.220	1.226	1.206	1.219	1.219	1.213	1.211	1.217	1.187	1.186	1.217	1.188
Singapore	1.152	1.179	1.120	1.151	1.163	1.147	1.145	1.161	1.110	1.104	1.161	1.112
Greece	1.214	1.220	1.201	1.213	1.214	1.208	1.207	1.212	1.189	1.187	1.212	1.189
Hong_Kong_China_(SAR)	1.152	1.173	1.113	1.151	1.155	1.137	1.132	1.150	1.081	1.076	1.150	1.084
Portugal	1.169	1.188	1.152	1.168	1.179	1.169	1.170	1.179	1.162	1.157	1.179	1.163
Slovenia	1.254	1.272	1.267	1.255	1.278	1.280	1.279	1.277	1.256	1.260	1.277	1.255
Korea_Rep_of	1.221	1.239	1.225	1.221	1.240	1.239	1.239	1.240	1.226	1.227	1.240	1.226
Czech_Republic	1.257	1.267	1.279	1.257	1.278	1.285	1.283	1.276	1.253	1.261	1.276	1.251
Argentina	1.012	1.011	0.959	1.010	0.985	0.965	0.968	0.988	1.000	0.983	0.989	1.004
Estonia	1.127	1.145	1.124	1.127	1.142	1.138	1.140	1.144	1.154	1.149	1.144	1.155
Poland	1.182	1.182	1.191	1.182	1.187	1.190	1.191	1.188	1.197	1.198	1.187	1.196
Hungary	1.238	1.260	1.272	1.239	1.276	1.286	1.286	1.277	1.268	1.277	1.276	1.267
Slovakia	1.227	1.237	1.252	1.227	1.249	1.257	1.255	1.248	1.231	1.240	1.247	1.230
Lithuania	1.165	1.168	1.175	1.165	1.173	1.176	1.178	1.174	1.187	1.188	1.174	1.187
Chile	0.972	0.959	0.902	0.970	0.926	0.901	0.902	0.926	0.925	0.906	0.927	0.930

Uruguay	1.070	1.045	1.036	1.069	1.030	1.021	1.022	1.031	1.053	1.043	1.031	1.055
Costa_Rica	1.065	1.035	1.021	1.064	1.015	1.003	1.001	1.013	1.008	0.999	1.013	1.010
Croatia	1.192	1.185	1.206	1.192	1.193	1.200	1.198	1.192	1.186	1.192	1.191	1.185
Latvia	1.137	1.143	1.150	1.138	1.149	1.153	1.155	1.151	1.168	1.169	1.151	1.167
Mexico	0.954	0.948	0.896	0.953	0.920	0.899	0.899	0.919	0.910	0.895	0.920	0.914
Trinidad_and_Tobago	1.060	1.067	1.048	1.059	1.061	1.055	1.056	1.061	1.052	1.049	1.061	1.053
Bulgaria	1.128	1.117	1.141	1.128	1.124	1.130	1.131	1.125	1.143	1.146	1.124	1.142
Malaysia	0.989	0.984	0.947	0.987	0.965	0.949	0.948	0.963	0.948	0.938	0.964	0.950
Russian_Federation	0.985	1.012	0.968	0.984	1.001	0.991	0.996	1.006	1.029	1.018	1.007	1.031
Macedonia_TFYR	1.170	1.143	1.189	1.171	1.154	1.165	1.163	1.152	1.161	1.169	1.152	1.159
Panama	0.936	0.903	0.871	0.935	0.872	0.853	0.853	0.872	0.886	0.869	0.873	0.890
Belarus	1.128	1.110	1.147	1.129	1.120	1.130	1.132	1.122	1.166	1.168	1.122	1.164
Albania	1.162	1.118	1.181	1.163	1.129	1.143	1.142	1.128	1.160	1.167	1.128	1.158
