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Abstract

It has been argued that rule of thumb consumers substantially alter the determinacy properties of simple interest rate rules and the dynamics of an otherwise standard New-keynesian model. In this paper we show that nominal wage stickiness helps reestablishing standard results. Key findings are that wage stickiness i) affects the shape of determinacy regions in the parameters space, restoring the relevance of the Taylor principle for the conduct of monetary policy; ii) implies that a rise in consumption in response to an innovation in government spending is not a robust feature of the model.

JEL classification: E52, E62.

Keywords: Rule of Thumb Consumers, Sticky Wages, Determinacy, Fiscal Shocks

1 Introduction

In the recent macroeconomic literature the paradigm of the forward looking, representative, agent is contaminated by "rule of thumb" consumers. Agents who cannot use financial markets to smooth consumption over time, but consume their available labor income in each period, stand next to standard forward looking agents. This framework was originally developed by Mankiw (2000) to account for the empirical relationship between consumption and disposable income, which seems stronger than suggested by the permanent income hypothesis.

Galì, Lopez-Salido and Valles (2004 and 2005; GLV (2004) and GLV (2005) henceforth), Bilbiie (2005) and Di Bartolomeo and Rossi (2005) show that considering rule of thumb, or non ricardian, consumers within the New Keynesian framework leads to substantially different predictions from those delivered by a standard model.[1]

In this paper we generalize the New Keynesian framework with capital accumulation and rule of thumb consumers, as developed by GLV (2004) and GLV (2005), to allow for nominal wage stickiness *a là* Calvo. Our key findings are that wage stickiness: i) alters the determinacy conditions of simple interest rate rules; ii) modify the impulse response function of the model economy after a government spending shock.

GLV (2004) study determinacy properties of interest rate rules in a sticky-price economy with a fraction of rule of thumb consumers and capital accumulation. The same issue is considered by Bilbiie (2005) and Di Bartolomeo and Rossi (2005) who provide an analytical treatment, but neglect capital accumulation. The general conclusion of these papers is that

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[1] The simple heterogeneity between households we have described, breaks the Ricardian Equivalence. For this reason rule of thumb consumers are also defined as non ricardian consumers and it what follows we will use the two definitions interchangeably. This terminology is due to Galì *et al* (2004). Simmetrically standard forward looking households are defined as ricardian households.

the Taylor principle may fail to guarantee a unique rational expectation equilibrium in the presence of non ricardian agents. In particular Bilbiie (2005) shows that when the importance of rule of thumb consumers in the economy is larger than a certain threshold, the determinacy of the rational expectation equilibrium is, in general, guaranteed by a so called *Inverted Taylor principle*. In this case the interest rate rule adopted by the central bank should be such to engineer a decrease in the real interest rate in response to positive variations in the, current or expected, inflation rate.

We find that even a mild degree of wage stickiness restores the Taylor principle as a necessary condition for equilibrium determinacy. Our analysis provides theoretical foundations to the results in Erceg *et al* (2005) who consider a New Keynesian model with rule of thumb consumers and sticky wages, but find no evidence of a failure of the Taylor principle.

Turning to the effect of fiscal shocks, GLV (2005) argue that rule of thumb consumers constitute a potential solution to the so called *Government Spending Puzzle*. Fatas and Mihov (2001) and Blanchard and Perotti (2002) use the VAR methodology to document that an innovation in government spending causes a persistent rise in private consumption. Nevertheless standard DSGE models predict that a positive shock to government purchases will have a contractionary effect on consumption.[2] The literature has identified this sharp contrast between the implications of the theory on one hand, and empirical results on the other, as a puzzle. GLV (2005) show that the interaction between rule of thumb consumers, sticky prices and deficit financing delivers a positive response of aggregate consumption to an innovation in government spending. However, in their model the crowding in of aggregate consumption is obtained through a strong response of the real wage to the fiscal shock which boosts consumption of non ricardian agents. Such a sharp increase in the real wage is at

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[2]In a nutshell, the reason is that an increase in government spending generates a negative wealth effect which induces forward looking households to consume less and to work more.

odds with the evidence. Burnside *et al* (2004) estimate a negative response of the real wage to a spending innovation, while Blanchard and Perotti and Fatas and Mihov (2002) identify a positive but limited response.

We find that nominal wage stickiness prevents the large increase in the real wage in the aftermath of a government spending shock which affected the GLV's model. For empirically plausible values of parameters, the positive response of aggregate consumption to an innovation in government spending vanishes.

Government purchase shocks are coupled with a raise in aggregate consumption when agents suffer a low cost of supplying labor in terms of utility. In such a case the increase in hours worked due to the government spending shock is enough to boost consumption of ricardian agents, and to compensate for the negative wealth effect, exerted by the shock, on consumption of ricardian agents.

Results are robust to various specifications of the Taylor rule used in the literature, including one which reacts to wage inflation.

The remainder of the paper is laid as follows. Section 2 and 3 outline the model and its log-linearized version. Section 4 contains the main results. Section 5 verifies the robustness of the results to alternative interest rate rules. Section 6 concludes.

2 The model

2.1 Households

There is a continuum of households indexed by $i \in [0, 1]$. As in GLV (2004) and GLV (2005), households in the interval $[0, \lambda]$ cannot access financial markets and do not have an initial capital endowment. The behavior of these agents is characterized by a simple rule of thumb: they consume their available labor income in each period. The rest of the households on the

interval $(\lambda, 1]$ is composed by standard ricardian households who have access to the market for physical capital and to a full set of state contingent securities. Ricardian households hold a common initial capital endowment. The period utility function is common across households and it has the following separable form

$$U_t = u\left[C_t\left(i\right)\right] - v\left[L_t\left(i\right)\right] \tag{1}$$

where $C_t(i)$ is agent i's consumption and $L_t(i)$ are labor hours.[3]

We assume a continuum of differentiated labor inputs indexed by $j \in [0,1]$. As in Schmitt-Grohé and Uribe (2004a), agent i supplies all labor inputs. Wage-setting decisions are taken by labor type-specific unions indexed by $j \in [0,1]$. Given the wage W_t^j fixed by union j, agents stand ready to supply as many hours on labor market j, L_t^j, as required by firms, that is

$$L_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t^d \tag{2}$$

where θ_w is the elasticity of substitution between labor inputs. Here L_t^d is aggregate labor demand and W_t is an index of the wages prevailing in the economy at time t. Formal definitions of labor demand and of the wage index can be found in the section devoted to firms. Agents are distributed uniformly across unions, hence aggregate demand of labor type j is spreaded uniformly between all households.[4] It follows that the individual quantity of hours worked, $L_t(i)$, is common across households and we will denote it with L_t. This must satisfy the time resource constraint $L_t = \int_0^1 L_t^j dj$. Combining the latter with (2) we obtain

$$L_t = L_t^d \int_0^1 \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj \tag{3}$$

The labor market structure allows to rule out differences in labor income between households

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[3]The function u is incresing and concave while the function v is increasing and convex.

[4]Thus a share λ of the associates of the unions are non ricardian consumers, while the remaining share is composed by non ricardian agents.

without the need to resort to contingent markets for hours. The common labor income is given by $L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj$.[5]

2.1.1 Ricardian households.

Ricardian Households' time t nominal flow budget reads as

$$P_t \left(C_t^o + I_t^o \right) + \left(1 + R_t \right)^{-1} B_t^o + E_t \Lambda_{t,t+1} X_{t+1} \tag{4}$$
$$\leq \quad X_t + L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj + R_t^k K_{t-1}^o + B_{t-1}^o + P_t D_t^o - P_t T_t^o$$

Ricardian agents have access to a full set of state contingent assets. More precisely, in each time period t, consumers can purchase any desired state-contingent nominal payment X_{t+1} in period $t+1$ at the dollar cost $E_t \Lambda_{t,t+1} X_{t+1}$. $\Lambda_{t,t+1}$ denotes a stochastic discount factor between period $t+1$ and t. $L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} dj$ denotes labor income and $R_t^k K_{t-1}^o$ is capital income obtained from renting the capital stock to firms at the nominal rental rate R_t^k. $P_t D_t^o$ are dividends due from the ownership of firms, while B_t^o is the quantity of nominally riskless bonds purchased in period t at the price $\left(1 + R_t \right)^{-1}$ and paying one unit of the consumption numeraire in period $t+1$. $P_t T_t^o$ represent nominal lump sum taxes. As in GLV, the household's stock of physical capital evolves according to:

$$K_t^o = \left(1 - \delta \right) K_{t-1}^o + \sigma \left(\frac{I_t^o}{K_{t-1}^o} \right) K_{t-1}^o \tag{5}$$

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[5] Erceg *et al* (2000), assume, as in most of the literature on sticky wages, that each agent is the monopolistic supplier of a single labor input. In this case, assuming that agents are spreaded uniformly across unions allows to rule out differences in income between households providing the same labor input (no matter whether they are ricardian or not), but it does not allow to rule out difference in labor income between non ricardian agents that provide different labor inputs. This would amount to have an economy populated by an infinity of different individuals, since non ricardian agents cannot share the risk associated to labor income fluctuations. Although this framework would be of interest, it would imply a tractability problem.

