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12 May 2009

Information Acquisition and Innovation under Competitive Pressure*

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March 6, 2012

Abstract

This paper studies information acquisition under competitive pressure and proposes a model to examine the relationship between product market competition and the level of innovative activity in an industry. Our paper offers theoretical support for recent empirical results that point to an inverted-U shape relationship between competition and innovation. The model presents an optimal timing decision problem where a firm endowed with an idea trades the benefits of waiting for additional information on whether this idea can be converted into a successful project against the cost of delaying innovation: a given firm's profit following innovation is decreasing in the number of firms that invested at earlier dates. By recognizing that a firm can intensify its innovative activity on two dimensions, a risk dimension and a quantitative dimension, we show that firms solve this trade-off precisely so as to generate the inverted-U shape relationship.

JEL Classification: D40, L10

Keywords: Innovation, Information Acquisition, Timing Games, Preemption.

*I would like to thank Alessandro Pavan and Marciano Siniscalchi for very helpful comments and discussions. I also thank comments from Andrew Daughety, Eddie Dekel, Silvana Krasteva, Asher Wolinsky, and from audiences at the University of South Florida, Florida State University, the 2010 International Industrial Organization Conference, and the 2010 Econometric Society World Congress. All errors are mine.

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1 Introduction

This paper studies information acquisition under competitive pressure and employs the resulting model to investigate the relationship between the degree of competition in an industry and the intensity of innovative activity. The clear policy implications of the nature of this relationship generated a large body of literature that investigated it. Beginning with the seminal work of Schumpeter (1943), the objective of these studies has been to determine whether there is an optimal market structure that results in the highest rate of technological advance. In particular, the literature tried to reconcile the intuitive appeal of Schumpeter's assertion that only large firms possessing a significant amount of monopoly power have the resources and incentive to engage in risky innovative activity, with a substantial amount of empirical literature that did not confirm it. More recent empirical papers, such as Aghion, Bloom, Blundell, Griffith and Howitt (2005) suggest *an inverted-U shape relationship* between product market competition and innovation.[1] According to these studies, for low levels of competition, an increase in competition induces more innovation, while for higher values of competition, as competition increases, firms become less innovative.

Our paper adopts a microeconomic approach and constructs a model that studies the innovation process at firm level by following a new project through its stages of development. The firms in our dynamic model become sequentially aware of an invention and decide on whether and when to undertake a costly investment in innovation. In making this decision, firms face a trade-off between seeking a first-mover advantage and waiting to acquire more information. Our study brings to main contributions to the literature. First, we identify the trade-off between information acquisition and competitive pressure as sufficient to generate the empirically observed inverted-U shape relationship. Second, we propose a new dimension, the risk dimension, on which a firm can intensify its innovative activity. Due to the strong intuitive nature of Schumpeter's assertion, the vast majority of theoretical models investigating the relationship between competition and innovation obtained a negative relationship. By disentangling the level of innovative activity along two dimensions, the quantitative and risk dimensions, we succeed in offering an explanation for the positive segment of the relationship.[2]

The model has a set of firms who, through their applied research activity, discover an invention or an idea that could generate future revenues for its investors, provided that it is a success from both a technological and a business standpoint. Once a firm is aware of the idea, it has the option to invest in the *innovation* of that project at any time. Innovation means the development of a marketable product; it is the stage in the R&D process where the first substantial financial commitment to the project is made. When the firm first learns of the idea, the knowledge about its feasibility is scarce, so investment is risky.[3] As time passes, the firm acquires new information

[1] Scherer (1967) is the first empirical paper to uncover this shape. See also Scott (1984).

[2] Also, our model can be seen as a study of *product* innovation where new products are introduced in the market. This differs from most of the theoretical literature on innovation, including Aghion et al. (2005), which focuses on *process* innovations where existing products are produced at a lower average cost.

[3] Mansfield et al. (1977, p. 9) found that the probability that an R&D project would result in an economically

and is able to better assess its chances of success. This additional information may lead the firm to decide not to invest in the project. This makes waiting beneficial as it can potentially aid avoiding the financial losses associated with the development of an unsuccessful product. On the other hand, in our model, earlier investors release the product earlier, and thus enjoy a natural first-mover advantage.

These two features of the model induce a trade-off in the firm's problem between investing early to enjoy the first-mover advantage, and waiting to acquire new information and reduce the risk of investment. Mansfield (1968, p. 105) emphasizes this trade-off in the firm's decision making process. As he states, on the one hand, "there are often considerable advantages in waiting, since improvements occur in the new product and process and more information becomes available regarding its performance and market." On the other, "there are disadvantages... in waiting, perhaps the most important being that a competitor may beat the firm to the punch." He concludes that "if the expected returns... justify the risks and if the disadvantages of waiting outweigh the advantages, the firm should innovate. Otherwise it should wait. Pioneering is a risky business; whether it pays off is often a matter of timing."

A more innovative industry is defined in our paper to be one in which firms allocate a larger budget to the innovative activity.[4] There are two channels for a firm to increase its innovative expenditures. First, the firm can pursue an increasing fraction of the projects that emerge from the applied research activity. Second, the firm can invest earlier in any given project, thereby undertaking riskier projects. Given a constant flow of ideas, this leads to more projects reaching the innovation stage where the substantial financial commitment to a project is made.

We show that when competition is low, firms invest in *all* projects that do not reveal themselves to be infeasible by the equilibrium time of investment. For these low values of competition, an increase in competition induces firms to invest earlier, and thus to undertake riskier projects. Therefore, as competition increases, firms become *more* innovative by intensifying their innovative activity along the risk dimension. In the literature, this is called the "escaping the competition effect". On the other hand, for high values of competition, as competition increases, firms hold constant the risk of investment, but decrease the fraction of projects in which they invest. Therefore, an increase in competition induces *less* innovation by having firms decrease the intensity of their innovative activity along the quantitative dimension. In the literature, this is called the "Schumpeterian effect".

The key driving force in our model is the effect of an increase in product market competition on the *marginal cost of waiting* for more information, as determined by the expected loss in first-mover advantage. Firms choose the optimal time of investment by comparing this marginal cost with the *marginal benefit of waiting* that is generated by the additional information. When competition is

successful product or process was about 0.12; the average probability of *technical* completion for a project was 0.57.

[4] Aghion et al. (2005) employ patent count data as a primary measure of innovation, but as a robustness check, they also use R&D expenditures as an alternative measure. The same inverted-U shape relationship emerges.

low, firms expect positive profits from innovation and invest in all projects. As competition increases over this range, the marginal cost of waiting increases, exceeding the marginal benefit earlier. Thus firms decrease the waiting time and invest in riskier projects. As competition increases and riskier projects are undertaken, firms end up just breaking even for a high enough level of competition. When competition further increases, to continue to sustain non-negative profits, firms become less innovative. They do so by investing in a decreasing fraction of projects. This leads to a decrease in the number of firms that invest in any given project, lowering the post-innovation level of competition, and allowing for non-negative expected profits from innovation.

In addition to the key comparative static result with respect to the level of competition, our model offers other predictions of interest. First, when innovation costs increase, firms invest later for all values of competition. Second, an increase in the speed of learning induces firms to invest in safer projects. Third, a stronger first-mover advantage induces firms to undertake riskier investments. Finally, the model is successful in supporting additional empirical regularities that Aghion et al. (2005) observed. More precisely, we show that a higher degree of levelness in an industry results in an inverted-U shape curve with a higher peak attained for a lower level of competition.

Most theoretical papers investigating the relationship between competition and innovation offered results consistent with the intuitive Schumpeterian view that the lower post-innovation rents associated with higher competition reduce the incentives to innovate. Yet, a substantial amount of empirical research did not support this hypothesis,[5] prompting a search for a theoretical model to explain these seemingly puzzling empirical findings. Inspired by the seminal work of Hart (1983), some papers focusing on managerial incentives, such as Schmidt (1997) or Aghion, Dewatripont and Rey (1999), suggested a positive correlation between competition and innovation. However, these results hinge on replacing the profit maximization assumption with a less appealing assumption of minimizing innovation costs, subject to the constraint that the firm does not go bankrupt.[6]

As Aghion et al. (2005) maintain, their theoretical model is the first to succeed in explaining the inverted-U shape relationship. Aghion et al. (2005) argue that the escaping-the-competition effect of an increase in innovation in response to an increase in competition is stronger in industries in which firms are at technological par, while the opposite Schumpeterian effect is stronger in industries that are technologically more dispersed. The inverted-U shape curve emerges because the steady-state fraction of industries in the economy that are at technological par adjusts in response to a change in competition. The results in Aghion et al. (2005) hinge on including in the definition of innovation the technological advancements made at no cost by laggard firms who duplicate the technology of the leader. If the definition of innovation does not include the zero-cost

[5] See for instance Nickell (1996) or Blundell, Griffith and Van Reenen (1995).

