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Bishnu, Monisankar and Ghate, Chetan and Gopalakrishnan, Pawan

Indian Statistical Institute - Delhi Center

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Distortionary Taxes and Public Investment in a Model of Endogenous Investment Specific Technological Change*

Monisankar Bishnu† Chetan Ghate‡
Pawan Gopalakrishnan §

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Abstract

We construct a model of endogenous growth in which public capital financed by distortionary taxes influences investment specific technological change. Our main result is that there exist infinitely many capital and labor tax-subsidy combinations that decentralize the planner's growth rate. Hence the optimal factor income tax mix is indeterminate which gives the planner the flexibility to choose policy rules from an infinitely large set. Accounting for administrative costs, however, reduces the set of optimal feasible tax mix of the planner. The size of this set shrinks as the convexity of the administrative costs increases, and eventually a unique factor income tax mix emerges as the only feasible solution. A numerical exercise shows that the growth effects of factor income tax changes are not large.

Keywords : Investment Specific Technological Change, Endogenous Growth, Factor Income Taxation, Public Policy, Administrative Costs, Indeterminacy.

JEL Codes: E2; E6; H2; O4

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†Planning Unit, Indian Statistical Institute, New Delhi – 110016, India. Fax: 91-11-4149-3981. E-mail: mbishnu@isid.ac.in. Tel: 91-11-41493935.

‡Corresponding Author: Planning Unit, Indian Statistical Institute, New Delhi – 110016, India. Tel: 91-11-4149-3938. Fax: 91-11-4149-3981. E-mail: cghate@isid.ac.in.

§Planning Unit, Indian Statistical Institute, New Delhi – 110016, India. Fax: 91-11-4149-3981. E-mail: pawan8ad@gmail.com. Tel: 91-11-27568306.

1 Introduction

A growing literature attributes the importance of investment specific technological change to long run growth (see Greenwood et al. (1997, 2000); Whelan (2003)). Investment specific technological change refers to technological change which reduces the real price of capital goods. Greenwood et al. (1997, 2000) show that once the falling price of real capital goods is taken into account, this explains most of the observed growth in output in the US, with relatively little being left over to be explained by total factor productivity. Other authors, such as Gort et al. (1999) distinguish between equipment specific technological change and structure specific technological change. These authors show that 15% of US economic growth rate can be attributed to structure specific technological change in the post war period, while equipment-specific technological progress accounts for 37% of US growth. This implies 52% of US economic growth can be attributed to technological progress in new capital goods.[1] However, investment specific technological change in these models is typically is assumed to be an exogenous process.

In a series of recent papers, Huffman (2007, 2008) builds upon this literature by explicitly modeling the mechanism through which the real price of capital falls when investment specific technological occurs. Such models are characterized by endogenous investment specific technological change. In Huffman (2008), the changing relative price of capital is driven by research activity, undertaken by labor effort. Higher research spending in one period lowers the cost of producing the capital good in the next period.[2] Agents equate the utility costs of raising employment in research with the benefits of doing so. The return to increasing research employment is the discounted value of the reduction in the real cost of investment in future periods. Extra research employment also leads to higher future consumption because of the reduction in cost of investing in capital goods because of the research.[3] Investment specific technological change is thus endogenous in the model, since employment can either be undertaken in a research sector or a production sector.

The specification of investment specific technological change in the above literature however has not incorporated two important empirical determinants: public capital and the effects of capital deepening. For instance, it is well known that the public sector can be a

[1] See Greenwood and Krusell (2007) for a careful discussion of growth accounting in the presence of investment specific technological change. Cummins and Violante (2002) show that "technological gaps", i.e., the comparative productivity of a new machine relative to an average machine, can explain the dynamics of investments in new technologies as well as returns to human capital.

[2] Krusell (1998) also builds a model in which the decline in the relative price of equipment capital is a result of R&D decisions at the level of private firms.

[3] However, if the utility cost of raising research employment is high, then all employment would be devoted to production of output, and there would be no research employment. There would be no output growth, and the economy converges to a steady state with constant levels of output.

source of manufacturing investment goods – which in turn – affects growth (see Schmitz, 2001). The point of departure of this paper is that we allow the stock of public capital (or public infrastructure stock) to *directly* affect the future real price of capital goods through investment specific technological change, and therefore balanced growth.[4] In particular, we extend the environment in Huffman (2008) to allow public capital to directly affect investment specific technological change. Our setup *also* allows investment specific technological change to enhance the accumulation of public capital.[5] Since the public capital stock is financed by distortionary taxes, we allow a role for factor income taxes in generating endogenous growth and raising welfare in the presence of investment specific technological change. This generates several interesting optimal factor income taxation results.

Second, Greenwood et al. (1997) show that the real price of capital equipment in the US – since 1950 - has fallen alongside a rise in the investment-GNP ratio. Greenwood et al. (1997, p. 342) say: "The negative comovement between price and quantity.....can be interpreted as evidence that there has been significant technological change in the production of new equipment. Technological advances have made equipment less expensive, triggering increases in the accumulation of equipment both in the short and long run." Given this, we assume that investment specific technological change also depends on aggregate capital accumulation. In particular, we assume that the aggregate capital-output ratio exerts a positive contribution on reductions in the future price of real capital goods because of aggregate investment activity. Our model can be therefore seen as providing an endogenous channel through which aggregate investment activity also enhances investment specific technological change.

Our baseline model has two sectors. The first sector produces a final good, using private capital, and two types of labor activities. One type of labor activity is devoted to final good production, and the other to research which directly reduces the real price of capital goods in the next period. The second sector captures the effect of public capital and research activity on reducing the real price of capital goods explicitly and not through the shadow price of

[4] Our paper is related to two strands of the literature on fiscal policy and long run growth in the neoclassical framework. Barro (1990), for instance, models public services as a flow. Futagami, Morita, and Shibata (1993), allow public capital to accumulate. However, in the large literature on public capital and its impact on growth spawned by these papers, the public input, whether it is modeled as a flow or a stock, doesn't *directly* influence the real price of capital goods. For instance, in Ott and Turnovsky (2006) - who use the flow of public services to model the public input - and Chen (2006), Fischer and Turnovsky (1998) - who use stock of public capital - the shadow price of private capital is a function of public and private capital. In our model, public capital affects the real price off capital explicitly. This means that the public input affects *future* output through its effect on *both* future investment specific technological change, as well as future private capital accumulation.