where δ denotes the physical rate of depreciation. Capital adjustment costs are introduced through the term $\sigma\left(\frac{I_t^o}{K_{t-1}^o}\right)K_{t-1}^o$, which determines the change in the capital stock induced by investment spending I_t^o. The function σ satisfies the following properties: $\sigma'(\cdot) > 0$, $\sigma''(\cdot) \geq 0$, $\sigma'(\delta) = 1$, $\sigma(\delta) = \delta$. Thus, adjustment costs are proportional to the rate of investment per unit of installed capital. Ricardian households face the, usual, problem of maximizing the expected discounted sum of istantaneous utility subject to constraints (4) and (5). Let ν_t and Q_t denote the Lagrange multipliers on the first and on the second constraint respectively. The first order conditions with respect to C_t^o, I_t^o, B_t^o, K_t^o, X_{t+1} are

$$u_c\left(C_t^o\right) = \nu_t P_t \tag{6}$$

$$\frac{1}{\phi'\left(\frac{I_t^o}{K_{t-1}^o}\right)} = q_t \tag{7}$$

$$\frac{1}{(1+R_t)} = \beta E_t \frac{\nu_{t+1}}{\nu_t} \tag{8}$$

$$Q_t = E_t\left\{\Lambda_{t,t+1}\left[R_{t+1}^k + Q_{t+1}\left((1-\delta) - \phi'\left(\frac{I_{t+1}^o}{K_t^o}\right)\frac{I_{t+1}^o}{K_t^o} + \phi\left(\frac{I_{t+1}^o}{K_t^o}\right)\right)\right]\right\} \tag{9}$$

$$\Lambda_{t,t+1} = \beta\frac{\nu_{t+1}}{\nu_t} \tag{10}$$

where $\beta = \frac{1}{1+\rho}$ represents the discount factor, ρ is the time preference rate and $q_t = \frac{Q_t}{P_t}$ is the real shadow value of installed capital, i.e. Tobin's Q. Substituting (6) into (10) we obtain the definition of the stochastic discount factor $\Lambda_{t,t+1} = \beta\frac{u_c\left(C_{t+1}^o\right)}{P_{t+1}}\frac{P_t}{u_c(C_t^o)}$ while combining (10) and (8) we recover the following arbitrage condition on the asset market

$$E_t\Lambda_{t,t+1} = (1+R_t)^{-1}$$

2.1.2 Non ricardian households.

Non ricardian agents do not hold physical capital neither enjoy firms' profits in the form of dividend income. The nominal budget constraint of a typical non ricardian household is given by

$$P_t C_t^{rt} = L_t^d \int_0^1 W_t^j \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} dj - P_t T_t^{rt} \tag{11}$$

Agents belonging to this class are forced to consume available income in each period and delegate wage decisions to unions. For these reasons there are no first order conditions with respect to consumption and labor supply. Similarly to GLV (2005) we let lump sum taxes (transfers) paid (received) by non ricardian households differ by those paid by ricardian.

2.2 Wage Setting

Nominal wage rigidities are modeled according to the Calvo (1983) mechanism. In each period a union faces a constant probability $1 - \xi_w$ of being able to reoptimize the nominal wage. We extend the analysis in GVL (2005) and assume that the nominal wage newly reset at t, $\widetilde{W}_t$, is chosen to maximize a weighted average of agents' lifetime utilities. The weights attached to the utilities of ricardian and non ricardian agents are $(1 - \lambda)$ and λ, respectively. The union problem is

$$\max_{\widetilde{W}_t} E_t \sum_{s=0}^{\infty} (\xi_w \beta)^s \left\{ \left[(1 - \lambda) u \left(C_{t+s}^o \right) + \lambda u \left(C_{t+s}^{rt} \right) \right] - v \left(L_{t+s} \right) \right\}$$

subject to (3), (4), and (11).[6] The FOC with respect to $\widetilde{W}_t$ is

$$E_t \sum_{s=0}^{\infty} (\beta \lambda_w)^{t+s} \Phi_{t,t+s} \left\{ \left[\lambda \frac{1}{MRS_{t+s}^{rt}} + (1 - \lambda) \frac{1}{MRS_{t+s}^o} \right] \frac{\widetilde{W}_t}{P_{t+s}} - \mu^w \right\} = 0 \tag{12}$$

where $\Phi_{t,t+s} = v_L \left(L_{t+s} \right) L_{t+s}^d W_t^{\theta_w}$ and $\mu^w = \frac{\theta_w}{(\theta_w - 1)}$ is the, constant, wage mark-up in the case of wage flexibility. MRS_{t+s}^{rt} and MRS_{t+s}^o are the marginal rates of substitution between labor and consumption of non ricardian and ricardian agents respectively. Notice

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[6] Many reasons have been provided to justify the presence of non ricardian consumers. A few of them are miopia, fear of saving and transaction costs on financial markets. None of these is, however, in contrast with rule of thumb consumers delegating wage decision to a forward looking agency, in this case a trade union.

that when wages are flexible (12) reduces to

$$\frac{W_t}{P_t} = \mu^w \left[\lambda \frac{1}{MRS_t^{rt}} + (1 - \lambda) \frac{1}{MRS_t^o} \right]^{-1} \tag{13}$$

which is identical to the wage setting equation in GLV (2005).

2.3 Firms

In each period t a final good Y_t is produced by a perfectly competitive firm, combining a continuum of intermediate inputs $Y_t(z)$, according to the following standard CES production function:

$$Y_t = \left(\int_0^1 Y_t(z)^{\frac{\theta_p - 1}{\theta_p}} dz \right)^{\frac{\theta_p}{\theta_p - 1}} \quad \text{with} \quad \theta_p > 1 \tag{14}$$

The producer of the final good takes prices as given and chooses the quantities of intermediate goods by maximizing its profits. This leads to the demand of intermediate good z and to the price of the final good which are respectively

$$Y_t(z) = \left(\frac{P_t(z)}{P_t} \right)^{-\theta_p} Y_t \quad ; \quad P_t = \left[\int_0^1 P_t(z)^{1-\theta_p} dz \right]^{\frac{1}{1-\theta_p}}$$

Intermediate inputs $Y_t(z)$ are produced by a continuum of size one of monopolistic firms which share the following technology:

$$Y_t(z) = [K_{t-1}(z)]^\alpha [L_t(z)]^{1-\alpha}$$

where $0 < \alpha < 1$ is the share of income which goes to capital in the long run, $K_{t-1}(z)$ is the time t capital service hired by firm z, while $L_t(z)$ is firm z's demand of the labor input. The latter is defined as $L_t(z) = \left(\int_0^1 \left(L_t^j(z) \right)^{\frac{\theta_w - 1}{\theta_w}} dj \right)^{\frac{\theta_w}{\theta_w - 1}}$ with $\theta_w > 1$. Firm's z demand for labor type j and the aggregate wage index are respectively

$$L_t^j(z) = \left(\frac{W_t^j}{W_t} \right)^{-\theta_w} L_t^d(z) \quad ; \quad W_t = \left(\int_0^1 \left(W_t^j \right)^{1-\theta_w} dj \right)^{1/(1-\theta_w)}$$

where $L_t^d(z)$ are units of labor bundle demand by firm z. The nominal marginal cost is given by

$$MC_t = \left(\frac{1}{\alpha}\right)^\alpha \left(\frac{1}{(1-\alpha)}\right)^{1-\alpha} W_t^{1-\alpha} \left(R_t^k\right)^\alpha$$

Price setting. We assume firms set prices according to the same mechanism assumed for wage setting. Firms in each period have a chance $1 - \xi_p$ to reoptimize their price. A price setter z takes into account that the choice of its time t nominal price, $\widetilde{P}_t$, might affect not only current but also future profits. The first order condition for price setting is:

$$E_t \sum_{s=0}^{\infty} \left(\beta\xi_p\right)^s \nu_{t+s} P_{t+s}^{\theta_p} Y_{t+s} \left[\widetilde{P}_t - \mu^p MC_{t+s}\right] = 0 \tag{15}$$

which can be given the usual interpretation.[7] Notice that $\mu^p = \frac{\theta_p}{\theta_p - 1}$ represents the markup over the price which would prevail in the absence of nominal rigidities.

