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14 July 2011

Online at https://mpra.ub.uni-muenchen.de/37701/ MPRA Paper No. 37701, posted 28 Mar 2012 12:27 UTC

# Endogenous Human Capital Formation, Distance to Frontier and Growth

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March 17, 2012

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#### Abstract

We examine human capital's contribution to economy-wide technological progress through two channels – imitation and innovation – innovation being more skilled-intensive than innovation. We develop a growth model considering an endogenous ability-driven skill acquisition decision of an individual. We show that skilled labor is growth enhancing in the "imitation-innovation" regime and the "innovation-only" regime whereas unskilled labor is growth enhancing in the "imitation-only" regime. Steady state exists and, in the long run, an economy may or may not converge to the world technology frontier, depending on its initial position and the growth rate of the frontier economy. In the diversified regime, technological progress raises the return to ability and generates an increase in wage inequality between and within groups – consistent with the pattern observed across countries.

Journal of Economic Literature Classifications: I24, J20, O30, O31, O33, O40.

Key Words: Economic Growth, Endogenous Labor Composition, Imitation-Innovation, Convergence, Wage Inequality.

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# 1 Introduction

#### 1.1 Background

The standard growth theory models fail to explain why European Union (EU) is persistently facing a slow growth rate (less than 2 percent per year against 3 percent in the United States (US) between 1995 and 2000) and why Latin America was leapfrogged or superseded by South-East Asia between 1970 and 2000 and then stagnated at a growth rate of 0.2 percent per year. Since the mid 1990's US has been growing faster than EU in spite of the fact that the average savings rate, and the average capital-labor ratio, of EU over the past decade has been higher than that of US and the capital-labor ratio has not noticeably decreased over the period. Not only do the neoclassical or AK models but even the innovation-based models (Romer (1990), Aghion and Howitt (1992)) cannot account for the recent growth gap between Europe and US, given that property rights and innovation subsidies stressed by these models are reasonably well established in Europe, and that Europe invests almost as large a fraction of GDP on research and development (R and D) as US (2 percent versus 2.5 percent). Moreover, given that R and D investments were higher in US than Europe throughout the 1960's and 1970's; these models do not seem to explain higher labor productivity growth in EU than in US during this period (3.5 percent versus 1.4 percent on an average during the 1970's).<sup>1</sup>

Krueger and Lindhal (2001) found that education is statistically significant and positively associated with subsequent growth only for the countries with the lowest level of education and are insignificant and negative for countries with middle and high levels of education. That is, an inverted U-shaped relationship is found to exist between the level of education and the growth rate of an economy.<sup>2</sup> The relationship is insignificant when the regression is run for the Organization of Economic Cooperation and Development (OECD) countries alone. This finding apparently looks puzzling – basic education helps growth but the relationship between the two dies down as the country progresses in terms of education.

A possible explanation of this phenomenon could be that education favors the adoption of new technologies but the aggregate level of human capital does not matter at the frontier, as noticed by Nelson and Phelps (1966). On the other hand, according to Romer (1990) education favors the innovation of new technologies. But Nelson et. al. (1960) seem to have ignored the fact that a country can also improve its technology level through innovation while Romer (1990) ignores the fact that technology improvement can be possible through imitation from the world technology frontier, especially for the technologically backward economy. Benhabib and Spiegel (1994) and Barro (1998) found that the catch-up component of growth is the dominant one for the technologically backward economy. To find a possible explanation to the puzzle posed by Krueger et. al. (2001), we need to focus on both – an economy's distance to the technology frontier and on the composition of its human capital (as much as on its level).

We consider Romer's (1990) knowledge-based technological progress. But, similar to Benhabib et. al. (1994) and Acemoglu, Aghion and Zilibotti (2006), technological progress depends on both innovation upon the local/ national technology frontier and the imitation of technology from the world technology frontier. Notably, however, in case of both of these models, technology improvement is dependent only on the total stock of human capital and not on the composition of human capital.<sup>3</sup> They assume that technology improvement is the same irrespective of whether skilled or unskilled laborers are engaged in imitation or innovation activities. By relaxing this assumption we consider a specification similar to Aghion, Boustan, Hoxby, Vandenbussche (2005) and Vandenbussche, Aghion,

<sup>&</sup>lt;sup>1</sup>Statistics from Aghion Howitt (2006)

<sup>&</sup>lt;sup>2</sup>This finding is similar to Durlauf and Johnson (1995), and Kalaitzidakis et. al. (2001)

<sup>&</sup>lt;sup>3</sup>Total stock of the human capital means the sum of the total number of skilled and unskilled labor in the economy.

Meghir (2006) where imitation and innovation activities require both skilled and unskilled labor as input, but with differing intensity of use.

Empirical data reveal that during 1999-2000, 37.3 percent of US population aged between 25-64 had a high education degree vis-a-vis only 23.8 percent in EU. A similar difference can be observed in respect of government spending on tertiary education – while US spends 3 percent of its GDP on tertiary education, EU spends only 1.4 percent. Thus, the question that arises is – what is the best channel to invest in education – primary, secondary or tertiary education – so as to enable convergence to the world technology frontier?<sup>4</sup>

The cross-country analysis of Vendabussche et. al. (2006) and the cross US-state analysis of Aghion et. al. (2005) brings out the role of appropriate institutions in analyzing the relationship between growth and the composition of human capital. By developing a theoretical model they show that skilled labor has a higher growth-enhancing effect closer to the technology frontier but unskilled labor is the main source of growth for a technologically backward economy. Also, Vendabussche et. al. (2006) provides evidence in favor of this prediction using a panel data set covering 19 OECD countries between 1960 and 2000. Unlike earlier research, they not only take into account the stock of human capital but also include the composition of human capital which makes a significant difference. They are able to solve the puzzle posed by Krueger et. al. (2001) and show that education has positive and significant impact on economic growth even for an economy with high level of education. Aghion et. al. (2005) confronts the same prediction in respect of cross-US-state panel data.

The main drawback of both of these studies is that they assume that there exists an exogenously given composition of skilled-unskilled human capital. They considered only the benefit of skilled labor and ignored the fact that skill acquisition is not cost less. Also the optimal composition of skilled labor can't be same for an economy, irrespective of its distance to frontier. We attempt to fill in this gap by endogenizing skill composition, based on an individual's decision to acquire education or not. The important features of our research are enumerated below.

- 1. Unlike Vandenbussche et. al. (2006), and Aghion et. al. (2005), we consider heterogeneous agents, and by endogenizing individual's schooling decision, the composition of skilled and unskilled human capital in any time period is ascertained.
- 2. In the cone of diversified economy, as an economy progresses, the catch-up effect is low and for technology improvement the economy relies more on innovation activity. Under the assumption that innovation is skilled labor-intensive, the number of skilled labor increases and that of unskilled labor decreases with the progress of the economy. Along with that, both skilled and unskilled labors shift from imitation to innovation activity. This result is in line with Aghion et. al. (2005), and Vandenbussche et. al. (2006). But the shift of both skilled and unskilled labors from imitation to innovation activity is relatively more pronounced in our case than their work. This is because, besides the effect of both skilled and unskilled labor force in the overall labor composition. Due to exogenously given composition of labor force, the second effect is ignored by Aghion et. al. (2005), and Vandenbussche et. al. (2006). On the other hand, there exists a fixed composition of skilled and unskilled human capital in the only-imitation and only-innovation regimes, since further technology improvement has similar impact on both the factors.

<sup>&</sup>lt;sup>4</sup>Unskilled labor, taken to be efficient in performing imitation activity, is produced through primary and secondary education whereas skilled labor, assumed to be efficient in performing innovation activity is produced by tertiary (specially graduate-level) education.

- 3. By considering both the cost and the benefit associated with choosing to be skilled, we show that skilled labor is growth enhancing for an economy which is performing both imitation and innovation activities and only-innovation activity while unskilled labor is growth enhancing for an economy which is performing only-imitation activity under the assumption similar to that of Vandenbussche et. al. (2006), and Aghion et. al. (2005). We go beyond the work of Vandenbussche et. al. (2006), and Aghion et. al. (2005) and also characterize the economies which are specialized in imitation and innovation activities. Our findings contradict the policy prescription of Vandenbussche et. al. (2006), and Aghion et. al. (2005), since according to them unskilled labor is growth maximizing for a relatively technologically backward economy which is performing both imitation and innovation activities. Also, we contradict the theory of Grossman and Helpman (1991) according to whom skilled labor is growth enhancing whereas unskilled labor is growth depressing.
- 4. The dynamics of the stylized economy show that even by implementing an appropriate policy a technologically very backward economy will never converge to the world technology frontier if the frontier is growing at a very fast rate. Otherwise, by implementing the appropriate policy, the economy will converge to the world technology frontier and at the frontier all the economies will grow at the same rate. This finding is different from Di Maria et. al. (2009) who show absolute convergence to the world technology frontier irrespective of its distance to the frontier.
- 5. Moreover, in the cone of diversified region, as an economy progresses technologically, the average income of skilled labor rises while that of unskilled labor falls and, consequently, this raises the income inequality between skilled and unskilled labor groups. Since technology improvement has a heterogeneous effect on individuals' incomes (depending on their cognitive ability), it tends to rise within group inequality among skilled labor. On the other hand, in the imitation-only region, due to diminishing marginal return to the imitation activity, as an economy progresses average income and consequently average consumption of skilled labor decreases. Further, aggregate consumption of the economy decreases as an economy progresses. There exists a constant income inequality between skilled and unskilled labor and within skilled labor groups. In the innovation-only region, as the technology level increases, the average income and consumption of skilled and unskilled and unskilled labor increase and so does aggregate consumption of the economy. In this region also there exists a constant income inequality between and with groups of skilled and unskilled labor.

#### 1.2 Structure of the Paper

The paper is organized as follows. In section 2, we provide the basic structure of the model, and give the economic intuition for the behavior of the economy over time.

