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# Prospects for a Unified Urban General Equilibrium Theory\*

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## Abstract

This is a short essay on open questions in urban economic theory.

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Essays predicting the future of a field are like cheap light bulbs: they are dull and have short shelf life. That will no doubt be true of this essay as well, but I shall at least be brief and focus on a very specific issue and research agenda, in the hope of stimulating a few filaments. If a light bulb (cheap or not) goes on somewhere, all the better. First, I shall describe the general context and problem, then hone in on the specifics of the models and the prospects for a solution to the problem.

Why should a unified urban general equilibrium theory be of interest? The current state of the literature is a scattered set of models set up to address specific questions. There is nothing wrong with this approach, provided that the models yield testable hypotheses so that competing models can be run against each other using data. One can argue about whether this actually happens, but in this limited space I won't address that issue. The purposes of a unified theory are:

- To make the commonalities between models and their differences absolutely stark. Here we mean to focus on both the assumptions, implicit or explicit, and the results.
- To allow new models of the same phenomena to be introduced and classified.
- To make the robustness of models clear.
- Such unified structures also give us new ways of thinking about and teaching material, much as general equilibrium theory, welfare economics, and the theory of the second best gave us new ways to see market failures, in contrast with partial equilibrium theory and classical one market cost-benefit analysis.

Isn't there already a unified urban general equilibrium theory? Actually, no, there isn't. There are many variations of models with perfect and imperfect competition, land modeled in various ways, pure exchange or production, a continuum or a finite number of agents, and various assumptions about agent mobility or location. There is little point to creating new combinations without sufficient motivation, namely questions that beg to be addressed, be they normative or positive. Many models, such as those of the New Economic Geography, seem tied to specific functional forms.<sup>1</sup>

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<sup>1</sup>My view of the New Economic Geography is provided elsewhere, in Berliant (2006).

What distinguishes urban models from others? Clearly, the use of mobile agents (in addition to commodities that might be mobile or immobile) is a distinguishing feature. One might think that what distinguishes urban models is the correlation of land use and the location of the agents using it, but this can be misleading.<sup>2</sup> Instead, I propose that it is the indivisibility of agents in terms of their choice of location (namely, each agent can only be at one place at any given time and state of the world) and the differentiation of commodities by this locational attribute that distinguishes urban models. But it is important to emphasize that in fact, the *field* of urban economics (as opposed to the models) is defined by a set of *questions*, not by a set of models.

The first order of business is to seek commonalities. The purpose is to prove (though not in this essay) theorems on existence of equilibrium, welfare, core, certain comparative statics, and so forth for all the models simultaneously. Such a unification would bring out the essential elements of the theory, including the underlying commonality in the commodity space, and thus the deeper and simpler structure of the mathematics common to urban models.

In this proposal, I shall attempt not to use many assumptions. The discussion might appear to be very abstract, but it can easily be made concrete by using the examples provided. I hope that it will subsume most known models, and a few unknown ones as well.

Urban general equilibrium models can be classified by their commodity spaces:

- There are a finite number of indivisible commodities (houses or parcels), plus perhaps divisible commodities modeled as a subset of  $\mathbb{R}^l$ , say  $\mathbb{R}_+^l$ . (It can be useful to think of this example in the other categories.)
- The commodity space is the finite or infinite union of copies of  $\mathbb{R}^l$ , representing consumption of mobile and immobile commodities, one for each location. Consumers can use positive amounts of commodities in only one location. The New Urban Economics and the New Economic Geography both fit here.
- Agents use intervals in  $\mathbb{R}$  or  $\mathbb{R}_+$ , and use mobile goods in  $\mathbb{R}^l$  as well. This is Alonso's (1964) model.
- Agents use measurable subsets of  $\mathbb{R}^k$ ; this also subsumes mobile commodities.

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<sup>2</sup>After all, at least in theory, agents could own bits of land everywhere, but this wouldn't look much like an urban model.

These are the models I know about, but I could be missing some. It might take some work, for instance if one has a model of networks, to see how it fits into the scheme. The intersection of each pair of classes can be nonempty, so the classification of a model might not be unique.

However much one might wish it, the equilibria of these models are rarely similar. Berliant and ten Raa (1991) provide a set of examples showing how equilibria differ in models with a continuum of agents and those with a finite number of agents. Berliant and Sabarwal (2006; henceforth BS) give a potentially testable comparative static that is different for models with a continuum of agents as opposed to those with a finite number. There is an older literature that gives more detail on how and why the equilibria and their properties generally differ.<sup>3</sup> More to the point, the allocations of these models live in different spaces, so it is silly to think that the equilibrium allocations will be similar or the same in general. Nevertheless, it's still possible that the basic theorems could be proved in general. For example, if a first welfare theorem can be proved in a universal commodity space, then it might also hold in each specific commodity subspace simply because the universal theorem is stronger. But this does not imply that the equilibria of the models are similar in respects other than the welfare properties of equilibrium allocations; Berliant *et al* (1990) show that even the latter might not be true. Under what circumstances are properties common to these models?