2.4 Government

The Government nominal flow budget constraint is

$$P_t T_t + (1 + R_t)^{-1} B_t = B_{t-1} + P_t G_t \tag{16}$$

where $P_t G_t$ is nominal government expenditure on the final good. We assume a fiscal rule of the form

$$t_t = \phi_b b_{t-1} + \phi_g g_t \tag{17}$$

where $t_t = \frac{T_t - T}{Y}$, $g_t = \frac{G_t - G}{Y}$ and $b_t = \frac{\frac{B_t}{P_t} - \frac{B_{t-1}}{P_{t-1}}}{Y}$. g_t is assumed to follow a first order autoregressive process $g_t = \rho_g g_{t-1} + \varepsilon_t^g$ where $0 \leq \rho_g \leq 1$ and ε_t^g is a normally distributed

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[7]Recall that ν_t is the value of an additional dollar for a ricardian household. It is the lagrange multiplier on ricardian households nominal flow budget constraint.

zero-mean random shock to government spending.[8]

2.5 Monetary Policy

An interest rate-setting rule is required for the dynamic of the model to be fully specified. Our baseline parameterization features the central bank setting the nominal interest rate as a function of current inflation according to the following log-linear rule

$$r_t = \tau_\pi \pi_t \tag{18}$$

where $r_t = \log \frac{(1+R_t)}{1+\rho}$ and $\pi_t = \log \frac{P_t}{P_{t-1}}$. In standard sticky prices models without capital accumulation, as in Woodford (2003) or Galì (2002), rule (18) ensures local uniqueness of the rational expectation equilibrium if it satisfies the *Taylor Principle*, i.e. if $\tau_\pi > 1$. Carlstrom and Fuerst (2005) show that when the central bank follows a contemporaneous rule the determinacy conditions are, in general, not altered by capital accumulation.

2.6 Aggregation

We denote aggregate consumption, lump sum taxes, capital, investment, dividends and bonds with C_t, T_t, K_t, I_t, D_t and B_t, respectively. These are defined as

$$C_t = \lambda C_t^{rt} + (1-\lambda) C_t^o; \quad D_t = (1-\lambda) D_t^o \quad I_t = (1-\lambda) I_t^o;$$

$$T_t = \lambda T_t^{rt} + (1-\lambda) T_t^o; \quad K_t = (1-\lambda) K_t^o; \quad B_t = (1-\lambda) B_t^o.$$

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[8] A sufficient condition for non explosive debt dynamics is

$$(1+\rho)(1-\phi_b) < 1$$

which is satisfied if

$$\phi_b > \frac{\rho}{1+\rho}$$

We assume this condition is satisfied throughout.

2.7 Market Clearing

The clearing of good and labor markets requires

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta_p} Y_t^d \quad \forall z \qquad Y_t^d = Y_t;$$

$$L_t^j = \left(\frac{W_t^j}{W_t}\right)^{-\theta_w} L_t^d \qquad \forall j \quad L_t = \int_0^1 L_t^j dj$$

where $Y_t^d = C_t + G_t + I_t$ represents aggregate demand, $L_t^j = \int_0^1 L_t^j(z)\,dz$ is the demand of labor input j and $L_t^d = \int_0^1 L_t(z)\,dz$ denotes firms' aggregate demand of the composite labor input. The clearing condition of the market for physical capital reads as

$$K_t = \int_0^1 K_t(z)\,dz$$

2.8 Steady State

As in GLV, steady state lump sum taxes are such that steady state consumption levels are equalized across agents. Variables without time subscript denote steady state values. Firm i's cost minimization implies

$$\frac{W}{P} = \frac{(1-\alpha)}{\mu^p}\frac{Y}{L}; \quad r^k = \frac{\alpha}{\mu^p}\frac{Y}{K}$$

where

$$\frac{K}{Y} = \frac{\alpha}{\mu^p(\rho+\delta)}$$

Since the ratio $\frac{G}{Y} = \gamma_g$ is, by assumption, exogenous, we can determine the steady state share of consumption on output, γ_c, as follows

$$\gamma_c = 1 - \frac{\delta\alpha}{\mu^p(\rho+\delta)} - \gamma_g$$

which, as noticed by GLV, is independent of λ. In what follows it will prove useful to know $\frac{W}{P}\frac{L}{C}$, which equals

$$\frac{W}{P}\frac{L}{C} = \frac{(1-\alpha)}{\mu^p}\frac{Y}{L}\frac{L}{C} = \frac{(1-\alpha)}{\mu^p\gamma_c}$$

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3 The Log-linearized model.

To make our results readily comparable to those in Bilbiie (2005) and GLV (2005) we adopt the same period utility function considered in their works:

$$u\left(C_t\right) = \log C_t \quad ; \quad v\left(L_t\right) = \frac{L_t^{1+\phi}}{1+\phi}$$

which features a unit intertemporal elasticity of substitution in consumption and a constant elasticity of the marginal disutility of labor $v_{LL} = \phi$.[9] In what follows lower case letters denote log-deviations from the steady state values. The log-deviation of the real wage, denoted by w_t, constitutes the only exception to this rule. The conditions which define the log-linear approximation to equations of the model are derived in GLV (2005) and we report them in the appendix. We provide, instead, a detailed derivation of the wage inflation curve and of the real wage schedule.

3.1 Wage inflation, the real wage schedule and the effect of economic activity on the real wage.

In the case of identical steady state consumption levels, agents have a common steady state marginal rate of substitution between labor and consumption. This implies that equation (12) can be given the following log-linear approximation

$$E_t \sum_{s=0}^{\infty} \left(\beta \xi_w\right)^{t+s} \left[w_{t+s} - mrs_{t+s}^A\right] = 0$$

where $mrs_t^A = \lambda mrs_t^{rt} + \left(1-\lambda\right) mrs_t^o$ is a weighted average of the log-deviations between the marginal rates of substitution of the two agents. In what follows we will refer to mrs_t^A as to

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[9] The selected period utility belongs to the King-Plosser-Rebelo class and leads to constant steady state hours.

the average marginal rate of substitution. Given the selected functional forms, the (log)wage optimally chosen at time t is defined as

$$\log \widetilde{W}_t = \log \mu_w + (1 - \beta \xi_w) E_t \sum_{s=0}^{\infty} (\beta \xi_w)^{t+s} \{\log P_{t+s} + \log C_t + \phi \log L_t\}$$

Combining the latter with the following, standard, log-linear approximation of the wage index

$$\log W_t = (1 - \xi_w) \log \widetilde{W}_t + \xi_w \log W_{t-1}$$

we obtain the desired wage inflation curve

$$\pi_t^w = \beta E_t \pi_{t+1}^w - \kappa_w \mu_t^w \tag{19}$$

where $\kappa_w = \frac{(1 - \beta \xi_w)(1 - \xi_w)}{\xi_w}$ and $\mu_t^w = (\log W_t - \log P_t) - (\log \mu_w + \log C_t + \phi \log L_t)$ is the wage mark-up that unions impose over the average marginal rate of substitution.[10] Notice that since unions maximize a weighted average of agents' utilities, the wage inflation curve takes a standard form. Equation (19) allows to obtain the log-deviation of time t real wage, which plays a prominent role in the determination of non ricardian agents consumption, as follows

$$w_t = \Gamma \left[w_{t-1} + \beta \left(E_t w_{t+1} + E_t \pi_{t+1} \right) - \pi_t \right] + \Gamma \kappa_w \left(\phi l_t + c_t \right) \tag{20}$$

where $\Gamma = \frac{\xi_w}{(1 + \beta \xi_w^2)}$. Γ determines both the degree of forward and backward lookingness.[11] Today's average real wage is a function of its lagged and expected value, expected and current inflation. The term $\phi l_t^d + c_t$ represents the average real wage that would prevail in the case of wage flexibility.

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[10] As pointed out by Schmitt-Grohe and Uribe (2004a), the coefficient κ_w is different form that in Erceg et al (2000), which is the standard reference for the analysis of nominal wage stickiness. The reason is that we have assumed that agents provide all labor inputs. In the more standard case in which each individual is the monopolistic supplier of a given labor input, κ_w would be equal to $\frac{(1 - \beta \xi_w)(1 - \xi_w)}{\xi_w (1 + \phi \theta_w)}$ hence lower than in the case we consider.

[11] The effect of discounting on the forward looking component is quantitatively negligible.

Substituting (29) into (20) we obtain:

$$w_t = \Gamma w_{t-1} + \Gamma\beta\left(E_t w_{t+1} + E_t\pi_{t+1}\right) + \Psi y_t - \Psi\alpha k_{t-1} + \Gamma\kappa_w c_t - \Gamma\pi_t \qquad (21)$$

where $\Psi = \Gamma\frac{\kappa_w}{(1-\alpha)}\phi$ determines the effect on the real wage due to changes in the level of real activity.

Comparative statics. $\frac{\partial\Gamma}{\partial\xi_w} > 0$: a longer average duration of wage contracts does not have a clear cut effect on real wage inertia. As ξ_w gets larger both forward and backward lookingness increase. $\frac{\partial\Psi}{\partial\phi} > 0$: the more elastic is the marginal disutility of labor, i.e. the higher is ϕ, the higher is the sensitivity of wages to an increase in economic activity. $\frac{\partial\Psi}{\partial\xi_w} < 0$: the higher is average duration of wage contracts, i.e. the higher is ξ_w, the lower is the sensitivity of wages to an increase in economic activity. The same can be said for what concerns the sensitivity of the real wage to hours.

Intuition goes as follows. A higher ξ_w implies that the nominal wage will be newly reset on a limited number of labor markets, thus the previous period average wage has a stronger influence on today's. At the same time those unions which optimally reset their wage will attach a higher weight on expected future variables.

