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Natural Disasters in a Two-Sector Model of Endogenous Growth*

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Abstract

Using an endogenous growth model with physical and human capital accumulation, this paper considers the sustainability of economic growth when the use of a polluting input (e.g., fossil fuels) intensifies the risk of capital destruction through natural disasters. We find that growth is sustainable only if the tax rate on the polluting input increases over time. The long-term rate of economic growth follows an inverted V-shaped curve relative to the growth rate of the environmental tax, and it is maximized by the least aggressive tax policy of those that asymptotically eliminate the use of polluting inputs. Unavailability of insurance can accelerate or decelerate the growth-maximizing speed of the tax increase depending on the relative significance of the risk premium and precautionary savings effects. Welfare is maximized under a milder environmental tax policy, especially when the pollutants accumulate gradually.

Keywords: human capital, global warming, environmental tax, nonbalanced growth path, precautionary saving

JEL Classification Codes: O41, H23, Q54

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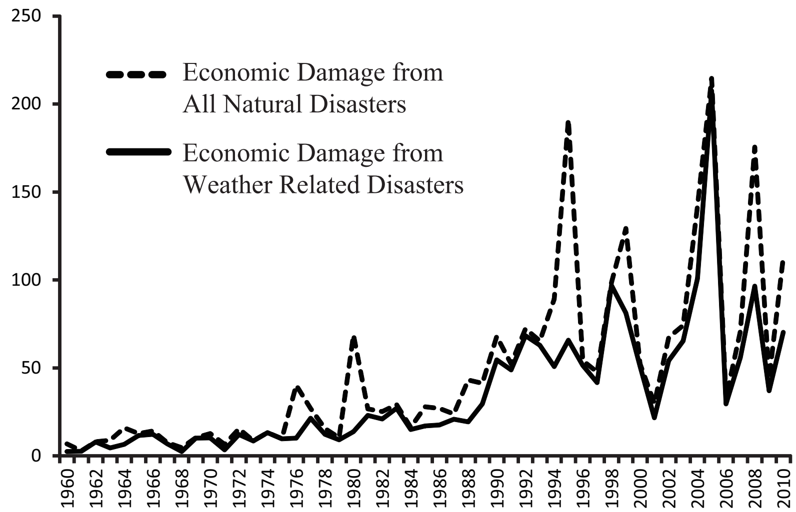

Figure 1: Economic Damage from Natural Disasters Worldwide (in billions of 2005 US dollars). The dashed line indicates the sum of damage from storms, droughts, extreme temperatures, floods, mass movements because of climate change, and wildfires. Source: Damage estimates in current US dollars are from EM-DAT, the International Disaster Database, CRED, the Université Catholique de Louvain. Present value estimates in 2005 US dollars are calculated using the implicit GDP price deflator from the Bureau of Economic Analysis.

1 Introduction

Natural disasters have a substantial impact on the economy, primarily through the destruction of capital stock. For example, Burton and Hicks (2005) estimated that Hurricane Katrina in August 2005 generated commercial structure damage of $21 billion, commercial equipment damage of $36 billion, and residential structure and content damage of almost $75 billion. These are not negligible values, even relative to the entire U.S. physical capital stock.[1] CRED (2012) reported that the floods in Thailand from August to December 2011 caused US$40 billion in economic damage, which is more than 12% of the nation's GDP. Figure 1 depicts the time series of the total economic damage caused by natural disasters throughout the world. Although the magnitude of damage caused by Hurricane Katrina

[1] In another study of the estimated costs of Hurricane Katrina, King (2005) reported that total economic losses, including insured and uninsured property and flood damage, were expected to exceed $200 billion. See Gaddis *et al.* (2007) for the full cost estimates.

may not appear typical, the figure clearly shows a steady and significant upward trend in economic damage arising from natural disasters.

One obvious reason behind this upward trend is the expansion of the world economy. As the world economy expands, it accumulates more capital, which means that it has more to lose from a natural disaster of a given physical intensity. However, this simple account cannot fully explain the overall growing trend in damages. To see this, we plot the ratio of the damage from natural disasters to world GDP in Figure 2. As shown, this ratio has been increasing since 1960. On this basis, the figure suggests that each unit of installed capital is facing an increasingly higher risk of damage and loss from natural disasters over time. This observation may then have serious implications for the sustainability of economic growth. Also, observe from Figures 1 and 2 that most economic damage is caused by weather-related disasters. Accordingly, if economic activity is to some extent responsible for climate change, and if climate change affects the intensity and frequency of weather-related disasters,[2] economic growth itself poses a threat to capital accumulation and the sustainability of future growth.

This paper theoretically examines the long-term consequences of the risk of natural disasters on economic growth in a setting where economic activity itself can intensify the risk of natural disasters. We introduce polluting inputs, such as fossil fuels, into a Uzawa–Lucas type endogenous growth model, and assume that the use of polluting inputs raises the probability that capital stocks are destroyed by natural disasters. In the model, we show that as long as the cost of using polluting inputs is constant, economic growth is not sustainable because the risk of natural disasters eventually rises to the point at which

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[2]There is an ongoing scientific debate about the extent to which natural disasters and global warming relate to human activity. The Intergovernmental Panel on Climate Change Fourth Assessment Report (IPCC 2007, p.6) notes, "Anthropogenic warming over the last three decades has likely had a discernible influence at the global scale on observed changes in many physical and biological systems." Emanuel (2005) found that the destructiveness of tropical cyclones is highly correlated with tropical sea surface temperature and predicted a substantial increase in hurricane-related losses in the future. Min *et al.* (2011) provided evidence that human-induced increases in greenhouse gases have contributed to the observed intensification of heavy precipitation events. There are also other explanations; e.g., Pielke *et al.* (2008) suggested that the increasing density of the population and property in coastal areas accounts for the trend of increasing hurricane damage in the U.S. We simply assume causality between the emission of greenhouse gases and the frequency of natural disasters. Scientific examination of the validity of this causality is beyond the scope of this paper.

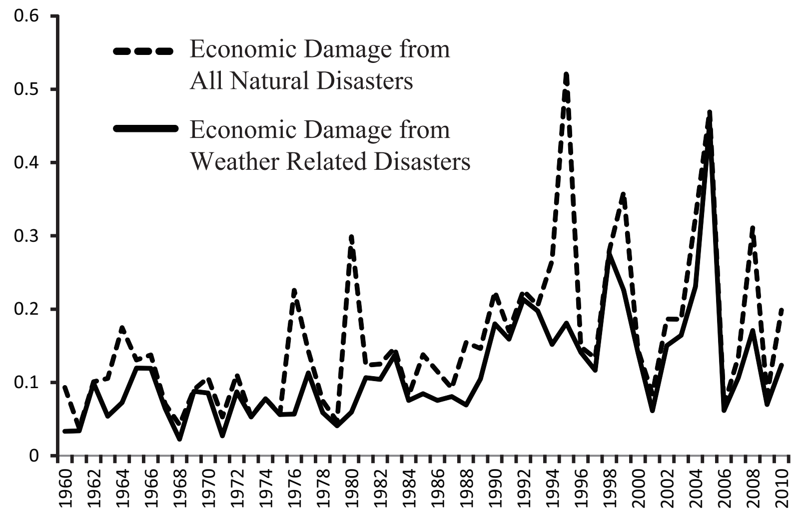

Figure 2: Ratio of Damage from Natural Disasters to World GDP (percent). Data source: World GDP (in current US dollars) is from World Development Indicators, World Bank Data Group.

agents do not want to invest in capital any further.

Given this result, we introduce a time-varying environmental tax on polluting input, which is shown to have both positive and negative effects on economic growth. On one hand, the faster the environmental tax rate increases, the lower the asymptotic amount of pollution and, therefore, the lower the probability of disasters. This gives households a greater incentive to save, which promotes growth.[3] On the other hand, the increased cost of using the polluting input by private firms reduces their (effective) productivity at each point in time, and this has a negative effect on growth. This paper shows that these opposing effects give rise to a non-monotonic relationship between the long-term rate of economic growth and the speed with which the environmental tax increases. We characterize the policy that maximizes the long-term growth rate and examine how it differs from the welfare-maximizing policy. We also examine how the market equilibrium and the optimal policy are affected by the way in which pollutants accumulate and by the extent to which disaster damages can be insured.

[3]In endogenous growth models of the Lucas (1988) type, increased savings and investments (which include the opportunity cost of education) promote growth primarily through faster human capital accumulation. This result depends on the assumption that the marginal productivity of human capital in the education sector is constant.

Relationship to the literature

The literature on the link between natural disasters and economic growth is relatively new. However, an increasing amount of work investigates both the theoretical and empirical relations between these events. There are mixed empirical results regarding whether natural disasters inhibit or promote growth. Empirical studies that use short-run data tend to find adverse effects of natural disasters on growth. Raddatz (2007) considered a vector autoregressive (VAR) model for low-income countries with various external shocks, including climatic disasters, and his estimates showed that climatic and humanitarian disasters result in declines in real per capita GDP of 2% and 4%, respectively. Using panel data for 109 countries, Noy (2009) found that more significant natural disasters in terms of direct damage to the capital stock lead to more pronounced slowdowns in production. In contrast, using cross-sectional data over a longer period of 1960–90, Skidmore and Toya (2002) found a positive correlation between the frequency of disasters and average growth rates. Although there is no general agreement on the overall effect of natural disasters on growth, the estimation performed by Skidmore and Toya (2002) suggested that the higher frequency of climatic disasters leads to a substitution from physical capital investment toward human capital. Consistent with this finding, our model shows that under appropriate environmental policies, agents accumulate human capital stock much faster than output and physical capital, enabling sustained growth with limited use of the polluting input.

The theoretical literature is still in its infancy.[4] For instance, Soretz (2007) explicitly introduced the risk of disasters into an AK-type one-sector stochastic endogenous growth model and considered optimal pollution taxation. Hallegatte and Dumas (2009) considered a vintage capital model and showed that under plausible parameter ranges, disasters never promote economic growth through the accelerated replacement of old capital. Lastly, using numerical simulations, Narita, Tol, and Anthoff (2009) quantitatively calculated the direct economic impact of tropical cyclones. Our analysis complements these studies by considering both human and physical capital accumulation in addition to the polluting

[4]Although not directly concerned with disasters, some previous studies have analytically examined the effect of environmental quality on economic growth. Bovenberg and Smulders (1995) and Groth and Schou (2007), for example, considered models where environmental quality affects productivity. Alternatively, Forster (1973), Gradus and Smulders (1993), John and Pecchenino (1994), Stokey (1998), and Hartman and Kwon (2005) introduced the disutility of pollution into endogenous growth models.

input. This is an important extension, not only because the substitution to human capital accumulation in the presence of disaster risk is empirically supported, but also because theoretically it is the key to sustained and desirable growth.[5] In addition, our methodology can analytically clarify the mutual causality between economic growth and the risk of natural disasters and how this relationship can be altered by environmental tax policy.[6] Rather than merely considering the optimal tax policy, we consider arbitrary dynamic tax policies and find both welfare-maximizing and growth-maximizing policies.

The rest of the paper is organized as follows. After presenting the baseline model in Section 2, Section 3 shows that in market equilibrium, growth cannot be sustained if the cost of (tax on) the polluting input is constant. We then derive the (asymptotically) balanced growth equilibrium path under a time-varying environmental tax in Section 4. The welfare analysis is in Section 5. Section 6 considers an extension of the model in which pollution accumulates gradually. Section 7 examines the case where the idiosyncratic risks to human capital cannot be insured. Section 8 concludes. The Appendix contains mathematical proofs and derivations.

2 The Baseline Model

Consider an Uzawa–Lucas growth model where the economy is populated by a unit mass of infinitely lived households $i \in [0, 1]$ holding human capital h_{it} and savings in the form of financial assets, s_{it}.[7] Production is performed by a unit mass of competitive firms $j \in [0, 1]$ with a homogenous production technology. One difference between our model and that of Lucas (1988) is that production at firm j requires not only physical capital k_{jt} and human capital n_{jt}, but also a polluting input p_{jt}, such as fossil fuels that emit pollutants and

[5] Using a growth model with pollution and physical capital, Stokey (1998) showed that sustained growth is not desirable even when it is technically feasible. However, Hartman and Kwon (2005) found that Stokey's (1998) result is overturned when human capital is introduced.

[6] Narita, Tol, and Anthoff (2009) assume that the savings rate is exogenous, while in our model it reacts endogenously to the risk of disasters. In Hallegatte and Dumas (2009), the long-term rate of growth is ultimately determined by the exogenous growth in total factor productivity (TFP), while in our model it is determined by endogenous human and physical accumulation.

[7] For compact notation, we employ subscript $_t$ rather than (t), even though time is continuous. We also omit 0 and 1 from the integrals $\int_0^1 \ldots di$ and $\int_0^1 \ldots dj$ when they are obvious.

greenhouse gases. Specifically, the output of firm j is

$$y_{jt} = Ak_{jt}^{\alpha} n_{jt}^{1-\alpha-\beta} p_{jt}^{\beta}, \tag{1}$$

where A is a productivity parameter of the production sector, $\alpha \in (0,1)$ represents the share of physical capital, and $\beta \in (0, 1-\alpha)$ is the share of the polluting input. All output is either consumed or added to the physical capital stock.

For simplicity, we consider neither resource limits nor extraction and/or production costs of the polluting input p_{jt}.[8] Rather, we focus on the possibility that the aggregate use of the polluting input $P_t \equiv \int p_{jt} dj$ increases the risk of natural disasters. Suppose that the economy consists of a continuum of small local areas, and both firms and households are dispersed across areas. In each area, natural disasters occur in a Poisson process. In this baseline model, we consider the simplest scenario where the use of the polluting input immediately increases the arrival rate per unit of time (the Poisson probability) such that $q_t = \bar{q} + \hat{q}P_t$, where $\bar{q}$ and $\hat{q}$ are positive constants. We will relax this assumption and consider accumulating pollution in Section 6. When a natural disaster occurs in an area, it causes damage to both physical and human capital. Specifically, it destroys a fraction $\epsilon_{jt}^K \in (0,1)$ of the physical capital stock installed to firms j located in that area and a fraction $\epsilon_{it} \in (0,1)$ of the human capital stock owned by households i in the area. The damage ratios ϵ_{jt}^K and ϵ_{it} are stochastic variables that are randomly drawn from the distribution functions $\Phi(\epsilon_{jt}^K)$ and $\Psi(\epsilon_{it})$, respectively. Both the occurrence and the damage ratios of natural disasters are assumed to be idiosyncratic across time and location.[9] Then, by the law of large numbers, the total damages to aggregate physical

[8]Although we ignore the finiteness of polluting inputs (e.g., fossil fuels), sustainability of growth under nonrenewable resources has been examined by, for example, Grimaud and Rougé (2003), Tsur and Zemel (2005), and Groth and Schou (2007). Elíasson and Turnovsky (2005) examined the growth dynamics with a resource that recovers only gradually. We also ignore extraction costs in our model because they would become increasingly small relative to the social marginal cost of pollution: section 5 will show that the social marginal cost of P_t (i.e., the expected marginal damage) increases exponentially in the long run.

[9]For simplicity of the analysis, we ignore the short-term fluctuations caused by large-scale (not idiosyncratic) disasters. In reality, the short-term fluctuations in investment and savings are not necessarily averaged out and may affect the long-term growth (see, for example, Hallegatte *et al.* 2007). In addition, as suggested by Denuit *et al.* (2011), the aggregate environmental risk also affects the optimal (social planner's) saving rate.

capital stock $K_t \equiv \int k_{jt}dj$ and aggregate human capital stock $H_t \equiv \int h_{it}di$ are written as:

$$\int q_t \epsilon_{jt}^K k_{jt} dj = \bar{\phi}(\bar{q} + \hat{q}P_t)K_t, \tag{2}$$

$$\int q_t \epsilon_{it} h_{it} di = \bar{\psi}(\bar{q} + \hat{q}P_t)H_t, \tag{3}$$

where $\bar{\phi}$ and $\bar{\psi}$ represent the expected values of distributions $\Phi(\epsilon_{jt}^K)$ and $\Psi(\epsilon_{it})$, respectively.