Section 3 contains the main analytical results. Here, we characterize the labor market equilibrium condition and the growth rate of the economy under the decentralized equilibrium. We find the equilibrium wage rate and the income of skilled and unskilled labor and between group and within group inequality of the economy. We derive the balanced growth path for the economy in a decentralized equilibrium. The equilibrium consumption basket for both skilled and unskilled labor and the consumption growth rate of the overall economy are also characterized.

Section 4 concludes our work and discusses some policy implications of this research. Moreover, we discuss some future areas of research. In what follows immediately, we describe the structure of the economy.

# 2 The Economic Environment

#### 2.1 Production

There are a finite number of small open economies. In each economy there is an entrepreneur who is engaged in the production of a final output, and there is a continuum of intermediate input producers. Each intermediate input producer also invests in R and D sector for productivity improvement, where the latter is characterized by free entry and exit. In any time period, each intermediate input producer possesses the locally available highest technology level; otherwise, he/ she will be thrown out of the market. There is size 1 population of workers. Workers have heterogeneous cognitive ability. We formulate the model in terms of effective labor and not in terms of workers. This allows us to aggregate across workers with heterogeneous schooling attainment, which is an important feature of the model. We also assume that once an individual becomes educated (or else decides not to go for education), the marginal return from education (or the lack of it), in terms of the wage rate, would be the same for all skilled labor (or unskilled labor) with different cognitive ability. We assume that among skilled workers *any one* organizes the production of final output, which basically means that he/ she is the entrepreneur. All other skilled laborers engage in the production of intermediate inputs. A small open economy implies that an individual can lend or borrow any amount from the world economy, and the economy is a price taker. Therefore, the rental rate for borrowing/ lending is fixed at  $\bar{r}_t$  in any period t.

We consider discrete time interval, where all the agents live for two time periods in an overlapping generation framework. In every time period and in each country the final output is produced competitively by using land and a continuum of mass 1 of intermediate inputs. For simplicity, we normalize the total supply of land to 1. We consider Cobb-Douglas production function of the form:

$$Y_t = l_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di, \quad 0 < \alpha < 1,$$

where  $Y_t$  is the final output in period t,  $l_t$  is the total amount of land  $(l_t = 1, \forall t)$ ,  $A_{it}$  is the level of technology in sector i in period t, and  $x_{it}$  is the amount of intermediate input 'i' in period t.

Since the final good sector produces under perfect competition, the price of the ' $i^{th}$ ' intermediate inputs in period t is equal to its marginal product. That is,

$$p_{it} = \frac{\partial Y_t}{\partial x_{it}} = \alpha A_{it}^{1-\alpha} x_{it}^{\alpha-1},$$

where  $p_{it}$  denotes the price of the intermediate input in sector *i* in period *t*. Each unit of the intermediate good is produced by using one unit of final good as capital. The monopolist chooses  $x_{it}$  by solving

$$\max_{x_{it}} \left( p_{it} x_{it} - x_{it} \right)$$

The monopolist produces the following amount of the intermediate good in sector i in period t:

$$x_{it} = \alpha^{\frac{2}{1-\alpha}} A_{it}.$$
 (1)

Accordingly, he/ she earns the following amount of profit in sector i in period t:

$$\pi_{it} = (p_{it} - 1)x_{it} = \left(\frac{1}{\alpha} - 1\right)\alpha^{\frac{2}{1-\alpha}}A_{it} = \delta_1 A_{it},$$
(2)

where  $\delta_1 = \left(\frac{1}{\alpha} - 1\right) \alpha^{\frac{2}{1-\alpha}}$ .

Note that both the optimal production and the profit in  $i^{th}$  intermediate input in period t is linearly dependent on the local (national) technology level in sector i in that period. That is, both the technology adjusted intermediate inputs and the profit are same for all the sectors in every time period.

#### 2.2 Dynamics of Productivity

A country can improve its technology level through two channels – imitating from the world technology frontier or innovating upon the local technology level. Since, we consider a knowledge based technology improvement scenario, the increment in technology is dependent on how efficiently skilled and unskilled labor imitate the world technology frontier and innovate upon the local technology frontier.<sup>5</sup> There is no uncertainty in technology improvement.

The technology improvement function is postulated as

$$A_{it} = A_{i,t-1} + \lambda \left[ u_{\min}^{\sigma} s_{\min}^{1-\sigma} (\overline{A}_{t-1} - A_{t-1}) + \gamma \ u_{\min}^{\phi} s_{\min}^{1-\phi} A_{t-1} \right], \quad \lambda > 0, \ \gamma > 0,$$
(3)

where  $\overline{A}_{t-1}$  is the world technology frontier at time t-1,  $A_{t-1}$  is the country's local technology frontier at time t-1,  $u_{m,i,t}$  (resp.  $s_{m,i,t}$ ) is the amount of unskilled (resp. skilled) labor input used in imitation at time t,  $u_{n,i,t}$  (resp.  $s_{n,i,t}$ ) is the amount of unskilled (resp. skilled) labor used in innovation at time t,  $\sigma$  (resp.  $\phi$ ) is the elasticity of unskilled labor in imitation (resp. innovation),  $\gamma$  measures the relative efficiency of innovation compared to imitation, and  $\lambda$  measures the efficiency of the overall process of technological progress.

Since imitation and innovation can perfectly substitute each other one can't ignore the possibility of the corner solution. Therefore, the specific technology improvement function for a sufficiently technologically backward economy is

$$A_{it} = A_{i,t-1} + \lambda u_{it}^{\sigma} s_{it}^{1-\sigma} (\overline{A}_{t-1} - A_{t-1}).$$

We call this 'imitation-only' regime. Similarly, the specific technology improvement function for a sufficiently technologically advanced economy is

$$A_{it} = A_{i,t-1} + \lambda u_{\text{nit}}^{\phi} s_{\text{nit}}^{1-\phi} A_{t-1}.$$

Henceforth this is referred to as the 'innovation-only' regime.

To satisfy our basic assumption that innovation is relatively skilled labor intensive than imitation, we make the following assumption:

A1. The elasticity of skilled labor is higher in innovation activity than in imitation activity or,  $\sigma > \phi$ . That is, the elasticity of unskilled labor is higher in imitation than innovation. In the imitation-only regime, imitation is unskilled labor intensive, that is,  $\sigma > \frac{1}{2}$  and in the innovation-only regime, innovation is skilled labor intensive, that is,  $\phi < \frac{1}{2}$ .

#### 2.3 Consumption

An individual devotes  $\tau$  fraction of his/ her first period time on education and the remaining  $(1-\tau)$  fraction to skilled labor force in the R and D sector. As a result, for education, a skilled worker has to sacrifice his

 $<sup>^{5}</sup>$ Here efficiency or productivity of skilled or unskilled labor is measured in terms of the elasticity of skilled or unskilled labor in imitation or innovation activity.

<sup>&</sup>lt;sup>6</sup>In the diversified regime our work does not require any assumption about the absolute intensity of skilled or unskilled labor in innovation or imitation activity respectively. These parametric restrictions are pertaining only to the specialized economy.

income as an unskilled labor during his education time. That is, the opportunity cost of education is the wage rate of unskilled labor in that period. Furthermore, we assume that getting educated would be easier for the more talented individual (individual with higher cognitive ability). So, the cost of education would be positively correlated with the wage rate of unskilled labor  $(w_{ut})$ , and negatively correlated with his cognitive ability  $(\theta)$ . That is, education is costly. As a result, the net income of skilled labor varies across the individuals, that is, it is an increasing function of the individual's cognitive ability. On the other hand, without bearing any additional cost, an individual can work as unskilled labor. One can also introduce this heterogeneity in different ways – a higher cognitive ability individual has higher ability to learn, which implies that  $\tau$ , the time required to learn a new knowledge, is a decreasing function of the cognitive ability. Else, one can assume that the performance of skilled labor would be dependent on his/ her cognitive ability, even if two individuals have the same educational background (that is, either both of them opt for education or neither of them opt for it). We assume that cognitive ability is uniformly distributed over the interval [0, 1]. A skilled labor works and studies simultaneously. The entrepreneur, intermediate input producers and both types of labor retire in the second period of their lives. Labor consumes in both the periods of his/ her life.

The cost of education of an individual with  $\theta$  cognitive ability is expressed as

$$E(\theta, A_{t-1}) = \frac{\tau \ w_{ut}}{\theta}, \qquad \qquad 0 < \tau < 1$$

Both skilled and unskilled labor maximizes their lifetime utility subject to the budget constraint. Each individual maximizes the following lifetime utility function

$$\mathbb{W}_k = \log c_{t,t} + (1+\rho)^{-1} \log c_{t,t+1}; \text{ where } k = s, u,$$

where  $\mathbb{W}_k$  is the lifetime utility of the  $k^{th}$  individual,  $c_{t,t}$  is the consumption of an individual in period t who is born in period t,  $c_{t,t+1}$  is the consumption of an individual in period t+1 who is born in period t, s denotes skilled labor and u denotes unskilled labor,  $\rho$  is the discount rate.

Along with his consumption in the first and second period of his/ her life, skilled labor has to bear the cost of education. Accordingly, the budget constraint for skilled labor is

$$c_{t,t} + \frac{\tau \ w_{ut}}{\theta} + \frac{c_{t,t+1}}{1 + \overline{r}_{t+1}} = (1 - \tau)w_{st},$$

where  $w_{st}$  is the wage rate of skilled labor in period t.

Since unskilled labor spends his/ her entire income on consumption, the budget constraint of an unskilled labor is

$$c_{t,t} + \frac{c_{t,t+1}}{1 + \overline{r}_{t+1}} = w_{ut}.$$

We assume a perfectly competitive labor market. Individuals have perfect foresight. For more mathematical tractability, we assume that there is no population growth. Each parent bears one child. At the end of the first period of the  $t^{th}$  generation, a new generation  $(t+1)^{th}$  appears.