[5] For instance, providing better infrastructure today reduces the cost of providing public capital in the future.

capital In the planner's problem, we assume that public capital is financed by a proportional income tax. We focus on the balanced growth path (BGP). We show that the balanced growth path is stable under a reasonable restriction. We characterize the growth and welfare maximizing tax rates. The growth and welfare maximizing tax rate are determined by the relative importance of the public capital output ratio vis-a-vis the private capital output ratio in the investment specific technological change function. The implication of this is that if a planner was to choose the tax rate, he could maximize long run growth as long as the tax rate equals the relative contribution of public capital to investment specific technological change. We show that welfare maximizing tax rate is smaller than the growth maximizing tax rate.

We then decentralize the planner's allocations. We assume that public capital is financed by distortionary factor income taxes on capital and labor income. Interestingly, we show that infinitely many combinations of factor income taxes can replicate the planner's allocations based on an optimal tax rule. The equilibrium factor income tax mix is therefore indeterminate. Intuitively, indeterminacy occurs because for any given tax rate on one factor income, changing the other factor income tax produces a different Laffer curve, with the optimal tax rule now satisfied under the new Laffer curve. The implication of indeterminacy in our model is that it gives the planner the flexibility to choose policy rules from an infinitely large set. We also show that indeterminacy remains robust to two natural variants of the model, i.e., in allowing just a single type of labor activity to augment final good production, and allowing agents to participate in the credit market.

We then incorporate administrative costs to tax collection. We show that accounting for administrative costs reduces the set of optimal feasible tax mix of the planner. In fact the size of this set shrinks as the convexity of the administrative costs increases and a unique factor income tax mix emerges as the only feasible solution. We also show that when administrative costs increase with increases in the tax rate there is a level reduction in the growth rate for all tax rates, and a lower growth maximizing tax rate. The indeterminacy in the factor income tax mix, and its robustness, is the main result of our paper.

From a growth-tax policy standpoint, we show that reducing the tax on labor, while increasing the tax on capital by an equi-proportionate amount, reduces growth marginally. However, a revenue neutral change - which takes into account the elasticity adjusted factor income tax changes - a rule that we characterize analytically – increases growth in comparison to the equi-proportionate case. This result holds for large changes in the labor income tax. However, if we reduce the tax on capital, the change required in terms of a revenue neutral increase in the labor income tax is less than the equi-proportionate case. Hence, a reduction in the tax on labor increases growth if we compare the equi-proportionate case to the revenue neutral case. These results contrast with the case where we increase the tax on capital. Our numerical results are consistent with some of the results in this literature that the growth effects of changes in the capital income tax rate are not large (see Stokey and Rebelo (1995)).[6]

The rest of the paper proceeds as follows. Section 2 develops the baseline model. Section 3 shows that our indeterminacy result is robust to two variants of the baseline model. Section 4 develops the model with administrative costs. Section 5 conducts numerous policy experiments. Section 6 concludes.

2 The Baseline Model

Consider an economy that is populated by identical representative agents, who at each period t, derive utility from consumption of the final good C_t and leisure $(1 - n_t)$.[7] The term n_t represents the fraction of time spent at time t in employment. The discounted life-time utility, U, of an infinitely lived representative agent is given by

$$U = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_t)]. \tag{1}$$

where $\beta \in (0, 1)$ denotes the period-wise discount factor. There is no population growth in the economy. The final good is produced by a standard Cobb-Douglas production function with a constant returns to scale technology. The production function is given by

$$Y_t = AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3}, \tag{2}$$

$A > 0$ is the productivity parameter. Output is produced using capital, K_t, and two different types of labor activity, n_{1t} and n_{2t}, both of which are essential to production. The first part of labor input, n_1, is devoted to direct production of final output. The second part of agent labor effort, n_2, can be thought of as a more specialized labor input required for research and development which affects the level of investment specific technological change, Z. In particular, we assume that a representative firm employs a fraction, $\delta_m \in (0, 1)$, of

[6] In the model with administrative costs, our policy experiments show that when we have concave administrative costs, it is easier to increase the tax on capital to re-establish optimal growth rates, compared to the model with convex administrative costs. We also show that there is virtually no change in growth or welfare for a significant increase in the tax on capital that matches a given reduction in the tax on labor. From a policy standpoint, this suggests that it may be easier to tax capital at a higher rate without changing growth or welfare when administrative costs are concave, when the tax on labor is reduced.

[7] See http://www.isid.ac.in/~cghate/chetanresearch.html for a detailed technical appendix of this paper.

n_2 in final production.[8] The remaining fraction of n_2 not used directly in production of the final good is devoted to research effort which augments the level of investment specific technological change (Z) in the subsequent time period.[9] Thus, n_2, is analogous to the research employment in Huffman (2008) that goes into raising Z.[10] In other words, the higher is the fraction of n_2 allocated for research effort, the higher is the future level of Z. The total supply of labor by an agent at time t is given by the following

$$n_t \equiv n_{1t} + n_{2t}. \tag{3}$$

The shares of capital K_t, n_{1t} and n_{2t} in final goods production are given by $\alpha_i \in (0,1), i = 1,2,3$ respectively. The assumption of constant returns to scale in this model ensures that they add up to unity, that is, $\alpha_1 + \alpha_2 + \alpha_3 = 1$.[11]

Private capital accumulation grows according to the standard law of motion augmented by investment specific technological change,

$$K_{t+1} = (1-\delta)K_t + I_t Z_t, \tag{4}$$

where $\delta \in [0,1]$ denotes the rate of depreciation of capital and I_t represents the amount of total output allocated towards private investment at time period t. Z_t represents investment-specific technological change. The higher the value of Z_t, the lower is the cost of accumulating capital in the future. Hence Z_t also can be viewed as the inverse of the price of per-unit private capital at time period t. Thus at every period t, Z_t augments investment I_t. $I_t Z_t$ thus represents the effective amount of investment driving capital accumulation in time period $t+1$.

