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# The Panel Conditionally Homogenous Vectorautoregressive Model\*

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## **Abstract**

In the panel conditionally homogenous vectorautoregressive model, the cross-sectional units' dynamics are generally heterogenous, but homogenous if units share the same structural characteristics. The panel conditionally homogenous vectorautoregressive model thus allows (i) to account for heterogeneity in dynamic panel data sets, (ii) to nevertheless exploit the panel nature of the data, and (iii) to analyze the relationship between the units' observed heterogeneities and structural characteristics. I show how standard least squares estimation can be applied, how impulse responses can be computed, how multivariate conditioning is implemented, and how polynomial order restrictions can be incorporated. Finally, I present an easy-to-use Matlab routine which can be used to estimate the panel conditionally homogenous vectorautoregressive model and produce impulse responses as well as forecast error variance decompositions.

*Keywords:* Panel VAR, Heterogeneity, Conditional Pooling.

*JEL-Classification:* C33.

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# 1 Introduction

Countries, regions and industries typically differ along various dimensions. Those differences are important because they affect the effectiveness of policies. Standard panel data models mostly assume cross-sectional homogeneity of the slope parameters which capture the effects of policies and economic mechanisms more generally. If these economic mechanisms differ across cross-sectional units, assuming slope homogeneity may seriously limit the potential of this type of models for policy advice. While it is true that one can control for heterogeneities by including additional regressor variables, many of the heterogeneities that are likely to affect the effects of policies display only rather limited or gradual variation over time. Once fixed effects are included in the panel data model, the role of these heterogeneities for policies cannot be identified separately from the fixed effects anymore. One could still argue that what one is interested in is only the average policy effect across a large number of cross-sectional units, and not the effect on a specific unit. While this might be a reasonable goal in a traditional panel data setting in which the units typically represent individuals and all units are subject to the same policy measures, there are important settings in which this does not appear to be the case. In particular, because in cross-country settings the policies analyzed are mostly country specific, it is of great importance to estimate the policy effects for a particular country. For example, even if there is evidence that the growth effects of trade integration are positive on average across a large sample of countries, this is only of limited help for national policymakers who have to decide whether their country should embrace trade openness or not. These policymakers need to know under what circumstances the effects of trade integration are positive, and why they differ across country settings. A natural way to estimate country-specific policy effects would be to discard the cross-sectional dimension of cross-country data sets and focus only a one country. However, this might be inefficient if the causes for the heterogeneities in the effects of policies and economic mechanisms can be modelled. In this case, the cross-sectional dimension of the data set can be exploited to identify the nature of economic mechanisms, and by that also to obtain country-specific policy effects. In this note, I generalize an empirical panel data framework that was originally proposed by Loayza and Raddatz (2007) and in which units' observed structural characteristics underly the observed heterogeneities in the units' dynamic behavior.<sup>1</sup> This model can be used to investigate the role of structural characteristics for policies and economic mechanisms more generally, while at the same time exploiting the panel structure of the data.

Consider a standard panel vectorautoregressive (VAR) model for cross-sectional units  $i = 1, 2, \dots, N$  with observations  $t = 0, 1, \dots, T$

$$\mathbf{y}_{it} = \mathbf{A} \cdot \mathbf{y}_{i,t-1} + \mathbf{u}_{it}, \quad \mathbf{u}_{it} \stackrel{i.i.d.}{\sim} (\mathbf{0}, \boldsymbol{\Sigma}_u), \quad (1)$$

where  $\mathbf{y}_{it}$  is a  $K \times 1$  vector of endogenous variables and  $\mathbf{A}$  represents a  $K \times K$  coefficient matrix. In this panel VAR framework, the dynamics of the endogenous variables  $\mathbf{y}_{it}$  are fully homogenous

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<sup>1</sup>Other paper that use the approach of Loayza and Raddatz (2007) include Abbritti and Weber (2010), Sa, Towbin and Wieladek (2011) as well as Towbin and Weber (2011).

across cross-sectional units, an assumption mainly maintained in order to gain degrees of freedom in estimation by pooling the data of many cross-sectional units, see Assenmacher-Wesche and Gerlach (2008), Goodhart and Hofmann (2008) and Ciccarelli, Maddaloni and Peydró (2010) for applications. If one admits that due to differences in various dimensions of the environment the cross-sectional units are operating in the dynamics of the endogenous variables are heterogeneous, the panel VAR model in Equation (1) boils down to a familiar cross-section specific VAR model

$$\mathbf{y}_{it} = \mathbf{A}_i \cdot \mathbf{y}_{i,t-1}, \quad \mathbf{u}_{it} \stackrel{i.i.d.}{\sim} (\mathbf{0}, \boldsymbol{\Sigma}_{u,i}), \quad (2)$$

which has been applied in an uncountable number of areas. Of course, as the environment the cross-sectional units are operating in need not be time invariant, a natural generalization is to consider a cross-section specific, time-varying VAR model

$$\mathbf{y}_{it} = \mathbf{A}_{it} \cdot \mathbf{y}_{i,t-1} + \mathbf{u}_{it}, \quad \mathbf{u}_{it} \stackrel{i.i.d.}{\sim} (\mathbf{0}, \boldsymbol{\Sigma}_{u,it}). \quad (3)$$

Bayesian techniques for the estimation of VAR models with time-varying parameters as in Equation (3) are available and have been applied, for example, by Primiceri (2005). Regime-switching VAR models have been introduced by Hamilton (1990).

The polar modeling extremes in Equations (1) and (3) are subject to drawbacks. On the one hand, the pooled panel VAR model in Equation (1) is designed to gain efficiency by exploiting common features in the dynamics of the endogenous variables across cross-sectional units. The model assumes that the dynamics are homogenous across cross-sectional units. While this might not appear to be a restrictive assumption if one interested in average policy effects, this model is clearly of limited value if one is interested in country-specific inference and if one admits that the policy effects may be heterogenous even upon controlling for time-invariant, unit-specific factors in the form of fixed effects. In addition, to the extent that the dynamics are, in fact, not homogenous, the panel VAR model entails inconsistency of the parameter estimates and will not be of help even if one is interested in average policy effects only, see Pesaran and Smith (1995). On the other hand, the cross-section specific, time-varying parameter VAR model in Equation (3) allows for entirely heterogenous and time-varying dynamics, but is unable to achieve any efficiency gain by exploiting common features of the dynamics of the endogenous variables across cross-sectional units. In fact, allowing for time variation in the parameters requires large amounts of data and/or renders estimation computationally non-trivial. In addition, it does not allow to learn about the sources of heterogeneity and time variation.

Approaches to allow for heterogeneity and time variation while at the same time exploiting the panel nature of the data have been put forward: The mean-group estimator of Pesaran and Smith (1995), which estimates the cross-sectional averages of the cross-section specific parameters, and the work on unobserved common factors by Canova and Ciccarelli (2009), which preserve consistency of parameter estimates by shifting heterogeneity and time variation into the unobserved factors. The mean-group and the unobserved factors approaches share a reduced-form treatment

of heterogeneity and time variation in the sense of thinking about non-constancy of parameters in terms of a decomposition of the form

$$\alpha_{it} = \alpha + \epsilon_i + \delta_t. \quad (4)$$

While these approaches certainly do have their merits, it appears that their value for policy advice might be limited. In particular, if policies are cross-section specific, are implemented at a specific point in time and/or if the cross-sectional dimension in macroeconomic applications is small, heterogeneity and time variation cannot be viewed as a nuisance, but are, in fact, of primary importance. Put differently, it is unclear whether knowledge of  $\alpha$  in Equation (4) is of practical relevance for policy advice in country  $i$  and period  $t$ . It would clearly be desirable to learn about the sources of heterogeneity and time variation, that is establish links between heterogeneities, time variation and observed cross-country differences in structural characteristics. In the panel conditionally homogenous VAR (PCHVAR) units' structural characteristics shape heterogeneities in units' observed dynamic behavior.