Bosnia_and_Herzegovi	1.180	1.143	1.203	1.181	1.156	1.171	1.168	1.153	1.160	1.171	1.153	1.157
Venezuela	0.985	0.950	0.943	0.984	0.931	0.921	0.921	0.931	0.951	0.940	0.932	0.953
Romania	1.129	1.118	1.149	1.130	1.129	1.138	1.138	1.128	1.141	1.147	1.128	1.140
Ukraine	1.127	1.105	1.152	1.128	1.118	1.131	1.133	1.120	1.168	1.172	1.120	1.166
Saint_Lucia	1.028	0.995	1.007	1.027	0.986	0.983	0.984	0.987	1.015	1.009	0.987	1.016
Brazil	0.867	0.883	0.807	0.865	0.852	0.827	0.831	0.856	0.863	0.842	0.857	0.868
Colombia	0.897	0.879	0.835	0.895	0.850	0.829	0.830	0.851	0.859	0.841	0.852	0.863
Thailand	1.005	1.005	0.988	1.004	0.997	0.991	0.992	0.998	1.004	0.998	0.998	1.005
Kazakhstan	1.085	1.090	1.110	1.086	1.101	1.110	1.113	1.105	1.139	1.142	1.104	1.138
Jamaica	1.071	1.005	1.057	1.071	1.002	1.007	1.005	0.999	1.029	1.029	0.999	1.028
Armenia	1.050	0.985	1.042	1.050	0.985	0.992	0.993	0.986	1.049	1.047	0.986	1.049
Philippines	0.970	0.937	0.945	0.970	0.926	0.921	0.924	0.928	0.973	0.964	0.928	0.974
Turkmenistan	0.998	0.984	0.994	0.998	0.982	0.982	0.987	0.986	1.039	1.033	0.986	1.039
Paraguay	0.880	0.851	0.822	0.878	0.824	0.807	0.809	0.826	0.854	0.836	0.827	0.858
Peru	0.934	0.914	0.899	0.933	0.898	0.887	0.890	0.900	0.927	0.915	0.900	0.929
Turkey	1.025	1.012	1.016	1.025	1.009	1.007	1.006	1.007	1.001	1.001	1.007	1.002
Azerbaijan	1.054	0.996	1.052	1.055	0.998	1.007	1.007	0.999	1.053	1.052	0.999	1.052
Jordan	1.053	1.015	1.055	1.053	1.018	1.025	1.025	1.018	1.053	1.054	1.018	1.052
Tunisia	1.031	1.012	1.018	1.031	1.007	1.005	1.001	1.003	0.976	0.978	1.003	0.976
China	0.979	0.950	0.957	0.979	0.940	0.936	0.936	0.940	0.961	0.955	0.940	0.962
Georgia	1.058	0.970	1.052	1.058	0.971	0.983	0.983	0.972	1.049	1.048	0.972	1.047
Dominican_Republic	0.936	0.946	0.912	0.935	0.934	0.924	0.925	0.935	0.934	0.925	0.936	0.935
Sri_Lanka	1.074	1.019	1.077	1.074	1.024	1.034	1.032	1.023	1.055	1.058	1.022	1.053
Ecuador	0.982	0.938	0.962	0.982	0.930	0.930	0.931	0.931	0.971	0.965	0.932	0.972
Iran_Islamic_Rep_of	0.985	0.979	0.969	0.985	0.972	0.967	0.964	0.969	0.943	0.943	0.970	0.944
El_Salvador	0.892	0.867	0.845	0.891	0.846	0.832	0.830	0.844	0.839	0.829	0.845	0.842
Guyana	0.937	0.942	0.934	0.937	0.939	0.937	0.942	0.944	0.986	0.979	0.945	0.987
Uzbekistan	1.096	1.012	1.128	1.097	1.031	1.056	1.058	1.033	1.132	1.139	1.033	1.128
Algeria	1.028	1.017	1.035	1.028	1.022	1.027	1.022	1.018	0.982	0.990	1.017	0.981