Before moving on to the analytical results, we discuss intuitively how the economy behaves over time. In period (t+1) a new generation appears. Depending on the available knowledge in period t, and his/ her cognitive ability, each individual decides whether he/ she opts for education or not. This determines the composition of skilled and unskilled labor in period (t + 1). A perfectly competitive labor market implies equalization of the wage rate of skilled and unskilled labor in imitation and innovation activities. Depending on the efficiency of both – imitation and innovation – activities in period (t + 1), market allocates the total stock of skilled and

unskilled labor in these two activities. For a sufficiently technologically backward (resp. advanced) economy, market allocates neither skilled nor unskilled labor in innovation (resp. imitation) activity; they concentrate only in the imitation (resp. innovation) activity. This constitute our "imitation-only" (resp. "innovation-only") regime. And the intermediate economies perform both the imitation and innovation activities. This is known as "imitation-innovation" regime. The efficiency of imitation and innovation activities determines the next period's technology level. The technology level in period (t + 1) determines the production of the intermediate inputs in sector *i* in period (t + 1), which in turn, determines the final output in period (t + 1). This generates the income and the consumption of skilled and unskilled labor force in period (t + 1). Again, the technology level in period (t + 1) determines the next period's technology level. This process continues and, in the long run, the economy may or may not converge to the world technology frontier, as we shall see below.

# 3 Analytical Results

We now formally derive the key results of our analysis.

#### 3.1 Labor Supply

The total supply of labor in period t is determined by an individual's maximization exercise. An individual chooses education if his/ her lifetime utility from being a skilled laborer is greater than from being an unskilled laborer. The cut-off or threshold value of the parameter  $\theta \ (\equiv \hat{\theta}_t)$  above which an individual chooses the status of being skilled is given by

$$\hat{\theta_t} \ge \frac{\tau \ w_{ut}}{(1-\tau)w_{st} - w_{ut}}.\tag{4}$$

For simplicity we assume that individuals who are indifferent between working as skilled or unskilled labor work as skilled labor.

#### 3.2 Imitation-only Regime

First, we consider an economy which performs only-imitation activity. Later in subsection (3.4.4) we explicitly characterize the parametric restriction when an economy performs only-imitation.

#### 3.2.1 Demand for Labor

An intermediate input producer chooses skilled and unskilled labor by maximizing its profit in the R and D sector. Since an intermediate input producer operates the production process for one period only, he/ she maximizes current profit net of labor costs. Profit maximizing exercise of the intermediate input producer is

$$\max_{U_{it},S_{it},} \delta_1 \lambda U_{it}^{\sigma} S_{it}^{1-\sigma} (\overline{A}_{t-1} - A_{t-1}) - w_{ut} U_{it} - w_{st} S_{it}.$$
(5)

Given that all intermediate input producers face the same maximization problem, in equilibrium we have

$$u_{mit} = u_{mt}, \qquad s_{mit} = s_{mt}, \qquad u_{nit} = u_{nt}, \qquad s_{nit} = s_{nt}.$$
 (6)

There is mass 1 of intermediate firms, so that labor market equilibrium condition is

$$S_t = s_{mt} + s_{nt},$$
  $U_t = u_{mt} + u_{nt}.$  (7)

From the first order condition in eq. (5), we get the relative demand curve of skilled-unskilled labor as

$$\left(\frac{w_{st}}{w_{ut}}\right)^M = \left(\frac{1-\sigma}{\sigma}\right) \left(\frac{U_t}{1-U_t}\right).$$

#### 3.2.2 Labor Market in Equilibrium

Under perfect competition, labor market always clear. By equating relative demand for skilled and unskilled labor with their respective relative supply level we get the equilibrium level of skilled and unskilled labor in a sufficiently technologically backward economy. The total number of unskilled labor in a relatively backward economy is

$$U^* = \frac{\tau}{[(1-\tau)w^* - 1]} = \frac{(1-\tau) + \sqrt{(1-\tau)^2 + 4\tau \left(1 + \frac{(1-\sigma)}{\sigma}(1-\tau)\right)}}{2\left(1 + \frac{(1-\sigma)}{\sigma}(1-\tau)\right)}.$$
(8)

The absolute and relative wage rates of skilled-unskilled labor are respectively given by

$$w^{*} = \frac{w_{st}}{w_{ut}} = \frac{(1-\sigma)}{\sigma} \left[ \frac{(1-\tau) + \sqrt{(1-\tau)^{2} + 4\tau \left[1 + \frac{(1-\sigma)}{\sigma}(1-\tau)\right]}}{2 \left[1 + \frac{(1-\sigma)}{\sigma}(1-\tau)\right] - (1-\tau) - \sqrt{(1-\tau)^{2} + 4\tau \left[1 + \frac{(1-\sigma)}{\sigma}(1-\tau)\right]}} \right];$$

$$w_{ut} = \lambda \ \sigma \ U^{*\sigma-1} S^{*1-\sigma} (\overline{A}_{t-1} - A_{t-1});$$

$$w_{st} = \lambda (1-\sigma) U^{*\sigma} S^{*-\sigma} (\overline{A}_{t-1} - A_{t-1}).$$
(9)

Due to diminishing marginal return of technology improvement in the imitation-only regime the marginal productivity of both skilled and unskilled labor decreases, and consequently the wage rate of skilled and unskilled labor decreases as an economy progresses. The first order condition in eq. (5) implies that this decrement has a similar effect on both skilled and unskilled labor. Therefore, there exists a constant relative wage rate of skilled-unskilled labor. As a result, there is no transitional dynamics in the composition of skilled-unskilled human capital and there exists a fixed composition of skilled and unskilled labor in the imitation-only equilibrium.

In this entire analysis we make comment on optimal education policy of an economy, that is, whether to invest in primary, secondary or tertiary education on the basis of the varying cost of education. Our main instrument for this is  $\tau$ . So, now we attempt comparative dynamics with respect to the cost of education (that is,  $\tau$ ). From, eq. (8), it is clear that with an increase in the cost of education there is an increase in the total number of unskilled labor and a decrease in the total number of skilled labor. Intuitively, an increment in  $\tau$  reduces the income of an individual as skilled labor, but does not impact the income of unskilled labor. As a result, individuals who are earlier marginally benefited by education will now prefer to work as an unskilled labor. This reduces the total number of skilled labor.

As unskilled labor works for one period alone and bears no additional cost of education, the absolute income of unskilled labor is identical to his/ her absolute wage rate. This leads us to conclude that the absolute income of unskilled labor behaves in the same manner as the wage rate of unskilled labor. On the other hand, as discussed earlier in section (2.3), average income of skilled labor is an increasing function of their cognitive ability. Since cognitive ability follows a uniform distribution over the interval [0, 1], one could define the average income of skilled labor with highest cognitive ability and income of skilled labor.

with lowest cognitive ability. Mathematically, the average income of skilled labor in the imitation-only regime is

$$\overline{I}_{s,t}^{M} = \frac{I_{st}|_{\theta=1} + I_{st}|_{\theta=\hat{\theta}_{t}}}{2} = \left(\frac{\lambda\delta_{1}}{2}\right) \left(\frac{S^{*}}{U^{*}}\right)^{-\sigma} \left(\overline{A}_{t-1} - A_{t-1}\right) \left[2(1-\tau)(1-\sigma) - \tau\sigma\left(\frac{S^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right)\right] + \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) \left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2}\left(\frac{1+U^{*}}{U^{*}}\right) = \frac{1}{2$$

where  $\overline{I}_{s,t}^{M}$  measures the average income of skilled labor in the imitation-only regime and  $I_{kt}$  denotes the income of  $k^{th}$  type of labor in time period t, where k = s, u. As discussed in eq. (9) the wage rate of skilled labor decreases in the imitation-only regime. For a similar reason the average income of skilled labor decreases as an economy moves to the technology frontier.

#### 3.3 Innovation-only Regime

Now, let us consider an economy which performs only-innovation. Later in subsection (3.4.4) we characterize the parametric restrictions under which an economy performs only-innovation. Similar to the imitation-only regime, here too we get a fixed composition of skilled and unskilled human capital and constant relative wage rate of skilled and unskilled labor. The total number of unskilled labor in the innovation-only regime is

$$\bar{U}^* = \frac{\tau}{(1-\tau)\bar{w}^* - 1} = \frac{(1-\tau) + \sqrt{(1-\tau)^2 + 4\tau \left(1 + \frac{(1-\phi)}{\phi}(1-\tau)\right)}}{2\left(1 + \frac{(1-\phi)}{\phi}(1-\tau)\right)}.$$

The relative wage rate of skilled-unskilled labor in innovation-only regime

$$\bar{w}^* = \frac{\bar{w_{st}}}{\bar{w_{ut}}} = \frac{(1-\phi)}{\phi} \left[ \frac{(1-\tau) + \sqrt{(1-\tau)^2 + 4\tau \left[1 + \frac{(1-\phi)}{\phi}(1-\tau)\right]}}{2\left[1 + \frac{(1-\phi)}{\phi}(1-\tau)\right] - (1-\tau) - \sqrt{(1-\tau)^2 + 4\tau \left[1 + \frac{(1-\phi)}{\phi}(1-\tau)\right]}} \right]$$

Now, marginal productivity of skilled and unskilled labor increases as the technology level increases and so does the wage rate and average income of skilled and unskilled labor. But, from the profit maximizing exercise of the R and D sector in the innovation-only regime, it is clear that the relative gain of higher knowledge is identical to both the factors. As a result, relative wage rate of skilled and unskilled labor is constant and this induces to a fixed composition of skilled and unskilled labor in the innovation-only regime. Performing the comparative static exercise, we understand that as the cost of education increases (that is, a rise in  $\tau$ ) there is an increase in the total number of unskilled labor and a decrease in the total number of skilled labor. The logic for the above argument is same as provided in the imitation-only regime.