[6] The only other theoretical papers in support of the positive relationship are Reinganum (1983), who shows that the existence of a potential entrant induces the incumbent to be more innovative when innovation is uncertain, Aghion, Harris and Vickers (1997), whose approach is close to the one from Aghion et al. (2005), and Aghion and Howitt (1996) who endogenize the rate at which firms switch from old technologies to new, and show that an increase in the substitutability between them induces firms to adopt the new technologies faster. Boone (2000) finds conditions under which more competitive pressure induces either more or less innovation, while Boone (2001) presents a model that can generate non-monotone relationships of any nature.

technological advancements, their main monotonicity result no longer holds unless the hazard rate of these events is insignificant. However, for small values of this hazard rate, it is straightforward to see that their model predicts that the industry structure will be such that the Schumpeterian effect will always dominate, and thus that innovation is always decreasing in competition. The definition in our model is closer to the more standard interpretation of innovation as being something new, different and usually better than what existed before.[7] As such, we isolate innovation from riskless technological progress. Kamien and Schwartz (1976) also obtain an inverted-U shape, but their model is decision theoretic, and thus does not account for the potential strategic considerations in the firms' decision processes.[8]

At a formal level, while our model is novel, the paper is related to the literature on timing of irreversible actions under uncertainty. Closer to our study, Jensen (1982) presents a model of information acquisition in which the incentive to innovate earlier is provided by the discounting of future revenues rather than the competitive pressure.[9] Chamley and Gale (1994) examine a model of endogenous information acquisition whereby firms learn about the profitability of a common value investment from the actions of the other players, while Decamps and Mariotti (2004) allow in addition for a private value component of the investment and for exogenous information. Caplin and Leahy (1993) develop a model in which investors learn of the profitability of new industries from the success of the earlier entrants. Unlike these papers, in our model, information is purely exogenous, but the incentive to invest early is determined endogenously. Finally, the experimentation literature (see for instance Bolton and Harris (1999)) studies the trade-off between current output and information that can help increase output in the future. In a different direction, the first-mover advantage is also present in the patent-race literature (see for instance the seminal paper by Reinganum (1982)). What distinguishes the current model from this literature is mainly the source of uncertainty. In the patent race literature, the uncertainty is generated by the randomness of the times of technological advancements or of the finish line. In contrast, in our model, the uncertainty stems from the fact that the firm does not know whether the project is feasible. Moreover, while this literature focuses on the firms' decision-making process prior to making a technological breakthrough, this paper analyzes the forces that determine the firms' investment decisions in the development of the product which, as anecdotal evidence suggests, accounts for the majority of R&D budgets.

The model is presented in section 2, while the analytical results and their discussion are presented in section 3. The conclusion is in section 4. Most proofs are relegated to the appendix. The paper also has a supplementary online appendix.[10]

[7] Schumpeter (1943) defines economic innovation as the introduction of a new good or new method of production, the opening of a new market, the use of a new input, or the implementation of a new organizational structure.

[8] In their model, while the firm under consideration changes its behavior by investing earlier or later as a response to the rival's expected time of innovation, the rival does not do so.

[9] See also Park and Smith (2008) or Argenziano and Schmidt-Dengler (2010).

[10] This supplementary appendix can be found at: http://sites.google.com/site/andreibarbos/research

2 The Model

2.1 The Framework

There is a continuum set of identical and risk-neutral firms who, sequentially, learn of an invention or idea.[11] A mass a of firms becomes aware at each instant $t \in [t_0, t_0 + \eta]$, with $\eta > 0$. Firms do not know t_0 but have a prior distribution on it that is uniform on $\mathbb{R}$.[12] We denote by t_i the time when firm i learns of the idea. Once firm i learns of the idea, it may invest in its innovation at any time $t_i + t$, with $t \geq 0$. There is a one-time sunk fixed cost c of innovating. Postponing innovation allows a firm to acquire at no cost additional information about the feasibility of the project by performing tests and investigating the potential technological or commercial problems that the project might encounter.[13] The information acquired may result in a negative signal, such as a failed safety test or an analysis revealing unfavorable market conditions. If the project is feasible, no negative signal is received. If the project is infeasible, a negative signal arrives with an instantaneous probability μ. At time t_i, firm i has a belief p_0 that the project is feasible. As time passes, if no negative signal is received, the belief is updated favorably and the risk of investment is reduced. If a negative signal is received, the firm learns that the project is infeasible and drops it.

Firms do not observe other firms' actions. The decision to invest in the development of a new drug, for instance, is taken many years prior to the releasing of the product in the market, and thus it is private rather than public information. In fact, as we will argue in section 3.4, the formal analysis does not change if the firms observed other firms' investment decisions with a delay longer than η. The formal analysis changes if the delay is shorter, but the intuition behind our results continues to hold. Also, in our model firms do not observe negative signals received by other firms. In the real world, this is private information acquired by each firm through its R&D activities.

2.2 The Payoffs

The definition of the firms' payoffs captures the first-mover advantage in the model. There is a large body of literature that investigates the determinants of the first-mover advantage in an industry. These numerous potential drivers have been classified into three main categories: preemption of scarce assets, technology leadership and switching costs.[14] We employ a reduced-form model of the post-innovation market that accounts for a first-mover advantage, but does not select a par-

[11] The sequential awareness assumption from our model is similar to the one used by Abreu and Brunnermeier (2003) in a model of financial bubbles and crashes.

[12] The continuum set of firms can be interpreted simply as the distribution of the unknown locations on the timeline of a finite number of firms. The nonstandard distribution of t_0 is used to avoid boundary effects. An alternative is to discard the common prior assumption. Thus, instead of having firm i's posterior belief about t_0 at t_i be derived from a common prior about t_0, we may consider directly that this belief is actually the firm's prior on t_0 at that moment.

[13] As we explain in section 3.4, the zero cost of information acquisition is without loss of generality.

[14] As argued in the literature, a first-mover could gain an advantage over its competitors through capturing valuable spaces or production resources, economies of scale, patenting, cost advantages via learning economies, switching costs generated by the buyers' habit formation, reputation advantages and high buyers' search cost.

ticular mechanism through which it is obtained. Thus, we assume that the expected value of the stream of *post-innovation profits* of firm i from investing at moment $t_i + t$ in a *feasible* project is $\Pi(n, m(t|t_i, t_0))$, where $m(t|t_i, t_0)$ is the measure of firms that innovate before firm i, and n is the total measure of firms that invest in the project. $\Pi(\cdot)$ is strictly decreasing in both arguments.[15] To isolate the effect of the competitive pressure in inducing firms to invest earlier, we assume no intertemporal discounting. If the project is infeasible, the post-innovation profits are zero.

In the model from this paper, we employ a quasi-linear functional form for Π

$$\Pi(n, m(t|t_i, t_0)) = A(n) - \theta \cdot m(t|t_i, t_0) \tag{1}$$

where $\theta \in \mathbb{R}_+$, and $A : \mathbb{R}_+ \to \mathbb{R}_+$ is a continuously differentiable and strictly decreasing function. This form allows for a more transparent intuition of the results and a clearer exposition because the marginal cost of waiting for one more period is essentially the expected measure of firms who invest in that period, rather than the corresponding effect on post innovation profits. In section S4 from the online appendix, we study the model with a general functional form for Π, and show that the salient results of this paper are preserved. We also show in appendix A1, that when Π is quasi-linear, the total amount of profits available from a successful innovation in the industry does *not* depend on the distribution of the moments when firms in the industry innovate.

2.3 The Measure of Competition

Competition has been modeled in the literature in several ways.[16] Boone (2008) argues that the salient feature common to all theoretical parameterizations of competition is that an increase in competition always raises the *relative* profit shares of the more advanced firms and reduces the profits of the least advanced firms active in the industry. In appendix A2 we show that in our model, in a symmetric equilibrium, these conditions are satisfied when competition is parametrized by a as long as the last firm to invest in a successful product makes nonpositive post-innovation profits.[17] Moreover, we also show in appendix A2 that an increase in a lowers the average and total profits in the industry. These are expected since in the setup of this model, the total measure of firms in the industry is $a\eta$. Note that in our model the *pre*-innovation level of competition is exogenous as it is considered to be the outcome of a policy maker's decision regarding the optimal market structure most conducive to innovation.

To simplify exposition, we make the following assumption that ensures an interior solution.

Assumption 1 *$a \in [a_m, a_M]$, where a_m is given by $\Pi(a_m\eta, a_m\eta) - c = 0$ and $a_M = \frac{c\mu(1-p_0)}{\theta}$.*

[15] One could also account for the the measure of firms that innovate at the same time as firm i. In the symmetric equilibrium under consideration, this measure is zero and thus omitting it is without loss of generality.

[16] For instance, Dasgupta and Stiglitz (1980) or Martin (1993) identify an increase in competition with an increase in the number of active firms in the industry. Aghion et al. (2005) or Aghion and Howitt (1992) identify it with a more aggressive interaction among firms and thus with decrease in the firms' rents.

[17] This condition is sufficient, but not necessary. An alternative sufficient condition is that $A(\cdot)$ is concave.

The upper bound ensures that firms do acquire some information about a project before investing. The lower bound eliminates the uninteresting case where firms have the option to wait so long that they innovate under almost certainty. It imposes that competition is high enough that the last firm to innovate makes negative profits even if the project is known to be feasible.