The remainder of this note is organized as follows: In Section 2, I introduce the PCHVAR framework. In Section 3, I discuss how standard errors of the PCHVAR estimates can be calculated. In Section 4, I augment the PCHVAR model by exogenous variables. In Section 5, I describe multivariate conditioning. In Section 6 I discuss the calculation of impulse responses in the PCHVAR. In Section 7, I show how polynomial order restrictions can be incorporated into the PCHVAR. In Section 8, I present a Matlab routine for the estimation of the PCHVAR model. Finally, Section 9 summarizes.

## 2 The PCHVAR Model

Consider a standard panel VAR model for cross-sections  $i = 1, 2, \dots, N$  with observations  $t = -p + 1, -p + 2, \dots, 0, 1, 2, \dots, T$ .<sup>2</sup> The PCHVAR model is a middle ground between the pooled panel VAR model in Equation (1) and the fully country-specific, time-varying VAR model in Equation (3), and features systematic parameter variation in the coefficient matrices by specifying

$$\mathbf{y}_{it} = \mathbf{A}_1(z_{it}) \cdot \mathbf{y}_{i,t-1} + \mathbf{A}_2(z_{it}) \cdot \mathbf{y}_{i,t-2} + \dots + \mathbf{A}_p(z_{it}) \cdot \mathbf{y}_{i,t-p} + \mathbf{u}_{it}, \quad \mathbf{u}_{it} \stackrel{i.i.d.}{\sim} (\mathbf{0}, \boldsymbol{\Sigma}_u). \quad (5)$$

so that the coefficient matrices vary with the realization of a scalar, exogenous *conditioning variable*  $z_{it}$  that reflects a unit's structural characteristics.<sup>3</sup> The functionals linking the dynamic properties of the model collected in  $\mathbf{A}_j(\cdot)$  to the underlying structural characteristics  $z_{it}$  are of course unknown. In order to operationalize the PCHVAR framework, I assume that each scalar coefficient  $a_{j,sm}(z_{it})$ ,  $j = 1, 2, \dots, p$ ,  $s = 1, 2, \dots, K$ ,  $m = 1, 2, \dots, K$ , in  $\mathbf{A}_j(\cdot)$  can be approximated by a

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<sup>2</sup>For convenience of exposition only, I assume that the time-series dimension is the same across all cross-sections.

<sup>3</sup>In Section 5 I describe the case of a multivariate conditioning variable.

scalar polynomial in the conditioning variable  $z_{it}$ , that is

$$a_{j,sm}(z_{it}) \simeq \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,sm}, \quad (6)$$

where  $\boldsymbol{\pi}(z_{it}) = [\pi_1(z_{it}), \pi_2(z_{it}), \dots, \pi_\tau(z_{it})]$  is a  $1 \times \tau$  vector with polynomials in  $z_{it}$ ,  $\pi_s(z_{it})$ , and  $\boldsymbol{\gamma}_{j,sm} = (\gamma_{j,sm1}, \gamma_{j,sm2}, \dots, \gamma_{j,sm\tau})'$  is a  $\tau \times 1$  vector of polynomial coefficients. With this approximation, observe that  $\mathbf{A}_j(\cdot)$  can be rewritten as

$$\begin{aligned} \mathbf{A}_j(z_{it}) &= \begin{bmatrix} \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,11} & \cdots & \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,1K} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,K1} & \cdots & \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,KK} \end{bmatrix} \\ &= \begin{bmatrix} \boldsymbol{\gamma}'_{j,11} & \boldsymbol{\gamma}'_{j,12} & \cdots & \boldsymbol{\gamma}'_{j,1K} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{\gamma}'_{j,K1} & \boldsymbol{\gamma}'_{j,K2} & \cdots & \boldsymbol{\gamma}'_{j,KK} \end{bmatrix} \cdot [\mathbf{I}_K \otimes \boldsymbol{\pi}'(z_{it})] \\ &= \boldsymbol{\Gamma} \cdot [\mathbf{I}_K \otimes \boldsymbol{\pi}'(z_{it})]. \end{aligned} \quad (7)$$

The model in Equation (5) can therefore be written as

$$\begin{aligned} \mathbf{y}_{it} &= \sum_{j=1}^p \boldsymbol{\Gamma}_j \cdot [\mathbf{I}_K \otimes \boldsymbol{\pi}'(z_{it})] \mathbf{y}_{i,t-j} + \mathbf{u}_{it} \\ &= \sum_{j=1}^p \boldsymbol{\Gamma}_j \cdot \mathbf{x}_{i,t-j} + \mathbf{u}_{it} = \boldsymbol{\Gamma}_j \cdot \mathbf{X}_{i,t-1} + \mathbf{u}_{it}, \end{aligned} \quad (8)$$

with

$$\mathbf{X}_{i,t-1} = (\mathbf{x}'_{i,t-1}, \mathbf{x}'_{i,t-2}, \dots, \mathbf{x}'_{i,t-p})', \quad \boldsymbol{\Gamma} = (\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \dots, \boldsymbol{\Gamma}_p).$$

The model in Equation (8) is a standard multiple equations panel time-series model with the same explanatory variables in every equation. Consequently, standard least squares estimation techniques can be applied. Once the polynomial coefficients  $\boldsymbol{\gamma}_{j,sm}$ ,  $j = 1, 2, \dots, p$ ,  $s, m = 1, 2, \dots, K$ , collected in the matrix  $\boldsymbol{\Gamma}$  have been estimated, the reduced form coefficient matrix  $\mathbf{A}_j(\cdot)$  can be calculated for different values of the conditioning variables  $z_{it}$ .

For the purpose of estimation, it is convenient to rewrite the model in Equation (8) as

$$\mathbf{Y} = \boldsymbol{\Gamma} \cdot \mathbf{X} + \mathbf{U}, \quad (9)$$

where

$$\begin{aligned} \mathbf{Y} &= [\mathbf{y}_{11}, \dots, \mathbf{y}_{1T}, \dots, \mathbf{y}_{N1}, \dots, \mathbf{y}_{NT}]_{(K \times NT)}, \\ \boldsymbol{\Gamma} &= [\boldsymbol{\Gamma}_1, \boldsymbol{\Gamma}_2, \dots, \boldsymbol{\Gamma}_p]_{(K \times pK\tau)}, \\ \mathbf{X} &= [\mathbf{X}_{10}, \mathbf{X}_{11}, \dots, \mathbf{X}_{1,T-1}, \dots, \mathbf{X}_{N0}, \mathbf{X}_{N1}, \dots, \mathbf{X}_{NT}]_{(pK\tau \times NT)}, \end{aligned}$$

and

$$\mathbf{U} = [\mathbf{u}_{11}, \mathbf{u}_{12}, \dots, \mathbf{u}_{1T}, \dots, \mathbf{u}_{N1}, \mathbf{u}_{N2}, \dots, \mathbf{u}_{NT}]_{(K \times NT)}.$$

Vectorization of Equation (9) yields

$$\mathbf{y} = (\mathbf{X}' \otimes \mathbf{I}_K) \cdot \boldsymbol{\gamma} + \mathbf{u}, \quad (10)$$

where  $\mathbf{y} = \text{vec}(\mathbf{Y})$ ,  $\mathbf{u} = \text{vec}(\mathbf{U})$  and  $\boldsymbol{\gamma} = \text{vec}(\boldsymbol{\Gamma})$ . The least squares estimators are given by

$$\hat{\boldsymbol{\Gamma}} = \mathbf{Y} \mathbf{X}' (\mathbf{X} \mathbf{X}')^{-1}, \quad (11)$$

$$\hat{\boldsymbol{\gamma}} = [(\mathbf{X} \mathbf{X}')^{-1} \mathbf{X} \otimes \mathbf{I}_K] \cdot \mathbf{y}. \quad (12)$$

Notice that when allowing for cross-sectional heteroskedasticity,  $\mathbf{u}_{it} \stackrel{i.i.d.}{\sim} (\mathbf{0}, \boldsymbol{\Sigma}_{u,i})$ , generalized least squares (GLS) estimation yields

$$\hat{\boldsymbol{\gamma}} = [(\mathbf{X} \otimes \mathbf{I}_K) \mathbf{S}^{-1} (\mathbf{X}' \otimes \mathbf{I}_K)]^{-1} (\mathbf{X} \otimes \mathbf{I}_K) \mathbf{S}^{-1} \mathbf{y}, \quad (13)$$

where

$$\mathbf{S} \equiv \begin{bmatrix} \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{u,1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{u,2} & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{I}_T \otimes \boldsymbol{\Sigma}_{u,N} \end{bmatrix}. \quad (14)$$

Notice that GLS estimation requires the inversion of a  $(\mathcal{T} \cdot K^2 \cdot p)$ -dimensional square matrix, with  $\mathcal{T}$  being the number of polynomials used for the approximation of the scalar functional  $a_{j,sm}(z_{it})$ . This may entail significant numerical inaccuracies in case of multivariate conditioning and/or high polynomial orders.