Let us state the resource constraint of the economy. Because we consider a closed economy where all savings are used as physical capital in the production sector, $\int s_{it}di = \int k_{jt}dj \equiv K_t$ holds. In contrast, human capital can be used for either production or education, and we denote by $u_t \equiv N_t/H_t \in [0,1]$ the aggregate fraction of human capital devoted to production, where $N_t \equiv \int n_{jt}dj$. To keep our model tractable, we ignore adjustment costs after a firm is hit by a disaster and assume that reallocation of physical capital across areas occurs instantly.[10] Because the production function (1) has constant returns to scale, this assumption implies that the firms have the same factor input ratios (both in market equilibrium and in the social planner's problem), so their amounts of production can be aggregated as $Y_t \equiv \int y_{jt}dj = AK_t^{\alpha}(u_tH_t)^{1-\alpha-\beta}P_t^{\beta}$. The remaining human capital stock $(1-u_t)H_t$ is used in the education sector to produce $B(1-u_t)H_t$ units of additional human capital, where B is a productivity parameter of the education sector. Let constants $\bar{\delta}_K$ and $\bar{\delta}_H$ denote the depreciation rates for physical and human capital stock, respectively, and define $\delta_K \equiv \bar{\delta}_K + \bar{\phi}\bar{q}$, $\phi \equiv \bar{\phi}\hat{q}$, $\delta_H \equiv \bar{\delta}_H + \bar{\psi}\bar{q}$, and $\psi \equiv \bar{\psi}\hat{q}$. Then, using (2) and (3), the resource constraints for the physical and human capital stocks can be summarized as:

$$\dot{K}_t = Y_t - C_t - (\delta_K + \phi P_t)K_t, \quad Y_t = AK_t^{\alpha}(u_tH_t)^{1-\alpha-\beta}P_t^{\beta}, \tag{4}$$

$$\dot{H}_t = B(1-u_t)H_t - (\delta_H + \psi P_t)H_t, \tag{5}$$

where $C_t \equiv \int c_{it}di$ represents the aggregate consumption of households. Equations (4) and (5) are very similar to Lucas (1988) except that the use of polluting input P_t in production effectively augments the depreciation rates of physical and human capital stocks.[11]

[10] Although we ignore the adjustment process, a number of studies have explicitly examined the cost of adjustment after a natural disaster. In a non-equilibrium dynamic model, Hallegatte *et al.* (2007) showed quantitatively that extreme events can entail much larger production losses than those analyzed in neoclassical growth models. In contrast, Rose (2004) has shown that the damage can be mitigated if agents take resilient (preventive or adaptive) actions in a computable general equilibrium model.

[11] In this respect, our model is closely related to that of Gradus and Smulders (1993, Section 4), who

Unlike standard endogenous growth models, the right-hand sides of Equations (4) and (5) are not homogenous of degree one in terms of quantities. Although the production function has constant returns to scale, the homothetic expansion of all of inputs (K_t, H_t and P_t) would result in increasingly frequent destruction of capital stocks. The following section will show that, without appropriate environmental policies, the intensification of natural disasters eventually makes further accumulation of capital impossible.

3 Market Economy

3.1 Environmental tax and behavior of firms

We start the analysis with the market economy, where markets are perfectly competitive but the government levies a per-unit tax of τ_t on the use of polluting inputs p_{jt} by firms (the numeraire is the final goods). Because we ignore the extraction cost and firms take the risk of natural disasters as given, the only private cost of using p_{jt} is τ_t. At the beginning of the economy, the government announces the tax rate τ_t for all t, and it is assumed that the government can commit to this tax policy. The tax revenue $T_t = \tau_t P_t$ is then distributed to consumers as a uniform transfer.

At each point in time, every firm j in the production sector chooses the employment of k_{jt} and n_{jt} and the amount of p_{jt} to maximize the expected profit by taking as given the interest rate r_t, the wage rate w_t, and τ_t. Similarly to (4), the sum of the depreciation and the expected natural disaster damage to firm j's physical capital is $(\delta + \phi P_t)k_{jt}$. Then, using the production function (1), the problem of firm j can be expressed as

$$\max_{k_{jt},n_{jt},p_{jt}} A k_{jt}^{\alpha} n_{jt}^{1-\alpha-\beta} p_{jt}^{\beta} - (r_t + \delta_K + \phi P_t)k_{jt} - w_t n_{jt} - \tau_t p_{jt}.$$

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extended Lucas (1988) to include air pollution, which causes human capital to depreciate at a faster rate through health problems. Aside from the difference in the focus, a notable distinction is that pollution can be abated by devoting goods in their model, whereas we consider P_t as a necessary input for production. They focused on the social planner's problem, whereas this paper examines a wider range of environmental tax policies.

The first-order conditions are[12]

$$r_t = \alpha \frac{y_{jt}}{k_{jt}} - \delta_K - \phi P_t, \quad w_t = (1-\alpha-\beta)\frac{y_{jt}}{n_{jt}}, \quad \tau_t = \beta \frac{y_{jt}}{p_{jt}}.$$

Because the above conditions are the same for all firms, we can replace y_{jt}/k_{jt}, y_{jt}/n_{jt} and y_{jt}/p_{jt} by their aggregate counterparts, yielding the aggregate use of the polluting input and factor prices as

$$P_t = \beta Y_t/\tau_t, \tag{6}$$

$$r_t = \alpha Y_t/K_t - \delta_K - \phi P_t, \quad w_t = (1-\alpha-\beta)Y_t/N_t. \tag{7}$$

Equation (6) shows that the environmental tax lowers the aggregate level of pollution. However, substituting this condition into the production function implies

$$Y_t = \left(\tilde{A}\tau^{-\frac{\beta}{1-\beta}}\right) K_t^{\hat{\alpha}} N_t^{1-\hat{\alpha}}, \tag{8}$$

where $\tilde{A} \equiv \beta^{\beta/(1-\beta)} A^{1/(1-\beta)}$ and $\hat{\alpha} \equiv \alpha/(1-\beta)$. Equation (8) clarifies that the environmental tax lowers the effective TFP, $\tilde{A}\tau^{-\beta/(1-\beta)}$.

The education sector has a representative competitive firm. It uses only human capital and has a linear production technology, where $B(1-u_t)H_t$ units of additional human capital are produced by employing $(1-u_t)H_t$ units of human capital. Under perfect competition, the price of one additional unit of human capital is determined by its marginal cost w_t/B.

3.2 Behavior of households

Every household i aims to maximize its expected utility:

$$\mathbb{E}\left[\int_0^\infty \frac{c_{it}^{1-\theta} - 1}{1-\theta} e^{-\rho t} dt\right], \tag{9}$$

where we assume that relative risk aversion θ is higher than 1 and that the rate of time preference satisfies $\rho < B - \delta_H$ so that households have sufficient incentive to invest in human capital.

[12]When these conditions are satisfied, the maximized expected profit is zero because the maximand in the problem is homogeneous of degree one with respect to production factors. In equilibrium, the aggregate profit will become zero because disaster damages are idiosyncratic.

At normal times, i.e., except at the moment the household is hit by a disaster, its savings and human capital evolve as

$$\dot{s}_{it} = r_t s_{it} + w_t h_{it} - (w_t/B)m_{it} - c_{it} + T_t, \tag{10}$$

$$\dot{h}_{it} = m_{it} - \bar{\delta}_H h_{it}, \tag{11}$$

where m_{it} is the purchase of additional human capital through the education sector at the unit price of w_t/B. One may also interpret m_{it} as including additions to own human capital through self or home training, in which case the opportunity cost of training (and not working) is w_t/B. T_t in (10) represents the amount of uniform transfer that each household receives. Because the total measure of households is unity, it is the same as the total revenue from environmental tax: $T_t = \tau_t P_t$.

It is convenient to express the budget constraint in terms of the total assets of household i, defined by $a_{it} \equiv s_{it} + (w_t/B)h_{it}$. Differentiating this definition with respect to time and then applying (10) and (11), we obtain

$$\frac{\dot{a}_{it}}{a_{it}} = (1-\eta_{it})r_t + \eta_{it}\left(B - \bar{\delta}_H + \frac{\dot{w}_t}{w_t}\right) - \frac{c_{it}}{a_{it}} + \frac{T_t}{a_{it}}, \tag{12}$$

where η_{it} is the fraction of human capital in the total assets, defined as

$$\eta_{it} \equiv \frac{(w_t/B)h_{it}}{s_{it} + (w_t/B)h_{it}} = \frac{w_t h_{it}}{B s_{it} + w_t h_{it}}. \tag{13}$$

When a household is hit by a disaster, its human capital shrinks from h_{it} to $\tilde{h}_{it} = (1-\epsilon_{it})h_{it}$, where ϵ_{it} is randomly drawn from distribution $\Psi(\epsilon_{it})$. The savings s_{it} are not significantly affected because they are invested in locationally dispersed firms. Thus, the total assets $a_{it} \equiv s_{it} + (w_t/B)h_{it}$ jump to

$$\tilde{a}_{it} = (1 - \eta_{it}\epsilon_{it})a_{it}, \quad \text{with Poisson probability } q_t = \bar{q} + \hat{q}P_t. \tag{14}$$

Because the households are risk averse, it would be optimal to insure against the possible loss of $\eta_{it}\epsilon_{it}a_{it}$ if such insurance is available. For the time being, we consider the case where such insurance is available with no transaction cost, and hence all households take out perfect insurance. The case without insurance will be analyzed in Section 7. The flow premium for this insurance is equal to the expected loss: $(\bar{q} + \hat{q}P_t)\,\mathbb{E}[\eta_{it}\epsilon_{it}a_{it}] = (\bar{q} + \hat{q}P_t)\eta_{it}\bar{\psi}a_{it}$. Subtracting this premium from the budget constraint (12), we obtain the budget constraint under perfect insurance, which holds for all t:

$$\frac{\dot{a}_{it}}{a_{it}} = (1-\eta_{it})r_t + \eta_{it}\left(B - \delta_H - \psi P_t + \frac{\dot{w}_t}{w_t}\right) - \frac{c_{it}}{a_{it}} + \frac{T_t}{a_{it}}, \tag{15}$$

where $\delta_H \equiv \bar{\delta}_H + \bar{\psi}\bar{q}$ and $\psi \equiv \bar{\psi}\hat{q}$ as in (5).

Given the time paths of r_t, w_t and P_t, each household chooses the path of consumption c_{it} and asset allocation η_{it} to maximize (9) subject to the budget constraint (15). The right-hand side (RHS) of the budget constraint (15) is linear in η_{it}. This linearity implies that households are willing to hold both savings (which will then be used as physical capital) and human capital only if the following arbitrage condition is satisfied:

$$B - \delta_H - \psi P_t + \frac{\dot{w}_t}{w_t} = r_t. \tag{16}$$

On the LHS of (16), $B - \delta_H - \psi P_t$ is the rate at which human capital is reproduced, and $\dot{w}_t/w_t$ is the capital gain (or loss if negative) in holding human capital when its value w_t/B changes. The sum of these must coincide with the interest rate r_t. Otherwise, all households would invest only in one type of capital stock, which would raise the value of the other type of capital due to scarcity, contradicting the decision of households not to invest in it.

Using the arbitrage condition (16), the budget constraint (15) reduces to a familiar form: $\dot{a}_{it} = r_t a_{it} - c_{it} + T_t$. The optimal solution to this problem is characterized by the Keynes-Ramsey Rule $-\theta(\dot{c}_{it}/c_{it}) = \rho - r_t$ and the transversality condition $\lim_{t\to\infty} a_{it} c_{it}^{-\theta} e^{-\rho t} = 0$.

3.3 Market equilibrium and sustainability of growth

Given the initial levels of K_0 and H_0 and the time path of τ_t, the aggregate variables in market equilibrium, K_t, H_t, P_t, u_t and C_t, are determined as follows. The dynamics for K_t and H_t are given by resource constraints (4) and (5).[13] Aggregate pollution P_t is given by (6). Substituting the time derivative of (7) into the arbitrage condition (16) gives the condition for the fraction of human capital devoted to production $u_t (\equiv N_t/H_t)$:

$$\frac{\dot{Y}_t}{Y_t} - \frac{\dot{H}_t}{H_t} - \frac{\dot{u}_t}{u_t} = \left(\alpha \frac{Y_t}{K_t} - \delta_K - \phi P_t\right) - (B - \delta_H - \psi P_t). \tag{17}$$

[13] We can confirm that aggregating the budget constraint (15) and then eliminating factor prices and $T_t = \tau_t P_t$ by (6) and (7) yields a weighted sum of resource constraints (4) and (5). Although each household is indifferent to the asset allocation η_{it} under (16), in aggregate η_{it}'s are determined to satisfy the equilibrium of the factor market, $\int (1-\eta_{it}) a_{it} di \equiv \int s_{it} di = K_t$ and $(B/w_t) \int \eta_{it} a_{it} di \equiv \int h_{it} di = H_t$.

Finally, aggregating the Keynes–Ramsey Rule and the transversality condition for all households gives the dynamics for aggregate consumption C_t:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}\left[\left(\alpha\frac{Y_t}{K_t} - \delta_K - \phi P_t\right) - \rho\right], \tag{18}$$

$$\lim_{t\to\infty} K_t C_t^{-\theta} e^{-\rho t} = 0, \quad \lim_{t\to\infty} (w_t/B) H_t C_t^{-\theta} e^{-\rho t} = 0, \tag{19}$$

where we used $\int a_{it} di = \int (s_{it} + (w_t/B)h_{it}) di = K_t + (w_t/B)H_t$. The market equilibrium is characterized by (4), (5), (6), (17), (18), and the transversality conditions (19).

Let us examine the long-run property of the market equilibrium in the simplest case, where the government sets a constant per-unit tax rate τ_0 on p_{jt}. From equation (6), pollution increases in proportion to output Y_t under this policy. Given that the increasing use of the polluting input makes natural disasters increasingly frequent, it appears that economic growth is not sustainable under such a static environmental policy. The following proposition formally shows that this insight is correct.

Proposition 1 *If the per-unit tax on the polluting input is constant ($\tau_t = \tau_0$ for all t), then economic growth is not sustainable in the sense that aggregate consumption cannot grow in the long run.*

Proof: The proof goes via *reductio ad absurdum.* Suppose that consumption grows in the long run (i.e., $\lim_{t\to\infty} \dot{C}_t/C_t > 0$). Using (6), the Keynes–Ramsey Rule (18) can be rewritten as:

$$\frac{\dot{C}_t}{C_t} = -\frac{\rho + \delta_K}{\theta} + \frac{1}{\theta}\left(\alpha - \frac{\phi\beta}{\tau_0}K_t\right)\frac{Y_t}{K_t}. \tag{20}$$

For the RHS to be positive, the sign of the value in the parentheses on the RHS must be positive. Hence, in the long run, physical capital K_t must be bounded above by a constant value at $\tau_0\alpha/\phi\beta$. Note that, when τ_t is constant, differentiating (8) with respect to time gives $\dot{Y}_t/Y_t = \hat{\alpha}\dot{K}_t/K_t + (1-\hat{\alpha})\dot{N}_t/N_t$. Using this and (18), the arbitrage condition (17) can be written as the following:

$$\frac{\dot{N}_t}{N_t} = \frac{\dot{K}_t}{K_t} - \frac{\theta}{\hat{\alpha}}\frac{\dot{C}_t}{C_t} + \frac{1}{\hat{\alpha}}\left(B - \delta_H - \psi P_t - \rho\right). \tag{21}$$

In the long run, the first term on the RHS is less than 0 because K_t is bounded. The second term is negative because we assumed consumption growth. The third term is also negative because consumption growth requires output growth, which implies $P_t = \beta Y_t/\tau_t \to \infty$

under the constant tax rate. Therefore, $\dot{N}_t/N_t$ is negative in the long run, implying that the human capital eventually shrinks. Given the boundedness of K_t and N_t, (8) means that production cannot grow in the long run. This result clearly contradicts the initial assumption that consumption grows in the long run. ■

Intuitively, the proof of the proposition explains that there is a barrier to capital accumulation under a constant environmental tax rate. As long as firms face a constant tax rate on the polluting input, the risk of disasters rises proportionally with output (see Equation 6). The rise in the expected damage to physical capital discourages firms from employing physical capital, which lowers the equilibrium interest rate in (7). Eventually r_t falls to ρ, at which point agents no longer want to save more. A higher environmental tax will expand this limit because the upper bound for K_t, i.e., $\tau_0\alpha/\phi\beta$, is increasing in τ_0.[14] Still, as long as the tax rate is constant, economic growth cannot be sustained forever. This result suggests that, to sustain economic growth, it is necessary to increase the rate of the environmental tax over time to prevent the risk of disasters from increasing excessively. In the remainder of the paper, we consider such a time-varying tax policy.