In addition to labor time deployed by the representative firm towards R&D, the public capital stock, G, plays a crucial role in lowering the price of capital accumulation. Typically, the public input is seen as directly affecting final production – either as a stock or a flow (e.g., see Futagami, Morita, and Shibata (1993), Chen (2006), Fischer and Turnovsky (1997, 1998), and Eicher and Turnovsky (2000)). Instead, we assume that the public input facilitates investment specific technological change. This means that the public input affects future output through future private capital accumulation directly. In the above literature, the public input affects current output directly.

[8] The results of the model are qualitatively similar when agents choose δ_m optimally.

[9] In Section (3), we assume that only n_1 enters in final good production, as in Huffman (2008). This does not change the results qualitatively.

[10] Other papers in the literature - such as Reis (2011) - also assume two types of labor affecting production. In Reis (2011), one form of labor is the standard labor input, while the other labor input is entrepreneurial labor. Our analysis considers n_2 as quality enhancing labor.

[11] In Huffman, n_2 doesn't enter into the production of final goods directly.

We assume that in every period, public investment is funded by a constant proportional tax, $\tau \in (0,1)$, on income. We assume that public capital evolves according to

$$G_{t+1} = (1-\delta)G_t + I_t^g Z_t, \tag{5}$$

where G_{t+1} denotes the public capital stock in $t+1$, and I_t^g denotes the level of public investment made by the government in time period t. As mentioned in the introduction, we assume that Z_t augments I_t^g in the same way as I_t since it enables us to analyze the joint endogeneity of Z and G. To derive the balanced growth path, we further assume that the period wise depreciation rate $\delta \in [0,1]$ is same for both private capital and public capital. The government budget constraint is given by

$$I_t^g = \tau Y_t, \tag{6}$$

where $\tau \in (0,1)$ is the tax rate imposed by the planner to finance public capital.

2.1 Investment Specific Technological Change

To capture the effect of public capital on research and development, we assume that Z grows according to the following law of motion,

$$Z_{t+1} = B((1-\delta_m)n_{2t})^{\theta} Z_t^{\gamma} \left\{ \left(\frac{G_t}{Y_{t-1}}\right)^{\mu} \left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu} \right\}^{1-\gamma}. \tag{7}$$

Here, B stands for an exogenously fixed scale productivity parameter, $1-\delta_m$ represents the fraction of labor input, n_{2t}, devoted towards R&D, and $\mu \in (0,1)$ captures the impact of public investments on investment specific technological change. We assume that the parameters, $\theta \in (0,1)$ and $\gamma \in (0,1)$, where θ stands for the weight attached to research effort and γ is the level of persistence the current year's level of technology has on reducing the price of capital accumulation in the future. The term $\frac{G_t}{Y_{t-1}}$ represents public capital's influence in affecting investment specific technological change in time period $t+1$. For a given μ, increases in G_t relative to Y_{t-1} leads to increases in the future level of Z. We further assume that aggregate investment activity, as captured by the aggregate capital-output ratio, $\frac{K_t}{Y_{t-1}}$, affects investment specific technological change. In particular, a higher aggregate stock of capital in t, K_t, relative to Y_{t-1}, raises Z_{t+1}. At this stage, we make the following remarks to compare our setup with that in Huffman (2008).

Remark 1 *Assuming $\gamma = 1$, $\delta_m \to 0$ and $\alpha_3 = 0$, in equation (7) yields Equation 2.9 in*

Huffman (2008) describing investment specific technological change.

Remark 2 *We require $\gamma \in (0,1)$ for the equilibrium growth rate to adjust to the steady state balanced growth path.*[12]

2.2 The Planner's Problem

We first solve the planner's problem. The resource constraint the economy faces in each time period t is given by

$$C_t + I_t \equiv Y_t(1-\tau) = AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3}(1-\tau) \tag{8}$$

where agents consume C_t at time period t and invest I_t at time period t. Aggregate consumption and investment add up to after-tax levels of output, $Y_t(1-\tau)$, in every time period.

The planner maximizes life-time utility of a representative agent – given by (1) – subject to the economy wide resource constraint given by (8), the law of motion of private capital in equation (4), the law of motion of public capital in equation, (5), the equation describing investment specific technological change (7), the identity for total supply of labor given by (3) and finally, the government budget constraint given by (6).[13]

2.2.1 First Order Conditions

The Lagrangian for the planner's problem is given by,

$$L = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_{1t} - n_{2t}) + \lambda_t \{AK_t^{\alpha_1} n_{1t}^{\alpha_2} (\delta_m n_{2t})^{\alpha_3}(1-\tau) - C_t - I_t\}]. \tag{9}$$

For simplicity, we assume that $\delta = 1$. The following first order conditions obtain with respect to C_t, K_{t+1}, n_{1t}, and n_{2t}, respectively[14]:

$$\tfrac{1}{C_t} = \lambda_t \tag{10}$$

$$\tfrac{1}{C_t Z_t} = \tfrac{\alpha_1 \beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \tfrac{\beta^2 I_{t+2}(1-\gamma)(1-\mu)}{C_{t+2}K_{t+1}} + \tfrac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \tfrac{\beta^j \gamma^j I_{t+j+3}}{C_{t+j+3}} \tag{11}$$

[12]This contrasts with Huffman (2008) where $\gamma = 1$ is required for growth rates of Z and output to be along the balanced growth path. In Huffman (2004), $\gamma < 1$ implies that the effect of research spending diminishes over time. This generates technical innovation having immediate productive effects that can be maintained only with more spending in the future. Therefore $\gamma = 1$ is not needed for balanced growth.

[13]Clearly, $C_t + I_t + I_t^g = Y_t$.