The parameter Ψ determines the size of the variation in real wage associated with a given variation in real economic activity. This is jointly determined by the probability that wages cannot be changed in a given period, ξ_w, and the elasticity of the marginal disutility of labor, ϕ. Woodford and Rotemberg (1997) report evidence suggesting that the output elasticity of real wage is in a neighborhood of 0.3. Figure 1 plots Ψ as a function of ϕ for alternative degrees of wage stickiness assuming the values $\beta = 0.99$ and $\alpha = \frac{1}{3}$. Empirical estimates suggest that wages have an average duration of an year ($\xi_w = 0.75$). In this case, a value of Ψ consistent with the estimates in Rotemberg and Woodford (1997) is obtained

by setting ϕ close to 5. In a model with a frictionless labor market this would lead to an intertemporal elasticity of substitution in labor supply equal to 0.2, which is in line with the micro-evidence in Card (1991) and Pencavel (1986). Thus, we obtain a output sensitivity of real wage consistent with the estimates using empirically plausible values of ϕ and ξ_w.

This is not the case under wage flexibility. When $\xi_w = 0$ equation (21) reduces to

$$w_t = \frac{\phi}{(1-\alpha)} y_t - \frac{\alpha}{(1-\alpha)} \phi k_{t-1} + c_t$$

which is the wage setting equation in GLV (2005). In order to be consistent with the afore-mentioned evidence on the output elasticity of real wage GLV (2005) set ϕ equal to 0.2. This value is, however, far from consistent with the microeconomic evidence on the elasticity of labor supply and from standard calibration of preferences.

4 Results

4.1 Calibration

We calibrate the parameters of the model since the analysis of equilibrium determinacy and equilibrium dynamics that follow draws on numerical results. The time unit is meant to be a quarter. In the baseline parametrization we set $\xi_w = 0.75$, which implies an average duration of wage contracts of one year as suggested by the estimates in Smets and Wouters (2003) and Levine *et al* (2005). α and β assume the standard values of $\frac{1}{3}$ and 0.99 respectively. Table 1 reports the output sensitivity of real wage Ψ as a function of ϕ. In column 2 we consider the baseline calibration for wage stickiness, while in column 4 we evaluate Ψ under the limiting case of wage flexibility. Table 1 shows that, under the baseline calibration for wage stickiness, setting $\phi = 4.84$ allows to match the output elasticity of real wage reported by Rotemberg and Woodford (1997), thus we take this value as the baseline. However, to evaluate the

dependence of the model's implications on the elasticity of the marginal disutility of labor, we consider two other values of ϕ beside the baseline. The first, $\phi = 0.2$, corresponds to the value employed by GLV (2005), the second $\phi = 3$ is chosen because commonly employed in the literature. Table 1, consistently with the discussion in the previous section, points out that when standard values are assigned to ϕ, the flexible wage scenario leads to extremely high output sensitivity of real wage. The baseline value for the share of non ricardian consumers, λ, is 0.5. This is consistent with the estimates in Campbell and Mankiw (1989) and Muscatelli *et al* (2003). Remaining parameters are displayed in Table 2, and the reader can refer to the references reported in GLV (2005) for empirical evidence supporting them. However, it is worth mentioning that in the baseline calibration τ_π is set to 1.5. Thus monetary policy is assumed to satisfy the standard Taylor Principle.

Table 1: Output sensitivity of real wage as a function of the elasticity of labor disutility and the calvo parameter on wages.

	Ψ		Ψ
$\phi=0.2$; $\xi_w=0.75$	0.011	$\phi=0.2$; $\xi_w=0$	0.3
$\phi=3$; $\xi_w=0.75$	0.116	$\phi=1$; $\xi_w=0$	4.5
$\phi=4.84$; $\xi_w=0.75$	0.300	$\phi=4.84$; $\xi_w=0$	7.26

4.2   Determinacy

Figure 2 depicts indeterminacy areas in the parameter space (τ_π, λ). Other parameters are set at their baseline values. A first result is visually evident:

Result 1. Determinacy and the Taylor Principle.   The Taylor Principle is a necessary and sufficient condition for equilibrium determinacy.

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To develop intuition behind this result, we build on the economic mechanism emphasized by Bilbiie (2005). To isolate the effect of wage stickiness on determinacy conditions it is initially convenient to assume that wages are flexible.

Suppose that the level of inflation starts increasing without any change in fundamentals that could justify it. To the extent that the central bank follows the Taylor Principle, the real interest rate increases in the aftermath of the rise in inflation. This has a contractionary effect on consumption of ricardian agents. Due to lower demand, some firms fix a lower price, while, firms which are prevented from doing it, reduce labor demand, putting a downward pressure on the real wage. As a result real marginal costs decrease and there is an increase in profits.[12] The latter implies a positive wealth effect on consumption of ricardian consumers, who own firms and enjoy profits in the form of dividend income. Notice that, due to aggregation, a one unit increase in profits leads to a $\frac{1}{1-\lambda}$ increase in individual dividend income. As a consequence, when the share of rule of thumb consumers is above a given threshold, the wealth effect stemming from a profits' increase may lead to a rise in aggregate demand.[13] If this is the case, the sunspot in inflation could become self-fulfilling through the positive relationship between output and inflation implied by the NKPC. As pointed out by Bilbiie (2005) an interest rate rule satisfied the *Inverted Taylor Principle*, would lead to a fall in profits, making the initial increase in inflation non compatible with a rational expectation equilibrium.

How does wage stickiness alter the adjustment process described above?

The key point is that wage stickiness dampens variation in the real wage associated to changes in hours. Thus, for a given reduction of labor demand, real marginal costs do not

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[12] The increse in profits becomes stronger as the marginal elasticity of labor disutility, ϕ, increases. In this case small variations in hours are accompanied by large variations in the real wage.

[13] As emphasized by Bilbiie (2005) when all agents hold assests this mechanism is irrelevant. The effect of the increse in profits on agents' income is exactly offset by the decrease in real wage.

decrease as they would if wages were flexible. In this case the wealth effect produced by the increase in profits does not offset the substitution effect exerted on demand of ricardian consumers by the initial real rate increase. This prevents the rise in demand that could ex-post rationalize the sunspot in inflation.

Similarly to GLV (2004) we find that when strong price stickiness coexists with a large share of non ricardian consumers the Taylor Principle, although necessary, needs to be strenghtened to enforce a unique rational expectation equilibrium. With respect to the process described above, the presence of extreme price stickiness may, in fact, lead to an increase in the real wage in the aftermath of the sunspot in inflation. This would boost consumption of non ricardian consumers. Since non ricardian agents' demand is insensitive to changes in the interest rate, the only way in which the central bank could control aggregate demand would be that of further depressing demand of ricardian agents engineering a stronger increase in the rate of interest. However, we raise an important qualification with respect to the analysis in GLV (2004). Namely, that the Taylor Principle remains a valid criterion to avoid sunspot fluctuations when the relevant parameters (ξ_w, ξ_p, λ) assume values compatible with the empirical estimates. In fact, under the baseline calibration, the Taylor Principle is a necessary and sufficient condition for determinacy for values of the price stickiness parameter $\xi_p \leq 0.79$. This threshold value corresponds to an average lifetime of price contracts of 4.8 quarters, which is sensibly larger than that estimated in empirical works.[14]

Figure 3 depicts indeterminacy areas in the case of alternative degrees of wage stickiness with respect to the baseline. In Panel a wages are perfectly flexible. When the share of non ricardian consumers is equal or above 20 percent, there are determinate equilibria which are compatible with an inflation response coefficient $\tau_\pi < 1$, i.e. with the Inverted Taylor

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[14]An analysis of the sensitivity of determinacy regions to the degree of price stickiness is reported in a companion appendix.

Principle. However, when the average duration of wage contracts reaches two quarters (panel b), the inverted Taylor Principle leads to equilibrium uniqueness just if the share of non ricardian consumers is larger than 70 percent. Notice that the latter value is well above the estimates of the importance of rule of thumb behavior reported above. For this reason cases where the Inverted Taylor Principle leads to equilibrium uniqueness can be regarded as of minor empirical relevance. Panel c shows, as expected, that our results are not altered when the average duration of wage contracts is increased to ten quarters ($\xi_w = 0.9$).

In sum, our analysis shows that rule of thumb consumers do not invalidate the relevance of the Taylor Principle when nominal wage stickiness, an uncontroversial empirical fact, is considered.

4.3   Consumption and Government Spending Shocks.

Figure 4 depicts the response of key variables to a government spending shock.

Result 2. Impact response of aggregate consumption. When wages are sticky aggregate consumption decreases in the aftermath of a, partially debt financed, government spending shock.