4 Asymptotically Balanced Growth Paths

In existing studies of endogenous growth, it is common to focus only on balanced growth paths (BGP), where the growth rates of all variables are constant for all t. However, in our model, the risk of capital destruction makes the system of the economy inevitably nonhomothetic, implying that any BGP may not exist. Following Palivos *et al.* (1997), we overcome this problem by considering a broader family of equilibrium paths that asymptote to a BGP only in the long run:

Definition 1 (NABGP) *An equilibrium path is said to be an asymptotically BGP if the growth rates of output, inputs, and consumption converge to finite constant values; that is, if* $g^* \equiv \lim_{t\to\infty} \dot{Y}_t/Y_t$, $g_K \equiv \lim_{t\to\infty} \dot{K}_t/K_t$, $g_H \equiv \lim_{t\to\infty} \dot{H}_t/H_t$, $g_u \equiv \lim_{t\to\infty} \dot{u}_t/u_t$, $g_P \equiv \lim_{t\to\infty} \dot{P}_t/P_t$, *and* $g_C \equiv \lim_{t\to\infty} \dot{C}_t/C_t$ *are well defined and finite. In addition, if*

[14] In contrast, if firms can use the polluting input almost freely ($\tau_0 \to 0$), the proof of Proposition 1 suggests that K_t and H_t will inevitably fall to zero. Even though using a massive amount of P_t might increase the output initially, the destruction of capital will overwhelm the production and collapse the economy.

$g_C \geq 0$, *it is said to be a nondegenerate, asymptotically balanced growth path (NABGP).*[15]

In this section, we seek to identify a tax policy that achieves positive long-run growth within the family of asymptotically BGP, referred to as a NABGP. From definition 1 and Equation (6), the asymptotic growth rate of the tax rate, which we denote by $g_\tau \equiv \lim_{t\to\infty} \dot{\tau}_t/\tau_t$, must also be well defined on any NABGP. The main task of this section is to examine the dependence of the long-term rate of economic growth g^* on the speed of increase of the environmental tax rate, g_τ. We first show that production cannot grow faster than the environmental tax rate:

Lemma 1 *On any NABGP, $g^* \leq g_\tau$.*
Proof: in Appendix A.1.

Intuitively, if production grew faster than the tax rate, the use of the polluting input $P_t = \beta Y_t/\tau_t$ would increase without bound, and natural disasters would be increasingly frequent. In such a situation, however, both physical and human capital deteriorate at an accelerating rate, contradicting the initial assumption that output can grow. One implication from Lemma 1 is that sustained growth (with $g^* > 0$) is possible only when $g_\tau > 0$; i.e., only when the per-unit tax rate increases at an asymptotically constant rate.

Another implication of $g^* \leq g_\tau$ is that P_t is not increasing with time in the long run ($g_P \equiv \lim_{t\to\infty} \dot{P}_t/P_t \leq 0$ from Equation 6). Given that the amount of polluting input P_t is nonnegative, this means that P_t converges to a constant value in the long run. We denote this asymptotic value by $P^* \equiv \lim_{t\to\infty} P_t$. In particular, if $g^* < g_\tau$, P_t falls with time ($g_P < 0$) and necessarily converges to $P^* = 0$. Even though we limit our attention to nondegenerate growth paths, we should not rule out this possibility. It is true that output Y_t is zero if $P_t = 0$ given the Cobb–Douglas production technology (1), where polluting inputs, such as fossil fuels, are necessary. However, in NABGPs where P_t *asymptotes* to P^*, P_t does not necessarily coincide with $P^* = 0$ at any date. Furthermore, $\lim_{t\to\infty} P_t = 0$ does not necessarily mean $\lim_{t\to\infty} Y_t = 0$ as the other production factors in (1), namely K_t and H_t, may be accumulated unboundedly.

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[15]Palivos *et al.* (1997) call an asymptotically BGP nondegenerate when every production input grows at a positive rate. Our definition of nondegeneration is weaker (broader) as we only require aggregate consumption not to fall. We will show that g_P can be negative in a NABGP.

Given the asymptotic constancy of P_t, the first-order and transversality conditions require u_t, $z_t \equiv Y_t/K_t$, and $\chi_t \equiv C_t/K_t$ to be asymptotically constant, which implies

$$g_u = 0, \quad g_K = g_C = g^*, \tag{22}$$

as formally confirmed in Appendix A.2. Although condition (22) means that physical capital and consumption grow in parallel with output, the growth rate of human capital cannot be the same as that of output. Differentiating the production function (8) logarithmically with respect to time gives $g^* = -\frac{\beta}{1-\beta} g_\tau + \hat{\alpha} g_K + (1-\hat{\alpha})(g_u + g_H)$, where we used $N_t = u_t H_t$. To be consistent with condition (22), g_H should satisfy

$$g_H = g^* + \frac{\beta}{1-\alpha-\beta} g_\tau. \tag{23}$$

Equation (23) says that on any NABGP, human capital must accumulate faster than physical capital and output, and the difference is larger when the growth rate of the environmental tax is higher. To see why agents are willing to accumulate human capital more quickly in equilibrium, observe that as the tax rate on the polluting input increases over time, the effective productivity of private firms $\tilde{A}\tau^{-\beta/(1-\beta)}$ gradually falls, as shown in (8). This means that if human capital were accumulated at the same speed as physical capital, output would only be able to grow slower than the speed of physical capital accumulation, and the marginal productivity of physical capital, $\alpha Y_t/K_t$, would fall. In this manner, raising the tax rate on the polluting input hinders physical capital investment, and consequently induces agents to choose human capital investment an alternate means of saving, as documented by Skidmore and Toya (2002).[16]

[16]Nonetheless, the marginal productivity of capital is kept constant on the NABGP. This is because as human capital becomes increasingly abundant relative to physical capital, it raises the marginal productivity of physical capital and eventually compensates for the decline in effective productivity.

Now we are ready to summarize the conditions that must be satisfied on any NABGP. Substituting (22) and (23) for equilibrium conditions (4), (5), (6), (17), and (18) gives

$$\text{Evolution of } K_t\text{: } g^* = z^* - \chi^* - (\delta_K + \phi P^*), \tag{24}$$

$$\text{Evolution of } H_t\text{: } g^* + \frac{\beta}{1-\alpha-\beta} g_\tau = B(1-u^*) - (\delta_H + \psi P^*), \tag{25}$$

$$\text{Arbitrage condition: } -\frac{\beta}{1-\alpha-\beta} g_\tau = (\alpha z^* - \delta_K - \phi P_t) - (B - \delta_H - \psi P^*), \tag{26}$$

$$\text{Keynes–Ramsey rule: } \theta g^* = (\alpha z^* - \delta_K - \phi P^*) - \rho, \tag{27}$$

$$\text{Asymptotic pollution: either} \begin{cases} P^* \geq 0 \text{ and } g^* = g_\tau, & \text{(Case 1)} \\ P^* = 0 \text{ and } g^* < g_\tau, & \text{(Case 2)} \end{cases} \tag{28}$$

where $u^* \equiv \lim_{t\to\infty} u_t \in [0,1]$, $z^* \equiv \lim_{t\to\infty} Y_t/K_t \geq 0$, and $\chi^* \equiv \lim_{t\to\infty} C_t/K_t \geq 0$. Given the tax policy $g_\tau \geq 0$, which is set by the government, the five conditions (24)-(28) determine five unknowns (g^*, z^*, χ^*, u^*, P^*) on the NABGP.

This problem can be solved as a system of linear equations once we determine which of the two cases in the complementary slackness condition (28) applies. To determine whether Case 1 applies under a given tax policy g_τ, we solve (24)-(27) with $g^* = g_\tau$ and then check if $P^* \geq 0$ holds. Similarly, Case 2 applies if the solution of (24)-(27) with $P^* = 0$ satisfies $g^* < g_\tau$. Appendix A.3 shows that this procedure yields a unique solution:

$$g^* = \begin{cases} g_\tau \text{ if } g_\tau \leq g^{\max}, \\ g^* = \frac{1}{\theta}\left(B - \delta_H - \rho - \frac{\beta}{1-\alpha-\beta} g_\tau\right) \text{ if } g_\tau \geq g^{\max}, \end{cases} \tag{29}$$

$$P^* = \begin{cases} P^* = \frac{1}{\psi}\left[B - \delta_H - \rho - \left(\theta + \frac{\beta}{1-\alpha-\beta}\right) g_\tau\right] \text{ if } g_\tau \leq g^{\max}, \\ 0 \text{ if } g_\tau \geq g^{\max}, \end{cases} \tag{30}$$

$$\text{where } g^{\max} \equiv \left(\theta + \frac{\beta}{1-\alpha-\beta}\right)^{-1} (B - \delta_H - \rho) > 0. \tag{31}$$

Equation (29) shows that the asymptotic rate of economic growth g^* is increasing in g_τ for $g_\tau \leq g^{\max}$ and thereafter decreases with g_τ. In particular, for the equilibrium path to be nondegenerate, the output must grow at a nonnegative rate, which requires the government to set g_τ between 0 and $g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B - \delta_H - \rho) > g^{\max}$. Given $g_\tau \in [0, g^{\lim}]$, we confirm in Appendix A.3 that the solutions to the other variables lie in the feasible range and that the transversality condition (19) is satisfied. In addition, this NABGP is saddle stable under a reasonable restriction of the parameter values, as stated below.

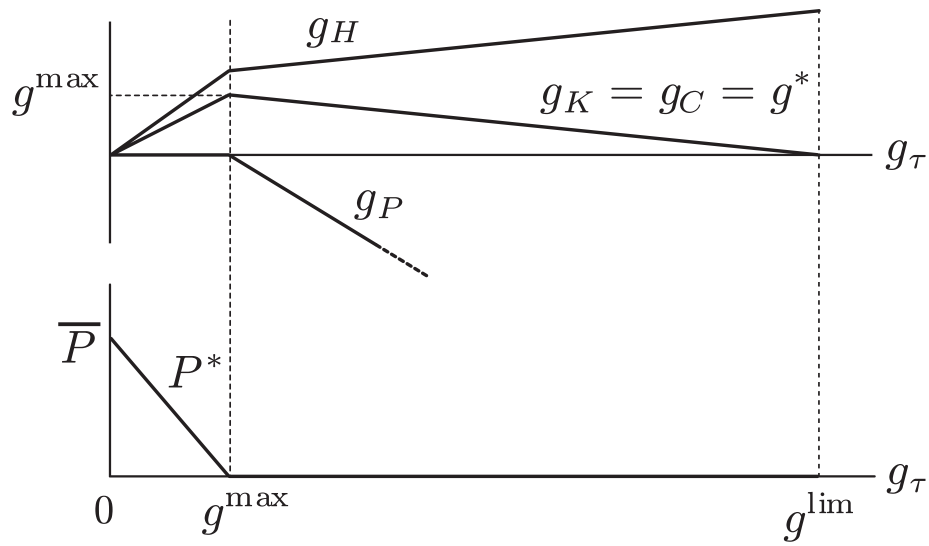

Figure 3: Growth rate of environmental tax and the NABGP. The upper panel shows the relationship between the growth rate of the environmental tax (g_τ) and that of human capital (g_H), physical capital (g_K), output (g^*), and pollution (g_P). The lower panel shows the level to which pollution converges in the long run ($P_t \to P^*$). Parameters: $\alpha = .3$, $\beta = .2$, $\theta = 2$, $\rho = .05$ $B = 1$, $\bar{\phi} = .5$, $\bar{\psi} = .25$, $\bar{q} = .1$, $\hat{q} = .02$, $\bar{\delta}_K = .05$, $\bar{\delta}_H = .065$ (these imply $\psi = .005$, $\phi = .01$, $\delta_H = .09$, and $\delta_K = .1$).

Proposition 2 *A NABGP uniquely exists if and only if the asymptotic growth rate of the per-unit tax on the polluting input, g_τ, is between 0 and $g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B-\delta_H-\rho)$. The long-term rate of economic growth follows an inverted V shape against $g_\tau \in [0, g^{\lim}]$ and is maximized at $g_\tau = g^{\max} \equiv (\theta + \frac{\beta}{1-\alpha-\beta})^{-1}(B-\delta_H-\rho)$. In addition, if $\psi/\phi < (1-2\alpha)/(1-\alpha-\beta)$, the equilibrium path is locally saddle stable.*[17]

Proof of stability: in Appendix A.4.

Once the environmental tax policy g_τ determines the asymptotic growth rate of output (29), the growth rates of human capital and pollution are obtained by (23) and $g_P = g^* - g_\tau$ from (6). Figure 3 illustrates the relationship between the environmental tax policy and the evolution of variables in the long run. When the environmental tax rate is asymptotically constant (i.e., when $g_\tau = 0$), the asymptotic growth rates of all endogenous variables are

[17]Given that the share of physical capital α is around 0.3 in reality, $(1-2\alpha)/(1-\alpha-\beta)$ is likely to be positive. (When $\alpha = 0.3$ and $\beta = 0.1$, for example, $(1-2\alpha)/(1-\alpha-\beta) = 2/3$.) In addition, the percentage of physical capital destroyed by a disaster, denoted by $\bar{\phi}$, is typically higher than that for human capital $\bar{\psi}$. This implies, $\psi/\phi = (\bar{\psi}\hat{q})/(\bar{\phi}\hat{q}) = \bar{\psi}/\bar{\phi}$, is typically low. Therefore, we reasonably assume that parameters satisfy condition $\psi/\phi < (1-2\alpha)/(1-\alpha-\beta)$ in Proposition 2

zero. This means that the economy settles to a no-growth steady state. In this steady state, the amount of pollution converges to $P^* = (B - \delta_H - \rho)/\psi \equiv \bar{P}$, which causes the probability of losing physical and human capital to be so high that agents lose the incentive to accumulate capital beyond a certain level. Interestingly, the asymptotic level of P_t does not depend on the level of the environmental tax rate, τ_t, as long as τ_t is asymptotically constant. Nonetheless, given $Y_t = \tau_t P_t/\beta$ from (6), a higher tax rate induces the economy to converge to a higher output level. This implies that a higher level of the environmental tax rate promotes growth in the transition, but not in the long run.

When the government raises the per-unit tax rate on polluting inputs at an asymptotically constant rate ($g_\tau > 0$), the asymptotic level of P_t can be kept below $\bar{P}$, which helps to overcome the barrier to capital accumulation. When g_τ is increased within the range of $[0, g^{\max}]$, the long-run amount of pollution P^* decreases, as does the risk of natural disasters. The reduced risk of natural disasters encourages agents to accumulate capital more quickly. As a result, the growth rate of physical capital g_K increases in parallel with g_τ (i.e., $g_K = g_\tau$). The growth rate of human capital, g_H, also increases with g_τ, and more than proportionately to physical capital. This makes possible sustained growth without increasing the use of the polluting input.

The long-term rate of economic growth is maximized at $g_\tau = g^{\max}$, under which the use of polluting inputs P_t converges asymptotically to the zero level ($P_t \to P^* = 0$). However, a further acceleration of the tax rate does not enhance economic growth: although it accelerates the convergence of the risk of natural disasters to the lowest level ($q_t = \bar{q}$), the acceleration in the decrease of the effective productivity of firms, $\tilde{A}\tau^{-\beta/(1-\beta)}$, has a dominant negative effect on growth in the long run. As a result, g^* is no longer increasing in parallel with g_τ, but is decreasing in g_τ. In particular, if $g_\tau > g^{\lim}$, the decrease of effective productivity is so fast that it cannot be compensated for by the faster accumulation of human capital or the quicker convergence of the disaster risk. This results in negative growth.

5 Welfare-maximizing Policy

In previous sections, we examined the relationship between the environmental policy and the feasibility of sustained economic growth. Even when production requires polluting inputs and the use of polluting inputs raises the risk of natural disasters, we showed that economic growth can be sustained in the long run if the government gradually increases the tax rate on the polluting inputs. We also found that an environmental policy maximizes the long-term rate of economic growth. However, this does not necessarily mean that such an environmental policy is desirable in terms of welfare. This section considers the welfare-maximizing policy and examines whether it differs from the growth-maximizing policy.