#### 3.4 Combined Imitation and Innovation Regime

Finally, we consider those economies that perform both imitation and innovation activities, or have diversified R and D sector.

#### 3.4.1 Demand for Labor

We start by determining the demand for skilled and unskilled labor in innovation and imitation activities. This depends on the efficiency of both types of labor force in imitation and innovation activities, as also the distance

of the economy from the world technology frontier. The total cost of labor productivity improvement in sector i in period t is given by

$$w_{it} = w_{ut}(u_{mit} + u_{nit}) + w_{st}(s_{mit} + s_{nit}),$$

where  $w_{it}$  be the total labor cost of the  $i^{th}$  R and D sector in period t.

Each intermediate producer chooses  $u_{mit}$ ,  $s_{mit}$ ,  $u_{nit}$ ,  $s_{nit}$  by solving the following maximization exercise

$$\max_{u_{mit}, u_{nit}, s_{mit}, s_{nit}} \delta_1 \lambda \left[ u_{mit}^{\sigma} s_{mit}^{1-\sigma} (\overline{A}_{t-1} - A_{t-1}) + \gamma u_{nit}^{\phi} s_{nit}^{1-\phi} A_{t-1} \right] - w_{it}.$$
(10)

From the first-order conditions of the maximization exercise in eq. (10) we get the following relative demand curves for labor in imitation and innovation activities respectively:

$$\frac{w_{st}}{w_{ut}} = \frac{(1-\sigma)}{\sigma} \frac{u_{mt}}{s_{mt}}; \qquad \qquad \frac{w_{st}}{w_{ut}} = \frac{(1-\phi)}{\phi} \frac{u_{nt}}{s_{nt}}. \tag{11}$$

Observe that the relative demand curve for skilled-unskilled labor in innovation is relatively more inelastic than in imitation activity. If the relative price of skilled labor increases (that is, either the wage rate of skilled labor increases or that of unskilled labor decreases or both), then by the law of demand, there would be a relative decrease in the demand of skilled labor force and an increase in the relative demand of unskilled labor. But, under **A1**, the fall in the demand for skilled labor is more in imitation than in innovation activity. This is why the substitution of unskilled labor for skilled labor is more in imitation than in innovation activity.

On account of perfectly competitive labor market, the marginal products of skilled and unskilled labor would be equalized across imitation and innovation. Using eq. (11)

$$\psi \ \frac{s_{mt}}{u_{mt}} = \frac{s_{nt}}{u_{nt}},\tag{12}$$

where  $\psi = \frac{\sigma(1-\phi)}{\phi(1-\sigma)} > 1$  by **A1**. The above equation implies that the ratio of skilled to unskilled labor is proportional across imitation and innovation activities. If the total demand for skilled labor force increases in imitation then also the demand for unskilled labor increases in that activity, given a fixed composition of human capital. Further, the increase is more pronounced in innovation than in imitation activity. By using eqs. (6), (7) and (12) in the first-order condition of the maximization exercise of eq. (10) we get the following expressions:

$$s_{mt} = \frac{h(a_{t-1}) U_t - S_t}{\psi - 1}; \qquad s_{nt} = \frac{\psi S_t - h(a_{t-1}) U_t}{\psi - 1}; u_{mt} = \frac{\psi [h(a_{t-1}) U_t - S_t]}{(\psi - 1) h(a_{t-1})}; \qquad u_{nt} = \frac{\psi S_t - h(a_{t-1}) U_t}{(\psi - 1) h(a_{t-1})}, \qquad (13)$$

where  $a_{t-1} = \frac{A_{t-1}}{\overline{A}_{t-1}}$ , and  $h(a_{t-1}) = \left[\frac{(1-\sigma)\psi^{\sigma}(1-a_{t-1})}{\gamma(1-\phi)a_{t-1}}\right]^{\frac{1}{\sigma-\phi}}$ . That is,  $a_{t-1}$  is the inverse measure of the country's distance to the world technology frontier. As a country moves to the world technology frontier, value of  $a_{t-1}$  increases and approaches 1. The ratio  $\frac{(1-a_{t-1})}{a_{t-1}}$  measures the relative distance of the economy from the world technology level and the world frontier decreases, thus reducing the relative distance of the economy from the frontier. Since  $h(a_{t-1})$  is a monotonic function of the relative distance of the economy from the frontier. Since technologically, the value of  $h(a_{t-1})$  decreases. That is,  $h(a_{t-1})$  is a decreasing function of  $a_{t-1}$ .

Eqs. (12) and (13) imply

$$\frac{s_{mt}}{u_{mt}} = \frac{h(a_{t-1})}{\psi}; \qquad \qquad \frac{s_{nt}}{u_{nt}} = h(a_{t-1}). \tag{14}$$

which is used later to determine the optimal level of skilled and unskilled labor and its allocation to both the imitation and innovation activities.

#### 3.4.2 Labor Market in Equilibrium

Under conditions of perfect competition labor market always clears at every point in time. The interaction of demand and supply curves of skilled and unskilled labor in each activity determines the optimal composition of human capital in period t. By substituting eqs. (6), (14) and first-order condition of eq. (10) in eq. (4) we get the following expressions of unskilled and skilled labor force in the economy

$$U_t = \frac{\tau \ \phi \ h(a_{t-1})}{\left[(1-\tau)(1-\phi) - \phi h(a_{t-1})\right]} = \frac{\tau \ \sigma \ h(a_{t-1})}{\left[\psi(1-\tau)(1-\sigma) - \sigma h(a_{t-1})\right]};$$
(15)

$$S_t = 1 - \frac{\tau \ \phi \ h(a_{t-1})}{\left[(1-\tau)(1-\phi) - \phi h(a_{t-1})\right]} = 1 - \frac{\tau \ \sigma \ h(a_{t-1})}{\left[\psi(1-\tau)(1-\sigma) - \sigma h(a_{t-1})\right]}.$$
(16)

For the existence of positive quantities of both skilled and unskilled labor in equilibrium, the following condition needs to be satisfied

$$U_t > 0 \quad \Rightarrow \quad [(1 - \tau)(1 - \phi) - \phi h(a_{t-1})] > 0. \tag{17}$$

Next, we attempt some comparative dynamics.

First, we discuss the impact of an increase (resp. a decrease) in the cost of education on the composition of the human capital. As can be seen from eq. (16) and eq. (17),  $\frac{dS_t}{d\tau} < 0$ , and  $\frac{dU_t}{d\tau} > 0$ . Intuitively, due to the similar logic as discussed in subsection 3.2.2, as cost of education increases in equilibrium total number of skilled laborers decreases and the total number of unskilled laborers increases in the economy.

Next, we study the transition path of skilled and unskilled human capital as the economy moves to the world technology frontier. We have

$$\begin{aligned} \frac{dU_t}{da_{t-1}} &= -\frac{(1-\tau)(1-\phi)\phi h(a_{t-1})}{(\sigma-\phi)a_{t-1}(1-a_{t-1})[(1-\tau)(1-\phi)-\phi h(a_{t-1})]^2} < 0;\\ \frac{dS_t}{da_{t-1}} &= -\frac{\partial U_t}{\partial a_{t-1}} > 0. \end{aligned}$$

From eq. (3), the catch up effect is high for a technologically backward economy. Under A1, the demand for unskilled labor is relatively high in a technologically backward economy. Moreover as a country moves towards the world technology frontier, imitation becomes relatively less important than innovation. Under A1, the demand for skilled (resp. unskilled) labor increases (resp. decreases) and associated with this, the marginal productivity of skilled (resp. unskilled) labor increases (resp. decreases) as an economy progresses. Therefore, the optimum level of skilled labor increases and unskilled labor decreases as a country gets closer to the world technology frontier. This indicates that, keeping a fixed amount of skilled and unskilled labor for any distance from the world technology frontier can not be an optimum decision, an assumption made by Aghion et. al. (2005), and Vandenbussche et. al. (2006). Our above discussion casts a doubt on their assumptions. Thus, we have

#### Lemma 1

Under A1,

i. The optimum stock of skilled human capital increases (resp. decreases) and unskilled human capital decreases (resp. increases) as a country moves to the world technology frontier, even as the aggregate stock of human capital remains unchanged.

ii. Any increment (resp. decline) in the cost of education decreases (resp. increases) the total stock of skilled human capital and increases (resp. decreases) the stock of unskilled human capital, for the country which is performing both imitation and innovation activities.

Further, we attempt to characterize the optimal allocation of human capital into both imitation and innovation activities. Substituting the values of (15) and (16) in (13), we get the solution of the optimum allocation of skilled and unskilled labor in imitation and innovation activities to be:

$$u_{mt} = \frac{\psi \tau \phi [1 + h(a_{t-1})]}{(\psi - 1)[(1 - \tau)(1 - \phi) - \phi h(a_{t-1})]} - \frac{\psi}{(\psi - 1)h(a_{t-1})};$$

$$u_{nt} = \frac{\psi}{(\psi - 1)h(a_{t-1})} - \frac{\tau \phi [\psi + h(a_{t-1})]}{(\psi - 1)[(1 - \tau)(1 - \phi) - \phi h(a_{t-1})]};$$

$$s_{mt} = \frac{\tau \phi h(a_{t-1})[1 + h(a_{t-1})]}{(\psi - 1)[(1 - \tau)(1 - \phi) - \phi h(a_{t-1})]} - \frac{1}{\psi - 1};$$

$$s_{nt} = \frac{\psi}{\psi - 1} - \frac{\tau \phi h(a_{t-1})[\psi + h(a_{t-1})]}{(\psi - 1)[(1 - \tau)(1 - \phi) - \phi h(a_{t-1})]}.$$
(18)

We again carry out a comparative dynamics exercise. We first analyze the impact of a relatively higher cost of education on the allocation of skilled and unskilled human capital in imitation and innovation activities. From eq. (18), we get  $\frac{ds_{mt}}{d\tau} > 0$ ,  $\frac{ds_{nt}}{d\tau} < 0$ ,  $\frac{du_{mt}}{d\tau} > 0$ ,  $\frac{du_{nt}}{d\tau} < 0$ . Intuitively, under **Lemma 1** as cost of education increases, total number of unskilled labor increases. From **A1**, imitation attracts more unskilled human capital and, due to the complementary effect, it also raises the demand for skilled labor in imitation activity. This again raises the marginal productivity of unskilled labor in imitation activity as a second round effect. This process continues entailing that the employment of both skilled and unskilled labor increases in imitation activity and falls in innovation activity.