Two forces act in the direction of reducing consumption of ricardian consumers. The first one is the negative wealth effect determined by the government purchase shock, while the second one is due to the positive response of the real interest rate to the shock. In fact, although wage stickiness dampens the variations in real marginal costs, and through this channel those of inflation, the response of monetary policy is such that the real interest goes up. To analyze the overall effect on aggregate consumption, we have to take into account the response of non ricardian agents' consumption to the unexpected rise in Government spending. Sticky wages prevent the large increase in real wage affecting the GVL's model. This,

jointly with a less prominent rise in hours worked, implies that consumption of non ricardian consumers does not grow as much as required to determine a positive impact response of aggregate consumption.

In what follows we assess the sensitivity of result 2 to alternative parameterization of the elasticity of marginal disutility of labor (ϕ) and to the share of non ricardian consumers (λ). In Figure 5 we evaluate the sensitivity to ϕ. Dotted lines correspond to the value chosen by GVL (2005), dashed lines to the case $\phi = 3$, while solid lines to the baseline value.

Result 3. Impact response of aggregate consumption and ϕ. The effect of a Government spending shock on private consumption is positive when the elasticity of marginal disutility of labor, ϕ, is low.

Consider the case where $\phi = 0.2$, which corresponds to the calibration adopted by GVL (2005). Under this parameterization (see Table 1), wage stickiness implies an extremely low sensitivity of the real wage to economic activity and to changes in hours. In this case the government spending shock leads to a negligible increase in the real wage. This causes a mild rise in real marginal costs and, thus, in inflation. As a consequence, the real interest rate grows modestly and the reduction in ricardian agents' consumption is well below those registered in the case where ϕ is larger. At the same time, when $\phi = 0.2$, the strong increase in hours brings about an increase in consumption of ricardian agents which is larger than under the other parameterizations.

In sum, when the elasticity of the marginal disutility of labor is low the impulse responses of both agents' consumption levels are favorable to a positive impact variation of aggregate consumption with respect to the baseline case.

However as the elasticity of marginal disutility of labor approaches the values supported by the empirical evidence the response of the real wage to the innovation in government

spending gets stronger, although it remains much lower than in GVL (2005). In this case the variation in inflation is such to imply a stronger reaction of the real interest rate which depresses consumption of ricardian consumers. Finally the joint movement of real wage and hours dampens the change in consumption of non ricardian agents and prevents an increase in aggregate consumption. Notice that monetary policy, plays a crucial role for this results since it impacts on ricardian agents consumption through its effect on the real interest rate. The robustness of our results to alternative interest rate setting rules in explored below.

We conclude this section assessing the role played by the share of non ricardian consumers, λ. A clear result emerges from figure 6.

Result 4. Impact response of aggregate consumption and λ. Aggregate consumption shows a positive response to a government spending shock for large values of the share, λ, of non ricardian consumers.

Figure 6 makes clear that aggregate consumption shows a positive, and mildly persistent, response for values of the share on non ricardian consumers which are above the upper interval of empirical estimates. As in GVL (2005) the effect of the spending shock on output is increasing in the share of non ricardian consumers. This implies also that the effect on labor demand and on the real wage are positive function of the importance of non ricardian agents in the economy. The pattern of the real wage is transmitted to price inflation. Since monetary policy obeys to the Taylor Principle, the real rate grows. For this reason consumption of ricardian consumers is lower the higher the share of non ricardian consumers. This effect partly counterbalances the increase in consumption of non ricardian agents, which is, instead, a positive function of λ.

5   Robustness to alternative interest rate rules.

In this section we discuss whether Results 1 and 2 are robust to simple variant of the Taylor rules proposed in the literature.

We consider rules which are specialization of the, general, instrumental rule

$$r_t = \rho_r r_{t-1} + \tau_\pi E_t \pi_{t+i} + \tau_y E_t y_{t+i} \tag{22}$$

When $i = -1$, (22) reduces to a backward looking rule, when $i = 0$ it corresponds to a contemporaneous rule and when $i = 1$ it becomes a forward looking rule. For each of the specifications mentioned we consider the case of inertia, with $\rho_r = 0.5$.

Determinacy.   Figure 7 depicts indeterminacy regions for each of the specification of the interest rate rule we consider. A key result is stated in the following.

Result 5. Determinacy and non ricardian consumers. Under most of the Taylor-type interest rate setting rules considered in the literature, the determinacy and indeterminacy regions for the model with non ricardian consumers featuring price-wage stickiness are similar to those identified for a representative agent economy.

The forward looking rule, depicted in panel *f,* shows a determinacy region which is severely restricted with respect to the case of a contemporaneous rule. As pointed out by Carlstrom and Fuerst (2005), forward looking rules increase the likelihood of sunspot fluctuations in the case of endogenous capital accumulation and should be implemented with care. Panels *a,c* and *e* suggest that nominal interest rate inertia makes indeterminacy less likely, no matter the rule followed by the central bank. Increasing the size of rule of thumb consumers does not determine variations of indeterminate regions in the contemporaneous and forward looking case. It affects, instead, the backward looking case. More precisely indeterminacy regions in

the inertial case are similar to those obtained for the non inertial case.[15]

Consumption and Government Spending Shocks. Figure 8 reports the response of aggregate consumption to a government spending shock under the various specifications of the general rule (22) we have analyzed. The response of the central bank to price inflation is kept at its baseline value, while we report impulse response functions for three different parameterizations of τ_y. We emphasize the following.

Result 6. Aggregate consumption and monetary rules. Backward looking monetary rules are more likely than contemporaneous and forward looking rules to deliver a positive impact response of aggregate consumption to a government spending shock. Reacting to deviations of output from its steady state level reduces, instead, the likelihood of a positive impact response of consumption.

The reason for which a backward looking rule helps obtaining a positive impact response is straightforward. Differently from what happens under the contemporaneous and the forward looking rule, when the central bank responds to lagged variables there is no positive impact increase in the real interest rate. This favours a mild reduction in consumption of ricardian consumers, while that of non ricardian is positively affected by the increase in hours worked and the real wage. However, as the effects of the shock are transmitted to inflation and output, the positive variation in the real rate of interest drives consumption of non ricardian agents below the steady state level and at the same time leads to a reduction in the level of

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[15]The interested reader can find a detailed analysis of alternative intrerest rate rules at the in a companion appendix, where we also consider a rule which reacts to wage inflation. In this case a necessary condition for determinacy is $\tau_p + \tau_w > 1$, where τ_w is the wage inflation coefficient response. It should not be, by now, surprising that this is equivalent to the condition which holds in a model without non ricardian consumers as shown by Schmitt-Grohe and Uribe (2004a).

output and hours. This negatively affects consumption of non ricardian agents. These effects are mirrored in the dynamic pattern of aggregate consumption, which exhibits a positive response on impact, but lacks of persistence. Notice that this stands in sharp contrast with what happens if the central bank follows, for example, a contemporaneous rule, where aggregate consumption decreases smoothly after the government spending shock (panel d). The contemporaneous and the forward looking rule do not, instead, differ relevantly for what concerns the likelihood of delivering a positive impact response of consumption, no matter whether we consider an inertial component in interest rate setting.

Reacting to output deviation determines a less marked increase in production in the aftermath of the shock, containing the variation in hours worked and, thus, in consumption of non ricardian consumers.[16]

6 Conclusions

We regard a framework where current income affects consumption possibilities as a promising step towards realism in economic modeling. In this case, however, it should be taken into account that labor markets and the wage setting process are subject to some form of imperfections. In an economy populated by an exogenous share of non ricardian consumers, wage stickiness affects both the response of aggregate variables to a government spending shock and the conditions for equilibrium determinacy. Once wage stickiness is considered, the positive effect of government spending on aggregate consumption reported by the empirical studies of, *inter alia,* Blanchard and Perotti (2002), is not a robust feature of the model with rule of thumb consumers. In particular, it can be replicated just when the marginal disutility of labor effort is low. Contrary to Bilbiie (2005) and GLV (2004) we have shown

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[16]The case of a central bank reacting to wage inflation is detailed in the companion appendix.

that, for a wide set of parameter configurations, the Taylor Principle implies equilibrium determinacy. Determinacy regions are similar to those obtained in a representative agent model under most interest rate setting rules considered in the literature.

Our results suggest that the determinacy properties of the model with non ricardian consumers strongly depends on the kind of nominal rigidities considered. For this reason, we warn against reappraisals of the conduct of monetary policy in specific past periods which are based on non ricardian consumers but neglect wage stickiness.

For what concerns the feature of welfare maximizing monetary policy, we conjecture that the optimality of a passive monetary rule, as advocated by Bilbiie (2005) in a sticky prices-flexible wages economy, could be altered by considering a modest degree of wage stickiness. The latter aspect is part of our ongoing research.

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Appendix

Log-linearized equilibrium conditions.

This appendix provides a log-linear approximation to the equlibrium conditions of the model economy described in the text. For a detailed derivation see also GVL.