3 Results

3.1 The Equilibrium for a Fixed Value of Competition

A strategy for firm i is a cumulative distribution function S over the augmented set of possible waiting times $\mathbb{R}_+ \cup \{\infty\}$. For $t \in \mathbb{R}_+$, $S(t)$ is the probability that firm i invests by time $t_i + t$, conditional on no negative signal having yet been received. With a slight abuse of notation, the mass point on $\{\infty\}$ denotes the probability that the firm does *not* invest in a project even if no negative signal is received. For $\tau \in \mathbb{R}_+$, we define a *simple strategy with waiting time* τ to be a strategy S_τ such that:

$$S_\tau(t) = \begin{cases} 0, & \text{for } t \in [0, \tau) \\ s_\tau \in (0,1], & \text{for } t \in [\tau, \infty) \\ 1, & \text{for } t = \infty \end{cases} \tag{2}$$

A simple strategy S_τ prescribes that firm i innovates at $t_i + \tau$ with probability s_τ if no negative signal has been received up to that time, and does not innovate at all with probability $1 - s_\tau$. We will focus on equilibria in simple strategies. Alternative symmetric equilibria may exist, but as we show in section 3.5, the firms' response to an increase in competition in these equilibria is similar to the one from the equilibria in simple strategies.

The analysis of the game is based on the comparison for a firm i of the marginal cost and the marginal benefit of waiting at $t_i + t$ for an infinitesimal amount of time, while keeping track throughout of the option value of waiting. We examine a firm's decision problem from the viewpoint of the time when it becomes aware of the invention. Thus, the marginal cost and marginal benefit for firm i will be evaluated *as of time* t_i. We define the *marginal cost* (MC) of waiting at $t_i + t$ to be the expected decrease in post-innovation rents due to the expected loss in first-mover advantage. The *marginal benefit* (MB) of waiting at $t_i + t$ is defined to be the expected value of the information acquired by waiting at that time. In monetary terms, the MB is measured as the expected *forgone* costs on an infeasible project generated by the additional information.

To be more precise, denote by F the event that the project is feasible, by N_t the event that a negative signal is received by firm i before moment $t_i + t$, and by E^c the complement of any event E. The constant rate of arrival of a negative signal from an infeasible project implies that the delay until a firm receives a negative signal has an exponential distribution of parameter μ. Thus,

$$\Pr(N_t | F^c) = 1 - e^{-\mu t} \tag{3}$$

Then, the MB of waiting at $t_i + t$ is defined as

$$MB(t) \equiv \lim_{\delta \to 0} c \cdot \frac{\Pr(N_{t+\delta} \cap N_t^c | F^c)}{\delta} \cdot \Pr(F^c) \tag{4}$$

Note that $\Pr(N_{t+\delta} \cap N_t^c | F^c)$ is the probability, as of time t_i, that an infeasible project reveals its quality to firm i in between $t_i + t$ at $t_i + t + \delta$. Since $N_t \subset N_{t+\delta}$, we have $\Pr(N_{t+\delta} \cap N_t^c | F^c) = \Pr(N_{t+\delta} \backslash N_t | F^c) = \Pr(N_{t+\delta} | F^c) - \Pr(N_t | F^c) = e^{-\mu t} - e^{-\mu(t+\delta)}$. By taking the limit and using the fact that $\Pr(F^c) = 1 - p_0$, it follows that

$$MB(t) = c\,(1 - p_0)\,\mu e^{-\mu t} \tag{5}$$

We define next the marginal cost of waiting in an equilibrium in which all firms adopt symmetric simple strategies S_τ, for some $\tau \in \mathbb{R}_+$. Note first that for a given value of t_0, if all firms adopt strategy S_τ, innovation starts in the industry at $t_0 + \tau$ and is completed at $t_0 + \tau + \eta$. Thus, conditional on t_0, the measure of firms who have invested by time $t_i + t$ from the viewpoint of firm i is[18]

$$m_{S_\tau}(t|t_i, t_0) \equiv s_\tau a \min(\eta, \max(t_i + t - \tau - t_0, 0)) \tag{6}$$

Since at t_i firm i's posterior of t_0 is uniform on $[t_i - \eta, t_i]$, the expected measure of firms who have invested by time $t_i + t$ is then

$$\lambda_{S_\tau}(t|t_i) \equiv E_{t_0}\left[m_{S_\tau}(t|t_i, t_0)\right] = \frac{1}{\eta}\int_{t_i-\eta}^{t_i} m_{S_\tau}(t|t_i, t_0)dt_0 \tag{7}$$

On the other hand, in the equilibrium under consideration, the total measure of firms that invest in the project is $s_\tau a\eta$. Since $\Pi(\cdot)$ is quasi-linear in the baseline model, conditional on a feasible project, firm i's expected post-innovation profit from investing at $t_i + t$ is

$$E_{t_0}\left[\Pi\left(s_\tau a\eta, m_{S_\tau}(t|t_i, t_0)\right)\right] = A\left(s_\tau a\eta\right) - \theta\lambda_{S_\tau}(t|t_i) \tag{8}$$

Then, firm i's MC as of time t_i of waiting at $t_i + t$ is defined as

$$MC_{S_\tau}(t) \equiv -p_0\frac{\partial}{\partial t}E_{t_0}\left[\Pi\left(s_\tau a\eta, m_{S_\tau}(t|t_i, t_0)\right)\right] = p_0\theta\frac{\partial}{\partial t}\lambda_{S_\tau}(t|t_i) \tag{9}$$

Throughout the paper, the subscripts such as in equations (6), (7) and (9) specify the particular equilibrium under consideration.

The next proposition, whose proof is in appendix B1, describes a symmetric equilibrium in which firms adopt a strategy S_τ and the value of competition is *fixed*.

[18] $m_{S_\tau}(t|t_i, t_0)$ is: (i) 0, when $t_i + t < t_0 + \tau$; (ii) $a\eta$, when $t_i + t > t_0 + \tau + \eta$; (iii) $(t_i + t) - (t_0 + \tau)$, when $t_0 + \tau \le t_i + t \le t_0 + \tau + \eta$. These can be written concisely as in (6).

Proposition 2 *(i) A symmetric equilibrium in simple strategies S_τ exists if and only if*

$$p_0 \Pi \left(s_\tau a\eta, \frac{1}{2} s_\tau a\eta \right) - c \left[p_0 + (1 - p_0) e^{-\mu\tau} \right] \geq 0, \text{ with equality when } s_\tau < 1 \tag{10}$$

$$MC_{S_\tau}(\tau) = MB(\tau) \tag{11}$$

$$\Pi (s_\tau a\eta, s_\tau a\eta) \leq c, \text{ for } s_\tau \in (0, 1) \tag{12}$$

(ii) A symmetric equilibrium in simple strategies is unique for any value of a.

To understand condition (10), note first that in the symmetric equilibrium under consideration, the expected post-innovation profit of any firm, conditional on a feasible project, is $\Pi \left(s_\tau a\eta, \frac{1}{2} s_\tau a\eta \right)$. This is because on one hand, the expected measure of firms that become aware of the invention before any firm i is precisely $\frac{1}{2} a\eta$, while on the other, all firms wait the same amount of time before investing with probability s_τ. Second, $\Pr (N_\tau^c) = p_0 + (1 - p_0) e^{-\mu\tau}$ is the unconditional probability that the investment is made when the firm adopts the simple strategy S_τ. Thus, $c \Pr (N_t^c)$ are the expected innovation expenditures. Therefore, the left hand side of (10) is the *equilibrium expected profit from innovation.* Condition (10) states that this expected profit is non-negative. Condition (11) states that a firm chooses the optimal waiting time by equating the MC and MB of waiting. Condition (12) ensures that firms do not deviate from the prescribed equilibrium strategies to invest after all uncertainty about the project is removed. When $s_\tau = 1$, this condition is implied by assumption 1.

The proof of proposition 2 examines firm i's expected payoff from innovating at all times $t_i + t$ when its competitors adopt strategy S_τ. Essentially, though, the proof amounts to showing a virtual single crossing property between the MC and MB curves. Thus, the MB curve is above the MC curve for $t < \tau$, and is below it for values of t immediately above τ. While the two curves may intersect again for some higher value $t > \tau$, the firm does not find it profitable to wait until that time. Condition (12) plays a key role in showing this second fact. Therefore, firms postpone investing as long as the MB of waiting exceeds the MC, and invest as soon as they are equal.

We close this section with a corollary that elicits some straightforward comparative statics. Its proof follows immediately from the precise characterizations of the equilibrium in proposition 2. Denote by $p_t = \Pr (F | N_t^c)$ the belief of a firm that the project is feasible after acquiring information for time t without having received a negative signal. Thus p_t is a measure of the risk of investment after waiting for a time t. It is straightforward to show that $p_t = \frac{p_0}{p_0 + (1 - p_0) e^{-\mu t}}$, for all $t \geq 0$.