Kyrgyzstan	1.064	0.984	1.092	1.066	1.001	1.024	1.027	1.004	1.109	1.114	1.004	1.106
Indonesia	1.008	0.979	1.024	1.009	0.988	0.999	0.999	0.989	1.019	1.024	0.988	1.017
Viet_Nam	1.003	0.941	1.009	1.004	0.947	0.959	0.959	0.947	1.004	1.006	0.947	1.002
Moldova_Rep_of	0.996	0.910	1.002	0.997	0.917	0.933	0.935	0.919	1.014	1.014	0.919	1.012
Bolivia	0.893	0.864	0.886	0.893	0.861	0.863	0.868	0.866	0.933	0.926	0.866	0.933
Honduras	0.830	0.777	0.782	0.829	0.756	0.746	0.746	0.756	0.785	0.773	0.757	0.788
Tajikistan	0.997	0.887	1.008	0.999	0.897	0.918	0.921	0.900	1.027	1.027	0.900	1.025
Nicaragua	0.833	0.777	0.783	0.832	0.756	0.746	0.745	0.755	0.782	0.770	0.756	0.784
Mongolia	0.889	0.839	0.884	0.889	0.838	0.844	0.849	0.844	0.940	0.931	0.844	0.939
South_Africa	0.693	0.824	0.674	0.691	0.807	0.779	0.790	0.817	0.798	0.779	0.818	0.803
Egypt	0.997	0.966	1.008	0.998	0.975	0.984	0.977	0.968	0.936	0.947	0.968	0.934
Guatemala	0.859	0.849	0.836	0.859	0.839	0.832	0.829	0.836	0.812	0.809	0.836	0.813
Morocco	0.928	0.901	0.925	0.928	0.903	0.906	0.898	0.895	0.836	0.847	0.894	0.835
Namibia	0.531	0.656	0.486	0.528	0.626	0.590	0.602	0.638	0.637	0.607	0.640	0.643

India	0.943	0.920	0.971	0.944	0.936	0.950	0.945	0.931	0.910	0.924	0.930	0.907
Botswana	0.565	0.725	0.552	0.563	0.709	0.679	0.691	0.721	0.696	0.675	0.722	0.701
Ghana	0.917	0.909	0.964	0.919	0.933	0.952	0.951	0.932	0.942	0.956	0.931	0.939
Cambodia	0.824	0.816	0.842	0.824	0.825	0.833	0.834	0.826	0.844	0.849	0.826	0.843
Papua_New_Guinea	0.721	0.722	0.709	0.720	0.716	0.712	0.711	0.715	0.702	0.700	0.715	0.703
Lao_People's_Dem_Rep	0.812	0.811	0.847	0.814	0.827	0.841	0.842	0.829	0.851	0.859	0.828	0.848
Swaziland	0.505	0.663	0.510	0.503	0.655	0.632	0.646	0.669	0.669	0.650	0.670	0.673
Bangladesh	0.874	0.837	0.909	0.876	0.858	0.877	0.869	0.850	0.815	0.835	0.849	0.811
Nepal	0.827	0.786	0.851	0.828	0.801	0.817	0.811	0.796	0.786	0.800	0.795	0.784
Cameroon	0.685	0.736	0.715	0.686	0.747	0.750	0.756	0.753	0.766	0.767	0.752	0.766
Pakistan	0.860	0.830	0.892	0.862	0.849	0.866	0.857	0.840	0.784	0.805	0.839	0.780
Lesotho	0.471	0.589	0.469	0.470	0.579	0.560	0.575	0.594	0.637	0.616	0.595	0.641
Uganda	0.689	0.725	0.726	0.690	0.740	0.749	0.757	0.748	0.800	0.800	0.748	0.798
Zimbabwe	0.508	0.637	0.529	0.507	0.638	0.626	0.643	0.655	0.706	0.689	0.655	0.708