We now turn towards an analysis of the transition path of the allocation of skilled and unskilled human capital as an economy progress to the world technology frontier.

From eq. (18), we have  $\frac{du_{mt}}{da_{t-1}} < 0$ ,  $\frac{ds_{mt}}{da_{t-1}} > 0$ ,  $\frac{ds_{nt}}{da_{t-1}} > 0$ . First of all, from **Lemma 1**, as an economy progresses total number of skilled labor increases and by A1, both skilled and unskilled labor shifts from imitation to innovation activity. Secondly, as the economy moves to the world technology frontier, the catch-up effect tends to get weaker and the economy relies more on innovation activity for further technological progress. Under A1, innovation requires more skilled labor. Due to the complementary effect, both skilled and unskilled labor moves from imitation to innovation activity. Both of these factors would induce a decline in the employment of skilled and unskilled human capital in imitation and an increase in innovation activities, as the economy closes its gap with the world technology frontier. Hence,

#### Lemma 2

#### Under A1,

i. The optimal amount of skilled and unskilled labor employment increases (resp. decreases) in innovation (resp. imitation) activity, as a country moves to the world technology frontier.

ii. A relatively more (resp. less) expensive education system shifts skilled and unskilled labor from innovation (resp. imitation) to imitation (resp. innovation) activity.

#### 3.4.3 Wage Rate and Income of Skilled and Unskilled Labor

We now try to derive the market clearing wage rate and, consequently, the income of both skilled and unskilled human capital inside the cone of diversification. In this region, we also analyze the behavior of the competitive wage rate and income as the economy moves towards the frontier.

First, we derive the wage rates of skilled and unskilled labor force from the first-order condition of eq. (10), together with eqs. (6) and (14), to be:

$$w_{st} = \lambda \delta_1 \gamma (1 - \phi) h^{-\phi}(a_{t-1}) a_{t-1} \overline{A}_{t-1} = \lambda \delta_1 (1 - \sigma) \psi^{\sigma} h^{-\sigma}(a_{t-1}) (1 - a_{t-1}) \overline{A}_{t-1};$$
  

$$w_{ut} = \lambda \delta_1 \gamma \phi h^{1-\phi}(a_{t-1}) a_{t-1} \overline{A}_{t-1} = \lambda \delta_1 \sigma \psi^{-(1-\sigma)} h^{1-\sigma}(a_{t-1}) (1 - a_{t-1}) \overline{A}_{t-1}.$$
(19)

We look at the transition path of wage rate of both kinds of labor force as an economy progresses to the world technology frontier. Differentiating eq. (19) w.r.t.  $a_{t-1}$  we get that

$$\frac{dw_{st}}{da_{t-1}} = \lambda \delta_1 \gamma (1-\phi) \left[ \frac{\phi + (\sigma - \phi)(1 - a_{t-1})}{(\sigma - \phi)(1 - a_{t-1})} \right] h^{-\phi}(a_{t-1}) \overline{A}_{t-1} > 0;$$
  
$$\frac{dw_{ut}}{da_{t-1}} = -\lambda \delta_1 \gamma \phi \left[ \frac{(1-\sigma) + (\sigma - \phi)a_{t-1}}{(\sigma - \phi)(1 - a_{t-1})} \right] h^{1-\phi}(a_{t-1}) \overline{A}_{t-1} < 0.$$

From eq. (3) it is clear that the contribution of imitation is lower in a relatively advanced economy. As we know from A1, the marginal productivity of skilled labor increases and unskilled labor decreases and, consequently, wage rate of skilled human capital also rises and unskilled human capital falls as a country moves to the world technology frontier. While Di Maria et. al. (2009) only concentrate on the relative movement of the wage rate, our analysis not only characterizes the relative wage rate but also the absolute wage of skilled unskilled labor.

Observe that, in the diversified regime as a country progresses to the frontier, the total stock of skilled labor rises, and so does its wage rate. That is, as the supply of skilled labor rises, the absolute wages are not lowered; rather these are raised. This might appear to be contradictory to traditional demand-supply theory. Intuitively, an increase in the stock of skilled human capital raises the next period's technology level. Since we consider knowledge-driven technological progress, by **A1**, this further raises the demand for skilled labor. That is, the relative demand for skilled labor is now proportionately higher than its relative supply. So, our finding that there will be an increment in the wage rate of skilled human capital, following an increase in the supply of skilled human capital, in the transition, is in conformity with standard theory. This finding is also in line with Galor and Moav (2000), Acemoglu (2002), and Acemoglu (1998).<sup>7</sup> In the standard literature this is known as skilled-biased technological progress.

Now, we analyze the average income of skilled and unskilled human capital.

As discussed earlier, the absolute income of unskilled labor behaves in the same manner as the wage rate of unskilled labor. Now, the average income of skilled labor in the cone of diversified region is

$$\overline{I}_{s,t} = \lambda \delta_1 \gamma (1-\tau) h^{-\phi}(a_{t-1}) \left[ (1-\phi) + \phi h(a_{t-1}) \right] \frac{a_{t-1} \overline{A}_{t-1}}{2},$$

where  $\overline{I}_{s,t}$  measures the average income of skilled labor in period t in the cone of diversified region. A decrement in the distance of an economy from the world technology frontier not only decreases the cost of education but also raises the return from education. As a result, the average income of skilled labor increases as an economy moves to the world technology frontier.

<sup>&</sup>lt;sup>7</sup>This finding is also empirically supported by Autor Katz Krueger (1998) and Katz Murphy (1992) for the United States, Machin (1998) for the OECD countries, and Himanshu (2007), Sarkar Mehta (2010) for India.

#### 3.4.4 Existence Possibility of Imitation or Innovation Activities

We now try to find out the private decision of the intermediate input producers to invest in imitation or innovation activity. Since, in our stylized economy, imitation and innovation can substitute perfectly for each other, corner solutions can not be ruled out. The existence of an interior solution requires that  $s_{mt} \leq S_t$ ,  $s_{nt} \leq S_t$ ,  $s_{mt} \geq 0$ , and  $s_{nt} \geq 0$ .

#### Lemma 3

For further technology improvement an economy performs both imitation and innovation activities if and only if

$$\frac{[(1-\phi)-\phi h(a_{t-1})]}{[(1-\phi)+\phi h(a_{t-1})(1+h(a_{t-1}))]} < \tau < \frac{\psi[(1-\phi)-\phi h(a_{t-1})]}{\phi h^2(a_{t-1})+\psi[(1-\phi)+\phi h(a_{t-1})]};$$
(20)

an economy performs only-imitation if and only if

$$\tau \ge \frac{\psi[(1-\phi) - \phi h(a_{t-1})]}{\phi h^2(a_{t-1}) + \psi[(1-\phi) + \phi h(a_{t-1})]};$$
(21)

An economy performs only-innovation if and only if

$$\tau \le \frac{\left[(1-\phi)-\phi h(a_{t-1})\right]}{\left[(1-\phi)+\phi h(a_{t-1})(1+h(a_{t-1}))\right]}.$$
(22)

For the existence of both imitation and innovation activities, condition in (20) is both necessary and sufficient. This condition is also similar to the condition derived by Aghion et. al. (2005), Vandenbussche et. al. (2006), and Di Maria et. al. (2009) for an interior solution. We endogenize labor allocation based on the individual's personal choice and the cost of education determines the optimal composition of human capital. Under **A1**, for any value of a, a relatively skilled (resp. unskilled) labor abundant economy (that is  $\tau$  is low (resp. high)) completely specializes in innovation (resp. imitation) activity. We can also interpret this observation in another way. Given the relative factor endowments of skilled-unskilled labor in a period, a country never opts for innovation (resp. imitation) if it is too far away from (resp. close to) the frontier. Instead, it specializes in imitation (resp. innovation) activity since the catch up effect is high (resp. low), implied by a high (resp. low) value of  $h(a_{t-1})$ .

#### 3.4.5 Existence of Skilled Labor

An individual does not opt for education if benefit from education is lower than the lack of it. Since net income of skilled labor is an increasing function of his/ her cognitive ability, if the individual with maximum cognitive ability is not induced to go for education, then it is not optimal for anyone to opt for education. Therefore, a sufficient condition for no individual opting for education is given by

$$I_{ut}|_{\theta=1} > I_{st}|_{\theta=1}$$

$$\Rightarrow \qquad \frac{(1-\phi) - \phi h(a_{t-1})}{(1-\phi) + \phi h(a_{t-1})} < \tau.$$
(23)

If (23) holds, then there is no skilled labor in the economy. The converse of this condition is necessary for the existence of skilled labor. The condition required for the existence of both imitation and innovation activities ensure the existence of skilled labor in an economy. That is, the reverse of condition (23) is already ensured by condition (22).

#### 3.5 Growth Rate of an Economy under Decentralized Outcome

In this subsection we characterize the growth rate of an economy in period t and its behavior depending on its distance to frontier. Also, we characterize the optimal education policy of an economy depending on its distance to the frontier.