Corollary 3 *(i) The equilibrium waiting time τ is increasing in c; (ii) p_τ is increasing in μ.*

Part *(i)* suggests that when the innovation costs are higher, firms wait more before innovating. Put differently, the higher the profits that innovations promise in case of success, the riskier the projects which are undertaken. Second, μ measures the speed of learning in our model. Thus, all else equal, when firms learn faster about the feasibility of new inventions, they end up investing in safer projects. The effect on the equilibrium waiting time is ambiguous. An increase in μ increases the equilibrium value of the belief that the project is successful, but it also increases the speed of learning and thus that belief level may be attained earlier.

3.2 The Relationship between Competition and Innovation

The main result of the paper describes the equilibrium of the model *as competition varies*. To identify the corresponding level of competition, we assign throughout superscripts to equilibrium strategies. Thus, τ^a and s_τ^a represent the equilibrium waiting time and probability of investment, respectively, when the level of competition is a.

Proposition 4 *(i) There exists a symmetric equilibrium in simple strategies for any* $a \in [a_m, a_M]$.

(ii) Moreover, there exists $\widehat{a}$ *such that:*

(1) For $a < \widehat{a}$, $s_\tau^a = 1$, $\frac{d}{da}\tau^a < 0$, *and firms expect strictly positive profits from innovation.*

(2) For $a \geq \widehat{a}$, $\frac{d}{da}s_\tau^a < 0$, $\tau^a = \tau^{\widehat{a}}$, *and firms expect zero profits from innovation.*

The proof of proposition 4, as well as more precise statements, with the exact conditions determining τ^a, s_τ^a, and the cutoff $\widehat{a}$ can be found in appendix B2.

Proposition 4 states that there is a threshold $\widehat{a}$, such that for $a < \widehat{a}$, firms invest with probability $s_\tau^a = 1$ in a project that does not reveal itself to be infeasible by time τ^a. For these values, as a increases, firms decrease the equilibrium waiting time τ^a. Therefore, when competition is low, an increase in competition induces firms to invest in riskier projects, and thus to become more innovative. On the other hand, for $a > \widehat{a}$, firms hold constant the waiting time at $\tau^a = \tau^{\widehat{a}}$, but as a increases, they invest in a project with a decreasing probability s_τ^a. Thus, for high values of competition, as competition increases, firms become less innovative.[19]

To understand the intuition for proposition 4, note first that from (6)-(9), it follows immediately that an increase in a shifts the MC curve upwards. On the other hand, the MB curve is unaffected by a change in competition. Therefore, for $a < \widehat{a}$, as a increases, the MC curve crosses the MB curve for a smaller value of t. From (11), it follows that τ^a decreases. On the other hand, since competition is low and relatively safer projects are undertaken, firms can sustain positive equilibrium expected profits from innovation while they *all* invest in the project. Therefore, $s_\tau^a = 1$.

[19] To understand these, assume that the budget allocated to the innovative activity in a given period is $zc\varphi$, where z is the number of projects emerging from the applied research activity, c is the cost of innovation per project, and φ is the probability of investing in any given project. Our analysis considers z and c to be fixed, and focuses on identifying the effect of a change in competition, denoted by a, on φ. Thus, note that the probability of investing in any given project is $\varphi^a = s_\tau^a \frac{p_0}{p_{\tau^a}}$. An increase in s_τ^a increases the level of innovative activity along the quantitative dimension. A decrease in τ^a, and as such in p_{τ^a}, corresponds to an increase along the risk dimension.

Let $\widehat{a}$ be the level of competition where, in equilibrium, condition (10) holds with equality for $s_{\tau}^{\widehat{a}} = 1$. For $a > \widehat{a}$ there is no symmetric equilibrium in simple strategies in which all firms invest in the project. To see this, note that as a increases above $\widehat{a}$, in order for condition (10) to continue to be satisfied while $s_{\tau}^{a} = 1$, firms would need to invest in safer projects. Thus, τ^{a} should increase. However, as a increases above $\widehat{a}$, if $s_{\tau}^{a} = 1$, the MC curve would continue to shift up. Thus, the trade-off between the MC and MB of waiting would actually be solved *earlier*, which is inconsistent with the fact that τ^{a} should be increasing.

Instead, for $a > \widehat{a}$, firms invest in a project with a decreasing probability s_{τ}^{a}. Effectively this implies that a firm pursues only a fraction of the projects emerging from the applied research activity, and thus becomes less innovative. In line with the Schumpeterian argument, the explanation is that in highly competitive industries, the potential revenues from a successful new product are divided among many firms and thus each firm's expected profit from the innovation is virtually zero. As competition increases, by innovating with a decreasing probability, the level of competition in the post-innovation market is endogenized, allowing firms to expect nonnegative profits. In particular, it allows for condition (10) to continue to hold. Therefore, when the pre-innovation competition is high, the competition in the post-innovation markets becomes *endogenous*. From a policy perspective, this finding implies that the positive welfare effects of increasing competition in the marketplace have only a limited scope when considered in a dynamic context.

The following corollary states that as a increases above $\widehat{a}$, $a \cdot s_{\tau}^{a}$ stays constant.

Corollary 5 $\frac{d}{da}\left(a \cdot s_{\tau}^{a}\right) = 0$ *for* $a \geq \widehat{a}$.

The result is intuitive since in order for profits to be kept at zero as a increases above $\widehat{a}$, the constant timing of innovation implies that the post-innovation level of competition must also stay constant. Thus, if a parameterizes the number of firms in an industry, as this number increases, while the firm level intensity of innovative activity decreases, the industry-wide level intensity stays constant. Note that in Aghion et al. (2005), the intensity of innovation is measured at firm level.[20] Also, the corollary implies that when $a \geq \widehat{a}$, the MC curve no longer shifts up, which explains why firms maintain a constant waiting time.

The final result of this section, whose proof is in appendix B3, presents the effect of an increase in θ on the equilibrium strategies. An increase in θ captures a stronger first-mover advantage in the industry, as the drop in the post-innovation profits from being a laggard firm becomes sharper. Denote by $\widehat{a}(\theta)$ the cutoff from proposition 4 as a function of θ.

Corollary 6 *(i) When* $a < \widehat{a}(\theta)$, $\frac{d}{d\theta}\tau^{a} < 0$; *(ii)* $\frac{d}{d\theta}\widehat{a}(\theta) < 0$.

[20] On the other hand, if an increase in competition is associated with a decrease in the ability of a fixed number of firms to collude, then the innovation at industry level decreases at the same rate as at the firm level.

As the corollary suggests, an increase in θ leads firms to become more innovative when competition is low. Intuitively, a stronger first-mover advantage increases the MC of waiting, inducing firms to invest earlier. On the other hand, recall that $\widehat{a}(\theta)$ is the level of competition at which firms make zero equilibrium expected profits from innovation while they all invest. Since part *(i)* of the corollary implies that the increase in θ induces firms to undertake riskier projects, the level of competition for which they make zero expected profits is lower when θ increases. Thus, $\widehat{a}(\theta)$ is decreasing in θ.

We close this section by noting that in this paper we investigated the level of competition that maximizes innovation, not social welfare. Clearly, more innovation may lead to a welfare loss when firms undertake too much risk or duplicate lines of research. In section S2 from the online appendix, we investigate the welfare effects of innovation under competitive pressure. A social planner who aims at designing the market structure most conducive to innovation has to take into account the effects of an increase in competition on the post-innovation social welfare, on the firms' risk taking behavior, on the timing of innovations, and on the degree of redundancy in parallel innovations. We find conditions that determine the level of competition that optimizes these welfare effects of innovation and argue that, generically, this level is different from the one that maximizes the industry-wide innovative activity.

3.3 The Innovation-Maximizing Level of Competition

To provide additional testable implications of our model, we examine the behavior of the peak of the inverted-U shape curve that maps competition into the level of innovative activity. More precisely, we argue that our model supports theoretically two additional empirical facts uncovered by Aghion et al. (2005). These facts describe the behavior of this peak in response to a change in the technological levelness of an industry.[21] In the empirical part of the paper, Aghion et al. (2005) show in Figure III that for the subsample of industries with a *higher* degree of levelness, the inverted-U curve has a *higher peak* which is attained at a *lower* level of competition than the curve corresponding to the entire sample of industries. However, while the theoretical model in Aghion et al. (2005) does support the first of these two results, it does not support the second one. We show next that our model supports both empirical regularities.

As a proxy for the technological levelness of an industry, we use the length of the awareness window η, i.e., the time it takes for all firms to learn of an invention.[22] Note that as η changes, the measure of firms in the industry also changes. To isolate the effect of the change in the η from

[21] The location of the peak of the curve is defined by the values of competition that maximize innovation, and by the corresponding level of the innovative activity.