### 3 Asymptotic Properties

Adapting the exposition in Lütkepohl (2006, chpt. 3), if  $\mathbf{G} = \text{plim}(\mathbf{X} \mathbf{X}' / NT)$  exists and is non-singular (meaning that if  $\mathbf{X}_{i,t-1}$  is stable and if  $\mathbf{u}_{it}$  is standard white noise so that

$$\begin{aligned} \frac{1}{\sqrt{NT}} \sum_{i=1}^N \sum_{t=1}^T \text{vec}(\mathbf{u}_{it} \mathbf{X}'_{i,t-1}) &= \frac{1}{\sqrt{NT}} \text{vec}(\mathbf{U} \mathbf{X}') \\ &= \frac{1}{\sqrt{NT}} (\mathbf{X} \otimes \mathbf{I}_K) \mathbf{u} \xrightarrow{d} N(\mathbf{0}, \mathbf{G} \otimes \boldsymbol{\Sigma}_u), \end{aligned}$$

then

$$\sqrt{NT} (\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \xrightarrow{d} N(\mathbf{0}, \mathbf{G}^{-1} \otimes \boldsymbol{\Sigma}_u). \quad (15)$$

Noting that

$$\begin{aligned}
\text{vec}[\mathbf{A}_j(z_{it})] &= \text{vec} \left\{ \begin{bmatrix} \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,11} & \dots & \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,1K} \\ \vdots & \ddots & \vdots \\ \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,K1} & \dots & \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,KK} \end{bmatrix} \right\} \\
&= \begin{bmatrix} \boldsymbol{\pi}(z_{it}) & 0 & \dots & 0 \\ 0 & \boldsymbol{\pi}(z_{it}) & & \vdots \\ \vdots & & \ddots & \vdots \\ 0 & \dots & \dots & \boldsymbol{\pi}(z_{it}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\gamma}_{j,11} \\ \boldsymbol{\gamma}_{j,21} \\ \vdots \\ \boldsymbol{\gamma}_{j,KK} \end{bmatrix} \\
&= [\mathbf{I}_{K^2} \otimes \boldsymbol{\pi}(z_{it})] \cdot \boldsymbol{\gamma}_j \\
&= \boldsymbol{\Pi}(z_{it}) \cdot \boldsymbol{\gamma}_j,
\end{aligned} \tag{16}$$

and that

$$\begin{aligned}
\mathbf{A}(z_{it}) &= [\mathbf{A}_1(z_{it}), \mathbf{A}_2(z_{it}), \dots, \mathbf{A}_p(z_{it})] \\
\text{vec}[\mathbf{A}(z_{it})] &= \boldsymbol{\alpha}(z_{it}) = \{\text{vec}[\mathbf{A}_1(z_{it})]', \text{vec}[\mathbf{A}_2(z_{it})]', \dots, \text{vec}[\mathbf{A}_p(z_{it})]'\}' \\
&= [\boldsymbol{\gamma}'_1 \boldsymbol{\Pi}'(z_{it}), \boldsymbol{\gamma}'_2 \boldsymbol{\Pi}'(z_{it}), \dots, \boldsymbol{\gamma}'_p \boldsymbol{\Pi}'(z_{it})]' \\
&= [\mathbf{I}_p \otimes \boldsymbol{\Pi}(z_{it})] \cdot \boldsymbol{\gamma},
\end{aligned} \tag{17}$$

it follows that

$$\begin{aligned}
&\sqrt{NT} [\hat{\boldsymbol{\alpha}}(z_{it}) - \boldsymbol{\alpha}(z_{it})] \\
&\xrightarrow{d} N \{ \mathbf{0}, [\mathbf{I}_p \otimes \boldsymbol{\Pi}(z_{it})] [\mathbf{G}^{-1} \otimes \boldsymbol{\Sigma}_u] [\mathbf{I}_p \otimes \boldsymbol{\Pi}'(z_{it})] \}.
\end{aligned} \tag{18}$$

## 4 The PCHVARX Model

Consider the model in Equation (5) augmented by a  $d \times 1$  vector  $\boldsymbol{\delta}_i$  of deterministic terms, such as fixed effects, cross-section specific time trends or seasonal dummies, the  $M \times 1$  vector of exogenous variables with homogenous effects  $\mathbf{w}_t$ , and the  $L_i \times 1$  vector of exogenous variables with cross-section specific effects  $\mathbf{v}_{it}$ :<sup>4</sup>

$$\begin{aligned}
\mathbf{y}_{it} &= \mathbf{A}_1(z_{it}) \cdot \mathbf{y}_{i,t-1} + \mathbf{A}_2(z_{it}) \cdot \mathbf{y}_{i,t-2} + \dots + \mathbf{A}_p(z_{it}) \cdot \mathbf{y}_{i,t-p} \\
&\quad + \boldsymbol{\delta}_i + \mathbf{B}_0 \cdot \mathbf{w}_t + \dots + \mathbf{B}_q \cdot \mathbf{w}_{t-q} + \mathbf{F}_{i0} \cdot \mathbf{v}_{it} + \dots + \mathbf{F}_{if_i} \cdot \mathbf{v}_{i,t-f_i} + \mathbf{u}_{it}.
\end{aligned} \tag{19}$$

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<sup>4</sup>Of course, also the exogenous variables can be specified to feature dynamics that vary with the units' structural characteristics.



After replacing the coefficient matrices  $\mathbf{A}_j(z_{it})$ ,  $j = 1, 2, \dots, p$  by interactions of the endogenous variables and the polynomials in the conditioning variables, the model can be rewritten compactly as

$$\mathbf{Y} = \mathbf{\Gamma} \cdot \mathbf{X} + [\boldsymbol{\delta}, \mathbf{B}, \mathbf{F}] \cdot \begin{bmatrix} \mathbf{d} \\ \mathbf{W} \\ \mathbf{V} \end{bmatrix} + \mathbf{U}, \quad (20)$$

where  $\mathbf{Y}, \mathbf{\Gamma}, \mathbf{X}$  and  $\mathbf{U}$  are defined as in the previous section, and

$$\boldsymbol{\delta} = [\boldsymbol{\delta}_1, \boldsymbol{\delta}_2, \dots, \boldsymbol{\delta}_N]_{(K \times d)},$$

$$\mathbf{d} = [\mathbf{d}_{11}, \mathbf{d}_{12}, \dots, \mathbf{d}_{1T}, \dots, \mathbf{d}_{N1}, \mathbf{d}_{N2}, \dots, \mathbf{d}_{NT}]_{(d \times NT)},$$

$$\mathbf{B} = [\mathbf{B}_0, \mathbf{B}_2, \dots, \mathbf{B}_q]_{(K \times Mq)}$$

$$\mathbf{W} = [\mathbf{w}, \mathbf{w}, \dots, \mathbf{w}]_{(Mq \times NT)}$$

$$\mathbf{w} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \dots & \mathbf{w}_T \\ \mathbf{w}_0 & \mathbf{w}_1 & \dots & \mathbf{w}_{T-1} \\ \vdots & \vdots & & \vdots \\ \mathbf{w}_{-q+1} & \mathbf{w}_{-q+2} & \dots & \mathbf{w}_{T-q} \end{bmatrix}_{(Mq \times T)}$$

$$\mathbf{F} = [\mathbf{F}_{10}, \dots, \mathbf{F}_{1f_1}, \dots, \mathbf{F}_{N0}, \dots, \mathbf{F}_{Nf_N}]_{(K \times \sum_i f_i L_i)}$$

$$\mathbf{V} = \begin{bmatrix} \mathbf{v}_1 & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{v}_2 & & \vdots \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \dots & \dots & \mathbf{v}_N \end{bmatrix}_{(\sum_i f_i L_i \times NT)}$$

$$\mathbf{v}_i = \begin{bmatrix} \mathbf{v}_{i1} & \mathbf{v}_{i2} & \dots & \mathbf{v}_{iT} \\ \mathbf{v}_{i0} & \mathbf{v}_{i1} & \dots & \mathbf{v}_{iT-1} \\ \vdots & \vdots & & \vdots \\ \mathbf{v}_{i-f_i+1} & \mathbf{v}_{i-f_i+2} & \dots & \mathbf{v}_{iT-f_i} \end{bmatrix}_{(L_i f_i \times T)}$$