Let us consider the social planner's problem. The social planner maximizes the representative household's expected utility (9) subject to resource constraints (4) and (5). From the first-order conditions for optimality, we show in Appendix A.5 that the dynamics of K_t, H_t, u_t and C_t in the welfare-maximizing path are exactly the same as those for the market equilibrium given by Equations (4), (5), (17) and (18). The transversality condition (19) is also the same. The remaining condition for the social planner's problem is that the amount of polluting input should be:

$$P_t = \beta \left(\phi \frac{K_t}{Y_t} + \psi \frac{(1-\alpha-\beta)}{B u_t} \right)^{-1}. \tag{32}$$

Recall that in the market economy, the government sets the tax rate τ_t and firms choose P_t according to $P_t = \beta Y_t / \tau_t$, as shown by Equation (6). Therefore, if the tax rate at each point in time satisfies:

$$\tau_t = \phi K_t + \psi H_t \frac{(1-\alpha-\beta) Y_t}{B u_t H_t}, \tag{33}$$

then the firms' decision on P_t in the market equilibrium exactly coincides with the optimality condition (32). Given that the remaining conditions for the social optimum are the same as those for the market equilibrium, this means that the welfare-maximizing allocation can be achieved as a market equilibrium when the government set the environmental tax rate using the following rule (33).[18] This policy rule has an intuitive interpretation as the RHS of (33) represents the social marginal cost of using P_t: the first term represents the marginal increase in the expected damage to physical capital with respect to P_t, whereas the second term represents that to human capital, both measured in terms of final goods (in particular, $(1-\alpha-\beta)Y_t/(B u_t H_t)$ is the shadow price of human capital in terms of final

[18]We assume that all private agents are price takers and do not behave strategically. In this setting, a time-varying policy (a function only of time, as considered in the previous section) and a policy rule (a function of state variables such as equation Equation 33) result in the same outcome.

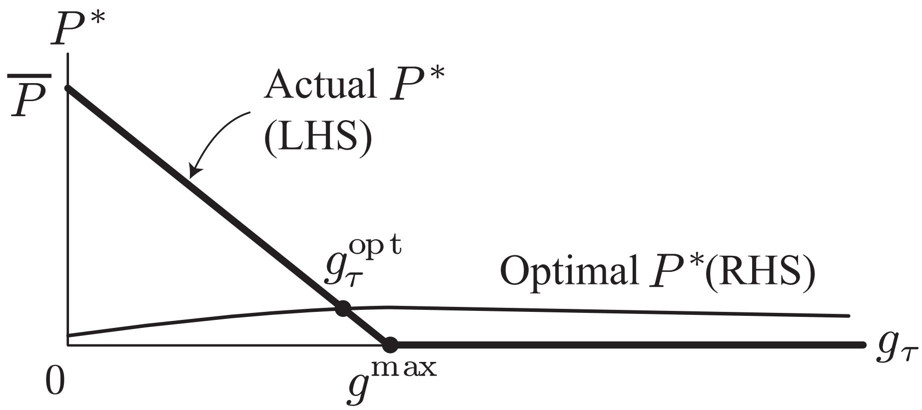

Figure 4: Determination of the optimal growth rate of the environmental tax. This figure plots the RHS and LHS of condition (34) against g_τ. The asymptotic growth rate of the optimal environmental tax is g_τ^{opt}, as given by the intersection, and is lower than the growth-maximizing rate, $g^{\max}$. The parameters are the same as in Figure 3.

goods). Thus, it is optimal to let firms pay the sum of these marginal expected damages on each use of P_t.

Let us characterize the equilibrium path under the optimal tax policy. Similarly to the previous section, we limit our attention to NABGP. Equation (30) shows that the asymptotic value of P_t on the NABGP is determined as a function of g_τ, which can be written as $P^*(g_\tau)$. Similarly, the asymptotic values of $z_t \equiv Y_t/K_t$ and u_t are determined as functions of g_τ from (58)-(63) in Appendix A.3, and hence we can write them as $z^*(g_\tau)$ and $u^*(g_\tau)$. For the welfare-maximizing condition (32) to hold in the long run, g_τ should satisfy:

$$P^*(g_\tau) = \beta \left(\phi \frac{1}{z^*(g_\tau)} + \frac{\psi(1-\alpha-\beta)}{Bu^*(g_\tau)} \right)^{-1}. \tag{34}$$

As illustrated in Figure 4, condition (34) can be interpreted as the coincidence of the actual amount of asymptotic pollution in equilibrium (the LHS) and the optimal amount of asymptotic pollution (the RHS), where both sides are determined by tax policy g_τ. The actual pollution is positive but decreasing in g_τ for $g_\tau \in [0, g^{\max})$, and is zero for $g_\tau \geq g^{\max}$. On the other hand, the optimal amount of pollution is positive for all $g_\tau \geq 0$, and at $g_\tau = 0$, is lower than $\bar{P} \equiv (B - \delta_H - \rho)/\psi$ given that parameters satisfy:[19]

$$(\alpha\phi/(\delta_K + \phi\bar{P} + \rho) + \psi(1 - \alpha - \beta)/\rho)\bar{P} > \beta. \tag{35}$$

Therefore, under condition (35), the two curves have an intersecting point $g_\tau^{\text{opt}} \in (0, g^{\max})$, at which point the optimality condition (34) is satisfied. The following proposition formally states this result.

Proposition 3 *Suppose the parameters satisfy condition (35). Then among the NABGP, there exists a path that maximizes the welfare of the representative household (9). This path can be realized by tax policy (33), and the asymptotic growth rate of the optimal per unit tax, g_τ^{opt}, is strictly positive but lower than the growth-maximizing rate, $g^{\max}$.*

Note that condition (35) is satisfied unless both ρ and β are large. Intuitively, it pays to enjoy a high level of consumption, production and, therefore, pollution today at the cost of accepting a higher risk of natural disasters only when the household heavily discounts the future (large ρ) and production substantially relies on polluting inputs (large β). If either the household values the future or the dependence of production on polluting inputs is limited, then sustained economic growth is not only feasible but also desirable. It is also notable, however, that the optimal policy does not coincide with the growth-maximizing policy ($g_\tau^{\text{opt}} < g^{\max}$). Thus, if the government cares about welfare, it should employ a milder policy for protecting the environment than when growth is their only concern. The difference between the growth-maximizing and welfare-maximizing policies is similar to the difference between the golden rule and the modified golden rule. Although an aggressive environmental policy that aims to eliminate the emission of pollutants in the long run (i.e., $P^* = 0$) may maximize the economic growth rate in the very long run, the cost in the form of the reduced effective productivity that must be incurred during the transition can overwhelm the benefit that can be reaped only far in the future.

6 Extension I: Stock of Pollution

In reality, the risk of natural disasters is often affected not only by how much current firms emit pollution, but also how much they emitted in the past. For example, the use of fossil

[19]When $g_\tau = 0$, Equations (30), (58), and (60) show that $P^* = (B - \delta_H - \rho)/\psi \equiv \bar{P}$, $z^* = (\delta_K + \phi\bar{P} + \rho)/\alpha$ and $u^* = \rho/B$. Substituting these into both sides of (34) shows that the intercept of the LHS is lower than that of the RHS if (35) holds.

fuels in the past increases the the stock of greenhouse gases in the atmosphere today, and this affects tropical sea surface temperature, and therefore the risk of disastrous hurricanes. To this point, for simplicity we do not distinguish between the flow of pollution and its stock. This section examines how the long-term properties obtained in previous sections change when pollution stocks affect the risk of natural disasters.

As before, we assume that firms use a polluting input (e.g., fossil fuels), causing them to emit pollution. Let e_{jt} denote the emission of pollution by firm j per unit of time. One unit of polluting input yields one unit of emission, so e_{jt} also represents the amount of polluting input used by firm j. The production function (1) is modified to:

$$y_{jt} = Ak_{jt}^{\alpha} n_{jt}^{1-\alpha-\beta} e_{jt}^{\beta}, \tag{36}$$

where we substituted e_{jt} for p_{jt}. The aggregate emission $E_t \equiv \int e_{jt} dj$ adds to the pollution stock P_t, which is now defined by:

$$P_t \equiv \gamma \int_{-\infty}^{t} E_s e^{-\delta_P (t-s)} ds. \tag{37}$$

There are two parameters in the accumulation process: γ represents the marginal impact of emissions on the pollution stock, and δ_P denotes the depreciation rate of the pollution stock (e.g., the fraction of greenhouse gases being absorbed by the oceans during a unit of time). If δ_P is smaller, use of a polluting input today has an impact on the environment for a longer period in the future. We assume the risk of natural disasters is affected by the pollution stock P_t, as described by (2) and (3). The law of motion for physical capital can then be written as:

$$\dot{K}_t = Y_t - C_t - (\delta_K + \phi P_t) K_t, \quad Y_t = AK_t^{\alpha} (u_t H_t)^{1-\alpha-\beta} E_t^{\beta}, \tag{38}$$

whereas that for human capital stock remains the same as (5). Note that P_t in these equations should now be interpreted as the pollution stock at t rather than the amount of polluting input used at t.

6.1 Market economy under stock pollution

In the market economy, the government levies an environmental tax τ_t on each unit of polluting input e_{jt} used by the firm. Similar to the analysis in Section 3.1, the first-order conditions for firms can be aggregated as (7) and:

$$E_t = \beta Y_t / \tau_t. \tag{39}$$

The behavior of households is exactly the same as described in Section 3.2. In this setting, the equilibrium dynamics of $\{K_t, H_t, u_t, C_t, E_t, P_t\}$ are characterized by (5), (17), (18), (37), (38), (39), and the transversality conditions (19).

Let us consider the NABGP, where the growth rates of all inputs, output, and consumption are asymptotically constant in the long run (recall Definition 1). The following proposition shows that the long-run property of the equilibrium is unaffected by the introduction of accumulated pollution.

Proposition 4 *In an economy where pollution accumulates through (37) and (39), a NABGP exists if and only if the asymptotic growth rate of the per-unit tax on polluting input, g_τ, is between 0 and $g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B-\delta_H-\rho)$. On the NABGP, the values of g^*, z^*, χ^*, u^*, and P^* are the same as the baseline model, where pollution does not accumulate. The level of emission asymptotically converges to $E^* = (\delta_P/\gamma)P^*$.*
Proof: in Appendix A.6.

The asymptotic growth rate of the economy is again an inverted V-shape against the growth rate of the environmental tax, as illustrated in Figure 3. Note that the long-run amount of pollution stock P^* does not depend on the parameters of pollution accumulation (γ and δ_P). This is interesting because if δ_P is smaller, the effect of emissions on the pollution stock remains for a longer time, and therefore P_t would become higher, provided that the amount of emissions is the same; i.e., independence of P^* from these parameters implies that the amount of emissions must change with the parameters. In fact, from (39) and Proposition 4, we see that the level of output asymptotes to $Y_t = \tau_t E_t/\beta \to \tau_t \delta_P P^*/(\beta\gamma)$, which is lower when the effect of pollution remains for a longer time. This means that the amount of production, and therefore the amount of emissions, is adjusted so that the pollution stock becomes asymptotically P^*, which depends on the growth rate of τ but not on δ_P and γ. A larger δ_P (or γ) might temporarily increase the pollution stock P_t, but higher P_t would cause more frequent natural disasters, which destroy capital stocks and eventually lower the demand for the polluting input to the initial level. As a result, the difference in the accumulation process (δ_P and γ) has level effects on output, but not growth effects.

6.2 Welfare-maximizing policy under stock pollution

Next, let us turn to welfare maximization. The social planner maximizes welfare (9) subject to resource constraints (5), (37), and (38). In Appendix A.7, we solve the dynamic optimization problem and again find that the dynamics of K_t, H_t, u_t, and C_t in the welfare-maximizing path are exactly the same as those for the market equilibrium (Equations 4, 5, 17, 18 and 19). The optimal amount of emissions is given by:

$$\begin{aligned} & E_t = -\frac{\beta Y_t C_t^{-\theta}}{\gamma \lambda_t}, \\ & \text{where} \quad \lambda_t = -\int_t^\infty C_s^{-\theta} \left(\phi K_s + \psi \frac{(1-\alpha-\beta)Y_s}{Bu_s} \right) e^{-(\rho+\delta_P)(s-t)} ds \end{aligned} \tag{40}$$

which represents the shadow value of one additional unit of polluting stock, which is, of course, negative. The optimal stock of pollution is obtained by substituting (40) into (37).

Observe that the only difference between the market equilibrium and the welfare-maximizing path is between (39) and (40). In particular, when the government sets the tax rate by:

$$\tau_t = \frac{-\gamma\lambda_t}{C_t^{-\theta}} = \gamma \int_t^\infty e^{-\delta_P(s-t)} \left(\phi K_s + \psi \frac{(1-\alpha-\beta)Y_s}{Bu_s} \right) \left(\frac{C_s^{-\theta} e^{-\rho(s-t)}}{C_t^{-\theta}} \right) ds, \tag{41}$$

the market economy coincides with the welfare-maximizing path; i.e., (41) gives the optimal policy when pollution accumulates. When a firm emits pollution in year t, it has negative effects on the environment for all years $s \geq t$. The integral on the RHS represents the cumulative negative effects of emissions for year t. More precisely, the first part of the integral, $e^{-\delta_P(s-t)}$, is the portion of emissions remaining by year s. The second part, $\phi K_s + \psi(1-\alpha-\beta)Y_s/(Bu_s)$, is essentially the same as (33), representing the marginal negative effect of the polluting stock in year s. The final part, $C_s^{-\theta} e^{-\rho(s-t)}/C_t^{\theta}$, is the intertemporal marginal rate of substitution between year s and t, and represents how we discount the future.

While equation (41) has a natural interpretation, the implementation of the optimal policy is not obvious because the optimal tax rate in year t depends on the whole time path of the economy in the future, which in turn depends on the whole path of the tax rate in the future. Following Section 5, we solve this problem by focusing on the family of NABGPs. In the NABGPs, $Y_s = Y_t e^{g^*(s-t)}$, $C_s = C_t e^{g^*(s-t)}$, $K_s = K_t e^{g^*(s-t)}$, $u_t = u^*$, $Y_s/K_s = z^*$ hold asymptotically. Substituting these for (41) and calculating the integral,

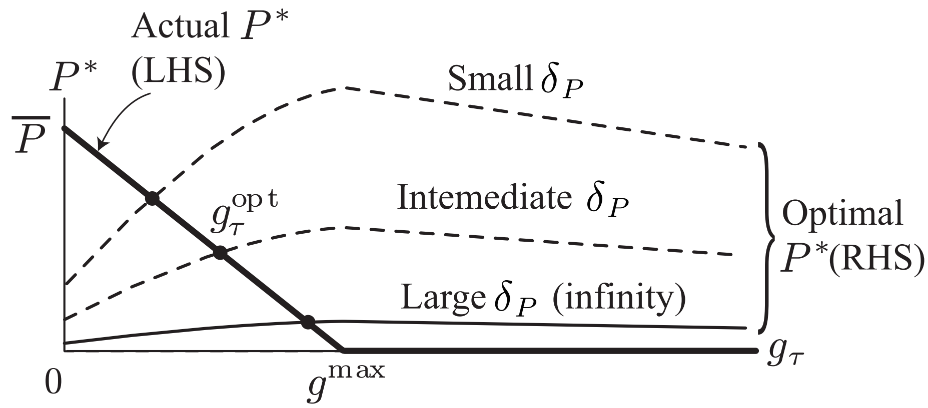

Figure 5: Optimal tax policy when pollution accumulates.

we can see that on a NABGP, the tax rate should be:

$$\tau_t = \frac{\gamma Y_t}{(\theta-1)g^* + \rho + \delta_P}\left(\frac{\phi}{z^*} + \psi\frac{1-\alpha-\beta}{Bu^*}\right). \tag{42}$$

From Equation (39) and Proposition 4, the environmental tax rate determines the amount of pollution as $P^* = \gamma E^*/\delta_P = \gamma\beta Y_t/\delta_P \tau_t$. Proposition 4 also implies that, in the market equilibrium with stock pollution, P^*, g^*, z^* and u^* are still determined by (29), (30) and (58)-(63) as functions of g_τ, and therefore can be represented as $P^*(g_\tau)$, $g^*(g_\tau)$, $z^*(g_\tau)$ and $u^*(g_\tau)$. Using these, the optimality condition (42) can be expressed as

$$P^*(g_\tau) = \beta\left(1 + \frac{(\theta-1)g^*(g_\tau) + \rho}{\delta_P}\right)\left(\frac{\phi}{z^*(g_\tau)} + \psi\frac{1-\alpha-\beta}{Bu^*(g_\tau)}\right)^{-1}. \tag{43}$$

The LHS of (43) is the actual amount of pollution stock under tax policy g_τ, while the RHS can be interpreted as the optimal amount of pollution stock. Both sides change with g_τ, and the optimal g_τ is such that the LHS and the RHS coincide. Figure 5 plots them against g_τ for the three different levels of δ_P. Observe that when δ_P is infinitely large, the term $((\theta-1)g^* + \rho)/\delta_P$ vanishes, and condition (43) coincides with (34). Thus, the optimal policy is the same as in Section 5. In fact, the baseline model in which the flow of pollution affects the disaster risk is a special case where both γ and δ_P are very large as the accumulation equation (37) reduces to $P_t = E_t$ when $\gamma = \delta_P \to \infty$. Intuitively, when the effect of emission depreciates very quickly, only the current use of the polluting input affects the risk of natural disasters. However, when δ_P is finite (i.e., when the effects of emissions remain for some time), the RHS is higher than in the previous case. Accordingly, the intersecting point in Figure 5 moves toward the upper left. The following proposition summarizes:

Proposition 5 *Suppose that pollution accumulates through (37) and (39), where δ_P is finite. Then, the asymptotic growth rate of the optimal tax rate, g_τ^{opt}, is lower than in Proposition 3. Moreover, as δ_P becomes smaller (i.e., when the effects of emissions remain for a longer time), g_τ^{opt} falls and the asymptotic pollution, P^*, rises. The optimal long-term rate of economic growth is also lower than in Proposition 3 and falls as δ_P becomes smaller.*

Previously, we have shown in Proposition 3 that in the case where pollution does not accumulate, the welfare-maximizing environmental policy is less strict than the growth-maximizing policy. Proposition 5 shows that, when emissions have a longer-lasting effect, it is optimal to adopt an even less strict environmental tax policy. This implies that the gap between the growth-maximizing policy and the welfare-maximizing policy is even larger when pollution accumulates.