[22] Aghion et al. (2005) use the total factor productivity of a firm as a measure of the firm's technological level. Thus, the underlying assumption made here is that the length of time it takes for all firms in an industry to make a technological breakthrough is correlated with the dispersion in efficiency levels accross the industry. In industries in which this assumption is not satisfied, the results of this section are meant to be just an additional testable implication of the model, rather than a confirmation of the empirical findings in Aghion et al. (2005).

that of the change in competition, we denote the total measure of firms in the industry by α and the length of the awareness window by σ. By substituting σ for η and $\frac{\alpha}{\sigma}$ for a in the results of the previous section, we ensure that as we increase the length of the awareness window, the measure of firms in the industry, and thus their equilibrium expected profits, stay constant. Therefore, in this section α *is the measure of competition*, and σ *is the measure of the technological levelness of the industry*. We analyze how the value of α that maximizes innovation and the corresponding intensity of innovative activity respond to a change in σ.

The maximum intensity of innovation is attained for the value of competition that induces investment by all firms and the minimum equilibrium waiting time. As the previous analysis suggests, this value of competition is precisely the cutoff from proposition 4. The following proposition, whose proof is in Appendix B4, is the main result of this section. Denote by $\widehat{\alpha}(\sigma)$ the threshold given by proposition 4 as a function of σ, and by $\tau^{\widehat{\alpha}(\sigma)}$ the corresponding waiting time.[23]

Proposition 7 *(i)* $\frac{d}{d\sigma}\widehat{\alpha}(\sigma) > 0$. *(ii)* $\frac{d}{d\sigma}\tau^{\widehat{\alpha}(\sigma)} > 0$.

Thus, as z increases, the value of competition that maximizes innovation moves to the right. On the other hand, the minimum equilibrium waiting time increases and thus the corresponding intensity of innovative activity decreases. Therefore, the peak of the inverted-U curve moves down and to the right. Conversely, when σ *decreases*, and thus the degree of technological levelness of the industry increases, the peak of the inverted-U curve moves up and to the left. This is consistent with what Aghion et al. (2005) uncovered. Intuitively, when σ decreases, each firm i expects that the times when the rest of the firms learned of the same invention are closer to the moment when firm i learned. In other words, it increases the density of firms in the awareness window. This increases the MC of waiting for more information at any time, and therefore induces firms to invest earlier for any value of competition. Moreover, since at the peak of the inverted-U curve firms make zero profits while investing with probability $s_{\tau}^{\alpha} = 1$, the lower equilibrium belief about the ultimate success of the investment that results from investing earlier must correspond to a value of competition which is also lower.

3.4 Discussion of the Modelling Choices

In our model, firms are not informed of the exact moment when other firms became aware of the same invention. Besides capturing the real world uncertainty that firms face, this assumption has the merit that it induces a smooth marginal cost of waiting and thus a smooth payoff function essential for equilibrium existence.

The exponential conditional distribution of the time of arrival of the first negative signal emerges

[23] The analysis in this section is reminiscent of that in Brunnermeier and Morgan (2010) who study the effect of an increase in the degree of clock desynchronization in preemption games with sequential awareness. Barbos (2012) studies the case when the increase in the number of players is associated with an increase in clock desynchronization.

naturally from the assumption of a constant rate of arrival of negative signals, and due to its tractability and intuitive appeal, is standard in models of experimentation with exponential bandits such as Cripps, Keller and Rady (2005).

Assuming a non-zero information acquisition cost adds a negative component to the expected profit from pursuing the project representing the expected lifetime information acquisition costs. This does not change the analysis in a meaningful way since these costs would need to be incurred anyway until uncertainty is removed, and thus would not impact the timing of innovation. Also, the specification of a *one-time* innovation cost is inessential. The cost c can be interpreted in the model as the *expected* present value of all future expenditures on the development of this new product.

If firms observed the investment decisions of other firms with a delay that is longer than η, the formal analysis would not change. Since the time it takes for all firms to invest is precisely η, when $a < \widehat{a}$, they would have all invested by the time the action of the first firm becomes public information. For $a \geq \widehat{a}$, condition (12) would ensure again that firms do not have an incentive to invest. If the delay after which the investment of the first firm becomes public information is smaller than η, the formal analysis would change. Once investment is observed, all information becomes public, and if there is still room for innovation, firms who have not yet invested engage in a second-stage game in which they compete for the remaining market share. Firms would have to account for that possibility in the first-stage incomplete information game. The analysis of the resulting game is intractable because the equilibrium waiting time from the first stage affects the equilibrium payoffs of the second stage as it determines the amount of information about the project that firms start with at the beginning of the second stage. However, the intuition behind the results from our paper would continue to hold. To see this, note that the alternative specification would imply that the MC of waiting also contains a component that is the difference between the equilibrium expected profits from the first-stage game minus by the equilibrium expected profits from the second-stage game. Since, as argued in section 2.3, the increase in a increases the wedge between the profits of the earlier investors and those of the latter investors, this component would also be increasing in a. Thus, the MB curve would continue to shift up in response to the increase in competition, which is the salient driving force behind our results.

Finally, while the parameter a was introduced for simplicity of exposition as being the mass of firms that become aware at any instant, it can also be interpreted as the inverse measure of the degree to which a fixed number of firms in an industry are able to collude. Notice that if the mass of firms that become aware of the invention at any instant is 1 rather than a, and instead a multiplies the two measures that are the arguments of Π, the quantitative results are identical. Under this alternative specification of the model, a can be interpreted as the counterpart of the Lerner index since an increase in a lowers the average profits in the industry, while keeping the measure of firms in the industry constant.

3.5 Alternative Equilibria

In the above, we focused the analysis on the equilibria in simple strategies. Next, we consider the possible alternative equilibria of the game and show that the inverted-U shaped relationship emerges again. We define a *discrete strategy* $S_{\mathcal{H}}$ to be a distribution function over $\mathbb{R}_+ \cup \{\infty\}$ with a discrete support. The following lemma states that any symmetric equilibrium strategy is a discrete strategy and provides a sufficient condition under which the only symmetric equilibrium of the game is in simple strategies.[24] The proof of the lemma is in section S3 of the online appendix.

Lemma 8 *(i) Any symmetric equilibrium strategy is discrete.*

(ii) If $\mu\eta < 1$, any symmetric equilibrium strategy is simple.

The next proposition describes the key comparative statics with respect to a. Its proof is in section S3 of the online appendix.

Proposition 9 *If $S^a_{\mathcal{H}}$ is the strategy in a symmetric equilibrium, with support $\mathcal{H}^a = \{\tau^a_1, \tau^a_2, ...\}$ and associated probabilities $Q^a \equiv \{q^a_1, q^a_2, ...\}$, then there exists $\widehat{a}$ such that:*

(i) For $a < \widehat{a}$, $\frac{d}{da}\tau^a_1 < 0$, $\frac{d}{da}\left(\tau^a_{j+1} - \tau^a_j\right) = 0$ and $\frac{d}{da}q^a_j = 0$ for all $j \geq 1$, and $\sum_j q^a_j = 1$.

(ii) For $a \geq \widehat{a}$, $\mathcal{H}^a = \mathcal{H}^{\widehat{a}}$, and $aq^a_j = q^{\widehat{a}}_j$ for all $j \geq 1$.

For $a < \widehat{a}$, as a increases, the points in $\mathcal{H}^a$ remain equidistant to each other, while all moving to the left. Since Q^a is independent of a, an increase in a induces an increase in the intensity of innovation in the industry. On the other hand, for $a \geq \widehat{a}$, since the support of $S^a_{\mathcal{H}}$ stays constant, while the associated probabilities are decreasing, an increase in a induces less innovation. Therefore, if alternative equilibria exist, the firms' behavior in these equilibria continue to generate the inverted-U shape relationship between competition and innovation.

4 Conclusion

The issue of innovation is complex and has many facets, some of which have been studied extensively in the industrial organization literature over the past half a century. Our model uncovers two of the main driving forces influencing the level of innovative activity in an industry. These two forces are not only sufficient to generate the empirically documented inverted-U shape relationship between competition and innovation, but as anecdotal evidence suggests, they are also some of the major

[24] As shown in the proof, the points in the support of $S_{\mathcal{H}}$ must be at least η units of time apart from each other. The condition $\mu\eta < 1$ implies that if either the speed of learning μ or the length of the awareness window is sufficiently low, then sustaining an equilibrium in which firms mix among waiting for distinct times is infeasible.

forces that influence a firm's innovation decisions. In order to isolate the effect of the trade-off that we focus on, we abstract away from other factors that may play a role in the firms' decision making process. Clearly, enriching the model to include some of these additional forces would improve the predictive power of the model.

The main implication of the results in our paper is that monotonic public policies of *always* decreasing the level of product market competition, as Schumpeter suggested, or always increasing it, as other economists who looked for a linear relationship concluded, are not necessarily optimal for stimulating the innovative activity in an industry. Instead, our paper argues that a more thorough empirical analysis needs to be performed to find the right way to design the market structure so as to promote innovation in the industry under consideration.