Kenya	0.669	0.688	0.701	0.670	0.701	0.711	0.720	0.711	0.791	0.788	0.711	0.790
Yemen	0.835	0.767	0.869	0.837	0.787	0.811	0.806	0.782	0.799	0.816	0.781	0.795
Madagascar	0.670	0.628	0.677	0.670	0.633	0.642	0.644	0.635	0.694	0.694	0.635	0.693
Nigeria	0.631	0.609	0.632	0.631	0.610	0.614	0.617	0.613	0.660	0.657	0.613	0.660
Mauritania	0.739	0.768	0.773	0.740	0.784	0.792	0.788	0.780	0.721	0.735	0.779	0.719
Gambia	0.746	0.748	0.779	0.747	0.765	0.778	0.772	0.760	0.712	0.728	0.759	0.709
Senegal	0.703	0.708	0.728	0.703	0.722	0.731	0.725	0.717	0.669	0.683	0.716	0.667
Guinea	0.686	0.728	0.722	0.687	0.745	0.752	0.747	0.740	0.670	0.686	0.740	0.669
Tanzania_U_Rep_of	0.657	0.652	0.713	0.659	0.678	0.699	0.706	0.684	0.762	0.769	0.683	0.758
Cote_d_Ivoire	0.583	0.646	0.622	0.583	0.662	0.667	0.669	0.664	0.646	0.652	0.664	0.644
Zambia	0.466	0.544	0.503	0.467	0.555	0.557	0.571	0.569	0.644	0.636	0.569	0.644
Malawi	0.508	0.536	0.541	0.509	0.548	0.556	0.567	0.559	0.652	0.646	0.559	0.651
Central_African_Rep	0.417	0.475	0.413	0.416	0.469	0.459	0.463	0.473	0.459	0.452	0.473	0.460
Ethiopia	0.704	0.705	0.778	0.706	0.742	0.770	0.767	0.739	0.731	0.755	0.738	0.726
Mozambique	0.584	0.642	0.647	0.586	0.669	0.684	0.687	0.672	0.667	0.680	0.672	0.665
Guinea-Bissau	0.557	0.557	0.582	0.558	0.570	0.579	0.578	0.569	0.567	0.575	0.568	0.565
Burundi	0.638	0.655	0.713	0.640	0.690	0.716	0.718	0.692	0.713	0.731	0.691	0.708
Mali	0.526	0.525	0.535	0.527	0.531	0.535	0.527	0.524	0.456	0.468	0.523	0.455
Burkina_Faso	0.521	0.543	0.542	0.522	0.554	0.558	0.550	0.546	0.452	0.469	0.545	0.451
Niger	0.493	0.498	0.507	0.494	0.506	0.511	0.503	0.499	0.429	0.442	0.498	0.428

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Table-5. Weights and Correlation obtained by Constituent Variables of Pena's Indicators

	Absolute weights				Relative weights				Correlation with Pena's Indicator			
P	LE	ED	PCI	EQ	LE	ED	PCI	EQ	LE	ED	PCI	EQ
1	1.0000	0.4684	0.3368	0.7433	0.3924	0.1838	0.1322	0.2917	0.9359	0.7913	0.8029	0.7020
2	0.4684	1.0000	0.3368	0.7433	0.1838	0.3924	0.1322	0.2917	0.8845	0.8770	0.8152	0.6621
3	0.3790	1.0000	0.4163	0.7433	0.1493	0.3939	0.1640	0.2928	0.7986	0.8919	0.8855	0.6653
4	1.0000	0.3790	0.4163	0.7433	0.3939	0.1493	0.1640	0.2928	0.9365	0.7794	0.8112	0.7038
5	0.4375	0.3606	1.0000	0.7433	0.1721	0.1419	0.3935	0.2925	0.8290	0.8746	0.8806	0.6555