[14]See Appendix A for details.

$$\frac{1}{1-n_t} + \frac{\beta^2\alpha_2(1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t(1-\tau)}{C_t n_{1t}} \tag{12}$$

and,

$$\frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau)}{C_t n_{2t}} + \frac{\beta\theta I_{t+1}}{C_{t+1} n_{2t}} + \frac{\beta^2(\gamma\theta-\alpha_3(1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+2}}{C_{t+j+2}}. \tag{13}$$

Equation (10) represents the standard first order condition for consumption, equating the marginal utility of consumption to the shadow price of wealth. Equation (11) is an augmented form of the standard Euler equation governing the consumption-savings decision of the household. The first term on the RHS of equation (11), $\frac{\alpha_1\beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}}$, corresponds to the after tax marginal productivity of capital in $t+1$. The second term, $\frac{\beta^2 I_{t+2}(1-\gamma)(1-\mu)}{C_{t+2}K_{t+1}} > 0$, is the (further) increment to the marginal productivity of capital that agents get in period $t+2$ by postponing consumption today. This is increasing in the investment-consumption ratio, but adjusted by the weight, $1-\mu$, of the aggregate capital-output ratio, in the investment specific technological change equation. The third term, $\frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+3}}{C_{t+j+3}}$, is the discounted increase in marginal productivity of investing in capital from period $t+3$ onwards. This expression is adjusted by the term $(\gamma(1-\mu)-\alpha_1)$, which can be either positive or negative – depending on the relative importance of capital in equation (7) vis-a-vis its direct contribution to increasing output, from (2). It is easy to see that when $\gamma = 1$, the additional terms in the Euler equation are equal to zero, yielding the standard Euler equation.

Equation (12) denotes the optimization condition with respect to labor supply (n_{1t}). If we reorganize (12), we get the following expression for the marginal utility of leisure,

$$\frac{1}{1-n_t} = \frac{\alpha_2 Y_t(1-\tau)}{C_t n_{1t}} - \frac{\beta^2\alpha_2(1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+2}}{C_{t+j+2}}. \tag{14}$$

Since $0 < \gamma < 1$, the second term in the RHS, $\frac{\beta^2\alpha_2(1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+2}}{C_{t+j+2}}$, is positive, which constitutes a reduction in the marginal utility of leisure. This reduces n_1 relative to the standard case in which there is no investment specific technological change.

Similarly, the terms of (13), are,

$$\frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau)}{C_t n_{2t}} + \frac{\beta\theta I_{t+1}}{C_{t+1} n_{2t}} + \frac{\beta^2(\gamma\theta-\alpha_3(1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \frac{\beta^j\gamma^j I_{t+j+2}}{C_{t+j+2}}. \tag{15}$$

The second and third terms in the RHS are the $t > 0$ increment to marginal utility of leisure that accrues in the future because of n_2's role in assisting *both* research effort and increasing output. However, because n_2 has a direct and indirect effect (through production and investment specific technological change, respectively), the future discounted gains are adjusted by the term, $\gamma\theta - \alpha_3(1-\gamma)$. Going forward we assume $\gamma\theta - \alpha_3(1-\gamma) > 0$ which implies that final good production is not n_2 intensive.

We now derive the closed form decision rules based on the above first order conditions.

2.2.2 Decision Rules

Lemma 1 *C_t, $I_t, n_t, n_{1t,}n_{2t}$ are given by (16), (17), (18), where $0 < \Phi < 1$ is given by (19), and $0 < x < 1$ given by (20) are constants. The supply of labor is constant over time and is independent of the tax rate. Then,*

$$C_t = \Phi Y_t(1-\tau), I_t = (1-\Phi)Y_t(1-\tau) \tag{16}$$

$$n_t = n = \frac{\alpha_2[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi)]}{(\alpha_2 + \Phi x)(1-\beta\gamma) - \alpha_2\beta^2(1-\gamma)(1-\Phi)}, \tag{17}$$

$$n_{1t} = n_1 = xn, n_{2t} = n_2 = (1-x)n, \tag{18}$$

where Φ is given by

$$\Phi = \frac{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu) - \alpha_1\beta] - \beta^3(1-\gamma)[\gamma(1-\mu) - \alpha_1]}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)] - \beta^3(1-\gamma)[\gamma(1-\mu) - \alpha_1]}, \tag{19}$$

and x is given by

$$x = \frac{\alpha_2(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\Phi)}{[\alpha_2 + \alpha_3 + \beta\theta(1-\Phi)](1-\beta\gamma) + \beta^2(1-\Phi)[\gamma\theta - \alpha_3(1-\gamma) - \alpha_2(1-\gamma)]}. \tag{20}$$

Proof. These expressions follow from the first order conditions as shown in (10), (11), (12) and (13). See appendix B for details. ■

How does a change in μ affect consumption and investment? While the decision rules for consumption and investment given by (16) suggest that the levels of consumption and investment would fall if the tax rates increases (because of the $1-\tau$ term), the actual share of after tax income spent on consumption given by (19) rises when μ rises, although for investment it falls.[15] Intuitively, the representative agent does not invest as much in private capital because of an enhanced role of public capital in augmenting investment specific technological change.

[15] See Appendix F.

We will show later that when $\tau \neq \mu$, the allocations from the planner's problem are sub-optimal, even though there is balanced growth. The labor supply is affected by μ. This can be shown by the following lemma

Lemma 2 *Increases in μ has an ambiguous effect on n_1 but has a strong negative effect on n_2 which leads to an overall reduction in n.*
Proof. *Shown in Appendix F.* ■

An increase in μ increases the share of n devoted to n_1, i.e., $\frac{\partial x}{\partial \mu} > 0$. Since $\frac{\partial \Phi}{\partial \mu} > 0$ from before, this implies $\frac{\partial n}{\partial \mu} < 0$. To see this, we can decompose the total change in n because of changes in μ by

$$\frac{\partial n}{\partial \mu} = \frac{\partial n_1}{\partial \mu} + \frac{\partial n_2}{\partial \mu}.$$

Given $\frac{\partial x}{\partial \mu} > 0$ and $\frac{\partial \Phi}{\partial \mu} > 0$ (and hence, $\frac{\partial(1-x)}{\partial \mu} < 0$) $\frac{\partial n_2}{\partial \mu} < 0$ will be true. Since the change in n_1 due to a change in μ can be written as

$$\frac{\partial n_1}{\partial \mu} = x \underbrace{\frac{\partial n}{\partial \mu}}_{<0} + n \underbrace{\frac{\partial x}{\partial \mu}}_{>0},$$

$\frac{\partial n_1}{\partial \mu}$ may or may not be negative. Hence, while an increase in μ has an ambiguous effect on n_1, it reduces n_2 and since the latter effect dominates, n falls.