A reduced-form version of the model in this paper would have the marginal cost of waiting and the marginal benefit of waiting curves satisfying two conditions. First, they would exhibit the single crossing property. Second, the marginal cost curve would shift up in response to an increase in competition, while the marginal benefit curve would stay fixed. Then, an increase in competition would decrease the time at which the two curves intersect, thus explaining the increase in innovation for the small values of competition. When this equilibrium waiting time is sufficiently low, firms would expect zero profits from innovation, and thus a further increase in competition would require firms to become less innovative. The need for the fully fledged model in this paper stems mainly from three considerations. First, the reduced-form model does not explain the link between the level of competition, which is a parameter with immediate empirical interpretation, and the marginal cost of waiting, whose interpretation is difficult in the absence of a well defined model. Second, the reduced model does not immediately suggest a way in which firms can become less innovative for higher values of competition. Simply stating that they would invest later is unsatisfactory since the marginal cost of waiting would continue to increase and thus the trade-off would be solved earlier rather than later. The model in this paper allows us to distinguish between decreases in innovation that lead to a delay in innovation and decreases in innovation that lead to a decrease in the number of projects undertaken. Finally, the model predicts additional testable regularities that a reduced form model would not uncover.

Appendix

Appendix A1.

For an arbitrary distribution of innovation times in the industry, denote by $G(t)$ the measure of firms who have invested by time t and let $n \equiv G(\infty)$ be the total measure of firms who invest.

Remark 10 *When* Π *is quasi-linear as in (1), the total amount of profits earned in the industry is independent of the distribution* $G(\cdot)$.

Proof. Since G is a cumulative distribution function, it is right-continuous and therefore the set of points of discontinuity is countable. Denote this set by $Y_d = \{k_1, k_2, k_3, ..., k_{|K_d|}\}$, where $|K_d|$ can be ∞, and let $k_0 \equiv t_0$. Also, for any $k \in Y_d$ denote by $G(k_-) \equiv \lim_{t \to s_-} G(k)$, and for any $t \in [t_0, \infty) \backslash K_d$ denote by $g(t)$ the probability distribution function associated with G. Then, the total amount of profits earned from the innovation in the industry is

$$\begin{aligned} \Pi^{ind} &= \sum\nolimits_{k \in K_d} \left[A(n) - \theta G(k_-) - \theta \frac{G(k) - G(k_-)}{2} - c \right] [G(k) - G(k_-)] + \\ &\quad + \int_{t \in [t_0, \infty) \backslash K_d} [A(n) - \theta G(t) - c] \, g(t) dt \\ &= n(A(n) - c) - \theta \sum\nolimits_{i=1,...,|K_d|} \left[\int_{k_{i-1}}^{k_{i-}} G(t) g(t) dt + \frac{G^2(k) - G^2(k_-)}{2} \right] \end{aligned}$$

where we used the fact that $G(\infty) = n$. Integrating by parts $\int_{k_{i-1}}^{k_{i-}} G(t)g(t)dt$, we obtain: $\int_{k_{i-1}}^{k_{i-}} G(t)g(t)dt = \frac{G^2(k_{i-}) - G^2(k_{i-1})}{2}$. Therefore, as claimed $\Pi^{ind} = n(A(n) - c) - \theta \frac{n^2}{2}$ does not depend on G. $\square$

Appendix A2.

Let s be the equilibrium probability that a firm innovates. Then, the total measure of firms who invest is $sa\eta$. The fact that the absolute profits of all firms decrease with competition is immediate. On the other hand, from the proof of remark 10, the total profits in the industry are $sa\eta [A(sa\eta) - c] - \theta \frac{(sa\eta)^2}{2}$, so the relative profit shares of the firm with rank ν is: $\beta(\nu) = \frac{A(sa\eta) - \theta\nu - c}{sa\eta[A(sa\eta) - c] - \theta \frac{(sa\eta)^2}{2}}$. By taking the derivative, we obtain that

$$\frac{\partial}{\partial a}\beta(\nu) = \frac{A'(sa\eta)sa\eta \left[A(sa\eta) - c - \theta \frac{sa\eta}{2}\right] - [A(sa\eta) - \theta sa\eta - c + sa\eta A'(sa\eta)] [A(sa\eta) - \theta\nu - c]}{(sa\eta)^2 \left[A(sa\eta) - c - \theta \frac{sa\eta}{2}\right]^2} s\eta \tag{13}$$

If the last firm to invest has nonpositive post-innovation profits or $A(\cdot)$ is concave, we have $A(sa\eta) - \theta sa\eta - c + sa\eta A'(sa\eta) < 0$. Thus, $\frac{\partial}{\partial a}\beta(\nu) > 0$ if and only if

$$\nu < A(sa\eta) - c + \frac{A'(sa\eta)sa\eta \left[A(sa\eta) - c - \theta \frac{sa\eta}{2}\right]}{-[A(sa\eta) - \theta sa\eta - c + sa\eta A'(sa\eta)]}$$

Therefore, as claimed, the relative profit shares of the most advanced firms increase in a. Finally, note also that $A(sa\eta) - \theta sa\eta - c + sa\eta A'(sa\eta) < 0$ implies that the total profits in the industry are decreasing in competition. The fact that the average profits are decreasing in competition is immediate. $\square$

Appendix B1. Proof of Proposition 2

We first compute the expected measure of firms who invest by time $t_i + t$ from firm i's viewpoint.

Lemma 11 *Consider a strategy profile under which each firm employs a simple strategy S_τ. Then, the expected measure of firms who invested by time $t_i + t$, from the viewpoint of firm i is*

$$\lambda_{S_\tau}(t|t_i) = \begin{cases} 0, & \text{for } t \in [0, \max(0, \tau - \eta)] \\ s_\tau a \frac{1}{2\eta}(t - \tau + \eta)^2, & \text{for } t \in [\max(0, \tau - \eta), \tau] \\ s_\tau a \left[\frac{\eta}{2} + (t - \tau) - \frac{1}{2\eta}(t - \tau)^2\right], & \text{for } t \in [\tau, \tau + \eta] \\ s_\tau a\eta, & \text{for } \ t \geq \tau + \eta \end{cases} \tag{14}$$

Proof. From (6) and (7), we have

$$\lambda_{S_\tau}(t|t) = \frac{1}{\eta}\int_{t_i-\eta}^{t_i} m_{S_\tau}(t|t_i, t_0)dt_0 = \frac{1}{\eta}\int_{t_i-\eta}^{t_i} s_\tau a \min(\eta, \max(t_i + t - \tau - t_0, 0))dt_0$$

Now, note that when $t \in [0, \max(0, \tau - \eta)]$ we have $t_i + t - \tau - t_0 < 0$ for all $t_0 \in [t_i - \eta, t_i]$. To see this, note that $t_i + t - \tau - t_0 < t_i + \max(0, \tau - \eta) - \tau - t_0 < \max(0, \tau - \eta) - \tau + \eta < 0$, where the first inequality follows from $t < \max(0, \tau - \eta)$, and the second one follows from $t_0 > t_i - \eta$. Therefore, when $t \in [0, \max(0, \tau - \eta)]$, we have $\int_{t_i-\eta}^{t_i} \min(\eta, \max(t_i + t - \tau - t_0, 0))dt_0 = 0$.

When $t \in [\max(0, \tau-\eta), \tau]$, we have (i) $t_i+t-\tau-t_0 < \eta$ for all $t_0 \in [t_i - \eta, t_i]$; (ii) $t_i+t-\tau-t_0 > 0$ if and only if $t_0 < t_i + t - \tau \in [t_i - \eta, t_i]$. To see (i), note that $t_i + t - \tau - t_0 < t_i - t_0 < \eta$, where the first inequality follows from $t < \tau$, and the second from $t_0 > t_i - \eta$. For (ii), $t_i + t - \tau \in [t_i - \eta, t_i]$ follows from $t \in [\max(0, \tau - \eta), \tau]$. Therefore,

$$\begin{aligned} \int_{t_i-\eta}^{t_i} \min(\eta, \max(t_i + t - \tau - t_0, 0))dt_0 &= \\ \int_{t_i-\eta}^{t_i+t-\tau} (t_i + t - \tau - t_0)\, dt_0 &= \\ \int_{-\eta}^{t-\tau} (t - \tau - t_0)\, dt_0 &= \frac{1}{2}(t - \tau + \eta)^2 \end{aligned}$$

When $t \in [\tau, \tau + \eta]$, we have (i) $t_i + t - \tau - t_0 < \eta$ if and only if $t_0 > t_i + t - \tau - \eta \in [t_i - \eta, t_i]$; (ii) $t_i + t - \tau - t_0 > 0$ for all $t_0 \in [t_i - \eta, t_i]$. For (i), note that $t_i + t - \tau - \eta \in [t_i - \eta, t_i]$ follows from $t \in [\tau, \tau + \eta]$. To see (ii), note that $t_i + t - \tau - t_0 > t - \tau > 0$, where the first inequality follows from $t_0 < t_i$ and the second from $t > \tau$. Therefore,

$$\begin{aligned} \int_{t_i-\eta}^{t_i} \min(\eta, \max(t_i + t - \tau - t_0, 0)) dt_0 &= \\ \eta \int_{t_i-\eta}^{t_i+t-\tau-\eta} dt_0 + \int_{t_i+t-\tau-\eta}^{t_i} (t_i + t - \tau - t_0)\, dt_0 &= \\ \eta\,(t-\tau) + \int_{t-\tau-\eta}^{0} (t - \tau - t_0)\, dt_0 &= \frac{\eta^2}{2} + (t-\tau)\eta - \frac{1}{2}(t-\tau)^2 \end{aligned}$$