The deterministic terms and the exogenous variables (as long as the aim of the analysis is not in investigating dynamic multiplier effects of changes in the exogenous variables) can be concentrated out by right-multiplying Equation (20) by

$$\mathbf{M} = \mathbf{I} - \mathbf{B}'(\mathbf{B}\mathbf{B}')^{-1}\mathbf{B}, \quad \mathbf{B} = [\mathbf{d}', \mathbf{W}', \mathbf{V}']', \quad (21)$$

yielding

$$\mathbf{Y}^* = \mathbf{\Gamma} \cdot \mathbf{X}^* + \mathbf{U}. \quad (22)$$

Estimation from this point on proceeds as described in the previous section.

## 5 Multivariate Conditioning

In case of multivariate conditioning, one can approximate the unknown functionals in  $\mathbf{A}_j(\cdot)$  by a multivariate polynomial  $\boldsymbol{\pi}(\mathbf{z}_{it})$  in the conditioning variable vector  $\mathbf{z}_{it}$ . To this end, in the bivariate conditioning case the multivariate polynomial  $\boldsymbol{\pi}(\mathbf{z}_{it})$  can be obtained as the tensor product of two univariate polynomials in the conditioning variables  $z_{1i,t-1}$  and  $z_{2i,t-1}$ , that is

$$\boldsymbol{\pi}(\mathbf{z}_{it}) \cdot \boldsymbol{\gamma}_{j,sm} = [\boldsymbol{\pi}_1(z_{1i,t-1}) \otimes \boldsymbol{\pi}_2(z_{2i,t-1})] \cdot \boldsymbol{\gamma}_{j,sm}, \quad (23)$$

$$\boldsymbol{\pi}_j(z_{ji,t-1}) = [\pi_j^{(0)}(z_{ji,t-1}), \pi_j^{(1)}(z_{ji,t-1}), \dots, \pi_j^{(\tau_j)}(z_{ji,t-1})], \quad j = 1, 2. \quad (24)$$

Alternatively, in order to conserve on degrees of freedom, one can drop the interaction terms between the conditioning variables and simply use

$$\tilde{\boldsymbol{\pi}}(\mathbf{z}_{it}) \cdot \boldsymbol{\gamma}_{j,sm} = [1, \tilde{\boldsymbol{\pi}}_1(z_{1i,t-1}), \tilde{\boldsymbol{\pi}}_2(z_{2i,t-1})] \cdot \boldsymbol{\gamma}_{j,sm}, \quad (25)$$

$$\tilde{\boldsymbol{\pi}}_j(z_{ji,t-1}) = [\pi_j^{(1)}(z_{ji,t-1}), \dots, \pi_j^{(\tau_j)}(z_{ji,t-1})], \quad j = 1, 2. \quad (26)$$

The order of the polynomials can of course be larger than two, which, however, complicates the exposition of the results and will in almost all practical cases lead to sparsely populated regions in the conditioning space, ensuing highly sensitive and imprecise parameter estimates.

## 6 Impulse Responses

Due to the dependence of the coefficient matrices on the conditioning variable and the dynamic properties of the model, the impulse responses in the PCHVAR are (i) a function of the conditioning variable and (ii) history dependent. To see this, rewrite the model in Equation (5) as

$$\begin{bmatrix} \mathbf{y}_{it} \\ \mathbf{y}_{i,t-1} \\ \vdots \\ \mathbf{y}_{i,t-p+1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1(z_{it}) & \mathbf{A}_2(z_{it}) & \dots & \mathbf{A}_{p-1}(z_{it}) & \mathbf{A}_p(z_{it}) \\ \mathbf{I}_K & \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_K & & \vdots & \mathbf{0} \\ \vdots & & \ddots & \mathbf{0} & \\ \mathbf{0} & \dots & \dots & \mathbf{I}_K & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{y}_{i,t-1} \\ \mathbf{y}_{i,t-2} \\ \vdots \\ \mathbf{y}_{i,t-p} \end{bmatrix} + \begin{bmatrix} \mathbf{u}_{it} \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix}$$

$$\mathbf{Y}_{it} = \mathbf{A}(z_{it}) \cdot \mathbf{Y}_{i,t-1} + \mathbf{U}_{it}. \quad (27)$$

Substituting recursively gives

$$\mathbf{Y}_{it} = \mathbf{A}(z_{it}) \cdot \mathbf{Y}_{i,t-1} + \mathbf{U}_{it} \quad (28)$$

$$= \mathbf{A}(z_{it}) \cdot \mathbf{A}(z_{i,t-2}) \cdot \mathbf{Y}_{i,t-2} + \mathbf{A}(z_{it}) \cdot \mathbf{U}_{i,t-1} + \mathbf{U}_{it}$$

$\vdots$

$$= \left[ \prod_{j=1}^s \mathbf{A}(z_{i,t-j}) \right] \cdot \mathbf{Y}_{i,t-s} + \sum_{j=0}^s \left[ \prod_{k=1}^j \mathbf{A}(z_{i,t-k}) \right] \cdot \mathbf{U}_{i,t-j}. \quad (29)$$

Left-multiplying by

$$\mathbf{J}_K = [\mathbf{I}_K, \mathbf{0}, \dots, \mathbf{0}] \quad (30)$$

yields

$$\begin{aligned} \mathbf{J}_K \cdot \mathbf{Y}_{it} &= \mathbf{J}_K \cdot \left[ \prod_{j=1}^s \mathbf{A}(z_{i,t-j}) \right] \cdot \mathbf{Y}_{i,t-s} + \mathbf{J}_K \cdot \sum_{j=0}^s \left[ \prod_{k=1}^j \mathbf{A}(z_{i,t-k}) \right] \cdot \mathbf{J}'_K \cdot \mathbf{J}_K \cdot \mathbf{U}_{i,t-j} \\ \mathbf{y}_{it} &= \mathbf{Y}_s \cdot \mathbf{Y}_{i,t-s} + \sum_{j=0}^s \mathbf{\Phi}_j(\underline{z}_{i,t-1}^{(j)}) \cdot \mathbf{u}_{i,t-j}, \end{aligned} \quad (31)$$

where

$$\underline{z}_{i,t-1}^{(j)} = [z_{it}, z_{i,t-2}, \dots, z_{i,t-j}]. \quad (32)$$

The impulse responses are therefore obtained from the matrices

$$\mathbf{\Phi}_j(\underline{z}_{i,t-1}^{(j)}) = \mathbf{J}_K \cdot \left[ \prod_{k=1}^j \mathbf{A}(z_{i,t-k}) \right] \cdot \mathbf{J}'_K. \quad (33)$$

In order to analyze the effects on the impulse responses of the structural characteristics under investigation, one can consider impulse responses for a grid of values of the conditioning variable. While this is straightforward for the case of univariate conditioning, it becomes more complicated once one moves to multivariate conditioning. In principle, even in the case of multivariate conditioning one can still investigate the effects of a specific structural characteristic on the impulse responses by fixing the remaining structural characteristics included into the model at specific values, for example the median. In practice, however, it typically turns out that many regions in the conditioning variable space are only sparsely populated. This means that when varying the values of one of the conditioning variables while fixing the values of the remaining ones, one is likely to enter regions in the conditioning variable space which are sparsely populated. In these regions, impulse responses tend to explode, or at least be estimated extremely imprecisely, simply because there are no data points available for identification.<sup>5</sup>

The PCHVAR approach developed so far does not allow to condition the contemporaneous effects on the structural characteristics. In effect, the PCHVAR model presented so far imposes uncon-

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<sup>5</sup>In Georgiadis (2011) I take a different approach and attempt to replicate the impulse responses estimated from unit-specific VAR models by the impulse responses implied by the PCHVAR model with trivariate conditioning.

ditional homogeneity of the contemporaneous effects. As the contemporaneous relationships are intimately linked to the variance matrix of the reduced-form disturbances, doing so would render volatility a function of the structural characteristics as well. This may be an interesting direction for future research.