This implies that an increased weight of public capital induces agents to supply lesser labor (n), particularly towards research effort (n_2).

2.3 Stability of the Balanced Growth Path

To obtain the balanced growth path (BGP), we substitute the above decision rules into the law of motion for investment specific technological progress, (7), to characterize the balanced growth path (BGP). Given the decision rules (16), (17), (18), (20) and (6), we can re-write the above law of motion as

$$Z_{t+1} = \widehat{M} Z_t^{\gamma} Z_{t-1}^{(1-\gamma)} \{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)} \tag{21}$$

where $\widehat{M}$ is a constant and is expressed as

$$\widehat{M} = B((1-\delta_m)(1-x)n)^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}.$$

If we define the absolute growth rate by $\frac{Z_{t+1}}{Z_t} = g_{z_{t+1}}$, then we can re-write (21) as

$$g_{z_{t+1}} = g_{z_t}^{\gamma-1}\widehat{M}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}. \tag{22}$$

Using (22) and the parameter restrictions in Remark 1, we will get the same constant growth rates along the balanced growth path (BGP) as in Huffman (2008). We re-write (22) in the following way,

$$g_{z_{t+1}} = \frac{\Omega}{g_{z_t}^{(1-\gamma)}}, \tag{23}$$

where Ω is a constant and is expressed as

$$\Omega = \widehat{M}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}.$$

Given the assumptions it is easy to show that we can obtain a constant growth rate for Z, K, G and Y. This condition necessarily implies $0 < \Phi < 1$ and $0 < x < 1$ (as shown in Appendix B). We therefore have the following lemma. Figure [1] shows the dynamic adjustment to the steady state balanced growth graphically.

Lemma 3 *On the steady state balanced growth path, the gross growth rate of Z, K, G and Y are given by (24), and (25)*

$$\widehat{g_z} = \Omega^{\frac{1}{1+(1-\gamma)}} = [\widehat{M}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}, \tag{24}$$

$$\widehat{g_k} = \widehat{g_g} = \widehat{g_z}^{\frac{1}{1-\alpha_1}}, \widehat{g_y} = \widehat{g_k}^{\alpha_1} = \widehat{g_z}^{\frac{\alpha_1}{1-\alpha_1}}. \tag{25}$$

Proof. *While $\widehat{g_z}$ can be computed directly from (23), the expressions for the remaining variables are derived in Appendix C.* ■

There are several aspects of the equilibrium growth rate worth mentioning. First, the growth rate is independent of the technology parameter, A, as in Huffman (2008). Second, the growth rate of output, $\widehat{g_y}$, is less than $\widehat{g_k}$ along the balanced growth path because equation (7) is homogenous of degree $1 + \theta$.

Finally, from expression (24), the tax rate exerts a positive effect on growth as well as a negative effect. This is similar to the equation characterizing the growth maximizing tax rate in models with public capital. The mechanism here is however different. For small values of the tax rate, a rise in τ leads to higher public capital relative to output, Y_{t-1}. This raises the future value of investment specific technological change, Z. An increase in Z reduces the real price of capital, stimulating investment and long run growth. However, for higher tax rates, further increases in the tax rate depresses after tax income, and investment. This reduces

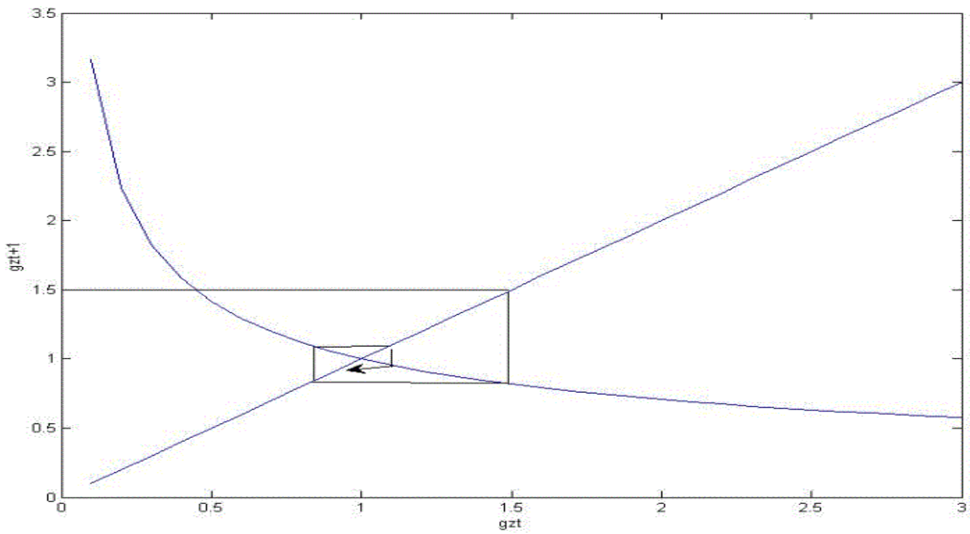

Figure 1: The dynamic adjustment process to a steady state BGP

G relative to Y, lowering Z, and depressing investment and long run growth. Hence, there is a unique growth maximizing tax rate.

Using the expression for $\widehat{g_z}$ in (24) we can characterize the growth maximizing tax rate as follows:

Proposition 1 *In the steady state, there exists a unique growth maximizing tax rate, given by* $\widehat{\tau} = \mu$.
Proof. *See appendix D.* ■

Proposition [1] sets a benchmark for the planner to set the optimal tax rate. If the planner wants to maximize growth, he should set the tax rate to μ. The higher the weight attached to $\frac{G_t}{Y_{t-1}}$ in the investment specific technological change equation, the higher should be the optimal tax rate set by the planner. This result is intuitive since it suggests that the government would have to impose a higher tax rate on income if public capital were to play a greater role in driving investment specific technological change.