6	0.3606	0.4375	1.0000	0.7433	0.1419	0.1721	0.3935	0.2925	0.8801	0.8081	0.8935	0.6607
7	0.3355	0.9197	1.0000	0.3801	0.1273	0.3490	0.3795	0.1442	0.7788	0.8279	0.7423	0.8598
8	0.9197	0.3355	1.0000	0.3801	0.3490	0.1273	0.3795	0.1442	0.7117	0.7921	0.8517	0.8652
9	1.0000	0.3355	0.7582	0.4610	0.3914	0.1313	0.2968	0.1805	0.9348	0.8005	0.7067	0.7880
10	0.3355	1.0000	0.7582	0.4610	0.1313	0.3914	0.2968	0.1805	0.7637	0.7805	0.8939	0.7515
11	0.4684	1.0000	0.3355	0.7462	0.1837	0.3921	0.1316	0.2926	0.8843	0.8765	0.8147	0.6631
12	1.0000	0.4684	0.3355	0.7462	0.3921	0.1837	0.1316	0.2926	0.9357	0.7909	0.8023	0.7029
13	1.0000	0.7490	0.3761	0.4163	0.3935	0.2947	0.1480	0.1638	0.9361	0.7056	0.7781	0.8103
14	0.7490	1.0000	0.3761	0.4163	0.2947	0.3935	0.1480	0.1638	0.6671	0.8911	0.7974	0.8852
15	0.3761	1.0000	0.4112	0.7582	0.1478	0.3928	0.1615	0.2979	0.7397	0.7823	0.7715	0.8942
16	1.0000	0.3761	0.4112	0.7582	0.3928	0.1478	0.1615	0.2979	0.9353	0.7766	0.8083	0.7084
17	0.3458	0.3761	1.0000	0.9017	0.1318	0.1434	0.3812	0.3437	0.8691	0.7771	0.8699	0.7096

18	0.3761	0.3458	1.0000	0.9017	0.1434	0.1318	0.3812	0.3437	0.7598	0.8616	0.7461	0.8429
19	0.2991	0.4349	0.9017	1.0000	0.1135	0.1650	0.3421	0.3794	0.8578	0.7699	0.8449	0.7414
20	0.4349	0.2991	0.9017	1.0000	0.1650	0.1135	0.3421	0.3794	0.7868	0.8651	0.7047	0.8716
21	0.9197	0.2991	0.4264	1.0000	0.3477	0.1131	0.1612	0.3781	0.7054	0.8586	0.8049	0.8550
22	0.2991	0.9197	0.4264	1.0000	0.1131	0.3477	0.1612	0.3781	0.8535	0.8315	0.7919	0.7362
23	0.8964	0.4375	0.2991	1.0000	0.3405	0.1662	0.1136	0.3798	0.7031	0.7879	0.8654	0.8725
24	0.4375	0.8964	0.2991	1.0000	0.1662	0.3405	0.1136	0.3798	0.8095	0.6984	0.8602	0.8588

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Table-6. Correlation Coefficients of HDI_1 and HDI_2 with Pena's Indicators obtained by Permutation of LE, ED, PCI and EQ

Indi-cators	P(1)	P(2)	P(3)	P(4)	P(5)	P(6)	P(7)	P(8)	P(9)	P(10)	P(11)	P(12)
HDI_1	0.9898	0.9912	0.9927	0.9895	0.9915	0.9930	0.9822	0.9857	0.9891	0.9767	0.9912	0.9897
HDI_2	0.9785	0.9837	0.9849	0.9780	0.9846	0.9856	0.9646	0.9719	0.9772	0.9550	0.9836	0.9783
Indi-cators	P(13)	P(14)	P(15)	P(16)	P(17)	P(18)	P(19)	P(20)	P(21)	P(22)	P(23)	P(24)
HDI_1	0.9893	0.9926	0.9763	0.9889	0.9879	0.9832	0.9843	0.9886	0.9867	0.9835	0.9889	0.9876
HDI_2	0.9775	0.9845	0.9543	0.9767	0.9745	0.9652	0.9669	0.9757	0.9736	0.9666	0.9762	0.9754