Can we find a universal commodity space? The first attempt is actually clear, both from the work over the years on general equilibrium existence theorems from general equilibrium theory, and from the natural structure of the space of subsets of a given set. What about a lattice structure? Unfortunately, models in categories two and three do not have a natural lattice structure. For example, the intersection of two intervals is an interval, but the union of two intervals is not necessarily an interval.<sup>4</sup>

It appears that we are at an impasse, so we should do something more productive like go watch the Cardinals. But there is actually a more subtle connection. The price space for all these classes of models is the set of con-

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<sup>3</sup>See Berliant (1985, 1991), Asami *et al* (1991), Kamecke (1993), Papageorgiou and Pines (1990) and Berliant and ten Raa (1991).

<sup>4</sup>Chiaki Hara has recently reminded us that fixed point theorems on partially ordered sets might be useful for proving a general theorem on existence of equilibrium for the fourth category, where the partial order is given by, for example, set inclusion. When the author and Thijs ten Raa looked into this some 22 years ago, they found that the order preservation requirement on excess demand maps would be too demanding.

tinuous functions on the set of locations. So if we call the underlying location space  $S$ , where depending on the model agents can locate at either a point or on a measurable subset of  $S$ , then the price space is  $C(S)$  (or if it has many dimensions in its range to accommodate many commodities at each location, the corresponding product space). Now since value is a bilinear form on the commodity space and the price space, it is useful to know the dual space of  $C(S)$ , the set of continuous linear functionals on  $C(S)$ , and its pre-dual, the space over which  $C(S)$  is the set of continuous linear functionals, for these essentially give us bounds on the universal commodity space. If  $S$  is a compact metrizable space, it is known that the dual of  $C(S)$  is the set of finite Borel measures on  $S$ , that we call  $\mathcal{M}$  (see Aliprantis and Border, 1999, Theorem 13.15 p. 466). This is essentially the duality used by Jones (1983, 1984). The pre-dual of  $C(S)$  is more problematic; however, it is larger than  $\mathcal{M}$ . In any case, one can embed the commodity spaces of all four classes of models in the set of finite Borel measures in rather obvious ways, for example by using indicator functions of sets. Thus, there is no need to worry about the pre-dual of  $C(S)$ , since  $\mathcal{M}$  is already large enough. This is the first commonality.

The set  $\mathcal{M}$  of finite Borel measures on  $S$  is the universal commodity space. One could begin by showing that it's the smallest space with certain properties (for example, it's a linear space) embedding all of the examples. Then one could proceed to construct universal theorems. For instance, a universal theorem on existence of equilibrium for urban models would begin: "For any economy that has as its commodity space a subset of  $\mathcal{M}$  that satisfies the following properties..."

But wait, there's more! At equilibrium in these models, one will generally find that a first order condition for consumer optimization is represented by an equation that says marginal utility is proportional to price. If there is a mobile consumption good that everyone likes, then this can be written as marginal rate of substitution is equal to price. Therefore marginal utility lies in the same linear space as price, namely  $C(S)$ . Thus, we have a second commonality. So we know that (possibly non-linear) utility functions must be smooth on at least a subspace of the set of finite Borel measures on  $S$  with derivatives defined in an appropriate way.

Another interesting question that naturally arises here is whether a nonlinear, continuously differentiable utility function defined on a subspace, namely the commodity space of a model, is extendable to the whole space, and whether this extension is unique. This might provide unique ways to relate the classes

of models to each other. If there is no extension, then there are models in one class that have no analog in another. If the extension is not unique, then a model in one class has many analogs in another. Of course, if the extension exists and is unique, this does not imply that the equilibria of the two models are the same, but only that consumer behavior is consistent across models. In other words, the same consumer can generate demand in the two models.

Arguments similar to those in the preceding two paragraphs will apply to production.

Finally, the “universal” analog of the classical Muth-Mills condition would be of interest. It should collapse to the appropriate condition in each model’s commodity subspace as a special case. The condition is already known for the models of the New Urban Economics (where it was discovered) and in Alonso’s model (see Berliant and Fujita, 1991). The form of the condition is actually quite different in these two models, so the universal analog should be quite interesting.

To the reader not adept at functional analysis, this essay might seem like technical drivel, and perhaps it is. But putting aside the introductory motivation, the reason these ideas could be useful is that there is no general theory of existence of equilibrium, beyond examples, for popular models such as those of the New Economic Geography. I believe that this train of thought is the most promising approach to such a theory. Alternatives, such as the arguments used by Lucas and Rossi-Hansberg (2002), are less promising due to indeterminacy of equilibrium (Berliant and Kung, forthcoming).<sup>5</sup>

How many urban economists does it take to replace a cheap light bulb with something better?

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<sup>5</sup>I should note that in spite of their apparent sophistication, existence of equilibrium in models where all agents are mobile can often turn out to be trivial; check to see if the uniform distribution of agents with prices and allocations constant across locations is a feature of one equilibrium. Of course, this presumes that there are no natural advantages to any particular location in the model. The formal context here is Starrett’s Spatial Impossibility Theorem; see Starrett (1978), Fujita (1986) or Fujita and Thisse (2002, chapter 2.3).

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