α_1	0.35*
α_2	0.4
α_3	0.25
β	0.95*
γ	0.67
δ	1*
δ_m	0.2
μ	0.5
θ	0.2*
χ	2 in the model with convex costs 0.1 in the model with concave costs
c	0.4 in the model with convex costs 0.01 in the model with concave costs

Table 1: Parameter Values

Figure [2] calibrates the impact of a change in μ on the long run growth rate for two arbitrary values of μ. Parameter values taken from Table 1.[16] When $\mu = 0.5$, the growth maximizing tax rate is given by $t_1 = 0.5$. When $\mu = 0.6$, the growth maximizing tax rate rises to $t_2 = 0.6$. However, because μ has increased, there is a level downward shift in the growth tax curve associated with the higher value of μ (the red line is lower than the blue line). This is because of the reduction in growth due to a reduction n_2. Due to this effect, the planner needs to raise taxes to maximize growth, given that now there is a higher weightage on $\frac{G_t}{Y_{t-1}}$. In sum, a higher value of μ tilts the growth-tax curve in a south-westerly direction, leading to a higher growth maximizing tax rate. The net effect on growth however depends on which effect (reduction in n_2 versus more weightage on $\frac{G_t}{Y_{t-1}}$) dominates. This is sensitive to the value of γ. In particular, the effect of n_2 on reducing growth is higher for higher values of γ. Likewise, if there is a reduction in μ, n_2 may increase sufficiently leading to higher growth depending on the value of γ.

Finally, the growth rate is decreasing in δ_m, the weight attached to n_2 in production. In other words, as δ_m increases, more n_2 (specialized R&D labor) is devoted to production of the final good, and less to investment specific technological change. This reduces the future value of Z, and in the long run lowers optimal growth although there is no change in the growth optimizing tax rate. This is because a higher δ_m leads to a level downward shift in the growth tax curve, for all tax rates. To illustrate the quantitative impact of higher δ_m on

[16] Without loss of generality we assume $A = 1$ and $B = 1$. The parameters which have a ($*$) against their values means that they have been borrowed from Huffman (2008). The rest of the parameters have been assigned in order to ensure feasibility.

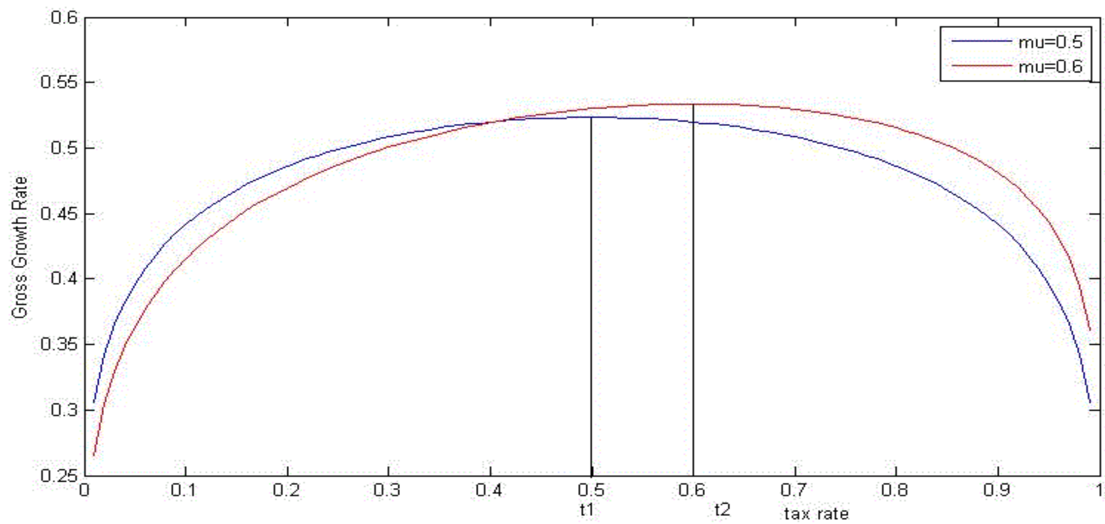

Figure 2: Comparing optimal taxes for different weights on the public input

growth, we first increase δ_m arbitrarily from 0.2 to 0.3. We find that growth falls from 0.491 to 0.4622 (the difference being 0.0288). For higher values of δ_m that is, increasing δ_m from 0.5 to 0.6, we find that the fall in growth is 0.3968 to 0.3587 (the difference is 0.0381). This shows that the fall in the growth rate is *higher* for higher values of δ_m. Therefore, a greater reduction in the share of n_2 available for future Z has a more detrimental, and non-linear effect on growth.

2.4 The Decentralized Equilibrium

Consider an economy that is populated by a set of homogenous and infinitely lived agents. There is no population growth and the representative firms are completely owned by agents, who supply labor for final goods production, n_1, and R&D, n_2. Agents derive utility from consumption of the final good and leisure given in (1). Agents fund consumption and investment decisions from their after tax wages w_1 and w_2, which they receive for supplying labor n_1 and n_2, profits Π_t earned from the final goods production, which they take as given, and the returns to capital lent out for production at each time period t .

The representative firm produces the final good based on (2) where the law of motion of private capital is given by (4). The government now funds public investment, I_t^g, at each time period t using a distortionary tax imposed on labor, $\tau_n \in (-1, 1)$, and capital, $\tau_k \in (-1, 1)$ respectively. Like Huffman (2008), it is assumed that profits are taxed according to the same rate as capital income.