From the technological progress eq. (3), one can define the growth rate of a decentralized economy in period t as

$$g_t = \int_0^1 \frac{A_{it} - A_{t-1}}{A_{t-1}} di.$$

#### 3.5.1 Imitation-only Regime

The growth rate of the economy in imitation-only regime is captured by

$$g_t^M = \int_0^1 \frac{A_{it} - A_{t-1}}{A_{t-1}} di = \lambda U^{*\sigma} S^{*1-\sigma} \left(\frac{1 - a_{t-1}}{a_{t-1}}\right),$$
(24)

where  $g_t^M$  denotes the growth rate of an economy in the imitation-only regime. From eq. (24), we have  $\frac{dg_t^M}{da_{t-1}} < 0$ . Due to diminishing marginal return of the imitation activity, the scope of further technology improvement decreases as an economy progresses. As a result, in the imitation-only regime, the growth rate of an economy decreases as the distance to frontier decreases.

Now, let us try to find out the optimal education policy of an economy which is in the imitation-only regime. Let us do some comparative static analysis. We get

$$\frac{d^2 g_t^M}{d \ a_{t-1} \ d\left(\frac{U^*}{S^*}\right)} = -\lambda \left(\frac{U^*}{S^*}\right)^{\sigma-1} \frac{1}{a_{t-1}^2} \left[\sigma - \frac{U^*}{S^*}\right] > 0.$$

Now,  $w^* > 1$ , implies  $0 < \sigma \frac{S^*}{U^*} < (1 - \sigma) < 1$ .

Under A1, the increment in technology level is higher when the relative endowment of unskilled labor rises. This induces a higher rate technological improvement in the imitation-only regime. That is, unskilled labor is growth enhancing in the imitation-only regime.

#### 3.5.2 Innovation-only Regime

The growth rate of an economy which is in the innovation-only regime can be expressed as:

$$g_t^N = \int_0^1 \frac{A_{it} - A_{t-1}}{A_{t-1}} di = \lambda \bar{U^*}^{\phi} \bar{S^*}^{1-\phi}, \qquad (25)$$

where  $g_t^N$  denotes the growth rate of an economy in the innovation-only regime. As discussed earlier there exists a fixed composition of the human capital in the innovation-only regime. Consequently a marginal increment in the technology level is linearly dependent on the country's own technology level. This induces a constant growth rate in the innovation-only regime. Thus all the economies which are performing only-innovation are growing at the same rate and hence it should be identical to the world technology frontier. We denote it as  $\bar{g}$ . Moreover, in this regime the optimal composition of skilled-unskilled labor and the absolute and relative wage rates of skilled and unskilled labors are identical across all the economies. This again shows that all the economies which are at the innovation-only regime are at the frontier. One can now derive the basis for policies toward promotion of higher education in the innovation-only regime. Under A1, as the relative endowment of skilled labor increases it raises the increment in the technology level and, consequently, the growth rate of the economy. This implies that skilled labor is growth enhancing in the innovation-only regime.

#### 3.5.3 Cone of Diversified Regime

Here we characterize the growth rate of an economy in the cone of diversified region in period t

$$g_t = \lambda \left[ \left( \frac{u_{mt}}{s_{mt}} \right)^\sigma \ s_{mt} \ \frac{(1 - a_{t-1})}{a_{t-1}} + \gamma \ \left( \frac{u_{nt}}{s_{nt}} \right)^\phi \ s_{nt} \right].$$
(26)

The above expression implies that the overall efficiency adjusted growth rate of the decentralized economy, which performs both imitation and innovation activities, is dependent on the factor intensity of unskilled-skilled labor in both imitation and innovation activities (that is,  $\left(\frac{u_{mt}}{s_{mt}}, \text{ and } \frac{u_{nt}}{s_{nt}}\right)$ ), the allocation of skilled labor in the two activities (namely, ( $s_{mt}$ , and  $s_{nt}$ )) and the distance of the economy from the world technology frontier  $\left(\frac{1-a_{t-1}}{a_{t-1}}\right)$ . From eq. (14) and **Lemma 2**, it is clear that both terms in the r.h.s of the above equation are an increasing function of  $a_{t-1}$ . Therefore, the growth rate of an economy in imitation-innovation regime in period t is found to be increasing in the inverse measure of the distance to the frontier. That is,  $\frac{dg_t}{da_{t-1}} > 0$ . This result is quite intuitive given the specification of our technological progress equation.

Next, substituting eq. (14) and (18) into (26), we can express the growth rate of an economy in period t entirely in terms of the parameters and the distance to the frontier.

$$g_t = \lambda \gamma (1 - \phi) h^{-\phi}(a_{t-1}) - \frac{\lambda \gamma \ \phi \ \tau \ h^{1-\phi}(a_{t-1}) [(1 - \phi) - \phi h(a_{t-1})]}{[(1 - \tau)(1 - \phi) - \phi h(a_{t-1})]}.$$
(27)

Let us look at some comparative static results.

$$\frac{dg_t}{d\tau} = -\lambda\gamma\phi h^{1-\phi}(a_{t-1})\frac{[(1-\phi)-\phi h(a_{t-1})]}{[(1-\tau)(1-\phi)-\phi h(a_{t-1})]} < 0$$

From eq. (17) it is clear that the denominator of the second term in the r.h.s of the eq. (27) is positive. Moreover, since  $\tau$  is a fraction, the numerator of the second term in the r.h.s is also positive. Hence, as an economy increases (resp. decreases) its cost of education it reduces (resp. raises) the growth rate of that economy. This is true for any country, whatever may be its distance from the frontier. Also, we have

$$\frac{d^2g_t}{da_{t-1}\partial\tau} = \frac{2\lambda\gamma \ \tau \ \phi^2(1-\phi)h^{2-\phi}(a_{t-1})[(1-\phi)-\phi h(a_{t-1})]}{(\sigma-\phi)a_{t-1}(1-a_{t-1})[(1-\tau)(1-\phi)-\phi h(a_{t-1})]^3} > 0.$$

The intensity of the effect of the higher cost of education can be understood separately for a technologically advanced economy relative to a technologically backward economy. With an increase in the cost of education, the growth rate of the economy is lowered, irrespective of its distance from the world technology frontier. Moreover, this decrease is more pronounced for a relatively technologically advanced economy. That is, the loss in the growth rate of an economy on account of a higher cost of education is more for a technologically advanced economy. Let us look at the intuition behind that. First, consider the case for a relatively less advanced economy. By **Lemma 2**, with an increase in the cost of education in period t, an economy shifts its relatively costly input (that is, skilled labor) in imitation activity. But, by **A1**, skilled labor is more efficient in innovation activity. Hence, the use of a relatively costly input (that is, skilled labor), in an activity where its utilization is relatively less efficient,

cannot be growth maximizing. Now, consider a relatively technologically advanced economy, for which innovation is the main source of technological progress. Here, by increasing the cost of education, an economy not only shifts its costly input (that is, skilled labor) in an activity where it is less productive, but it also shifts its labor force in an activity where its comparative advantage is low. This reduces the increment to the technology gain and, in turn, reduces the growth rate of an advanced economy in period (t + 1) in the decentralized equilibrium.

#### **Proposition 1:**

#### Under A1,

i. Skilled labor is growth maximizing in the innovation-only regime.

ii. Skilled labor is growth enhancing and unskilled labor is growth depressing in the cone of diversified regime. Furthermore, unskilled labor is more growth depressing for the relatively advanced economy.

iii. Unskilled labor is growth maximizing in the imitation-only regime.

Proposition 1 is a contradiction to the findings of Aghion et. al. (2005), Vandenbussche et. al. (2006). Under A1, both these papers show that in the diversified region a marginal increase in total number of unskilled labor is growth enhancing for the relatively technologically backward economy. A key assumption of their work is that the composition of labor force is exogenously specified. They only focus on the gains from higher skilled labor but ignore the costs associated with acquiring skills/ education. Therefore, when a country is farther away from the world technology frontier, shifting skilled labor from innovation to imitation is costless in their model. This is somewhat a very restrictive assumption. We improve upon this by explicitly characterizing the cost of education of an individual and endogenously determining the composition of human capital. Therefore, if an economy shifts its costly input into a relatively less efficient activity, there is a reduction in the growth rate of the economy, irrespective of its distance from the world technology frontier.

Now, let us try to define the path or transition of the growth rate of an economy as it moves to the frontier. To study the behavior of the economy, we need to know the change in the growth rate as it progresses, that is,

$$\frac{dg_t}{da_{t-1}} = \frac{\lambda\gamma(1-\phi)h^{-\phi}(a_{t-1})}{(\sigma-\phi)a_{t-1}(1-a_{t-1})} + \frac{\lambda\gamma\tau\phi(1-\phi)h^{1-\phi}(a_t-1)[(1-\phi)-\phi h(a_{t-1})]}{(\sigma-\phi)a_{t-1}(1-a_{t-1})[(1-\tau)(1-\phi)-\phi h(a_{t-1})]} + \frac{\lambda\gamma\tau^2\phi^2(1-\phi)h^{2-\phi}(a_{t-1})}{(\sigma-\phi)a_{t-1}(1-a_{t-1})[(1-\tau)(1-\phi)-\phi h(a_{t-1})]^2} > 0.$$
(28)

As we have already discussed above and also eq. (28) implies, it is clear that the growth rate of an economy increases as the economy moves to the world technology frontier. Intuitively, from **Lemma 1**, as an economy closes its gap with the world technology frontier it raises the stock of skilled human capital; and from **Proposition 1**, skilled labor is growth enhancing for an economy, whatever may be the distance of the economy from the world technology frontier. This leads to a higher growth rate for the economy. Further, since we consider knowledge based model, an economy innovates upon technology depending on its previously available local/ national technology frontier. This concept is similar to the 'building on the shoulders of the giant'. Knowledge is also high for a technologically advanced economy. This generates a higher growth rate for a technologically advanced economy. This generates a higher growth rate for a technologically advanced economy. This generates a higher growth rate for a technologically advanced economy. This generates an higher growth rate for a technologically advanced economy. This generates an higher growth rate for a technologically advanced economy. This generates an higher growth rate for a technologically advanced economy. This result is contrary to the one Di Maria et. al. (2009). They measure the total number of labor force in terms of their working hour. As a result, there is no change in the total payment to skilled-unskilled human capital as an economy progresses. In their framework, producers are grabbing the entire benefit of higher technology level to skilled labor is relatively low in their work. As a result, the total number of skilled labor is relatively low in their work than ours. **Proposition 1** implies that the increment in technology level and, consequently, the growth rate is relatively low in their research than ours.