We can again interpret this apparently paradoxical result in terms of time preference. When emissions have a longer effect, the larger part of the social cost of using the polluting input comes long after the benefit of using the polluting input (i.e., larger output) is realized. Thus, as long as the agent discounts the future, there is more social gain in accepting a high level of pollution stock and lower growth in the long run than where pollution does not accumulate. Specifically, observe that $(\theta - 1)g^* + \rho$ in condition (43) represents the rate of decrease in the marginal utility $C_t^{-(\theta-1)}e^{-\rho t}$. Because this expression is always positive on the NABGP (recall $\theta > 0$, $\rho > 0$ and $g^* \geq 0$), there is a benefit from frontloading output, which makes the optimal pollution in (43) higher than (34). As a result, it is optimal to increase the environmental tax more slowly.

7 Extension II: Non-insurable Risks

In most developed countries, life insurance is available to compensate for the loss of expected income when a household member dies or is disabled permanently. However, partial and temporary losses of human capital are generally more difficult to insure against, mainly because there is no objective and verifiable way to measure human capital. When a natural disaster hits an area and destroys some firms or an industry (or forces them to close for an extended period), it damages the firm-specific or even industry-specific human capital of workers in that area. Although the lifetime incomes of those workers would be significantly affected in such an event, insurance for this type of risk is rarely available. While previous sections assumed that the damages to human capital are fully insured, this section explores how non-insurable disaster risks to human capital affect the relationship between economic growth and the environmental tax policy. For simplicity, we ignore the accumulation of pollution.

Without insurance, households explicitly consider the possibility that they may lose a part of their human capital stock according to the stochastic process (14). Because natural disasters occur idiosyncratically, the unavailability of insurance also means that there are non-trivial ex-post distributions in the asset holdings and consumption among households. To make the analysis clear and tractable, we slightly change the way in which the revenue from environmental tax is distributed: this section assumes that the tax revenue, $\tau P_t = \beta Y_t$ from (6), is distributed as a consumption subsidy σC_t (or a reduction in consumption tax, if one exists), rather than a uniform transfer, so that the redistribution does not affect the intertemporal consumption decisions among households.[20] The constant subsidy rate σ is determined so that the government runs a balanced budget in the long run; i.e., $\sigma = \lim_{t\to\infty} \beta Y_t / C_t$, which is well defined in the NABGP, as we confirm later.[21] In this setting, the evolution of household assets a_{it}, except at the time when the household is hit by a natural disaster, is modified from (12) to

$$\frac{\dot{a}_{it}}{a_{it}} = (1-\eta_{it})r_t + \eta_{it}\left(B - \bar{\delta}_H + \frac{\dot{w}_t}{w_t}\right) - \bar{\sigma}\frac{c_{it}}{a_{it}}, \quad \bar{\sigma} \equiv 1 - \sigma. \tag{44}$$

7.1 Optimization of households under non-insurable risks

Every household i maximizes its lifetime expected utility (9) subject to budget constraints (14) and (44). In Appendix A.8, we show that this problem can be solved as a dynamic

[20]If perfect insurance is available, the uniform transfer and the constant rate consumption subsidy yield the same equilibrium outcome. Without insurance, however, the uniform transfer has a side effect of directly reducing the income risk of households by providing a stable flow of income. A constant-rate consumption subsidy does not affect households' intertemporal consumption decisions, as is confirmed by (47).

[21]If there is a government surplus in the transition, we assume that the government uniformly distributes the present value of the surplus $T_0 = \int_0^\infty (\beta Y_t - \sigma C_t) \exp(\int_0^t r_{t'} dt') dt$ at the beginning in a lump-sum fashion by issuing debts so that there are no government savings or debts in the long run. If the surplus is negative, the government levies a lump-sum tax $-T_0$ at the beginning.

programming (DP) problem in continuous time. From the first-order condition for the asset allocation η_{it}, we obtain:

$$B - \delta_H - \psi P_t + \dot{w}_t / w_t = r_t + (\bar{q} + \hat{q} P_t) R(\eta_{it}), \quad \text{where} \tag{45}$$

$$R(\eta_{it}) \equiv \mathbb{E}\Big[(1 - \eta_{it}\epsilon_{it})^{-\theta}\epsilon_{it}\Big] - \bar{\psi}, \ R(0) = 0, \ R'(\eta_{it}) > 0. \tag{46}$$

Condition (45) resembles the arbitrage condition (16), but it states that the expected return from holding human capital (represented by the LHS) should now be higher than the interest rate by $(\bar{q} + \hat{q}P_t)R(\eta_{it})$ to compensate for the exposure to the non-insurable risk. When a household is hit by a natural disaster, it loses a fraction $\eta_{it}\epsilon_{it}$ of its total assets and reduces consumption from c_{it} to $(1 - \eta_{it}\epsilon_{it})c_{it}$. As a result, the marginal utility increases by a factor of $(1 - \eta_{it}\epsilon_{it})^{-\theta} > 1$. Function $R(\eta_{it})$ shows that, in terms of utility, the cost of disaster damage of a given size ϵ_{it} is multiplied by $(1 - \eta_{it}\epsilon_{it})^{-\theta}$, compared to the case where the household is able to pay an insurance premium to avoid such a change in marginal utility. Because this additional loss is incurred with probability $(\bar{q} + \hat{q}P_t)$ per unit time, households require a "risk premium" of $(\bar{q} + \hat{q}P_t)R(\eta_{it})$ to hold human capital.

The risk premium function (46) depends only on the damage distribution $\Psi(\epsilon_{it})$ and the relative risk aversion θ. Figure 6(i) and (ii) depict various density functions for $\Psi(\epsilon_{it})$[22] and the corresponding shapes of the function $R(\eta_{it})$. Observe that $R(\eta_{it})$ is upward sloping and convex because increased exposure to the non-insurable risk raises the risk premium. In addition, even when $\bar{\psi} \equiv \mathbb{E}[\epsilon_{it}]$ is the same, a more dispersed damage distribution increases the risk premium because it enhances the extreme possibilities in which the household loses most of its human capital. Because $R(\eta_{it})$ is monotonic in η_{it}, there exists a unique value of η_{it} that satisfies the condition (45), given prices and pollution. Because this optimal allocation is the same for all households, we simply write it as η_t.[23]

Next, from the envelope condition for the DP problem, we obtain the evolution of

[22] We choose the Beta distribution as an example because it take various shapes depending on its parameters, and also because its support is the interval $(0, 1)$, which is consistent with our assumption for the damage distribution $\Psi(\epsilon_{it})$. Its probability density function is proportional to $\epsilon_{it}^{a-1}(1 - \epsilon_{it})^{b-1}$, where we choose parameters a and b to match the specified mean and standard deviation.

[23] If a household loses a portion of human capital due to a natural disaster, its η_{it} might temporarily fall below the optimal value. However, the household then regains the optimal asset allocation through intensive education by spending its savings. For simplicity, we assume that this adjustment occurs quickly so that (almost) all households share the same η_t.

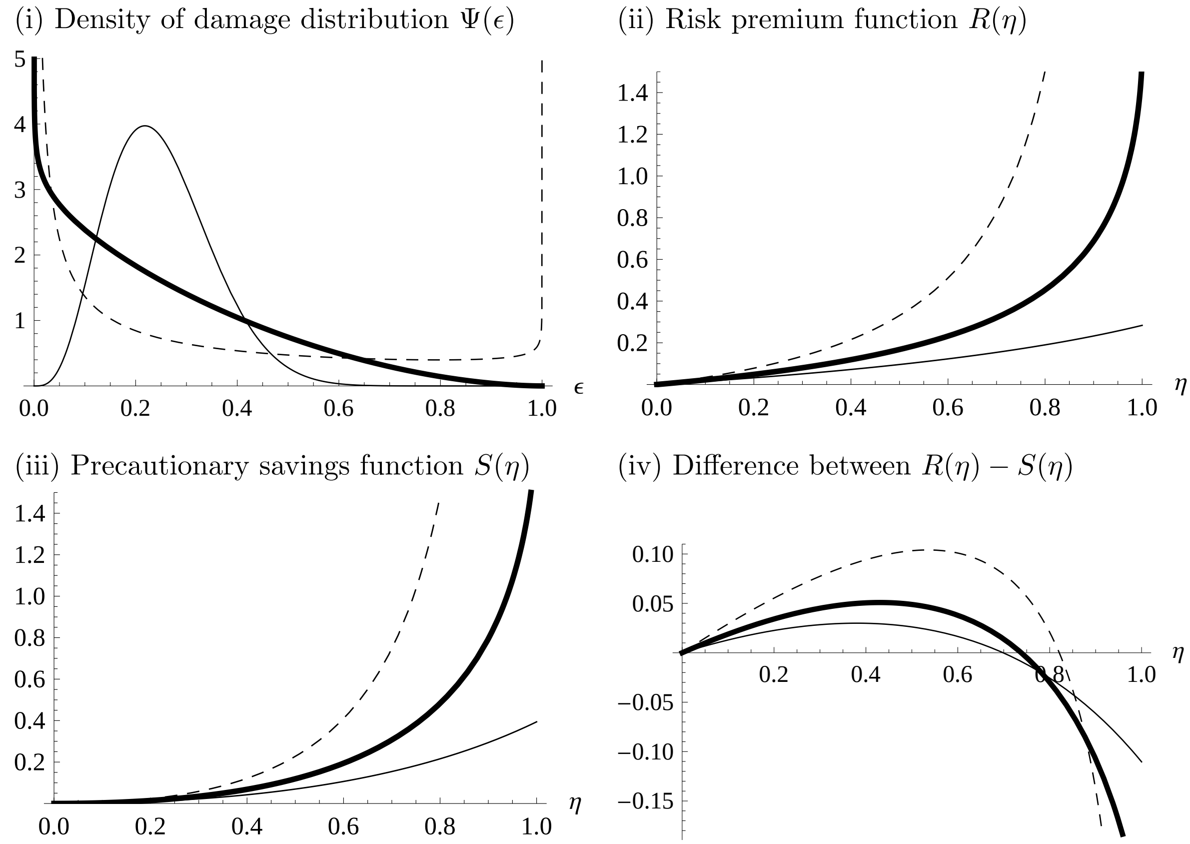

Figure 6: Examples of risk premium and precautionary savings functions. Damage distribution $\Psi(\epsilon)$ is specified as Beta distributions with mean $\bar{\psi} = .25$ and three different standard deviations: 0.1 (thin curve), 0.2 (thick curve), and 0.3 (dashed curve). Risk aversion parameter θ is set to 2.

consumption c_{it} for each household:

$$-\theta \frac{\dot{c}_{it}}{c_{it}} + (\bar{q} + \hat{q}P_t)\Big\{\mathbb{E}\big[(1 - \eta_t \epsilon_{it})^{-\theta}\big] - 1\Big\} = \rho - r_t, \tag{47}$$

which must hold for all i and t except the time when household i is hit by natural disasters. When compared to the standard Keynes-Ramsey rule, $-\theta \dot{c}_{it}/c_{it} = \rho - r_t$, (47) has an extra term (the second term on the LHS) that represents the expected change in the marginal utility due to the risk of natural disasters. As explained above, each household is hit by a disaster with probability $q_t = (\bar{q} + \hat{q}P_t)$ per unit time, and at that time consumption drops from c_{it} to $(1 - \eta_t \epsilon_{it})c_{it}$. Because natural disasters occur idiosyncratically, we can calculate the aggregate fall in consumption due to natural disasters per unit time as: $\int q_t\big\{c_{it} - (1 - \eta_t \epsilon_{it})c_{it}\big\}di = (\bar{q} + \hat{q}P_t)\bar{\psi}\eta_t C_t$. Aggregating the individual evolution of consumption (47) and then subtracting the above fall, we obtain the evolution of the aggregate consumption:

$$\frac{\dot{C}_t}{C_t} = \frac{1}{\theta}\Big[r_t - \rho + (\bar{q} + \hat{q}P_t)S(\eta_t)\Big], \quad \text{where} \tag{48}$$

$$S(\eta_t) \equiv \mathbb{E}\big[(1 - \eta_t \epsilon_{it})^{-\theta}\big] - 1 - \theta\bar{\psi}\eta_t, \;\; S(0) = 0, \;\; S'(\eta_t) > 0. \tag{49}$$

When compared to the case where perfect insurance is available (see equation 18, where $r_t = \alpha Y_t/K_t - \delta_K - \phi P_t$), condition (48) implies that the non-insurable risks lead to more savings, so the aggregate consumption growth is faster by $(1/\theta)(\bar{q} + \hat{q}P_t)S(\eta_t)$. This is "precautionary saving" in the sense that the non-insurable risk induces households to save more as a precaution against possible losses of human capital by natural disasters.[24] Thus, we call $S(\eta_t)$ the precautionary saving function. Figure 6(iii) shows the shapes of function $S(\eta_t)$ for three examples of damage distributions. The shapes are similar to $R(\eta_t)$, although they tend to have higher curvatures. Naturally, a higher exposure to risk (a higher η_t) and a more dispersed damage distribution will lead to more precautionary savings.

7.2 NABGP for market equilibrium with non-insurable risks

Similarly to Section 4, let us focus on the nondegenerate, asymptotically balanced growth paths (NABGP) where the growth rates of Y_t, K_t, H_t, C_t and τ_t are asymptotically constant and $u^* \equiv \lim_{t\to\infty} u_t$, $z^* \equiv \lim_{t\to\infty} Y_t/K_t$ and $\chi_t \equiv \lim_{t\to\infty} C_t/K_t$ are well defined. Because all households have the same η_t in the presence of non-insurable risks, the definition of η_{it} in (13) can be aggregated for all i. Then, using the market clearing conditions,[25] $\int s_{it} di = K_t$ and $\int h_{it} di = H_t$, and substituting w_t from (7), we see that η_t is asymptotically constant at

$$\eta^* \equiv \lim_{t\to\infty} \eta_t = \frac{(1 - \alpha - \beta)z^*}{Bu^* + (1 - \alpha - \beta)z^*}. \tag{50}$$

The behavior of firms is not affected by unavailability of insurance because firms only care for expected profits. The resource constraints are also the same as the benchmark

[24] Lord and Rangazas (1998) quantitatively examined the extent to which the riskiness of human capital investment increases the saving rate, although they did not explicitly consider natural disasters.