Under the assumed functional forms, the Euler equation for Ricardian households takes

the log-linear form

$$c_t^o - E_t c_{t+1}^o = -E_t \left(r_t - \pi_{t+1} \right) \tag{23}$$

Log-linearization of equations (7) and (9) leads to the dynamic of (real)Tobin's Q

$$q_t = \left(1 - \beta \left(1 - \delta \right) \right) E_t r_{t+1}^k + \beta E_t q_{t+1} - \left(r_t - E_t \pi_{t+1} \right) \tag{24}$$

and its relationship with investment:

$$\eta q_t = i_t - k_{t-1}$$

Equation (11) determines the following log-linear form for consumption of non ricardian agents

$$c_t^{rt} = \frac{(1 - \alpha)}{\mu_p \gamma_c} \left(l_t + \omega_t \right) - \frac{1}{\gamma_c} t_t^{rt} \tag{25}$$

while the assumption that consumption level are equal at the steady state implies that aggregate consumption is

$$c_t = \left(1 - \lambda \right) c_t^o + \lambda c_t^{rt} \tag{26}$$

The stock of capital evolves according to

$$\delta i_t = k_t - \left(1 - \delta \right) k_{t-1} \tag{27}$$

Log-linearization of the aggregate resource constraint around the steady state yields

$$y_t = \gamma_c c_t + g_t + \left(1 - \widetilde{\gamma}_c \right) i_t \tag{28}$$

where $\widetilde{\gamma}_c = \gamma_c + \gamma_g$. As in shown by Woodford (2003) a log-linear approximation to the aggregate production function is given by

$$y_t = \left(1 - \alpha \right) l_t^d + \alpha k_{t-1} \tag{29}$$

Assuming that steady state stock of debt is zero and a steady state balanced government budget, the dynamic of debt around the steady state yields the following law of motion for

the stock of debt

$$b_t = (1 + \rho)(b_{t-1} + g_t - t_t) \tag{30}$$

The New Keynesian Phillips Curve (NKPC) is obtained through log-linearization of condition (15) and reads as

$$\pi_t = \kappa_p mc_t + \beta E_t \pi_{t+1} \tag{31}$$

where $\kappa_p = \frac{(1 - \beta\xi_p)(1 - \xi_p)}{\xi_p}$ and $mc_t = (1 - \alpha) w_t + \alpha r_t^k$ is the real marginal cost.

Equations (23) through (31), equation (21) together with the policy rules (17) and (18) determine the equilibrium path of the economy we have outlined.

Tables

<div align="center">

Table 2: Baseline calibration

Parameter	Value	Description
β	0.99	subjective discount factor
λ	0.5	share of non Ricardian consumers
α	1/3	share of capital
δ	0.025	depreciation rate
ξ_p	0.75	Calvo parameter on prices
ξ_w	0.75	Calvo parameter on wages
θ_p	6	implies a steady state price mark-up of 0.2
θ_w	6	implies a steady state wage mark-up of 0.2
γ_g	0.2	steady state share of government purchase
τ_π	1.5	Monetary policy response to π
ϕ_b	0.33	debt feedback coefficient
ϕ_g	0.1	public expenditure feedback coefficient
ρ_g	0.9	autoregressive coefficient for g process

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Figures
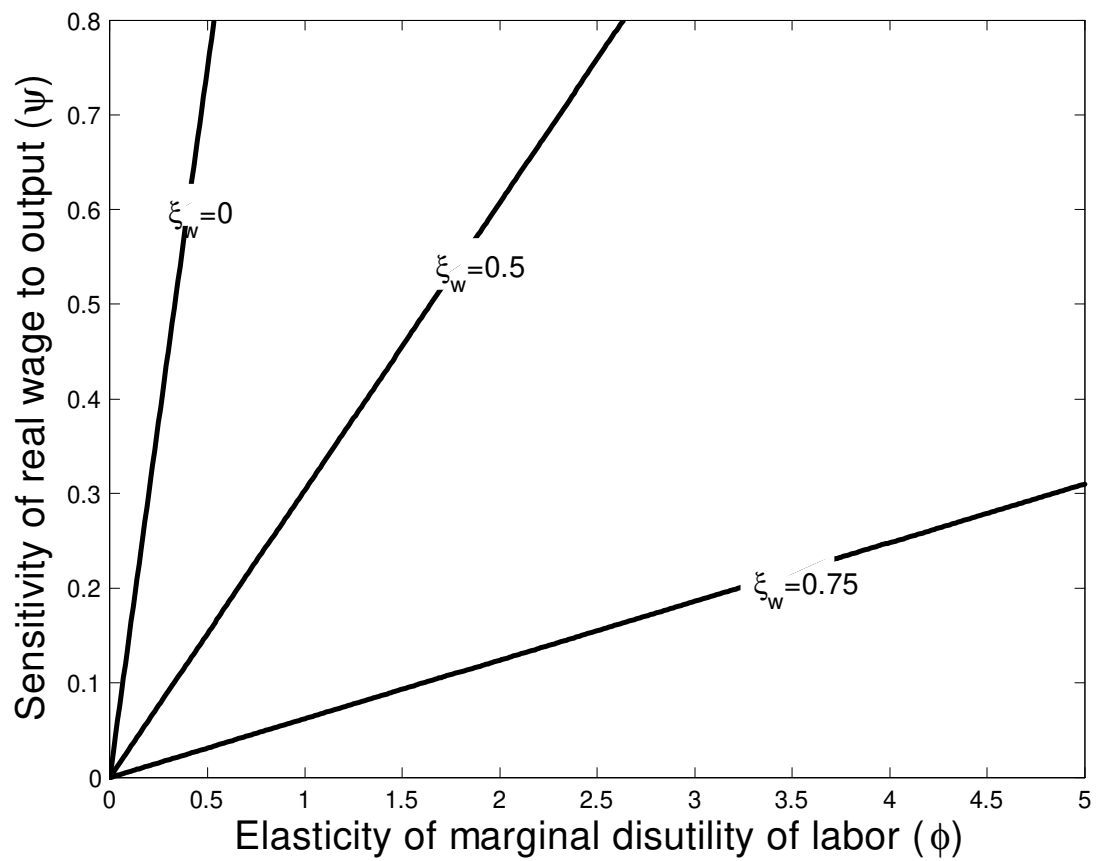
Figure 1: Sensitivity of real wage with respect to output as a function of the elasticity of marginal disutility of labor.
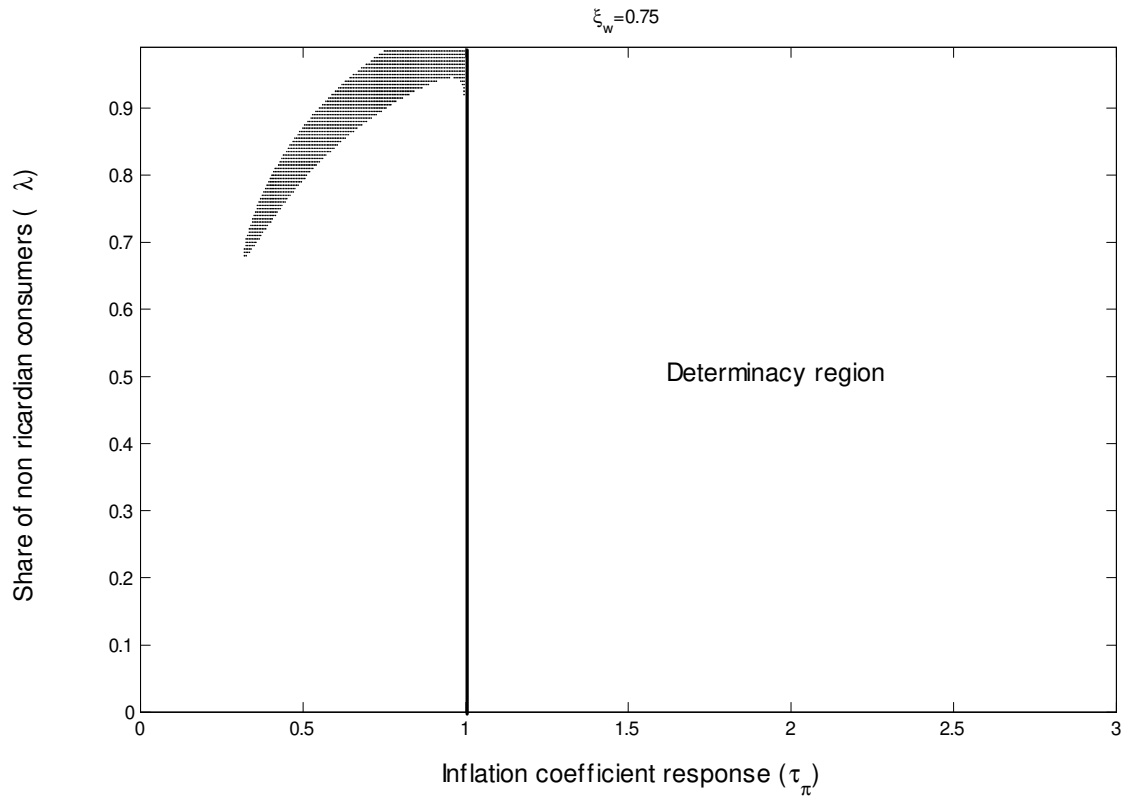
Figure 2: Determinacy region when wages have an average duration of 4 quarters ($\xi_w = 0.75$).
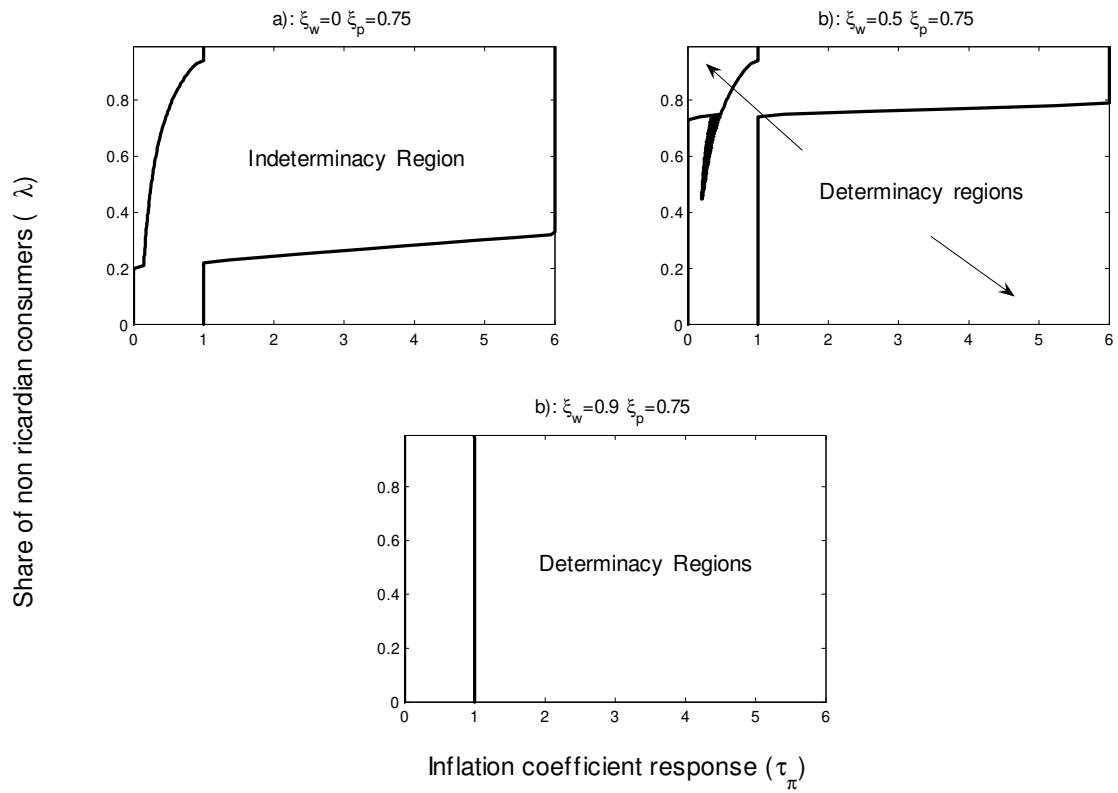
Instability area in black.