Finally, when $t \geq \tau + \eta$, we have $t_i + t - \tau - t_0 > \eta$ for all $t_0 \in [t_i - \eta, t_i]$, so $\int_{t_i-\eta}^{t_i} \min(\eta, \max(t_i + t - \tau - t_0, 0)) dt_0 = \eta$. Collecting these results, we obtain (14). □

Corollary 12 *$\lambda_{S_\tau}(\tau|t_i) = \frac{1}{2} s_\tau a \eta$.*

Proof. The corollary follows immediately from lemma 11. □

Corollary 13

$$MC_{S_\tau}(t) = \begin{cases} 0, \;\; \textit{for } t \in [0, \max(0, \tau - \eta)] \cup [\tau + \eta, \infty) \\ \theta s_\tau a \frac{1}{\eta}\,(t - \tau + \eta)\,, \qquad \textit{for } t \in [\max(0, \tau - \eta), \tau] \\ \theta s_\tau a \left[1 - \frac{1}{\eta}\,(t - \tau)\right], \qquad\qquad \textit{for } t \in [\tau, \tau + \eta] \end{cases} \tag{15}$$

Proof. The corollary follows from (9) and (14). □

Denote now by

$$\Psi_{S_\tau}(t) \equiv p_0 \left[A\,(s_\tau a \eta) - \theta \lambda_{S_\tau}(t|t_i)\right] - c\left[p_0 + (1 - p_0)\, e^{-\mu t}\right], \text{ for } t \geq 0. \tag{16}$$

Thus, $\Psi_{S_\tau}(t)$ is firm i's expected profit from innovation, as of moment t_i, if it invests at $t_i + t$ and all other firms adopt simple strategies S_τ. The next lemma shows that under the conditions defined in the text of proposition 2, if all other firms but firm i adopt a simple strategy S_τ, then firm i's best response is to adopt the same strategy.

Lemma 14 *If (10), (11) and (12) are satisfied, then $\Psi_{S_\tau}(\tau) \geq \Psi_{S_\tau}(t)$ for any $t \in \mathbb{R}_+$.*

Proof. Note first that if $\tau > \eta$, then $\lambda_{S_\tau}(t|t_i) = 0$ on $[0, \tau - \eta]$ so firm i does not have an incentive to invest before $t_i + \tau - \eta$. Also, by (12), clearly it does not have an incentive to invest after $t_i + \tau + \eta$ since at that time all other firms have already invested. Therefore, it is sufficient to show that $\Psi_{S_\tau}(\tau) \geq \Psi_{S_\tau}(t)$ for any $t \in [0, \tau + \eta]$.

Now, the condition $p_0 \left[A\,(s_\tau a \eta) - \frac{1}{2} s_\tau \theta a \eta\right] - c\left[p_0 + (1 - p_0)\, e^{-\mu \tau}\right] \geq 0$ from the text of the proposition, ensures that $\Psi_{S_\tau}(\tau) \geq 0$ since by corollary 12, we have $\lambda_{S_\tau}(\tau|t_i) = \frac{1}{2} s_\tau a \eta$. Therefore, firm i would expect nonnegative profits from adopting the strategy S_τ. Second, $\Psi'_{S_\tau}(\tau) = 0$ if and only if $MC_{S_\tau}(\tau) = MB(\tau)$.

By straightforward calculations using (14), it follows that for $t \in [\max(0, \tau - \eta), \tau]$, we have $\Psi''_{S_\tau}(t) = -p_0\theta\lambda''_{S_\tau}(t|t_i) - \mu^2 c\,(1-p_0)\,e^{-\mu t} = -p_0\theta a \frac{s_\tau}{\eta} - \mu^2 c\,(1-p_0)\,e^{-\mu t} < 0$, Therefore, Ψ_{S_τ} is concave for $t \leq \tau$. On the other hand, when $t \in [\tau, \tau+\eta]$, we have $\Psi'''_{S_\tau}(t) = \mu^3 c\,(1-p_0)\,e^{-\mu t} > 0$.

Since $\Psi'''_{S_\tau}(t) > 0$ for $t \in [\tau, \tau+\eta]$, it follows that if $\Psi''_{S_\tau}(t^*) = 0$ for some $t^* \in [\tau, \tau+\eta]$, we have $\Psi''_{S_\tau}(t) > 0$ for all $t > t^*$. Since, $\Psi'_{S_\tau}(\tau) = 0$, $\Psi_{S_\tau}(t)$ can start increasing only after it becomes convex. Therefore, after $\Psi_{S_\tau}(t)$ starts increasing, it will increase forever. Since (12), for the case $s_\tau < 1$, and assumption 1 for the case $s_\tau = 1$, ensure that $\Psi_{S_\tau}(\tau+\eta) \leq 0$, it means that $\Psi_{S_\tau}(t) < 0$ for $t \leq \tau + \eta$. Therefore, as desired, $\Psi_{S_\tau}(\tau) \geq \Psi_{S_\tau}(t)$ for all $0 \leq t \leq \tau + \eta$. Moreover, condition (10) ensures that $\Psi_{S_\tau}(\tau)$ when $s_\tau < 1$, and thus that firm i is also willing to mix between investing and not investing. This completes the proof of the lemma. □

Lemma 14 proves the sufficiency of conditions (10), (11) and (12) for a symmetric equilibrium in simple strategies. The necessity of these conditions is straightforward. Note that when $s_\tau < 1$, (10) is necessary to be satisfied with equality to have the firms willing to mix, while (12) is necessary because otherwise the firms could deviate and invest after they remove all uncertainty.

To prove uniqueness of the symmetric equilibrium in simple strategies, note first that (11) implies $p_0\theta s_\tau a = c\,(1-p_0)\,\mu e^{-\mu\tau}$. If $s_\tau = 1$, this implies that τ is uniquely given by $p_0\theta a = c\,(1-p_0)\,e^{-\mu\tau}$.

On the other hand, if $s_\tau < 1$, then (10) must be satisfied with equality, and thus $p_0\Pi\left(s_\tau a\eta, \frac{1}{2}s_\tau a\eta\right) - c\left[p_0 + (1-p_0)\,e^{-\mu\tau}\right] = 0$. Since $\Pi\left(s_\tau a\eta, \frac{1}{2}s_\tau a\eta\right) = A(s_\tau a\eta) - \frac{1}{2}\theta s_\tau a\eta$, by substituting $c\,(1-p_0)\,e^{-\mu\tau}$ from the equality $p_0\theta s_\tau a = c\,(1-p_0)\,\mu e^{-\mu\tau}$, we have then that s_τ must satisfy

$$A(s_\tau a\eta) - \frac{1}{2}\theta s_\tau a\eta - \left[c + \frac{1}{\mu}\theta s_\tau a\right] = 0 \tag{17}$$

Since $A'(\cdot) < 0$, there is at most one value of s_τ satisfying this equation. Therefore, for a given value of s_τ, τ must satisfy $p_0\theta s_\tau a = c\,(1-p_0)\,e^{-\mu\tau}$, it follows that there is also a unique pair τ, s_τ with $s_\tau < 1$ satisfying (10) and (11).

Assume now that there exist two pairs $(\tau, s_\tau = 1)$ and $(\tau', s'_\tau < 1)$ satisfying (10) and (11). Then, since $c\,(1-p_0)\,e^{-\mu\tau} = p_0\theta a > p_0\theta a s'_\tau = c\,(1-p_0)\,e^{-\mu\tau'}$, we have that $\tau' > \tau$. Also, since $\frac{d}{dx}\Pi\left(x, \frac{x}{2}\right) = A'(x) - \frac{1}{2}\theta < 0$, we have $\Pi\left(s'_\tau a\eta, \frac{1}{2}s'_\tau a\eta\right) > \Pi\left(a\eta, \frac{1}{2}a\eta\right)$. Therefore, $p_0\Pi\left(s'_\tau a\eta, \frac{1}{2}s'_\tau a\eta\right) - c\left[p_0 + (1-p_0)\,e^{-\mu\tau'}\right] > p_0\Pi\left(a\eta, \frac{1}{2}a\eta\right) - c\left[p_0 + (1-p_0)\,e^{-\mu\tau}\right]$ which is strictly positive by (10). Thus, $p_0\Pi\left(s'_\tau a\eta, \frac{1}{2}s'_\tau a\eta\right) - c\left[p_0 + (1-p_0)\,e^{-\mu\tau'}\right] > 0$ contradicting (10). This completes the proof of proposition 2. □

Appendix B2. Proof of Proposition 4

To show that an equilibrium in simple strategies always exists, it is sufficient to show that conditions (10), (11) and (12) from the text of proposition 2 are satisfied for some values τ^a and s_τ^a for any value of a. Consider first equation (11) and note that by (5) and (15), this can be rewritten as

$$p_0 \theta s_\tau a = c\,(1-p_0)\,\mu e^{-\mu\tau} \tag{18}$$

Note first that since by assumption 1 we have $a \leq a_M = \frac{c\mu(1-p_0)}{\theta}$, there always exists a value $\widetilde{\tau}\,(a) \equiv \frac{1}{\mu} \ln \frac{c\mu(1-p_0)}{p_0 \theta a}$ satisfying (18) for $s_\tau^a = 1$. Moreover, $\widetilde{\tau}\,(a)$ is decreasing in a. Let $\widetilde{S}_\tau^a$ be the simple strategy with waiting time $\widetilde{\tau}\,(a)$ and probability of investment $s_\tau^a = 1$. We have two cases to consider depending on the sign of $\Psi_{\widetilde{S}_\tau^{a_m}}(\widetilde{\tau}\,(a_m))$.