#### 3.6 Steady State Analysis for the Decentralized Economy

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Now we try to understand the behavior of the economy in steady state. Is there absolute or conditional convergence? What is the growth rate of an economy in steady state?

The absolute convergence of an economy implies that, in the long run, the economy will be at the frontier. That is,  $(1 - a_{t-1}) \rightarrow 0$  for sufficiently large values of t. If there is conditional convergence then the economy may not reach the world technology frontier, but instead it will converge to some value of a characterized by its own technological and preference parameters. Thus, let us define the growth rate of an economy in terms of its distance to the world technology frontier, which helps us to examine whether the economy will converge to the frontier or to some in between value of a in the long run.

Eq. (27) can be re-expressed as

$$g_{t} = \frac{a_{t}(1+\bar{g}) - a_{t-1}}{a_{t-1}}$$

$$\Rightarrow \qquad a_{t} = \frac{(1+g_{t})}{(1+\bar{g})} a_{t-1}.$$
(29)

Now, if the growth rate of the domestic economy is greater than the growth rate of the world technology frontier for all the future time periods, there will be an absolute convergence of the economy to the world technology frontier. This is the sufficient for absolute convergence to the world technology frontier.

Let us try to define the transition path of the economy as its moves to the frontier. First consider the imitation-only regime. By using eqs. (29) and (24), we get

$$\frac{da_t}{da_{t-1}} = \frac{1}{(1+\bar{g})} \left[ 1 - \lambda U^{*\sigma} S^{*1-\sigma} \right]$$
  
Therefore,  $\frac{da_t}{da_{t-1}} > 0$  if  $\lambda < \frac{1}{U^{*\sigma} S^{*1-\sigma}}$   
 $< 0$  otherwise

Now,  $\frac{da_t}{da_{t-1}} < 0$  implies that as time progresses the gap between the the country's own technology level and world technology frontier increases. This implies, if an economy is technologically very backward and if this is coupled with a very first growth of the world technology frontier, then there exists the possibility of an economy getting into a trap. This finding is opposite to the findings of Di Maria et. al. (2009), where they show the possibility of absolute convergence to the world technology frontier, whatever may be its distance to the world technology frontier. On the other hand, if  $\frac{da_t}{da_{t-1}} > 0$  then the distance of the economy's own technology level and the world technology level decreases and by **Lemma 3**, the economy shifts from the imitation-only regime to the diversified regime.

Now, let us try to characterize the transition path of the economy which performs both imitation and innovation. By using (3) and (28) we get that  $\frac{da_t}{da_{t-1}} > 0$ , for all such economies. Therefore, once an economy is in the imitation-innovation regime it will reduce its gap from the world technology frontier and with certainty in the long run will shift to the innovation-only regime.

As we already discussed above in eq. (25), all the economies which are in the innovation-only regime is at the frontier. Therefore, if  $\lambda < \frac{1}{U^{*\sigma} S^{*1-\sigma}}$ , then in the long run all the economies will converge to the world technology frontier.

Analytically as well, it is proven below that in steady state all the economies will grow at the same rate, and that their growth rate is identical to that for the frontier economy. We know that in steady state  $a_t$  will converge to  $a^*$ , that is,  $a_t \to a^*$  and the growth rate of the economy will converge to the  $g^*$ , that is,  $g_t \to g^*$ . Therefore, in steady state one can express eq. (29) as

$$6a^*(1+\bar{g}) = a^*(1+g^*)$$
  

$$\Rightarrow \text{ either } g^* = \bar{g} \text{ or } a^* = 0.$$

Thus, in the long run, either all the economies will grow at the same rate.

#### Proposition 2

Under A1, and A2,

i. An economy which is performing only-imitation in steady state may or may not converge to the world technology frontier.

ii. An economy which is performing both imitation and innovation activities will converge to the world technology frontier in steady state, and in the long run, all the economies will grow at the same rate.

iii. An economy which is performing only-innovation is at the technology frontier.

#### 3.7 Income Inequality

In this subsection we try to analyze the implications for the level of inequality (in terms of between group and within group inequality), and the changes therein (rise or fall) as an economy progresses towards the frontier.

We define the extent of income inequality between the group of skilled and unskilled labor force by comparing the average income of skilled and that of unskilled labor. Income inequality between skilled and unskilled labor in the imitation-only regime is captured by

$$IN_t^{S/U^M} = \frac{\overline{I}_{st}^M}{\overline{I}_{ut}^M} = \frac{1}{2\sigma} \frac{U^*}{S^*} \left[ 2(1-\tau)(1-\sigma) - \tau\sigma\left(\frac{S^*}{U^*}\right) \left(\frac{1+U^*}{U^*}\right) \right],$$

where  $IN_t^{S/U^M}$  measures income inequality between skilled and unskilled labor in period t for the imitation-only regime. From our previous discussion it is clear that the average income of both skilled and unskilled labor decreases as an economy moves to the frontier. As already discussed above, this decline is the same for both: the income of skilled and unskilled labor, irrespective of the distance of the economy to the frontier. Therefore, there exists a constant income inequality between skilled and unskilled labor as long as the economy is in the imitation-only regime.

$$IN_t^{S/U^N} = \frac{\overline{I_{st}^N}}{\overline{I_{ut}^N}} = \frac{2(1-\tau)(1-\phi) - \tau \ \phi\left(\frac{\bar{S}^*}{\bar{U}^*}\right)\left(\frac{1+\bar{U}^*}{\bar{U}^*}\right)}{2\left(\frac{\bar{S}^*}{\bar{U}^*}\right)},$$

where  $IN_t^{S/U^N}$  measures the income inequality between skilled and unskilled labor in the innovation-only regime. Due to the same logic as in the imitation-only regime, here too there exists a constant level of between group income inequality in the innovation-only regime. For the diversified economy, we have

$$IN_t^{S/U} = \frac{\overline{I}_{st}}{\overline{I}_{ut}} = \frac{(1-\tau)[(1-\phi) + \phi h(a_{t-1})]}{2\phi h(a_{t-1})}$$

where  $IN_t^{S/U}$  is a measure of income inequality between skilled and unskilled labor in period t for the economy which is performing both imitation and innovation. From our earlier analysis, it is clear that the average income of skilled labor increases and that of unskilled labor decreases as an economy moves to the frontier. This leads to a rise in the income inequality between skilled and unskilled human capital as the economy progresses.

Now, let us look at the within group inequality. Due to the heterogeneous income of skilled labor, an economy always faces some degree of income inequality within skilled labor force. Since we assume that an individual's cognitive ability is uniformly distributed over the interval [0, 1], one can define the income inequality within skilled labor by comparing the incomes of skilled labors with highest and lowest cognitive ability. Accordingly, income inequality within skilled labor group in the imitation-only regime is captured by

$$IN_{t}^{SM} = \frac{I_{st}|_{\theta=1}}{I_{st}|_{\theta=\hat{\theta}_{t}}} = \frac{(1-\tau)(1-\sigma) - \tau\sigma\frac{S^{*}}{U^{*}}}{(1-\tau)(1-\sigma) - \tau\sigma\frac{S^{*}}{U^{*}}}$$

where  $IN_t^{S^M}$  measures the income inequality within skilled labor in period t where for technology improvement economy performs only-imitation. Now the question is, in the imitation-only regime, whether the income inequality within skilled labor force increases or decreases in transition to the world technology frontier. Since the total stock of skilled labor remains unchanged in the transition to the world technology frontier, the additional benefit from the higher technology level is same for all skilled laborers. This leads to a constant income gap among skilled labor. Consequently, there exists a constant within group inequality in the imitation-only regime.

Further, we get

$$IN_t^{SN} = \frac{I_{st}^N|_{\theta=1}}{I_{st}^N|_{\theta=\hat{\theta}_t}} = \frac{(1-\tau)(1-\phi) - \tau\phi\left(\frac{\bar{S}^*}{\bar{U}^*}\right)}{(1-\tau)(1-\phi) - \tau\phi\left(\frac{\bar{S}^*}{\bar{U}^{*2}}\right)}$$

where  $IN_t^{S^N}$  measures the income inequality in the innovation-only regime. Again, by the same logic as in the imitation-only regime, there is constant level of inequality among skilled labor in the innovation-only regime.

$$IN_t^S = \frac{I_{st}|_{\theta=1}}{I_{st}|_{\theta=\hat{\theta}_t}} = \frac{(1-\tau)(1-\phi) - \tau\phi h(a_{t-1})}{\phi h(a_{t-1})},$$

where  $IN_t^S$  measures the income inequality within skilled labor in period t in the cone of diversified region. Let us check whether the income inequality within skilled labor force increases or decreases in transition to the world technology frontier. As the technology level increases, the return from education is much higher for individuals with high cognitive ability. From **Lemma 1**, the total stock of skilled labor increases as an economy moves to the world technology frontier. That is, diversity (in terms of cognitive ability) among skilled labor increases. This leads to a higher income gap within skilled labor group. Consequently, the within group inequality rises in the diversified regime.