[25] In transition, the equilibrium of the credit market requires $\int s_{it} di - D_t = K_t$, where the government debt D_t evolves according to $\dot{D}_t = \sigma C_t - \beta Y_t + r_t D_t$. On the NABGP, $Y_t/C_t = (Y_t/K_t)/(C_t/K_t)$ converges to a constant value z^*/χ^*, and the government can achieve a balanced budget by setting $\sigma = \beta z^*/\chi^*$. In addition, because T_0 ($\equiv -D_0$) is chosen to match the present value of government surplus during the transition, D_t converges to zero in the long run. Therefore, $\int s_{it} di = K_t$ holds on the NABGP.

model. Therefore, the equilibrium conditions for the NABGP are the same as (24)–(28), except that, from (45) and (48), the arbitrage condition (26) and the Keynes–Ramsey rule (27) should be replaced, respectively, by

$$-\frac{\beta}{1-\alpha-\beta}g_\tau = (\alpha z^* - \delta_K - \phi P^*) - (B - \delta_H - \psi P^*) + (\bar{q} + \hat{q}P^*)R(\eta^*), \tag{51}$$

$$\theta g^* = (\alpha z^* - \delta_K - \phi P^*) - \rho + (\bar{q} + \hat{q}P^*)S(\eta^*). \tag{52}$$

The six conditions (24), (25), (28), and (50)–(52) determine six unknowns (g^*, z^*, χ^*, u^*, P^*, η^*) on the NABGP as a function of the tax policy $g_\tau \equiv \lim_{t\to\infty} \dot{\tau}_t/\tau_t \geq 0$.

Let us illustrate how the unavailability of insurance influences the relationship between the environmental tax policy g_τ and the asymptotic growth rate g^* under a given value of η^*. From conditions (28), (51), and (52), the asymptotic economic growth rate on the NABGP g^* can be calculated as:

$$G^*(g_\tau;\eta^*) = \begin{cases} g_\tau \text{ if } g_\tau \leq G^{\max}(\eta^*) \\ \frac{1}{\theta}\left[B - \delta_H - \rho - \frac{\beta}{1-\alpha-\beta}g_\tau - \bar{q}\left\{R(\eta^*) - S(\eta^*)\right\}\right] \text{ if } g_\tau \geq G^{\max}(\eta^*), \end{cases} \tag{53}$$

$$\text{where } G^{\max}(\eta^*) = \left(\theta + \frac{\beta}{1-\alpha-\beta}\right)^{-1}\left(B - \delta_H - \rho - \bar{q}\left\{R(\eta^*) - S(\eta^*)\right\}\right). \tag{54}$$

We also obtain $P^* = 0$ when $g_\tau \geq G^{\max}(\eta^*)$. This result resembles the case of perfect insurance (equations 29-31), except that (53) and (54) depend on the difference between the risk premium and precautionary saving functions. Note also that the solution depends on η^*, which is endogenously determined in equilibrium. Let us focus on the range of g_τ under which the NABGP uniquely exists, where η^* should be representable as a function $\eta^*(g_\tau)$. When $\eta^*(g_\tau)$ is substituted for η^* in the second line of equation (53), it is clear that the relationship between g_τ and g^* for the case of $g_\tau \geq G^{\max}(\eta^*)$ is no longer linear. However, as long as

$$-\bar{q}\left[R'(\eta^*(g_\tau)) - S'(\eta^*(g_\tau))\right]\frac{d\eta^*(g_\tau)}{dg_\tau} < \frac{\beta}{1-\alpha-\beta} \text{ whenever } g_\tau > G^{\max}(\eta^*(g_\tau)), \tag{55}$$

function $G^*(g_\tau;\eta^*(g_\tau))$ is decreasing in g_τ for $g_\tau > G^{\max}(\eta^*(g_\tau))$, and hence the tax policy that attains $g_\tau = G^{\max}(\eta^*(g_\tau))$ maximizes the long-term growth. We found that condition (55) is likely to be satisfied under reasonable parameter values.[26]

[26] Because $P^* = 0$ for all $g_\tau > G^{\max}(\eta^*)$, the marginal effect of environmental tax policy on equilibrium is limited. In addition, as shown in figure 6(iv), the absolute value of $R'(\eta^*) - S'(\eta^*)$ is not very large as long as η^* is reasonably far from 1, which is true in equilibrium.

Under condition (55), function $G^{\max}(\eta^*)$ in (54) simultaneously represents the growth-maximizing rate of tax increase and the highest attainable long-run growth rate. Equation (54) can be interpreted intuitively: the risk premium effect $R(\eta^*)$ skews the investments away from the human capital, whereas the precautionary saving effect $S(\eta^*)$ increases the overall investment. If the risk premium effect is stronger, the absence of insurance lowers the human capital investment,[27] and hence the highest attainable long-run growth rate $G^{\max}(\eta^*)$. Because slower output growth implies fewer uses of P_t, the growth-maximizing environmental policy should also be milder (i.e., a lower g_τ). To the contrary, if the precautionary saving effect $S(\eta^*)$ is stronger, the absence of perfect insurance makes possible higher long-term economic growth. However, even when $G^{\max}(\eta^*)$ is higher, the first line of (53) implies that the higher growth is realized only when the government implements a stricter environmental policy (i.e., a higher g_τ). If g_τ is unchanged, the increased investments will induce firms to use more P_t until the increased damages to physical and human capital eventually nullify the increased savings.

7.3 Relative significance of risk premium and precautionary savings

In the following, we examine the relative significance of the two effects under a given set of parameters and damage distribution. From the definitions of $R(\eta^*)$ and $S(\eta^*)$ in (46) and (49), we can show that the risk premium effect dominates the precautionary saving effect if and only if η is smaller than a critical value $\bar{\eta}$:

Lemma 2 *For any damage distribution of $\epsilon_{it} \sim \Psi(\epsilon_{it})$, whose support is within interval $(0, 1)$, and for any risk aversion parameter $\theta > 1$, there exists a unique value of $\bar{\eta}$ such that $R(\bar{\eta}) = S(\bar{\eta})$ holds. In addition, $R(\eta) > S(\eta)$ holds for $\eta \in (0, \bar{\eta})$, and $R(\eta) < S(\eta)$ holds for $\eta \in (\bar{\eta}, 1]$.*
Proof: in Appendix A.9.

Figure 6(iv) depicts the representative shapes of $R(\eta) - S(\eta)$, which confirms that the precautionary savings effect is stronger only if η^* is larger than a certain threshold. We next derive the value of η^* under a growth-maximizing policy through a guess-and-verify method. Let us start with a guess $\tilde{\eta} \in (0, 1)$ of unknown η^*, and suppose that

[27] Although the risk premium effect may increase the physical capital investment, it contributes to economic growth only in a transitory manner because the production sector is subject to decreasing marginal product with respect to physical capital.

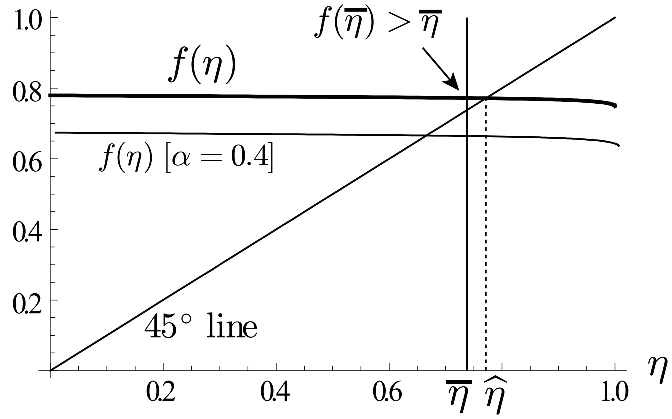

Figure 7: The fraction of human capital in the total assets under the growth-maximizing policy (represented as $\hat{\eta}$). Under condition (57), function $f(\eta)$ and the 45-degree line have a unique intersection at $\hat{\eta}$. If $f(\bar{\eta}) > \bar{\eta}$, then the intersection must be to the right of $\bar{\eta}$, and vice versa. Parameters are the same as in Figure 3, and $\Psi(\epsilon_{it})$ is specified as a Beta distribution with mean $\bar{\psi} = .25$ and standard deviation 0.2. In this setting, we obtain $\bar{\eta} = .738$, $\hat{\eta} = .772 > \bar{\eta}$. If α is higher, at 0.4, we obtain $\hat{\eta} = .672 < \bar{\eta}$.

the government sets the implied growth-maximizing policy $g_\tau = G^{\max}(\tilde{\eta})$, which means $g^* = g_\tau = G^{\max}(\tilde{\eta})$ and $P^* = 0$. Suppose also that households take $\tilde{\eta}$ as given. Then, u^* and z^* are calculated from (25) and (52), where $\tilde{\eta}$ is substituted for η^*. By substituting these results into (50), we obtain the actual asset allocation η^* on the NABGP as a function of the initial guess $\tilde{\eta}$:

$$\eta^* = f(\tilde{\eta}) \equiv \kappa \left(\kappa + \frac{\bar{\omega} + \bar{q}\omega\left[R(\tilde{\eta}) - S(\tilde{\eta})\right]}{\bar{\xi} - \bar{q}\left[(1-\xi)R(\tilde{\eta}) + \xi S(\tilde{\eta})\right]} \right)^{-1}, \tag{56}$$

where we define constants by $\omega \equiv (1-\alpha)/((1-\alpha-\beta)\theta+\beta) \in (0,1)$, $\xi \equiv \beta/((1-\alpha-\beta)\theta+\beta) \in (0,1)$, $\bar{\omega} \equiv (1-\omega)(B-\delta_H) + \omega\rho > 0$, $\bar{\xi} \equiv (1-\xi)(B-\delta_H) + \xi\rho + \delta_K > 0$, and $\kappa = (1-\alpha-\beta)/\alpha > 0$, all of which depend only on parameters. If $f(\tilde{\eta})$ coincides with $\tilde{\eta}$, then the initial guess $\tilde{\eta}$ was correct. Formally stated, a tax policy g_τ attains the highest long-term growth if, and only if, $\eta^*(g_\tau) \in (0,1)$ is a fixed point of function $f(\eta)$.

Now we need to check whether the above fixed point is lower or higher than $\bar{\eta}$. Definition (56) implies that as long as $\bar{q}$ is reasonably small, function $f(\eta)$ is continuous in η and satisfies[28]

$$f(\eta) \in (0,1) \text{ and } f'(\eta) < 1 \text{ for all } \eta \in [0,1]. \tag{57}$$

Given condition (57), the intermediate value theorem implies that the fixed point of (56) uniquely exists, which we denote by $\hat{\eta}$. In addition, as depicted in Figure 7, $\hat{\eta}$ is larger (or smaller) than $\bar{\eta}$ if and only if $f(\bar{\eta}) = \kappa(\kappa + \bar{\omega}/(\bar{\xi} - \bar{q}R(\bar{\eta})))^{-1}$ is larger (or smaller) than $\bar{\eta}$. Combining this result with Lemma 2, we obtain the following proposition.

Proposition 6 *Suppose that the disaster damages to human capital are not insurable and that conditions (55) and (57) are satisfied. Then, the long-term rate of growth is unimodal with respect to g_τ and maximized at $g_\tau = G^{\max}(\hat{\eta}) \equiv (\theta + \frac{\beta}{1-\alpha-\beta})^{-1}(B - \delta_H - \rho - \bar{q}\{R(\hat{\eta}) - S(\hat{\eta})\})$, where $\hat{\eta}$ is the fixed point of function $f(\eta)$ in (56). If $f(\bar{\eta}) = \kappa(\kappa + \bar{\omega}/(-\bar{q}R(\bar{\eta}) + \bar{\xi}))^{-1}$ is larger than $\bar{\eta}$ defined in Lemma 2, the precautionary savings effect $S(\hat{\eta})$ dominates the risk premium effect $R(\hat{\eta})$, so the growth-maximizing rate of tax increase, $G^{\max}(\hat{\eta})$, is higher than $g^{\max}$ in Proposition 2. The opposite holds if $f(\bar{\eta}) < \bar{\eta}$.*

Proposition 6 states that the basic relationship between the environmental tax and the long-term growth is preserved under the presence of non-insurable risks, but the precise growth-maximizing tax policy can be either more or less strict than the benchmark case. As depicted in Figure 7, we confirmed that both cases are possible depending on the parameter values. In particular, if the parameters satisfy $f(\bar{\eta}) > \bar{\eta}$, the maximized rate of growth is higher than the case with perfect insurance. However, this only means that the unavailability of insurance widens the discrepancy between the equilibrium allocation under growth-maximizing environmental policy and the welfare-maximizing allocation because the latter is unchanged given the resource constraint (see Proposition 3). Thus, even when the unavailability of insurance creates an additional possibility of higher growth through a stricter environmental policy, it is not likely to improve the welfare of agents.

[28]For a reasonably small $\bar{q} > 0$, both the numerator and the denominator of the fraction in (56) become positive, which is a sufficient condition for $f(\eta) \in (0,1)$ and the continuity of $f(\eta)$ for all $\eta \in [0,1]$. In addition, η affects the value of $f(\eta)$ only through $R(\eta)$ and $S(\eta)$, and both are multiplied by $\bar{q}$. Thus, the gradient of $f(\eta)$ is unlikely to exceed 1.

8 Conclusion

In this paper, we analyzed the sustainability of economic growth in a two-sector endogenous growth model when taking into account the risk of natural disasters. Here, polluting inputs are necessary for production, though they also intensify the risk of natural disasters. In this setting, we obtained the following results.

First, economic growth can be sustained in the long run only if the per unit tax on the polluting input increases over time. Although economic growth *ceteris paribus* induces private firms to use more of the polluting input, this environmental policy can lead firms to use more human capital (e.g., by investing in alternative technologies), which decreases their reliance on polluting inputs, and thereby prevents the risk of disaster from rising to a critical level. However, it should be noted that we do not consider the cost associated with extracting resources or the finiteness of these inputs. If the cost is significant and changes for some reason, the environmental tax rate must be adjusted to absorb these changes. A next step in our research agenda would be to integrate the analysis of natural disasters with a study of the finiteness of natural resources. This is clearly beyond the scope of this first attempt.

Second, the long-term rate of economic growth follows an inverted V-shaped curve relative to the growth rate of the environmental tax. When the rate of environmental tax is currently slowly growing, its acceleration will reduce the asymptotic level of emissions and the risk of natural disasters. This process enhances the incentive to save and hence promotes economic growth. When the rate of environmental tax is already fast growing, the asymptotic level of pollution is fairly small so that further acceleration of the environmental tax excessively impairs the productivity of private firms. This works against economic growth. Therefore, economic growth can be maximized with the choice of the most gradual increase in the environmental tax rate that minimizes the amount of pollution in the long run. We also find that, if the disaster damages to human capital are not insurable, the growth-maximizing environmental tax policy is affected by the relative strength of the risk premium effect and the precautionary savings effect: the former skews the investment away from human capital, whereas the latter increases the overall savings as a precaution. If the precautionary savings effect dominates, economic growth can be further accelerated by raising the speed of the tax increase.

Third, social welfare is maximized under a less strict (i.e., more slowly increasing) environmental tax policy than the growth-maximizing policy. This may appear paradoxical in that welfare considerations justify more pollution than when growth is the foremost policy concern. This is because maximization of the long-term rate of growth requires the minimization of the asymptotic level of pollution, but this can only be achieved only in the long run. As long as people discount the future, aiming for this ultimate goal would be too costly in terms of the efficiency loss that must be incurred in the transition. Thus, a less strict environmental policy is more desirable in terms of the discounted sum of expected utility. Moreover, when pollutants accumulate gradually and remain in the air for longer, the transition process takes more time and, therefore, the welfare-maximizing environmental tax policy is milder.

Appendix

A.1 Proof of Lemma 1

Suppose that $g^* > g_\tau$ (i.e., $\lim_{t\to\infty} \dot{Y}_t/Y_t > \lim_{t\to\infty} \dot{\tau}_t/\tau_t$). Then, $P_t = \beta Y_t/\tau_t \to \infty$. From (5) and $u_t \leq 1$, this means $\dot{H}_t/H_t \leq B - \delta_H - \psi P_t \to -\infty$. This contradicts with the definition of the NABGP, in which $g_H \equiv \lim_{t\to\infty} \dot{H}_t/H_t$ is finite.

A.2 Derivation of (22)

We first establish the following lemma.