## 7 Polynomial Order Restrictions

Recall that up to this point it was assumed that in

$$a_{j,sm}(z_{it}) = \boldsymbol{\pi}(z_{it}) \cdot \boldsymbol{\gamma}_{j,sm},$$

where  $\boldsymbol{\gamma}_{j,sm}$  is a  $\tau_{j,sm} \times 1$  vector of polynomial coefficients,  $\tau_{j,sm} = \tau$  for  $j = 1, 2, \dots, p$ ,  $s, m = 1, 2, \dots, K$ . Now suppose that  $\tau_{j,sm}$  generally differs across  $j = 1, 2, \dots, p$ ,  $s, m = 1, 2, \dots, K$ . The only necessary adjustment is to set  $\tau = \max_{j,s,m}(\tau_{j,sm})$  and  $\gamma_{j,sm\ell} = 0$  for  $\ell = \tau_{j,sm} + 1, \dots, \tau$ . With this adjustment,

$$\text{vec}(\boldsymbol{\Gamma}) = \mathbf{R} \cdot \boldsymbol{\gamma}, \quad (34)$$

where  $\mathbf{R}$  represents a suitable selection matrix featuring zero rows for all  $\gamma_{j,sm\ell}$ ,  $\ell = \tau_{j,sm} + 1, \dots, \tau$ , and

$$\mathbf{y} = (\mathbf{X}' \otimes \mathbf{I}_K) \mathbf{R} \cdot \boldsymbol{\gamma} + \mathbf{u}. \quad (35)$$

The least squares and the generalized least squares estimators are given by

$$\hat{\boldsymbol{\gamma}} = [\mathbf{R}'(\mathbf{X}\mathbf{X}' \otimes \mathbf{I}_K)\mathbf{R}]^{-1} \mathbf{R}'(\mathbf{X} \otimes \mathbf{I}_K) \cdot \mathbf{y} \quad (36)$$

$$\hat{\boldsymbol{\gamma}}_{GLS} = [\mathbf{R}'(\mathbf{X}\mathbf{X}' \otimes \boldsymbol{\Sigma}_u^{-1})\mathbf{R}]^{-1} \mathbf{R}'(\mathbf{X} \otimes \boldsymbol{\Sigma}_u^{-1}) \cdot \mathbf{y}, \quad (37)$$

with asymptotic distributions

$$\sqrt{NT}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma}) \xrightarrow{d} N\left\{\mathbf{0}, [\mathbf{R}'(\mathbf{G} \otimes \mathbf{I}_K)\mathbf{R}]^{-1} \mathbf{R}'(\mathbf{G} \otimes \boldsymbol{\Sigma}_u)\mathbf{R} [\mathbf{R}'(\mathbf{G} \otimes \mathbf{I}_K)\mathbf{R}]^{-1}\right\}, \quad (38)$$

$$\sqrt{NT}(\hat{\boldsymbol{\gamma}}_{GLS} - \boldsymbol{\gamma}) \xrightarrow{d} N\left\{\mathbf{0}, [\mathbf{R}'(\mathbf{G} \otimes \boldsymbol{\Sigma}_u^{-1})\mathbf{R}]^{-1}\right\}. \quad (39)$$

Finally, the asymptotic distributions of the reduced form coefficients are given by

$$\begin{aligned} & \sqrt{NT} [\hat{\boldsymbol{\alpha}}(z_{it}) - \boldsymbol{\alpha}(z_{it})] \\ & \xrightarrow{d} N\left\{\mathbf{0}, [\mathbf{I}_p \otimes \boldsymbol{\Pi}(z_{it})] \text{Var}[\sqrt{NT}(\hat{\boldsymbol{\gamma}} - \boldsymbol{\gamma})] [\mathbf{I}_p \otimes \boldsymbol{\Pi}'(z_{it})]\right\}, \end{aligned} \quad (40)$$

and

$$\begin{aligned} & \sqrt{NT} [\hat{\boldsymbol{\alpha}}_{GLS}(z_{it}) - \boldsymbol{\alpha}(z_{it})] \\ & \xrightarrow{d} N\left\{\mathbf{0}, [\mathbf{I}_p \otimes \boldsymbol{\Pi}(z_{it})] \text{Var}[\sqrt{NT}(\hat{\boldsymbol{\gamma}}_{GLS} - \boldsymbol{\gamma})] [\mathbf{I}_p \otimes \boldsymbol{\Pi}'(z_{it})]\right\}. \end{aligned} \quad (41)$$

## 8 A Matlab Code for Estimation of the PCHVAR

While estimation of the PCHVAR model is conceptually straightforward, it is not readily available in standard econometrics software packages. I therefore set up a Matlab code that carries out estimation of the PCHVAR model as well as impulse response analysis.<sup>6</sup> The routine is easy to use, written in a such a way that the user has to modify only one m-file in which she enters the name of the data set, the model specification, details of model estimation and impulse response analysis. In addition, the routine returns the impulse responses, confidence bands, forecast error variance decompositions in the form of matrices, so that the user can present the results in the way that suits her best. In this section, I illustrate how to use this Matlab code using the example of a three-variable (output, prices and a short-term interest rate) PCHVAR model estimated on data for 20 industrialized countries. The impulse responses in this example are specified to vary with (that is, to be conditioned on) countries' industry mix (the share of output accounted for by durable goods manufacturing) and degree of labor market rigidities (strictness of employment protection indicator).

First, the user has to add some utilities to her Matlab library

```
addpath('D:\Research Projects\PCHVAR MATLAB DISTRIBUTION\pchvar_libraries')
```

Second, in the m-file `PCHVARsettings.m` the user has to specify the path, the name and the file type (Excel `.xls` or Stata `.dta`) of the data set to be used<sup>7</sup>

```
crit.filepath = 'D:\Research Projects\PCHVAR MATLAB DISTRIBUTION\';  
crit.filename = 'pchvardataset';  
crit.filetype = 'dta'; % type 'xls' for Excel and 'dta' for Stata data set
```

Then, she has to specify the unit and time-series identifiers (it is best to use three-letter unit identifiers, for example country codes) as well as the data frequency

```
crit.csvar = 'Countrykey';  
crit.tvar = 'year';  
crit.frequency = 'm'; % 'm': monthly data; 'q': quarterly data; 'a': annual data;  
% This is just to label the axis in the data plots, so leave empty '[]' if data  
% are other frequency.
```

Then, the user specifies the sample she would like to investigate. She can select a subset of cross-sectional units from the full sample, restrict the sample period for the entire set of cross-sectional units, or restrict the sample period for individual cross-sectional units.

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<sup>6</sup>The codes are available at [www.jorgo-georgiadis.de](http://www.jorgo-georgiadis.de).