That is,

$$\frac{dIN_t^S}{da_{t-1}} = \frac{(1-\tau)(1-\phi)}{\phi(\sigma-\phi)a_{t-1}(1-a_{t-1})h(a_{t-1})} > 0.$$

Thus,

#### **Proposition 3**

Under A1 and A2,

i. In the imitation-only regime, wage rate of skilled and unskilled labor decreases as an economy progresses and consequently, average income of skilled and unskilled labor decreases as distance to the frontier decreases.

ii. In the imitation-innovation regime, wage rate and, consequently, the income of skilled labor increases and unskilled labor decreases as an economy moves to the world technology frontier.

iii. In the innovation-only regime, wage rate and the average income of skilled and unskilled labor increases as technology level increases.

iv. In both the imitation-only and the innovation-only regimes there exists a constant income inequality between skilled and unskilled labor and among skilled labor force.

v. Income inequality between skilled and unskilled labor and also income inequality within skilled labor increases as an economy moves to the world technology frontier.

#### 3.8 Consumption Path of the Economy

We now analyze the consumption levels of both skilled and unskilled labor in period t and also the average consumption of the economy in period t. In addition, we characterize the transition path of consumption of both types of labor as an economy closes its gap with the world technology frontier.

As discussed in subsection 2.3, an individual maximizes his/her life time utility subject to the budget constraint to optimally choose the consumption level in both the periods of his/her life.

The average consumption of an economy in period t is the overall average of the mean consumption levels of skilled and unskilled labor in period  $t^8$ . In period t, there are two generations – old and young. As a result, the average consumption in period t is the average of the consumption of four groups of individuals – first, individuals who worked as skilled labor in period (t-1), second, individuals who are working as skilled labor in period t, third, people who worked as unskilled labor in period (t-1), finally, those individuals who are working as unskilled labor in period t. Let us first define the average consumption of skilled labor in period t. This is the average of the mean level of consumption of the individuals who worked as skilled labor in period t. Since individuals' cognitive ability is uniformly distributed over the interval [0, 1], we can define the average consumption of skilled labor in period t in the following way

$$C_{st} = \frac{c_{s,t,t} + c_{s,t-1,t}}{2},$$

where  $C_{st}$  measures the average consumption of skilled labor in period t. From **Proposition 3**, in the imitationonly regime as an economy progresses to the frontier average income of skilled labor decreases, and, consequently, the average consumption of skilled labor decreases. On the other hand, in the imitation-innovation regime and in the innovation-only regime as an economy progresses technologically, the average income of skilled labor increases. This raises the average consumption of skilled labor in period t in both of these regimes.

The average consumption of unskilled labor is also defined in a similar way as for skilled labor. It is the average of the average consumption of individuals who worked as unskilled labor in  $(t-1)^{th}$  period and the individuals

<sup>&</sup>lt;sup>8</sup>In period t, there are two generations; one is old and the other is young. Old generation refers to the individuals who were born in period (t-1), and now enjoying their retirement life, and also consuming in period t. Their consumption is dependent on what they had saved in period (t-1). The new generation refers to the individuals who are born in period t, and either working or acquiring skills. In each generation there are two types of consumers; depending on their working status

who are working as unskilled labor in t<sup>th</sup> period. Since, in any particular period, each unskilled labor earns an equal amount, the average consumption of unskilled labor will be:

$$C_{ut} = \frac{c_{u,t,t} + c_{u,t-1,t}}{2},$$

where  $C_{ut}$  measures the average consumption by unskilled labor in period t. The average consumption of unskilled labor in period t decreases as the economy progresses, due to the decline in the average income of unskilled labor in both type of economies: where economy performs only-imitation and the economies are engaged in both imitation and innovation activities. On the other hand, in innovation-only regime average consumption of unskilled labor increases as technology level increases.

Thus, the average consumption of the economy in period t is defined as the mean of the average consumption of skilled labor and unskilled labor in period t.

That is,

$$C_t = \frac{C_{s,t} + C_{u,t}}{2},$$

where  $C_t$  measures the average consumption of the economy in period t. The average consumption of skilled and unskilled labor decreases in the imitation-only regime as an economy progresses. This leads to a decline in the aggregate consumption of the economy as it moves to the frontier in the imitation-only regime. Now, given the fact that in the diversified regime average consumption of unskilled labor decreases and skilled labor increases as an economy moves to the frontier, the average consumption of the aggregate economy may or may not increase in the imitation-innovation regime. On the other hand, aggregate consumption of the economy increases in innovation-only regime, since the consumption of the both skilled and unskilled labor increases as an economy moves to the frontier.

Now, let us try to define the consumption growth rate of the overall economy in period t. The consumption growth rate of the economy in period t can be defined as  $\frac{C_{t+1}-C_t}{C_t}$ . As discussed earlier, in the imitation-only regime the increment in the technology level (that is, the growth rate of the economy) is decreasing as an economy progresses to the world technology frontier and so does the income and the consumption growth of the aggregate economy. Similarly, technology level is increasing at a constant rate in the innovation-only regime, so do the income and consumption level of the aggregate economy. On the other hand, in the diversified region growth rate of an economy is increasing as an economy closes the gap to the world technology frontier. But since wage rate and consequently income and consumption of skilled labor increases and unskilled labor decreases as an economy progresses, we can not make any conclusion about the overall consumption growth rate of the economy.

#### **3.9** Production of the Economy

From eqs. (1) and (2), it is clear that the production of the intermediate inputs and the monopoly profits of the intermediate input producers rise as an economy progresses to the world technology frontier. This leads to an increase in the final output of the economy as it converges to the world technology frontier.

$$Y_t = l_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^{\alpha} di = \alpha^{\frac{2}{1-\alpha}} A_t.$$

Observe that the final output is a linear function of the aggregate technology level in period t. That is, technology adjusted final output is same in every period.

#### **Proposition 4**

#### Under A1 and A2,

i. In the imitation-only regime as time progresses average consumption of skilled and unskilled labor decreases and, consequently, aggregate consumption of the economy also decreases.

ii. As an economy progresses to the world technology frontier, the average consumption level of skilled labor increases and that of unskilled labor decreases as an economy moves to the frontier in the imitation-innovation regime.

iii. In the innovation-only regime as an economy progresses to the world technology frontier, the average consumption of skilled and unskilled labor and also the aggregate consumption of the economy increases as technology level increases.

iv. As an economy progresses to the world technology frontier, the production of the final output and the intermediate inputs also increases; this increase occurs at the same rate as the growth in the technology level.

## 4 Conclusions

Technological progress is a dual phenomenon. A country can improve its technology level by imitating from the frontier or by innovating new knowledge. An economy which is lagging far behind the world technology frontier can improve its technology level by allocating its labor force mainly into imitation. Similarly, an advanced economy can progress technologically by innovating new knowledge. Under the assumption that different types of labor are efficient in different activities, Vandenbussche et. al. (2006) and Aghion et. al. (2005) show that unskilled labor is the main source of growth for the technologically backward economy and skilled labor is the main source of growth for the technologically advanced economy among the economies which are performing both imitation and innovation activities. By relaxing their assumption of composition of the labor force being exogenously given, and by utilizing an endogenous growth model, we show that skilled labor is the engine of growth in the diversified economy. We also characterize the economies which are performing only-innovation activity and show that unskilled labor is growth enhancing in the imitation-only regime and skilled labor is growth enhancing in the imitation-only regime.

There exists an optimal composition of skilled and unskilled human capital and by implementing that appropriate policy an economy can converge to the world technology frontier if the world technology frontier is not growing at a very faster rate. That is, an optimal education policy is crucial for both technologically developing and developed economies. But, as we have seen, this might raise the income inequality between skilled and unskilled labor and within the group of skilled labor force, since due to skilled biased technological change reward to skilled labor is much higher in a technologically advanced economy. The finding of the possibility of an economy getting into a trap is also different from Di Maria et. al. (2009).

Our work can be extended in several directions. First, we assume that labor is not mobile across countries. If one allows for cross-country labor mobility, then due to higher wage rate in the advanced economy, it is possible that the stock of skilled labor in the backward economy moves to the relatively advanced economy. On account of this brain drain effect, the spending on higher education is a waste for a technologically backward economy. Aghion et. al. (2005) work in this area across US states; but they assume that the composition of human capital is exogenously given, which is a relatively restrictive assumption. So, one can try to examine the possibility of an optimal education policy by endogenizing the individual's schooling decision and also by allowing for mobility of skilled labor within the countries. Second, one can allow the possibility of outsourcing of the R and D activity by a developed economy to a less developed economy. Wage rate of skilled labor is relatively lower and the average cognitive ability of skilled labor is higher in the less developed economy. This increases the profit of the R and D producer in the developed economy and also raises the income of skilled and unskilled labor in the underdeveloped economy. Third, in this entire work we assume that knowledge is freely available to all the economies. One can characterize the growth path and the convergence condition of the economy if we rule out the assumption that world technology level is freely accessible. Fourth, till now all the work in this area has abstracted from international trade. One can develop a dynamic Ricardian model of international trade around the core idea of our work and can study cross-sectoral allocation of skilled and unskilled labor in the context of international specialization. This would certainly yield further insights on the relationship between distance to frontier and composition of human capital and economic growth. Fifth, both Aghion et. al. (2005) and Vandenbussche et. al. (2006) empirically tested this result for the developed economy. One can empirically test this hypothesis for a developed economy like India, and also for an underdeveloped economy such as in Sub-Saharan Africa.

#### Acknowledgements

This is my M.Phil paper under the supervision of Dr. Meeta Keswani Mehra at Centre for International Trade and Development, Jawaharlal Nehru University. I am grateful to Mausumi Das, Ashok Guha, Manas Ranjan Gupta and Debasis Mondal for their comments and suggestions which has helped this paper take its present forms. I am also thankful for the comments received during the presentation of this paper at a seminar in Indian Statistical Institute, Kolkata (August 2011) and Centre for Development Economics, Winter School 2011, Delhi School of Economics (December 2011), XXIst Annual Conference, Economics, Jadavpur University (December 2011), Centre for Economic Studies and Planning Young Scholars' Seminar (March 2012).

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