Lemma 3 *In the model with perfect insurance, a sufficient condition for the TVC (19) is that* $\lim_{t\to\infty}((1-\alpha)(Y_t/K_t)-(C_t/K_t))$ *and* $\lim_{t\to\infty} -Bu_t$ *are strictly negative. A necessary condition is that they are not strictly positive.*

Proof: From (4) and (18), the growth rate of $k_t c_t^{-\theta} e^{-\rho t}$ is $(1-\alpha)(Y_t/K_t) - (C_t/K_t)$. Similarly, from (5), (16) and the Keynes-Ramsey condition $-\theta(\dot{C}_t/C_t) = \rho - r_t$, the growth rate of $h_t(w_t/B)c_t^{-\theta}e^{-\rho t}$ is $-Bu_t$. The TVC (19) is necessarily satisfied if these two growth rates are strictly negative, and it cannot be satisfied if they are strictly positive. ■

We use Lemma 3 to show (22) and asymptotic constancy of u_t, Y_t/K_t, C_t/K_t. Because $\dot{H}_t/H_t$ and P_t are asymptotically constant on the NABGP, Equation (5) implies that u_t must also be asymptotically constant ($g_u \leq 0$). In addition, the TVC requires g_u

not to be strictly negative from Lemma 3. Therefore, $g_u = 0$. Next, as $\dot{C}_t/C_t$ and P_t are asymptotically constant, Equation (18) implies that the value of Y_t/K_t must also be constant in the long run. This means that the growth rate of Y_t/K_t is zero or negative ($g^* \leq g_K$). However, if $Y_t/K_t \to 0$, Equation (4) implies $\dot{K}_t/K_t < 0$, and thus $Y_t = (Y_t/K_t)\cdot K_t \to 0$, which is inconsistent with our definition of a NABGP ($g_C \geq 0$). Therefore, $g_K = g^*$. Finally, given that $\dot{K}_t/K_t$ and Y_t/K_t are asymptotically constant, Equation (4) in turn implies that C_t/K_t must also be asymptotically constant ($g_C \leq g_K$). However, if $C_t/K_t \to 0$, $C_t/K_t < (1-\alpha)(Y_t/K_t)$ will hold in the long run (recall that Y_t/K_t will not converge to zero), which violates the TVC from Lemma 3. Therefore, $g_C = g_K$ $(= g^*)$.

A.3 Derivation of the NABGP in the benchmark model

Given g_τ, we first examine the possibility of Case 1 ($P^* \geq 0$ and $g^* = g_\tau$). Substituting $g^* = g_\tau$ into (26) and (27), we obtain $P^* = \frac{1}{\psi}\Big[B - \delta_H - \rho - \Big(\theta + \frac{\beta}{1-\alpha-\beta}\Big)g_\tau\Big]$. From this, it turns out that $P^* \geq 0$ holds if and only if $g_\tau \leq g^{\max}$, where $g^{\max}$ is defined in (31). Substituting $g^* = g_\tau$ into (24)-(27), we also obtain:

$$z^* = \frac{1}{\alpha}\left(\theta g_\tau + \delta_K + \phi P^* + \rho\right), \tag{58}$$

$$\chi^* = \frac{1}{\alpha}\Big((\theta-\alpha)g_\tau + (1-\alpha)(\delta_K + \phi P^*) + \rho\Big), \tag{59}$$

$$u^* = \frac{1}{B}\left((\theta-1)g_\tau + \rho\right). \tag{60}$$

Because the NABGP requires $g_C \geq 0$, we need $g_\tau = g^* \geq 0$. Therefore, a NABGP in Case 1 is possible only if $g_\tau \in [0, g^{\max}]$. Substituting the above P^* into (58)-(60) shows that $z^* > 0$, $\chi^* > 0$, $u^* \in (0,1)$, and $(1-\alpha)z^* - \chi^* < 0$ holds for all $g_\tau \in [0, g^{\max}]$. From Lemma 3 in Appendix A.2, the last two inequalities imply that the transversality condition (19) is satisfied.

Next, we examine the possibility of Case 2 ($P^* = 0$ and $g^* < g_\tau$). Substituting $P^* = 0$ for (26) and (27) yields $g^* = \frac{1}{\theta}\Big(B - \delta_H - \rho - \frac{\beta}{1-\alpha-\beta}g_\tau\Big)$. It satisfies condition $g^* < g_\tau$ only if $g_\tau > g^{\max}$. Substituting $P^* = 0$ and the above g^* into (24)-(27), we obtain:

$$z^* = \frac{1}{\alpha}\Big(B + \delta_K - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big), \tag{61}$$

$$\chi^* = \Big(\frac{1}{\alpha} - \frac{1}{\theta}\Big)\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big) + \frac{1-\alpha}{\alpha}\delta_K + \frac{\rho}{\theta}, \tag{62}$$

$$u^* = \frac{1}{B\theta}\Big[(\theta-1)\Big(B - \delta_H - \frac{\beta}{1-\alpha-\beta}g_\tau\Big) + \rho\Big]. \tag{63}$$

Because a NABGP requires $g^* \geq 0$, we need $g_\tau \leq g^{\lim} \equiv (1-\alpha-\beta)\beta^{-1}(B-\delta_H-\rho)$. Therefore, a NABGP in Case 2 is possible only if $g_\tau \in (g^{\max}, g^{\lim}]$. Note that (61)-(63) implies $z^* \geq 0$, $\chi^* > 0$, $(1-\alpha)z^* - \chi^* < 0$, and $u^* \in (0,1)$ for all $g_\tau \in (g^{\max}, g^{\lim}]$. From Lemma 3, the latter two inequalities imply that the transversality condition (19) is satisfied.

Observe that those two possibilities are mutually exclusive—a NABGP in Case 1 exists if and only if $g_\tau \in [0, g^{\max}]$, whereas a NABGP in Case 2 exists if and only if $g_\tau \in (g^{\max}, g^{\lim}]$. Therefore, a NABGP uniquely exists whenever $g_\tau \in [0, g^{\lim}]$.

A.4 Proof of stability in Proposition 2

The equilibrium path is characterized by a four-dimensional dynamics system of $\{K_t, H_t, u_t, C_t\}$, where the laws of motion for these variables are given by (4), (5), (17), and (18). By making use of (6), (8), and $N_t = u_t H_t$, the values of Y_t and P_t appearing in these laws of motion can be expressed in terms of K_t, H_t, u_t and τ_t, where the motion of τ_t is given exogenously by the government. In this dynamic system, K_t and H_t are predetermined state variables, whereas u_t and C_t are jumpable. Therefore, the system is both stable and determinate when it has a stable manifold of dimension two. For convenience, we transform this system into another four-dimensional system in $\{u_t, \chi_t, z_t, P_t\}$, where $\chi_t \equiv C_t/K_t, z \equiv Y_t/K_t$ and $P_t \equiv \beta Y_t/\tau_t$. This transformed system is equivalent to the original system, because $\{K_t, H_t, u_t, C_t\}$ can be represented in terms of $\{u_t, \chi_t, z_t, P_t\}$ as $K_t = \tau_t P_t/(\beta z_t)$, $C_t = \tau_t P_t \chi_t/(\beta z_t)$, and $H_t = \left(\tau^{1/(1-\beta)-\hat{\alpha}}/\tilde{A}\right)^{1/(1-\hat{\alpha})} z_t^{\hat{\alpha}/(1-\hat{\alpha})} P_t/(\beta u_t)$. Therefore, saddle stability (and determinacy) can be established by confirming that this transformed system has a two-dimensional stable manifold. Using equilibrium conditions (4), (5), (6), (17) and (18), we can write the dynamics of the system as:

$$\dot{u_t} = u_t \left(B u_t - \chi_t + \beta z_t + \Lambda P_t + \frac{1-\alpha-\beta}{\alpha}(B+\delta_K-\delta_H) - \frac{\beta}{\alpha} g_\tau \right), \tag{64}$$

$$\dot{\chi_t} = \chi_t \left(\chi_t - \frac{\theta-\alpha}{\theta} z_t + \frac{\theta-1}{\theta}\phi P_t - \frac{\rho}{\theta} + \frac{\theta-1}{\theta}\delta_K \right), \tag{65}$$

$$\dot{z_t} = z_t \left(-(1-\alpha-\beta) z_t + \Lambda P_t + \frac{1-\alpha-\beta}{\alpha}(B+\delta_K-\delta_H) - \frac{\beta}{\alpha} g_\tau \right), \tag{66}$$

$$\dot{P_t} = P_t \left(-\chi_t + \frac{\alpha + (1-\alpha-\beta)\beta}{1-\beta} z_t + \Omega P_t + \frac{1-\alpha-\beta}{\alpha} B - \frac{\alpha+\beta}{\alpha} g_\tau + \frac{(1-2\alpha-\beta)\delta_K - (1-\alpha-\beta)\delta_H}{\alpha} \right) \tag{67}$$

where Λ and Ω are constants defined by $\Lambda \equiv (1-\alpha-\beta)(\phi-\psi)/\alpha$ and $\Omega \equiv ((1-2\alpha-\beta)\phi-(1-\alpha-\beta)\psi)/\alpha$.

We first examine the stability of the NABGP for the case of $g_\tau \in [0, g^{\max}]$. In this case steady state of the transformed system, denoted by $\{u^*, \chi^*, z^*, P^*\}$, is given by the first line of (30) and (58)–(60). Applying a first-order Taylor expansion of equations (64)–(67) around this steady-state yields:

$$\begin{bmatrix} \dot{u}_t \\ \dot{\chi}_t \\ \dot{z}_t \\ \dot{P}_t \end{bmatrix} \simeq \left[\begin{array}{c|ccc} u^*B & -u^* & \beta u^* & \Lambda u^* \\ \hline 0 & & & \\ 0 & & J_1 & \\ 0 & & & \end{array}\right] \begin{bmatrix} u_t - u^* \\ \chi_t - \chi^* \\ z_t - z^* \\ P_t - P^* \end{bmatrix}, \tag{68}$$

where,

$$J_1 \equiv \begin{bmatrix} \chi^* & -\frac{\theta-\alpha}{\theta}\chi^* & \frac{(\theta-1)\phi}{\theta}\chi^* \\ 0 & -(1-\alpha-\beta)z^* & \Lambda z^* \\ -P^* & \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta}P^* & \Omega P^* \end{bmatrix}.$$

We want to show that the Jacobian matrix of (68) has two positive and two negative eigenvalues. From the block-triangular structure of the matrix, one eigenvalue is $u^*B > 0$, and the other three are given by the eigenvalues of the submatrix J_1. The characteristic equation for J_1 is:

$$-\lambda^3 + \mathrm{tr}(J_1)\lambda^2 - \mathrm{M}(J_1)\lambda + \det(J_1) = 0, \tag{69}$$

where $\mathrm{tr}(J_1)$ is the trace of J_1, $\mathrm{M}(J_1)$ the sum of the principal minors, and $\det(J_1)$ the determinant. These are given by:

$$\begin{aligned}
\mathrm{tr}(J_1) &= \left\{\frac{\theta+\beta-\alpha}{\alpha} - \frac{(1-2\alpha)\phi}{\alpha\psi}\Big(\theta + \frac{\beta}{1-\alpha-\beta}\Big)\right\} g_\tau + \frac{\beta}{\alpha}\delta_K + \frac{\alpha+\beta}{\alpha}\rho \\
&\quad + \left\{\frac{(1-2\alpha)\phi}{\alpha\psi} - \frac{1-\alpha-\beta}{\alpha}\right\}(B-\rho-\delta_H), \\
\mathrm{M}(J_1) &= \begin{vmatrix} \chi^* & -\frac{\theta-\alpha}{\theta}\chi^* \\ 0 & -(1-\alpha-\beta)z^* \end{vmatrix} + \begin{vmatrix} -(1-\alpha-\beta)z^* & \Lambda z^* \\ \frac{\alpha+\beta(1-\alpha-\beta)}{1-\beta}P^* & \Omega P^* \end{vmatrix} + \begin{vmatrix} \chi^* & \frac{(\theta-1)\phi}{\theta}\chi^* \\ -P^* & \Omega P^* \end{vmatrix} \\
&= -\frac{1-\alpha-\beta}{\alpha}\left\{(\phi-\psi)(z^*-\chi^*) + \frac{\alpha\phi\chi^*}{\theta(1-\alpha-\beta)}\right\} \\
&\quad - \frac{1-\alpha-\beta}{\alpha}\cdot\frac{z^*}{P^*}\big\{(\theta-\alpha)g_\tau + (1-\alpha)\delta_K + (1-2\alpha)\phi P^* + \rho\big\}, \\
\det(J_1) &= \frac{\psi(1-\alpha-\beta)}{\theta} z^*\chi^*P^*,
\end{aligned}$$

We determine the sign of the real parts of the roots of (69) based on Theorem 1 of Benhabib and Perli (1994).

Theorem 1 (Benhabib-Perli) *The number of roots of the polynomial in (69) with positive real parts is equal to the number of variations of sign in the scheme*

$$-1 \qquad \mathrm{tr}(J_1) \qquad -\mathrm{M}(J_1) + \frac{\det(J_1)}{\mathrm{tr}(J_1)} \qquad \det(J_1).$$

Noting that $\mathrm{tr}(J_1)$ is linear in g_τ and that it is positive at both ends (i.e., $\mathrm{tr}(J_1) > 0$ at $g_\tau = 0,\ g^{\max}$) under the assumption that $\psi/\phi < (1-2\alpha)/(1-\alpha-\beta)$, we have $\mathrm{tr}(J_1) > 0$, $\mathrm{M}(J_1) < 0$, and $\det(J_1) > 0$. Thus, the above theorem implies that there is only one eigenvalue with positive real parts in the matrix J_1. Combined with $Bu^* > 0$ obtained before, we have two positive eigenvalues in total. This completes the stability analysis for the case of $g_\tau \in [0, g^{\max}]$.

Turning to the case of $g_\tau \in (g^{\max}, g^{\lim}]$, the (asymptotic) steady state of the transformed system for this case is given by $P^* = 0$ and (61)–(63). The Taylor expansion of Equations (64)–(67) around this steady state yields essentially the same expression as (68), with the only difference that submatrix J_1 is replaced by:

$$J_2 = \begin{bmatrix} \chi^* & \cdots & \cdots \\ 0 & -z^*(1-\alpha-\beta) & \cdots \\ 0 & 0 & g^* - g_\tau \end{bmatrix},$$

where g^* is the asymptotic growth rate of output, which is defined by (29). As J_2 is a triangular matrix, its eigenvalues are simply given by its diagonal elements. Observe that $g^* - g_\tau$ represents the asymptotic growth rate of $P_t = \beta Y_t/\tau_t$, which is negative in this case. Therefore, J_2 has one positive eigenvalue (χ^*) and two negative ones ($-z^*(1-\alpha-\beta)$ and $g^* - g_\tau$). This completes the stability analysis for the case of $g_\tau \in (g^{\max}, g^{\lim}]$.

A.5 Details of welfare maximization

The current value Hamiltonian for the social planner's problem is:

$$\mathcal{H} = \frac{C_t^{1-\theta} - 1}{1-\theta} + \nu_t[AK_t^\alpha (u_t H_t)^{1-\alpha-\beta} P_t^\beta - C_t - (\delta_K + \phi P_t)K_t]$$
$$+ \mu_t[B(1-u_t)H_t - (\delta_H + \psi P_t)H_t],$$

where ν_t and μ_t are the planner's shadow prices associated with the accumulation of physical capital and human capital, respectively. The first-order conditions are:

$$\nu_t = C_t^{-\theta}, \tag{70}$$

$$\frac{\dot{\nu}_t}{\nu_t} = \rho + \phi P_t + \delta_K - \alpha \frac{Y_t}{K_t}, \tag{71}$$

$$\mu_t = \frac{(1-\alpha-\beta)Y_t}{Bu_t H_t}\nu_t, \tag{72}$$

$$\frac{\dot{\mu}_t}{\mu_t} = \rho - \frac{\nu_t}{\mu_t}(1-\alpha-\beta)\frac{Y_t}{H_t} - B(1-u_t) + \delta_H + \psi_t P_t. \tag{73}$$

$$\frac{\beta Y_t}{P_t} = \phi K_t + \psi_t(\mu_t/\nu_t)H_t, \tag{74}$$

The resource constraints for the social planner's problem are (4) and (5). Differentiating the log of (72) with respect to time, eliminating $\dot{\nu}_t/\nu_t$ and $\dot{\mu}_t/\mu_t$ by (71) and (73), and then eliminating ν_t/μ_t by (72) gives condition (17). Similarly, differentiating the log of (70) with respect to time and eliminating $\dot{\nu}_t/\nu_t$ by (71) gives (18). The transversality conditions for this problem are $\lim_{t\to\infty} K_t\nu_t e^{-\rho t} = 0$ and $\lim_{t\to\infty} H_t\mu_t e^{-\rho t} = 0$. Eliminating ν_t and μ_t using (70) and (72), and then introducing w_t from (7) shows that these TVCs are the same as (19). Finally, eliminating (μ_t/ν_t) from (74) by (72) yields condition (32).