2.4.1 The Firm's Problem

Firms solve a dynamic optimization problem which, at time t, has capital stock, K_t, and Z_t. Let $v(K_t, Z_t)$ denote the value function of the firm at time t. Like Huffman (2008), the firm's optimization problem, assuming full depreciation, is given by,

$$v(K_t, Z_t) = \max_{K_{t+1}, n_{1t}, n_{2t}} \left\{ (Y_t - w_{1t}n_{1t} - w_{2t}n_{2t})(1-\tau_k) - \frac{K_{t+1}}{Z_t} + \beta v(K_{t+1}, Z_{t+1}) \right\}, \quad (26)$$

which it maximizes subject to (5) and (7). We assume that firms don't borrow or lend in the credit market and hence they assume that future payoffs are discounted at the rate β.[17]

From the firm's maximization exercise, we get the following first order conditions,

$$\{K_{t+1}\} : \tfrac{1}{Z_t} = \beta \left(\tfrac{\alpha_1 Y_{t+1}(1-\tau_k)}{K_{t+1}} + \tfrac{\beta(1-\gamma)(1-\mu)K_{t+3}}{K_{t+1}Z_{t+2}} + \tfrac{\beta^2(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \beta^j \gamma^j \tfrac{K_{t+j+4}}{Z_{t+j+3}} \right)$$

$$\{n_{1t}\} : w_{1t}(1-\tau_k) = \underbrace{\tfrac{\alpha_2 Y_t(1-\tau_k)}{n_{1t}}}_{MPn_1(1-\tau_k)} - \underbrace{\tfrac{\alpha_2\beta^2(1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \tfrac{\beta^j\gamma^j K_{t+j+3}}{Z_{t+j+2}}}_{D}$$

$$\{n_{2t}\} : w_{2t}(1-\tau_k) = \underbrace{\tfrac{\alpha_3 Y_t(1-\tau_k)}{n_{2t}}}_{MPn_2(1-\tau_k)} + \underbrace{\tfrac{\beta\theta K_{t+2}}{Z_{t+1}n_{2t}} + \tfrac{\beta^2(\gamma\theta-\alpha_3(1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \tfrac{\beta^j\gamma^j K_{t+j+3}}{Z_{t+j+2}}}_{E}.$$

Because of investment specific technological change, factor prices are no longer equal to the standard marginal products. In particular, the wage paid to n_1 is reduced by the term $\frac{D}{(1-\tau_k)}$, while the wage paid to n_2 is increased by the term $\frac{E}{(1-\tau_k)}$. In particular, $w_1 = MP_{n_1} - \frac{D}{(1-\tau_k)}$ and $w_2 = MP_{n_2} + \frac{E}{(1-\tau_k)}$.The general point to note is that, $w_1 = w_2$ does not imply that $MPn_1 = MPn_2$ unless the restriction, $D + E = 0$.[18] Figure [3] shows how investment specific technological change acts like a tax on n_1 and a subsidy to n_2. This effect is magnified by the changes in τ_k.

[17] In Section (3), we allow firms to borrow and lend in the credit market, which implies that firms discount future payoffs by $\frac{1}{1+r}$.

[18] Intuitively, because investment specific technological change has a dynamic effect on the marginal productivity of factor inputs, we cannot restrict ourselves to the static marginal productivities to calculate factor prices.

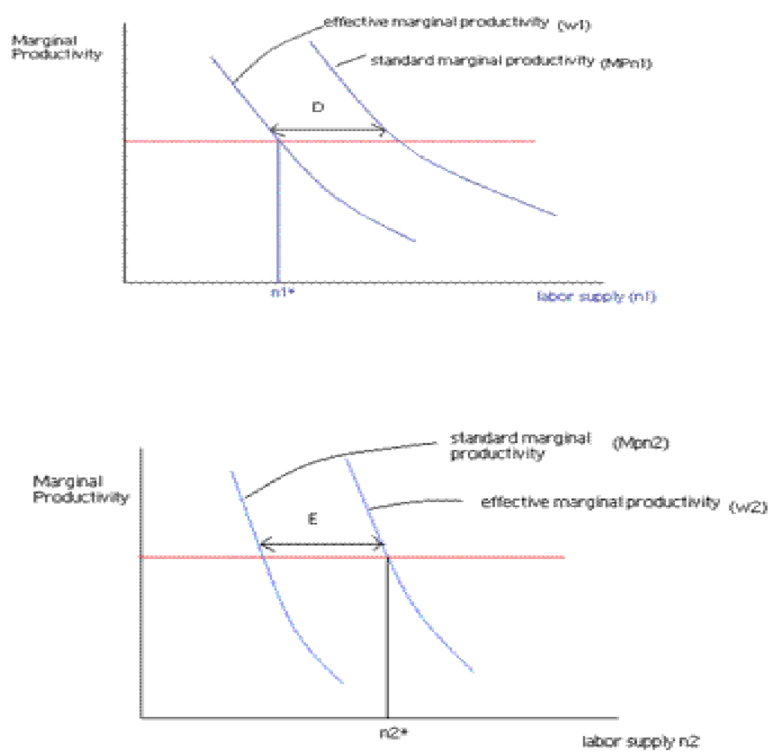

Figure 3: The effect of investment specific technological change on n_1 and n_2

2.4.2 The Agents Problem

A representative agent maximizes (1) subject to the following consumer budget constraint (CBC),

$$C_t + \frac{K_{t+1}}{Z_t} = [w_{1t}n_{1t} + w_{2t}n_{2t}](1-\tau_n) + [Y_t - (w_{1t}n_{1t} + w_{2t}n_{2t})](1-\tau_k), \tag{27}$$

the laws of motion given by (4), (5) and (7), total labor supply given by (3), and takes factor prices and profits as given. The first term on the right hand side denotes after tax wage income. The second term is the firm's capital income plus profits, which is taxed at the rate, τ_k. The following restriction required for decentralizing the planner's allocations.

Remark 3 *Suppose that $\gamma\theta - \alpha_3(1-\gamma) > 0$, which ensures that $w_{1t} = w_{2t} > 0$. Then, $Dn_1 = En_2$ is necessary and sufficient to decentralize the planner's allocation at every time period, t.*

The condition that, $Dn_1 = En_2$, means a reduction in the total wage bill paid to n_1 due to investment specific technological change gets offset by an equivalent increase in the wage bill paid to n_2. This ensures that the total wage bill paid by the firm to n does not depart from the marginal productivities paid to each type under investment specific technological change for any combination of factor income taxes.

Under *Remark* (3), the CBC – from equation (27) – can be written as

$$C_t + \frac{K_{t+1}}{Z_t} = \Theta Y_t$$

where $\Theta = (1-\alpha_1)(1-\tau_n) + (\alpha_1)(1-\tau_k)$. This implies the government budget constraint (GBC) is given by

$$I_t^g = (1-\Theta)Y_t. \tag{28}$$

In general, any factor income tax combination decentralizes the planner's allocations as long as *Remark* (3) is imposed, because of the offsetting effects on the total wage bill of the firm.