<sup>7</sup>Notice that results may differ slightly depending on whether a Stata or Excel data set are used because of different treatments of floating number precision.

```

crit.useunit    = [];           % '[]': Use all cross-sections in the data set
                                % {'AUS','AUT'}: use only Australia and Austria

crit.useperiod  = [];           % '[]': Use all available periods. If not all
                                % time periods shall be used, you need to
                                % specify 'crit.useperiod = [startdate,enddate]'.
                                % Notice that in this case, you can only select
                                % years; For example, if you have monthly data running
                                % from 1995 to 2009, you can only select to use all
                                % months from 1996 to 2008, but not to use data from
                                % 1996:7 to 2008:10.

crit.usecountryperiod = []; % You can specify to use only specific periods for
                                % specific cross-sectional units; for example, if you
                                % want to use only data for 1997 to 2008 for CAN, then
                                % specify 'crit.usecountryperiod.can = [1997,2008].
                                % You must specify the start and end period, and you
                                % can only select full years, see above. If you do not
                                % want to use this option, just specify
                                % 'crit.usecountryperiod = []'. Notice also that the
                                % general sample selection period above has priority
                                % over the cross section-specific sample period
                                % selection.

```

After that, the user enters the model specification including

- the endogenous variables (as a  $1 \times K$  cell array):

```
crit.yvar = {'logy','logp','r'};
```

- the conditioning variables (as a  $1 \times K^z$  cell array):

```
crit.zvar = {'soep','manufshare'};
```

- the common exogenous variables (as a  $1 \times M$  cell array):

```
crit.exogxvar = {'lpcomm'};
```

- the exogenous variables variables specific to each cross section (as a  $1 \times L_i$  cell array each, as a structure field for each cross section where the field name is the countrykey):

```
crit.exogxvar_i.can = {'fx'};
crit.exogxvar_i.gbr = {'fx'};
```

- the lag order of the endogenous variables:

```
crit.p          = 3;
```

- the lag orders of the common and the cross section-specific exogenous variables:

```
crit.exogxvar_q      = 0;
crit.exogxvar_qi.can = 0;
crit.exogxvar_qi.gbr = 0;
```

- the deterministic specification:

```
crit.determin = 1; % 0: neither intercept nor trend
                % 1: fixed effects
                % 2: fixed effects and heterogeneous time trends
```

- augmentation of common factors along the lines of Pesaran (2006):

```
crit.ccea = 0; % 0: no augmentation
            % 1: augmentation with cross-sectional averages
```

Then, still in the m-file PCHVARsettings.m, the user specifies several options for model estimation such as

- whether to use GLS estimation

```
crit.pchvargls = 0; % 1: cross-section specific variance matrices
```

- whether to use the tensor product or just individual polynomials in case of multivariate conditioning

```
crit.polyntype = 2; % 1: tensor product between polynomials;
                % 2: no interactions between individual z_j's
```

- the polynomial orders for the approximation of each autoregressive coefficient

```
crit.tao      = 2;
crit.Aorder = repmat(ones(max(size(crit.yvar)))*crit.tao, [1,crit.p,max(size(crit.zvar))]);
% row of Aorder:      equation for endogenous variable;
% column of Aorder:   lag;
% 3-rd dimension of Aorder: conditioning variable;
```

Finally, she chooses some options for impulse response analysis such as

- the number of grids over which to draw impulse responses for alternative values of the conditioning variables

```
crit.gridnum = 60;
```

- the impulse response horizon

```
crit.irhor = 48;
```

- whether to compute (asymptotic) confidence bands

```
crit.confbands = 0; % 0: do not obtain confidence bands
                % 1: get them
```

- the identification scheme

```
crit.irtype = 1; % 0: naive ir's
              % 1: orthogonalized ir's
              % 2: generalized ir's
```

- the shock size

```
crit.shocksize = 1; % 0: one standard deviation shock
                 % 1: unit shocks
```

- the horizons over which to compute forecast error variance decompositions

```
crit.fevdhor = [1,6,12,18,24,30,36,42,48]; % must be a 1 by X vector,
                                                    % leave '[]' if do not want to investigate FEVD
```

In order to obtain estimation results and impulse responses, the user has to run

```
crit = PCHVARsettings;
```

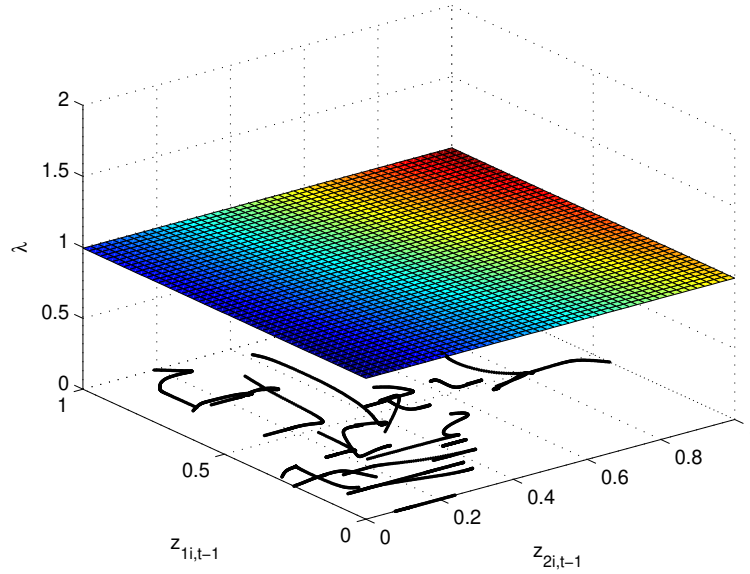
```
[IR,PCHVARres] = PCHVAR(crit);
```

Upon execution, the user is first asked whether she would like to plot the data. This is an important first step in the analysis to ensure that the data set has been set up properly. After estimation, a figure displaying the largest eigenvalue of the PCHVAR model at all possible combinations of the conditioning variables is produced, see Figure 1. In case there are two conditioning variables (in our example we have industry mix and labor market rigidities), the surface in Figure 1 depicts the magnitude of the largest eigenvalue of the companion matrix in the PCHVAR model for different values of the first conditioning variable on the left-hand side horizontal axis and the second conditioning variable on the right-hand side horizontal axis. Each dot at the bottom of Figure 1 represents an actual observation on a conditioning variable pair in the data set.

The user is also provided with two structures containing the following fields with results:



Figure 1: The Largest Eigenvalue of the Companion Matrix in the PCHVAR Model at Different Combinations of the Conditioning Variables



```
PCHVARres =
    Ghat: [3x27 double]
    Ghattvals: [3x27 double]
    Sigmahat: [1x1 struct]
    crit: [1x1 struct]
    z: [2x3412 double]
    K: 3
```

```
IR =
    zgrid: [60x2 double]
    point: [4-D double]
    accum: [4-D double]
    fevd: [5-D double]
    upper: [4-D double]
    lower: [4-D double]
    upper_accum: [4-D double]
    lower_accum: [4-D double]
```

PCHVARres.Ghat is the estimate of the matrix  $\Gamma$  in Equation (9) and PCHVARres.Ghattvals contains the corresponding  $t$ -values. PCHVARres.Sigmahat contains the estimated variance-covariance matrices of the matrix  $\hat{\Gamma}$  (PCHVARres.Sigmahat.Ghat), the reduced-form errors  $\mathbf{u}_t$  (PCHVARres.Sigmahat.u)

and its maximum likelihood version (`PCHVARres.Sigmahat.uml`), the estimates of the unit-specific variance-covariance matrices of the reduced-form errors (`PCHVARres.Sigmahat.Sigmahathetero`), the variance-covariance matrix of the estimate of the variance-covariance matrix of the reduced-form errors (`PCHVARres.Sigmahat.sigma`). `PCHVARres.z` contains the data on the conditioning variables and `PCHVARres.K` is the number of endogenous variables  $K$ . `IR.zgrid` is a `crit.gridnum`  $\times$   $K^z$  matrix containing the grid of the conditioning variables over which the impulse responses in `IR.point` are computed with  $K^z$  being the number of conditioning variables considered. `IR.point` contains the point estimates of the impulse responses over the grid `IR.zgrid` and `IR.upper` the corresponding upper confidence band. `IR.point` is of dimension  $K^2 \times \text{crit.irhor} \times \text{crit.gridnum} \times \text{crit.gridnum}$ . So for example, in a three-dimensional system the impulse response over all horizons of the first variable to a shock in the third variable arising at the value of the first (second) conditioning variable assumed at the fifth (ninth) grid point in `IR.zgrid` is accessed by typing `IR.point(7, :, 5, 9)`. `IR.accum` contains the accumulated impulse responses. Finally, `IR.fevd` is a  $X \times K \times K \times \text{crit.gridnum} \times \text{crit.gridnum}$  matrix of forecast error variance decompositions, where  $X$  is the number of horizons over which the decompositions shall be calculated as specified in `crit.fevdhor`. For example, the contribution of the second endogenous variable to the variance of the forecast error variance of the first endogenous variable at the third horizon specified in `crit.fevdhor` arising at the value of the first (second) conditioning variable assumed at the fifth (ninth) grid point in `IR.zgrid` is accessed by typing `IR.fevd(3, 2, 1, 5, 9)`.