(i) If $\Psi_{\widetilde{S}_\tau^{a_m}}(\widetilde{\tau}\,(a_m)) > 0$, where a_m is defined in assumption 1, let $\widehat{a}$ be the value of a satisfying $\Psi_{\widetilde{S}_\tau^a}(\widetilde{\tau}\,(a)) = 0$. Note from (16) that $\frac{\partial}{\partial a}\Psi_{\widetilde{S}_\tau^a}(\widetilde{\tau}\,(a)) < 0$ because $A'(\cdot) < 0$, $\lambda_{\widetilde{S}_\tau^a}(\widetilde{\tau}\,(a)\,|t_i) = \frac{1}{2}a\eta$ and $\frac{\partial}{\partial a}\widetilde{\tau}\,(a) < 0$. Thus $\widehat{a} > a_m$. Then for any $a \in [a_m, \widehat{a}]$, let $\tau^a \equiv \widetilde{\tau}\,(a)$ and $S_\tau^a \equiv \widetilde{S}_\tau^a$ and note that all conditions of proposition 2 are satisfied and that τ^a is decreasing in a, and $s_\tau^a = 1$.

Now, for $a \in [\widehat{a}, a_M]$, let $s_\tau^a \equiv \frac{\widehat{a}}{a} < 1$ and let S_τ^a be the simple strategy with waiting time $\tau^{\widehat{a}}$ and probability of investment s_τ^a. Note then first that (18) is satisfied for $s_\tau = s_\tau^a$ and $\tau = \tau^{\widehat{a}}$, because $s_\tau^a a = \widehat{a}$ and $p_0\theta\widehat{a} = c\,(1-p_0)\,\mu e^{-\mu\tau^{\widehat{a}}}$. Second, $\Psi_{S_\tau^a}(\tau^a) = p_0\left[A\left(s_\tau^a a\eta\right) - \theta\lambda_{S_\tau^a}(\tau^a|t_i)\right] - c\left[p_0 + (1-p_0)\,e^{-\mu\tau^a}\right] = p_0\left[A\left(\widehat{a}\eta\right) - \frac{1}{2}\theta\widehat{a}\eta\right] - c\left[p_0 + (1-p_0)\,e^{-\mu\tau^a}\right] = \Psi_{S_\tau^{\widehat{a}}}(\tau^{\widehat{a}}) = 0$, and thus (10) is also satisfied. Finally, (12) is satisfied because $\Pi\left(s_\tau^a a\eta, s_\tau^a a\eta\right) = \Pi\left(\widehat{a}\eta, \widehat{a}\eta\right) < c$ because $\widehat{a} > a_m$. Also note that s_τ^a is decreasing in a.

(ii) Assume now that $\Psi_{\widetilde{S}_\tau^{a_m}}(\widetilde{\tau}\,(a_m)) < 0$. In this case competition is already too high at a_m for firms to expect non-negative profits from innovation if they all invest in the project. In this case, let first $\widehat{a} \equiv a_m$. Second, for any $a \in [a_m, a_M]$, let S_τ^a and τ^a be such that they satisfy: $\Psi_{S_\tau^a}(\tau^a) = 0$ and $p_0\theta s_\tau^a a = c\,(1-p_0)\,\mu e^{-\mu\tau^a}$. To see that a solution to these equations exists, note first that $\Psi_{S_\tau^a}(\tau^a) = p_0\left[A(s_\tau^a a\eta) - \frac{1}{2}\theta s_\tau^a a\eta\right] - c\left[p_0 + (1-p_0)\,e^{-\mu\tau^a}\right]$ and substituting $c\,(1-p_0)\,\mu e^{-\mu\tau^a} = p_0\theta s_\tau^a a$, we get that s_τ^a needs to satisfy $A(s_\tau^a a\eta) - \frac{1}{2}\theta s_\tau^a a\eta - \left[c + \frac{1}{\mu}\theta s_\tau^a a\right] = 0$. Since $\Psi_{\widetilde{S}_\tau^{a_m}}(\widetilde{\tau}\,(a_m)) < 0$, we have

$$A(a_m\eta) - \frac{1}{2}\theta a_m\eta - \left[c + \frac{1}{\mu}\theta a_m\right] < 0 \tag{19}$$

On the other hand, the condition $\Pi(a_m\eta, a_m\eta) - c = 0$ implies that $A(a_m\eta) = \theta a_m\eta + c$, and therefore that $A(0) > A(a_m\eta) > c$.

From (19), $A(0) - c > 0$ and the fact that $a \geq a_m$, it follows by the continuity of $A\,(\cdot)$ that there exists $s_\tau^a \in \left(0, \frac{a_m}{a}\right)$ such that $A(s_\tau^a a\eta) - \frac{1}{2}\theta s_\tau^a a\eta - \left[c + \frac{1}{\mu}\theta s_\tau^a a\right] = 0$. It is straightforward to see that s_τ^a is decreasing in a because $s_\tau^a a$ must be constant as a increases. Finally, note that since $s_\tau^a a$ is constant as a increases, from $p_0\theta s_\tau^a a = c\,(1-p_0)\,\mu e^{-\mu\tau^a}$, it follows that τ^a must also be constant.

This completes the proof of proposition 4. $\square$

Appendix B3. Proof of Corollary 6

From the proof of proposition 4, it follows that when $a < \widehat{a}(\theta)$, we have $\tau^a \equiv \frac{1}{\mu} \ln \frac{c\mu(1-p_0)}{p_0 \theta a}$. Clearly, $\frac{d}{d\theta}\tau^a < 0$ and thus part (i) of the corollary is proved. On the other hand, also as in the proof of proposition 4, $\widehat{a}(\theta)$ is the solution to the equation in a: $A(a\eta) - \frac{1}{2}\theta a\eta - \left[c + \frac{1}{\mu}\theta a\right] = 0$. By denoting $K(a,\theta) \equiv A(a\eta) - \frac{1}{2}\theta a\eta - \left[c + \frac{1}{\mu}\theta a\right]$, note that we have $\frac{d}{da}K(a,\theta) = \eta A'(a\eta) - \frac{1}{2}\theta\eta - \frac{1}{\mu}\theta < 0$ and $\frac{d}{d\theta}K(a,\theta) = -\frac{1}{2}a\eta - \frac{1}{\mu}a < 0$. Therefore, indeed $\frac{d}{d\theta}\widehat{a}(\theta) < 0$. $\square$

Appendix B4. Proof of Proposition 7

The proof is similar to that of corollary 6. First, by substituting α for $a\eta$ and $\frac{\alpha}{\sigma}$ for a in $A(a\eta) - \frac{1}{2}\theta a\eta - \left[c + \frac{1}{\mu}\theta a\right] = 0$, it follows that $\widehat{\alpha}(\sigma)$ is the solution to the equation in α: $A(\alpha) - \frac{1}{2}\theta\alpha - \left[c + \frac{1}{\mu}\theta\frac{\alpha}{\sigma}\right] = 0$. Therefore, by denoting $L(\alpha,\sigma) \equiv A(\alpha) - \frac{1}{2}\theta\alpha - \left[c + \frac{1}{\mu}\theta\frac{\alpha}{\sigma}\right]$, we have $\frac{d}{d\alpha}L(\alpha,\sigma) = A'(\alpha) - \frac{1}{2}\theta - \frac{1}{\mu\sigma}\theta < 0$, and $\frac{d}{d\sigma}L(\alpha,\sigma) = \frac{1}{\mu}\theta\frac{\alpha}{\sigma^2}$. Therefore, $\frac{d}{d\sigma}\widehat{\alpha}(\sigma) > 0$. On the other hand, since $\tau^{\widehat{\alpha}(\sigma)}$ satisfies $p_0\left[A(\widehat{\alpha}(\sigma)) - \frac{1}{2}\theta\widehat{\alpha}(\sigma)\right] - c\left[p_0 + (1-p_0)e^{-\mu\tau^{\widehat{\alpha}(\sigma)}}\right] = 0$, when $\widehat{\alpha}(\sigma)$ increases, $\tau^{\widehat{\alpha}(\sigma)}$ must increase as well. Therefore, $\frac{d}{d\sigma}\tau^{\widehat{\alpha}(\sigma)} > 0$. $\square$

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