Figure 3: Indeterminacy regions under alternative degree of wage stickiness. Instability areas in black. Panel a $(\xi_w = 0)$, panel b $(\xi_w = 0.5)$ panel c $(\xi_w = 0.9)$.
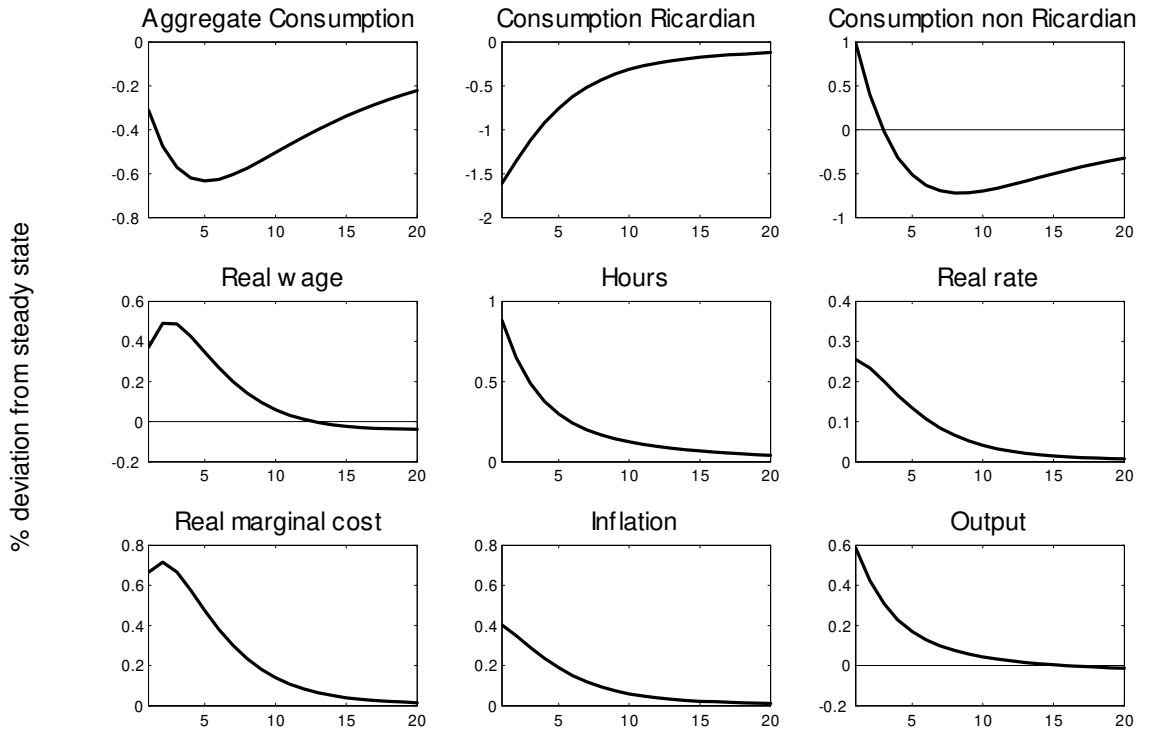
Figure 4: Impulse response functions to a government spending shock. Baseline parameterization.
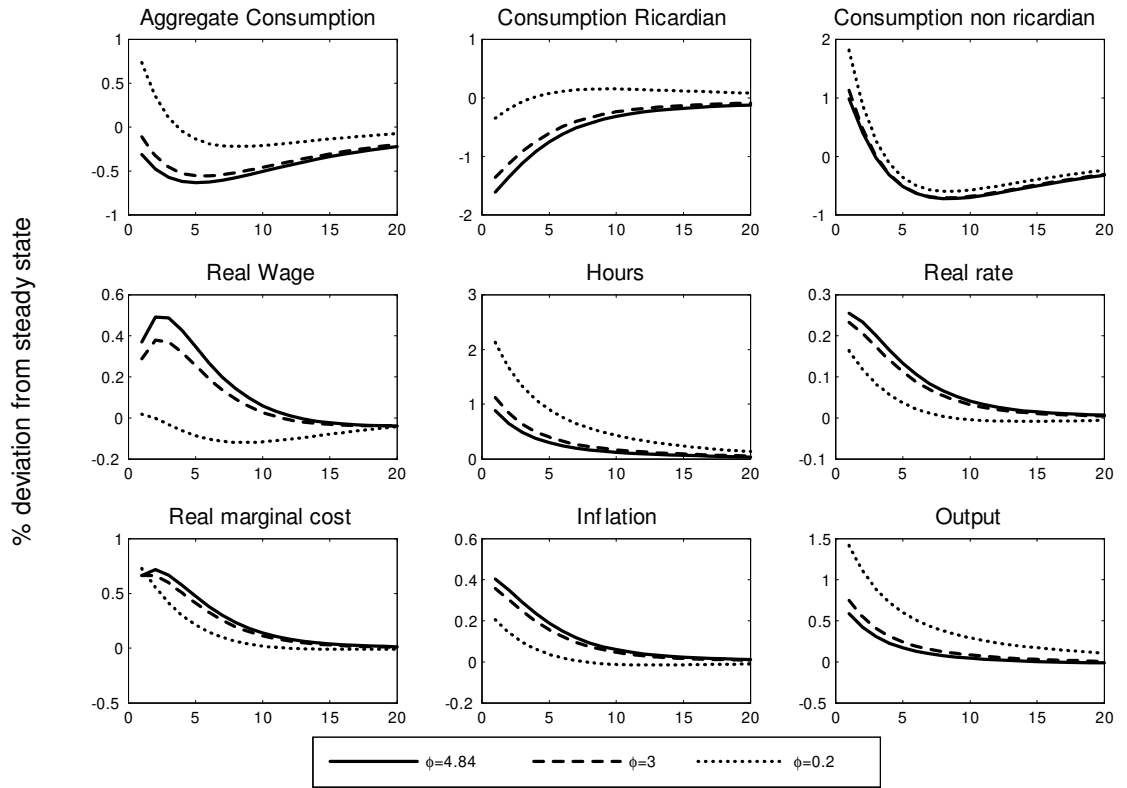
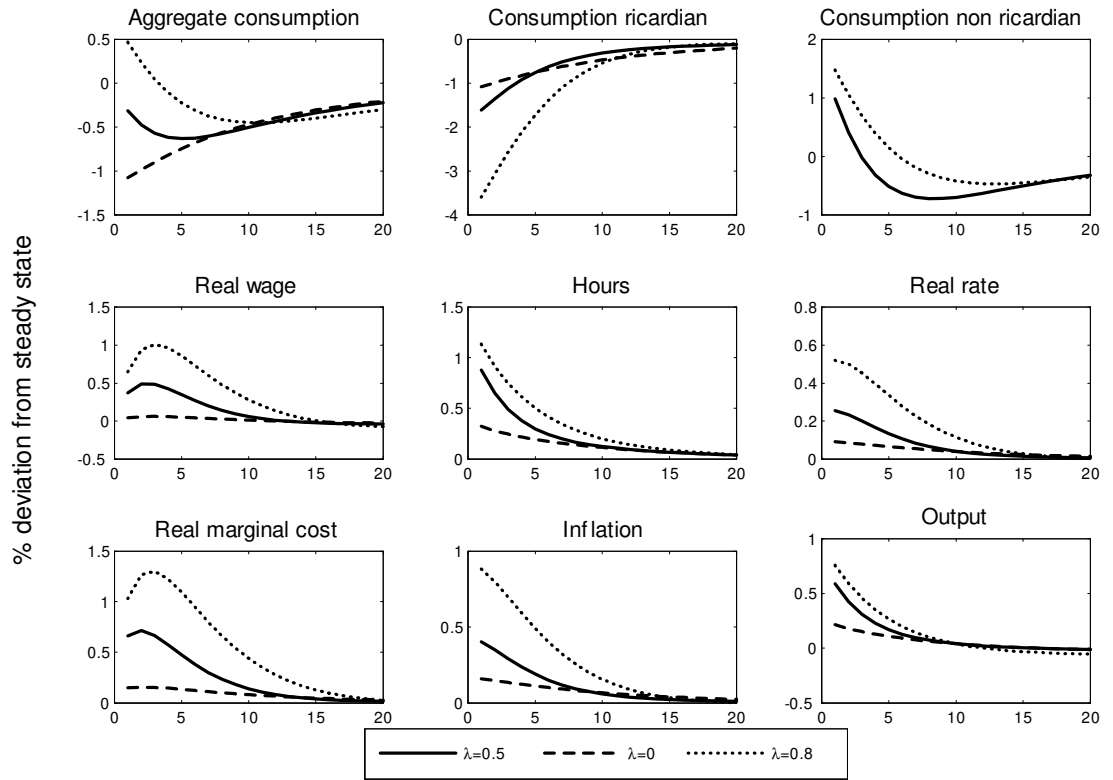Figure 5: Impulse response functions to a government spending shock. Sensitivity to ϕ.