The impulse responses for our example with output, prices and a short-term interest rate as endogenous variables and industry mix as well as labor market rigidities as conditioning variables are shown in Figures 2 to 3<sup>8</sup>. Since in this example the impulse responses vary both with a country's degree of labor market rigidity and industry mix, one of the two conditioning variables has to be fixed in order to draw the impulse responses over a grid of the other conditioning variable. Figure 2 shows the impulse responses for different degrees of labor market rigidity (right-hand side horizontal axis) while fixing industry mix at the 10% (50%, 90%) percentile. Conversely, Figure 3 shows the impulse responses for different values of industry mix (right-hand side horizontal axis) while fixing the degree of labor market rigidity at the 10% (50%, 90%) percentile. Finally, Figure 4 displays the impulse response of output to a contractionary monetary policy shock arising at the value of the first (second) conditioning variable assumed at the fifth (ninth) grid point in `IR.zgrid`, that is `IR.point(7, :, 5, 9)`, together with 95% confidence bands. The confidence bands are rather tight because of the large number of observations available for estimation in this example with  $T = 180$  and  $N = 20$ .

The results indicate that the output and the price level responses to a monetary policy shock are stronger the more rigid a country's labor markets. Moreover, the results also suggest that output responds faster to a monetary policy shock the more important durable goods manufacturing. These results reveal that there is a large amount of heterogeneity that would invalidate the use of

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<sup>8</sup>Notice that Figures 2 to 3 are not returned in this form from the PCHVAR routine. The figures produced by the PCHVAR routine are, however, very similar. The code for producing Figures 2 to 3 is given in `MAIN_PCHVAR_SAMPLE_FILE.m`

a traditional fixed effects panel VAR model, which would imply identical impulse responses for all the countries included, see Figure 5. The fixed effects panel data model can be estimated simply by running

```
crit = PCHVARsettings;
crit.tao      = 1; % interactions with polynomials of order zero
crit.Aorder  = repmat(ones(max(size(crit.yvar)))*crit.tao,[1,crit.p]);
crit.zvar    = {' '};
[IR,PCHVARres] = PCHVAR(crit);
```

One can also inspect the forecast error variance decompositions in Figure 6. For the sake of brevity, I only present the forecast error variance decompositions varying along the degree of labor market rigidities at the mean of the share of durable goods manufacturing and vice versa.

The code can easily be adjusted to work with other data sets. Only three important restrictions apply:

1. The data set must be a Stata `.dta`<sup>9</sup> or an Excel `.xls` data set. The data set in Stata need not be `xtset`.
2. The data set must be balanced, that is, the first and the last entry in the Stata/Excel spreadsheet must be for the same date for all countries. However, notice that this does not imply that there must be an observation in this entry; rather, it can be a missing observation. Therefore, unbalanced panels can be investigated.
3. Within a country data must not feature breaks at which data are unavailable.

## 9 Summary

This note lays out the PCHVAR model framework in which the dynamics of the cross-sectional units are generally heterogenous, but homogenous if units share the same structural characteristics. The PCHVAR model thus allows (i) to account for heterogeneity, (ii) to nevertheless exploit the panel nature of the data, and (iii) to analyze the relationship between the units' observed heterogeneities and structural characteristics. These features appear to be all the more important as empirical cross-country applications typically focus on a relatively small number of countries, and analyze policies that are country specific. In such settings, it is important to be able to foresee under which circumstances and why policy effects potentially differ across units. I also present an easy-to-use Matlab routine that can be used to estimate the PCHVAR model and produce impulse responses as well as forecast error variance decompositions for the endogenous variables.

---

<sup>9</sup>I am indebted to Daniel Schneider for making available to me his routine for inputting Stata data sets into Matlab.

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Figure 2: Impulse Responses of the Output/Prices/Interest Rate PCHVAR Model - Varying Labor Market Rigidities...

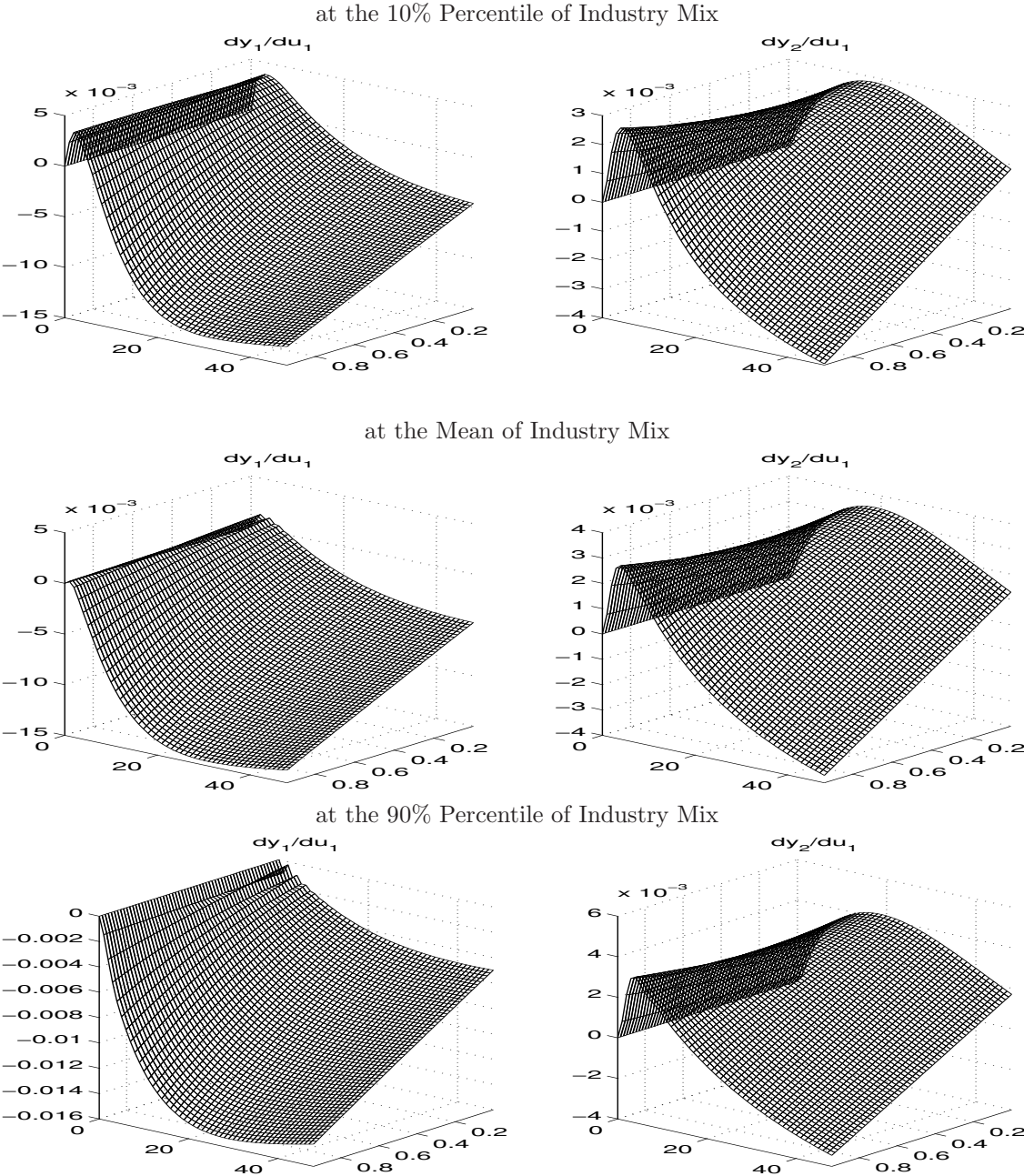
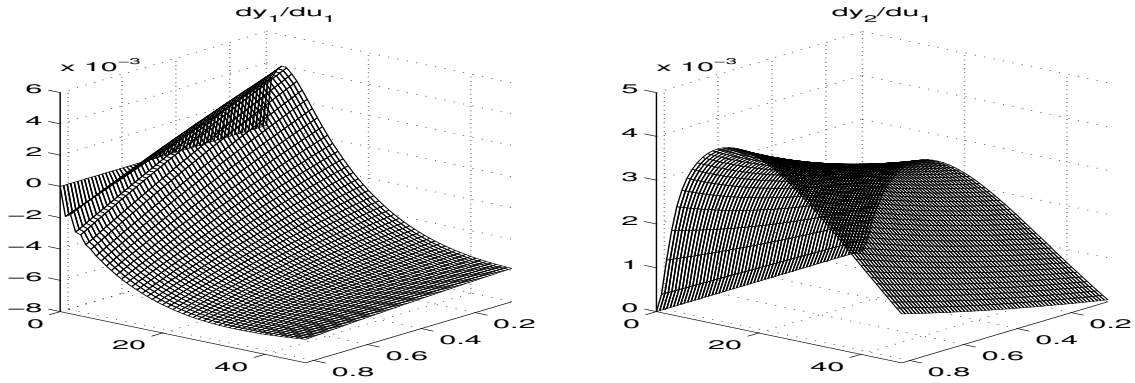
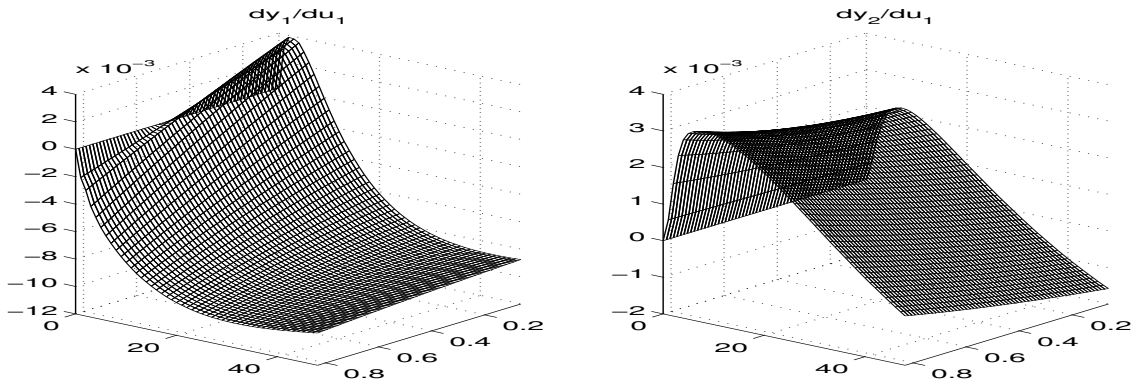