A.6 Proof of Proposition 4

The proof is essentially similar to the discussion in Section 4. Note that equation (39) implies $\dot{\tau}_t/\tau_t = \dot{Y}_t/Y_t - \dot{E}_t/E_t$, the RHS of which is asymptotically constant from the definition of NABGPs. Thus, the growth rate of τ_t is also asymptotically constant and written as $g_\tau = g^* - g_E$, where g_E is the asymptotic growth rate of emission. From this, we can show that the asymptotic growth rate of economy g^* cannot exceed g_τ. Observe that if $g^* > g_\tau$, the previous equation implies $g_E > 0$. This means emission E_t grows without bound, stock P_t also grows without bound from (37), natural disasters occur increasingly frequently, and physical and human capital are destroyed at an ever-increasing rate. As this is obviously incompatible with NABGPs, $g^* \le g_\tau$ must hold (See the proof of Lemma 1 in Appendix A.1).

Given $g^* \le g_\tau$, it results that the asymptotic growth rate of emissions is zero or negative ($g_E = g^* - g_\tau \le 0$). In fact, $E_t > 0$ and $g_E \le 0$ means that the amount of emissions E_t is asymptotically constant: $E_t \to E^* \ge 0$. Moreover, from (37), the stock of pollution is also asymptotically constant: $P_t \to P^* \equiv (\gamma/\delta_P)E^* \ge 0$. It is easy to see that $P^* = 0$

holds when $g^* < g_\tau$, because $g_E < 0$ and therefore $P^* = (\gamma/\delta_P)E^* = 0$. Thus, condition (28) in Section 4 holds also for the case of stock pollution. The remaining conditions that characterize the NABGP are also the same (conditions 24 to 27) because they are derived from (5), (17), (18), and the first equation of (38), none of which were changed by the introduction of pollution stocks. Therefore, the analysis in Appendix A.3 is still valid and yields the same values of g^*, z^*, χ^*, u^*, and P^* as in the baseline model.

A.7 Details of welfare maximization with stock pollution

From the definition of pollution stock (37), P_t evolves according to $\dot{P}_t = \gamma E_t - \delta_P P_t$. Using this, the current value Hamiltonian for the social planner's problem can be written as:

$$\begin{aligned}\mathcal{H} = \frac{C_t^{1-\theta} - 1}{1-\theta} &+ \nu_t[AK_t^\alpha (u_t H_t)^{1-\alpha-\beta} E_t^\beta - C_t - (\delta_K + \phi P_t)K_t] \\ &+ \mu_t[B(1-u_t)H_t - (\delta_H + \psi P_t)H_t] + \lambda_t[\gamma E_t - \delta_P P_t],\end{aligned}$$

where λ_t is the shadow price of pollution stock. The first-order conditions are given by (70)–(73) and:

$$\frac{\beta Y_t}{\gamma E_t} = \frac{-\lambda_t}{\nu_t} \tag{75}$$

$$\frac{\dot{\lambda}_t}{\lambda_t} = \frac{\nu_t}{\lambda_t}\phi K_t + \frac{\mu_t}{\lambda_t}\psi H_t + \rho + \delta_P. \tag{76}$$

The TVCs are $\lim_{t\to\infty} K_t \nu_t e^{-\rho t} = 0$, $\lim_{t\to\infty} H_t \mu_t e^{-\rho t} = 0$, and $\lim_{t\to\infty} P_t \lambda_t e^{-\rho t} = 0$. Similar to the analysis for Section 5 (see Appendix A.5), it can be shown that conditions (70)–(73) and the first two TVCs are the same as the market equilibrium. Note that $P_s \geq P_t e^{-\delta_P(s-t)}$ holds for all $s \geq t$ from $\dot{P}_t = \gamma E_t - \delta_P P_t$ and $E_t \geq 0$. This inequality and the TVC for P_t jointly imply:

$$0 = \lim_{s\to\infty} P_s \lambda_s e^{-\rho s} \geq \lim_{s\to\infty} \lambda_s P_t e^{-\delta_P(s-t)} e^{-\rho s} = P_t e^{-\rho t} \lim_{s\to\infty} \lambda_s e^{-(\rho+\delta_P)(s-t)} \geq 0.$$

Note that P_t cannot become 0 in finite t, although it may asymptote to 0. Therefore, the above means:

$$\lim_{s\to\infty} \lambda_s e^{-(\rho+\delta_P)(s-t)} = 0. \tag{77}$$

In the following, we derive the value of λ_t from (76) and (77). Substituting s for t in (76) and multiplying both sides by $\lambda_t e^{-(\rho+\delta_P)(s-t)}$ gives:

$$\dot{\lambda}_s e^{-(\rho+\delta_P)(s-t)} - (\rho + \delta_P)\lambda_s e^{-(\rho+\delta_P)(s-t)} = (\nu_s \phi K_s + \mu_s \psi H_s)\, e^{-(\rho+\delta_P)(s-t)}. \tag{78}$$

Observe that the LHS of (78) is the derivative of $\lambda_s e^{-(\rho+\delta_P)(s-t)}$ with respect to s. Thus, we can calculate the definite integral of the LHS from $s = t$ to $s \to \infty$, which becomes:

$$\Big[\lambda_s e^{-(\rho+\delta_P)(s-t)}\Big]_{s=t}^{\infty} = \lim_{s\to\infty} \lambda_s e^{-(\rho+\delta_P)(s-t)} - \lambda_t = -\lambda_t,$$

where the second equality follows from (77). As this must coincide with the definite integral of the RHS of (78), we obtain:

$$-\lambda_t = \int_t^{\infty} (\nu_s \phi K_s + \mu_s \psi H_s)\, e^{-(\rho+\delta_P)(s-t)} ds. \tag{79}$$

Eliminating ν_t and μ_t from (75) and (79) then using (70) and (72) gives (40) in the text.

A.8 Details of household optimization with non-insurable risk

Consider the problem of maximizing the lifetime expected utility (9) subject to budget constraints (14) and (44) with initial assets $a_{i0} = s_{i0} + (w_0/B)h_{i0} + T_0$ (see footnote 21). Let $V_t(a)$ be the value function representing the lifetime expected utility of a household that holds total assets of a at time t, given the market prices r_t and w_t and pollution P_t. Let us first consider an approximated problem where time is discretized by infinitesimally small intervals of size Δt.

Then, the optimization problem of the household between time t and $t + \Delta t$ can be set up using the Bellman Equation:

$$\begin{aligned} V_t(a) = \max_{c,\eta} \frac{c^{1-\theta}-1}{1-\theta}\Delta t + e^{-\rho\Delta t}(\bar{q}+\hat{q}P_t)\Delta t\ \mathbb{E}\Big[V_{t+\Delta t}\big((1-\eta\epsilon)a\big)\Big] + \\ e^{-\rho\Delta t}\big(1-(\bar{q}+\hat{q}P_t)\Delta t\big)V_{t+\Delta t}\Big(a + \big[(1-\eta)r_t a + \eta(B - \bar{\delta}_H + \dot{w}_t/w_t)a - \bar{\sigma}c\big]\Delta t\Big), \end{aligned}$$

where the stochastic variable ϵ follows the distribution $\Psi(\epsilon)$. Observe that $(\bar{q}+\hat{q}P_t)\Delta t$ and $\big(1-(\bar{q}+\hat{q}P_t)\Delta t\big)$ in the second and third terms of the RHS represent the probabilities that a natural disaster will occur and will not occur, respectively, between t and $t + \Delta t$. The arguments in function $V_{t+\Delta t}(\cdot)$ following these are the amount of assets at time $t+\Delta t$ conditional on the household being hit and not being hit by a natural disaster, respectively.

Although the above Bellman Equation holds only approximately, we can obtain a precise Hamilton-Jacobi-Bellman (HJB) equation by Taylor-expanding the RHS and then taking the limit of $\Delta t \to 0$:

$$\begin{aligned} \rho V_t(a) = \max_{c,\eta} \frac{c^{1-\theta}-1}{1-\theta} + \frac{\partial V_t(a)}{\partial t} + (\bar{q}+\hat{q}P_t)\Big\{\mathbb{E}\big[V_t\big((1-\eta\epsilon)a\big)\big] - V_t(a)\Big\} \\ + V_t'(a)\Big\{(1-\eta)r_t a + \eta(B - \bar{\delta}_H + \dot{w}_t/w_t)a - \bar{\sigma}c\Big\}, \end{aligned} \tag{80}$$

where we ignore terms of the order of $(\Delta t)^2$ and higher because they vanish in the limit of $\Delta t \to 0$. The second term of (80) represents the change in the functional form of $V_t(\cdot)$ due to evolutions in prices and pollution. The first-order conditions with respect to c and η, respectively, are: $V_t'(a) = c^{-\theta}/\bar{\sigma}$, and

$$V_t'(a)\big\{(B - \bar{\delta}_H + \dot{w}_t/w_t) - r_t\big\} = (\bar{q} + \hat{q}P_t)\,\mathbb{E}\big[V_t'\big((1-\eta\epsilon)a\big)\epsilon\big]. \tag{81}$$

Recall that the objective function (9) is homothetic in c_{it} and that the budget constraints (14) and (44) are homogenous of degree zero in c_{it} and a_{it}. These jointly imply that the optimal solution is also homothetic in c_{it} and a_{it} in the sense that if the value of a_{it} is multiplied by $(1-\eta_{it}\epsilon_{it})$, then the optimal plan for $\{c_{it'}\}_{t'=t}^{\infty}$ thereafter is also multiplied by $(1-\eta_{it}\epsilon_{it})$. Because $V_t'(a) = c^{-\theta}/\bar{\sigma}$, this means that $V_t'\big((1-\eta\epsilon)a\big) = \big((1-\eta\epsilon)c\big)^{-\theta}/\bar{\sigma} = (1-\eta\epsilon)^{-\theta}V_t'(a)$. Substituting it into (81) and then using $\bar{\psi} \equiv \mathbb{E}[\epsilon]$, $\psi \equiv \bar{\psi}\hat{q}$, and $\delta_H \equiv \bar{\delta}_H + \bar{\psi}\bar{q}$, we obtain the arbitrage condition (45) and the risk premium function (46) in the text.

Next, differentiating both sides of HJB equation (80) by total assets a gives the envelope condition:

$$\begin{aligned}\rho V_t'(a) = {} & \frac{\partial V_t'(a)}{\partial t} + V_t''(a)\Big\{(1-\eta)r_t a + (B - \bar{\delta}_H + \dot{w}_t/w_t)\eta a - \bar{\sigma}c\Big\} \\ & + V_t'(a)\Big\{(1-\eta)r_t + (B - \bar{\delta}_H + \dot{w}_t/w_t)\eta\Big\} \\ & + (\bar{q} + \hat{q}P_t)\Big\{\mathbb{E}\big[V_t'\big((1-\eta\epsilon)a\big)(1-\eta\epsilon)\big] - V_t'(a)\Big\}.\end{aligned} \tag{82}$$

On the RHS, the sum of the first and second terms can be written as $\partial V_t'(a)/\partial t + V_t''(a)\dot{a} = dV_t'(a)/dt = d(c^{-\theta}/\bar{\sigma})/dt = -\theta c^{-\theta-1}\dot{c}/\bar{\sigma} = -\theta(\dot{c}/c)V_t'(a)$, where $\dot{c}$ and $\dot{a}$ are movements in c and a when the household is not hit by a disaster. From (81), the sum of the third and fourth terms is $r_t V_t'(a) + (\bar{q} + \hat{q}P_t)\big\{\mathbb{E}\big[V_t'\big((1-\eta\epsilon)a\big)\big] - V_t'(a)\big\}$. Therefore, using $V_t'\big((1-\eta\epsilon)a\big) = (1-\eta\epsilon)^{-\theta}V_t'(a)$, the envelope condition (82) simplifies to (47) in the text.

A.9 Proof of Lemma 2

From (46) and (49), the difference between $R(\eta)$ and $S(\eta)$ can be written as

$$R(\eta) - S(\eta) = -\,\mathbb{E}\left[(1-\eta\epsilon)^{-\theta}(1-\epsilon)\right] + \theta\bar{\psi}\eta + 1 - \bar{\psi} \equiv Q(\eta;\theta,\Psi), \tag{83}$$

where we denote the difference by $Q(\eta;\theta,\Psi)$ to show explicitly its dependence on parameter θ and the distribution of ϵ (i.e., $\Psi(\epsilon)$), and on variable η. It is easy to confirm $Q(0;\theta,\Psi) = 0$

by substituting $\eta = 0$ into (83) and then using $\mathbb{E}[\epsilon] = \bar{\psi}$. The first and second derivatives of $Q(\eta; \theta, \Psi)$ with respect to η are

$$Q'(\eta; \theta, \Psi) = -\theta\, \mathbb{E}\left[(1-\eta\epsilon)^{-\theta-1}\epsilon(1-\epsilon)\right] + \theta\bar{\psi}, \tag{84}$$

$$Q''(\eta; \theta, \Psi) = -\theta(\theta+1)\, \mathbb{E}\left[(1-\eta\epsilon)^{-\theta-2}\epsilon^2(1-\epsilon)\right]. \tag{85}$$

By substituting $\eta = 0$ into (84), we obtain $Q'(0; \theta, \Psi) = \theta\, \mathbb{E}\left[\epsilon^2\right] > 0$. In addition, because the support of distribution $\Psi(\epsilon)$ is $\epsilon \in (0, 1)$ and η is between 0 and 1, $(1 - \eta\epsilon)$ and $(1 - \epsilon)$ are always positive. Therefore, the expression in the expectation operator in (85) is always positive, from which we obtain $Q''(\eta; \theta, \Psi) < 0$ for all $\eta \in [0, 1]$.

Next, we show that $Q(1; \theta, \Psi)$ is negative for all $\theta > 1$. Substituting $\eta = 1$ into (83) and then differentiating it with respect to θ gives:

$$Q(1; \theta, \Psi) = -\,\mathbb{E}\left[(1-\epsilon)^{1-\theta}\right] + (\theta-1)\bar{\psi} + 1, \tag{86}$$

$$\frac{\partial Q(1; \theta, \Psi)}{\partial \theta} = \mathbb{E}\left[(1-\epsilon)^{1-\theta}\log(1-\epsilon)\right] + \bar{\psi}. \tag{87}$$

Note that $(1 - \epsilon)^{1-\theta} > 1$ holds because $(1 - \epsilon) \in (0, 1)$ and $1 - \theta < 0$. In addition, we can confirm $\log(1 - \epsilon) < -\epsilon$ for all $\epsilon \in (0, 1)$ from the graph of the logarithmic function. From these, the expression in the expectation operator in (87) is always lower than $-\epsilon$, and hence the RHS of (87) is negative. In addition, (86) implies that $Q(1; \theta, \Psi) \to 0$ when $\theta \to 1$. Thus, $\partial Q(1; \theta, \Psi)/\partial\theta < 0$ means that $Q(1; \theta, \Psi) < 0$ for all $\theta > 1$.

We have shown that the smooth function $Q(\eta; \theta, \Psi)$ starts from origin ($Q(0; \theta, \Psi) = 0$), has a positive gradient at origin ($Q'(0; \theta, \Psi) > 0$), is strictly concave for all $\eta \in [0, 1]$ ($Q''(\eta; \theta, \Psi) < 0$), and becomes negative when $\eta = 1$ ($Q(1; \theta, \Psi) < 0$), as depicted in Figure 6(iv). From these, we can conclude that the graph of function $Q(\eta; \theta, \Psi)$ crosses the horizontal axis exactly once in the region of $\eta \in (0, 1)$; i.e., there exist unique $\bar{\eta} \in (0, 1)$ such that $R(\eta) - S(\eta) = Q(\bar{\eta}; \theta, \Psi) = 0$ holds. It is also obvious that $R(\eta) - S(\eta) = Q(\eta; \theta, \Psi) > 0$ for $\eta \in (0, \bar{\eta})$ and $R(\eta) - S(\eta) = Q(\eta; \theta, \Psi) < 0$ for $\eta \in (\bar{\eta}, 1]$.

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