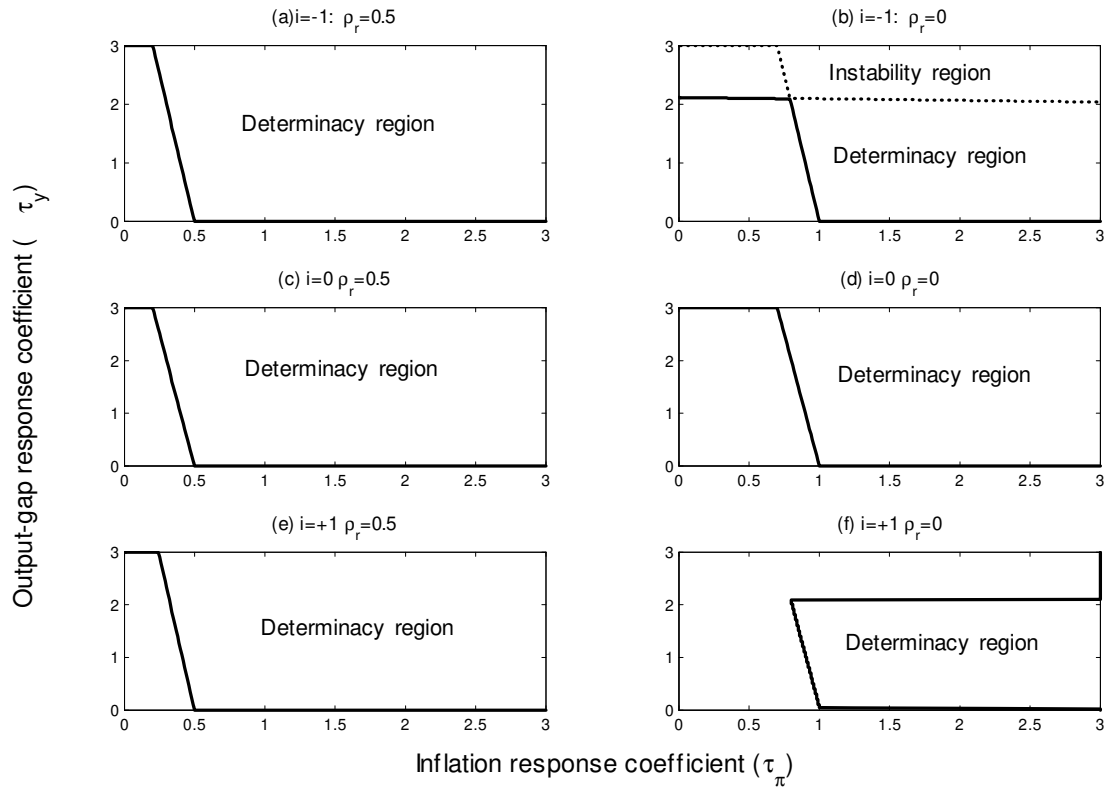Figure 6: Impulse response functions to a government spending shock. Sensitivity to λ.

Figure 7: Indeterminacy regions under alternative monetary rules. $i = -1$: backward looking rule; $i = 0$ contemporaneous rule; $i = +1$ forward looking rule
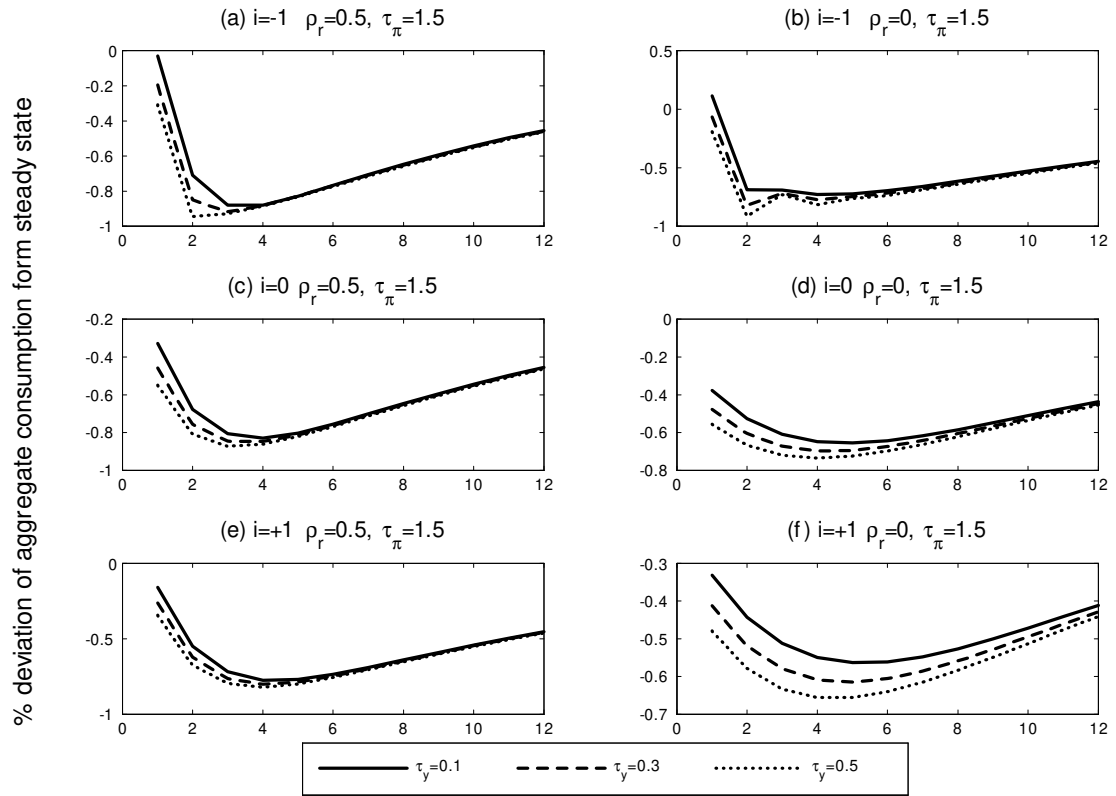
Figure 8: Response of aggregate consumption to a government spending shock under alternative monetary policy rules. $i = -1$: backward looking rule; $i = 0$ contemporaneous rule; $i = +1$ forward looking rule