Figure 3: Impulse Responses of the Output/Prices/Interest Rate PCHVAR Model - Varying Industry Mix...

at the 10% Percentile of Labor Market Rigidities



at the Mean of Labor Market Rigidities



at the 90% Percentile of Labor Market Rigidities

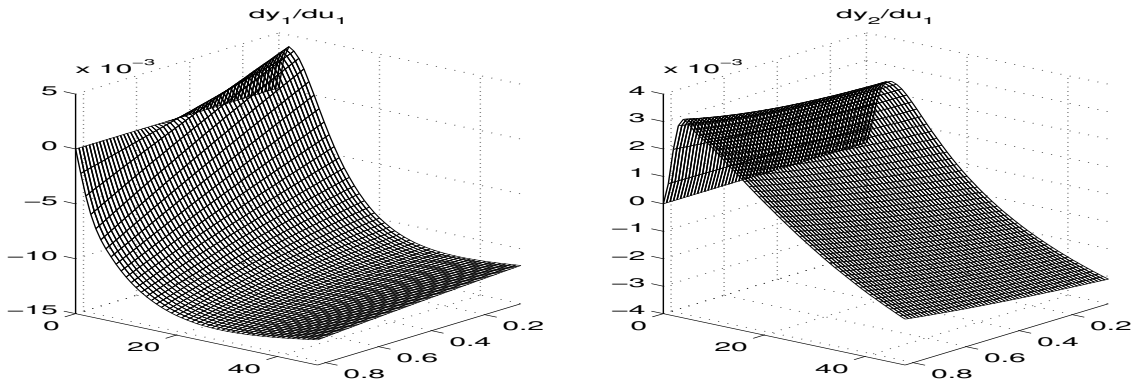


Figure 4: Impulse Response of Output to a Contractionary Monetary Policy Shock at low Levels of Industry Mix and Labor Market Rigidities

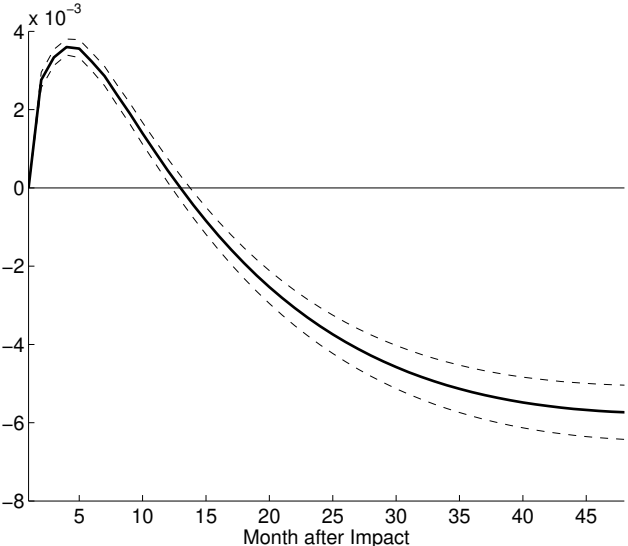


Figure 5: Impulse Responses of the Output/Prices/Interest Rate Fixed Effects Panel VAR Model

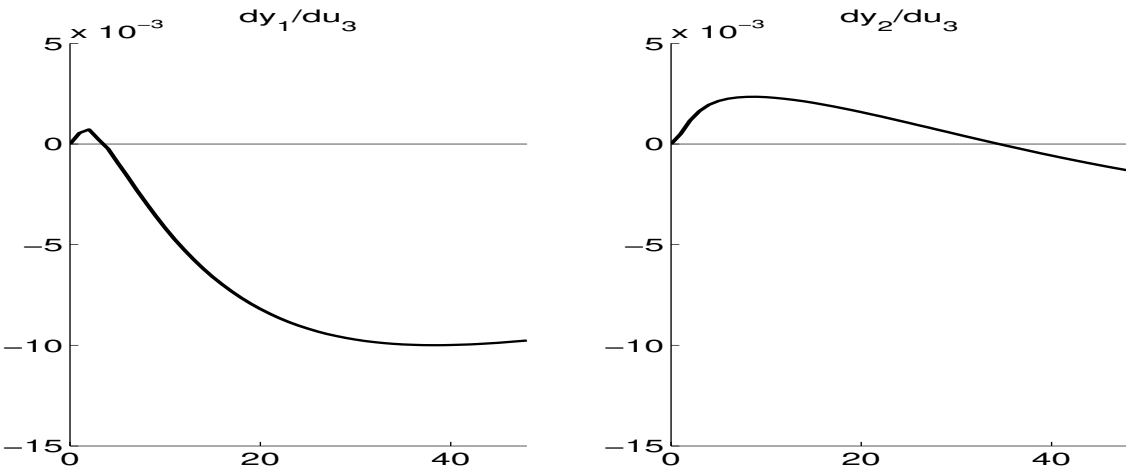




Figure 6: Forecast Error Variance Decompositions