2.4.3 The Agent's First Order Conditions

The Lagrangian given below for the agent's problem is given by

$$L = \sum_{t=0}^{\infty} \beta^t [\log C_t + \log(1 - n_{1t} - n_{2t}) + \lambda_t \{\Theta Y_t - C_t - I_t\}]. \tag{29}$$

The optimization conditions with respect to C_t, K_{t+1}, n_{1t}, and n_{2t}, are given by equations (30), (31), (32) and (33) respectively:

$$\tfrac{1}{C_t} = \lambda_t \tag{30}$$

$$\tfrac{1}{C_t Z_t} = \tfrac{\alpha_1 \beta Y_{t+1} \Theta}{C_{t+1} K_{t+1}} + \tfrac{\beta^2 (1-\gamma)(1-\mu) I_{t+2}}{C_{t+2} K_{t+1}} + \tfrac{\beta^3 (1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}} \sum_{j=0}^{\infty} \tfrac{\beta^j \gamma^j I_{t+j+3}}{C_{t+j+3}} \tag{31}$$

$$\tfrac{1}{1-n_t} + \tfrac{\beta^2 \alpha_2 (1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \tfrac{\beta^j \gamma^j I_{t+j+2}}{C_{t+j+2}} = \tfrac{\alpha_2 Y_t \Theta}{C_t n_{1t}} \tag{32}$$

$$\tfrac{1}{1-n_t} = \tfrac{\alpha_3 Y_t \Theta}{C_t n_{2t}} + \tfrac{\beta \theta I_{t+1}}{C_{t+1} n_{2t}} + \tfrac{\beta^2 (\gamma\theta - \alpha_3(1-\gamma))}{n_{2t}} \sum_{j=0}^{\infty} \tfrac{\beta^j \gamma^j I_{t+j+2}}{C_{t+j+2}}. \tag{33}$$

The above first order conditions can be derived in the same way as the planner's version as shown in Appendix A. Their interpretation is also the same as before, except that $1-\tau$ is replaced by Θ. The allocations that result from *Remark* (3) are also constrained Pareto Optimal. The closed form decision rules are characterized by the following lemma.

Lemma 4 *In the decentralized equilibrium, the expressions for* C_t*,* $I_t, n_t, n_{1t,} n_{2t}$ *are given by the same decision rules derived in (34), and (17), (18) respectively, where* $0 < \Phi < 1$ *is* *given by (19) and* $0 < x < 1$ *is given by (20), with*

$$C_t = \Phi Y_t \Theta \text{ and } I_t = (1-\Phi) Y_t \Theta. \tag{34}$$

Proof. The above expressions can be constructed from the first order conditions given by equations (30), (31), (32) and (33), as explained in Appendix B for the planner's version. ■

The first order conditions governing the planner's allocations can be easily be seen to be replicated by the decentralized equilibrium, once we assume $\tau_k = \tau_n = \tau$. In this case, $\Theta = 1 - \tau$, and the first order conditions characterizing the planner's allocations obtain. Hence, the comparative statics of consumption, investment, and labor input vis-a-vis changes in μ remain unchanged. The gross growth rate of Z, K, G and Y at the steady state can also be derived in a similar fashion. As in the planner's version, the condition for dynamic stability is the same, i.e., $0 < \gamma < 1$. We therefore have the following lemma

Lemma 5 *In the steady state of the decentralized economy, the gross growth rate of* Z*, is given by (35) while the gross growth rates for* K*,* G *and* Y *are given by (36)*

$$\widehat{g_z} = \Omega^{\frac{1}{2-\gamma}}, \text{ where } \Omega = \widehat{M}\{(1-\Theta)^{\mu}(\Theta)^{1-\mu}\}^{(1-\gamma)}, \; \widehat{M} = B((1-\delta_m)(1-\widehat{x})\widehat{n})^{\theta}(1-\widehat{\Phi})^{(1-\mu)(1-\gamma)} \tag{35}$$

$$\widehat{g_k} = \widehat{g_g} = \widehat{g_z}^{\frac{1}{1-\alpha_1}}, \widehat{g_y} = \widehat{g_k}^{\alpha_1} = \widehat{g_z}^{\frac{\alpha_1}{1-\alpha_1}}. \tag{36}$$

Proof. *These are derived in the same way as in the planner's version.* ■

Note that the expressions for the equilibrium long run growth rate are identical to those in the planned economy, except that the growth rates differ because Θ need not equal $1 - \tau$.

We want to check under what conditions the growth maximizing allocations from the planner's problem can be replicated by the decentralized economy. From the steady state growth rate given by (35), this leads to a second proposition.

Proposition 2 *If factor income taxes are chosen such that a linear combination of factor income taxes is equal to* μ*,i.e.,*

$$(1-\alpha_1)(\widehat{\tau_n}) + \alpha_1(\widehat{\tau_k}) = \mu = \widehat{\tau}. \tag{37}$$

then the decentralized allocations can replicate the growth maximizing allocations from the planner's problem.

Proof. *Shown in Appendix E.* ■

The proposition above suggests that there is no *unique* combination of labor and capital income taxes which maximizes growth. In particular, pick a value of μ and one of the two tax rates. *Lemma (5)* yields a Laffer curve for the other factor income tax in terms of μ and the given tax. Changes in the value of the given tax rate causes a horizontal shift in the Laffer curve. With the new Laffer curve, optimality is restored when $\Theta = \mu$. Further, if the condition specified in the proposition holds, the planner's allocations can be achieved from the solutions of the agent's version. Since there does not exist any unique growth optimizing tax rate combination for $\widehat{\tau_n}$ and $\widehat{\tau_k}$ the planner could choose multiple tax/subsidy schemes (subject to the restriction placed in *Remark* (3) such that the convex combination will always equal the planner's growth optimizing tax rate of μ. Under this condition, the solution to the agent's version replicates the planners optimization solution. We therefore have the following proposition

Proposition 3 $\widehat{\tau_n} \gtreqless \widehat{\tau} = \mu \Rightarrow \widehat{\tau_n} \gtreqless \widehat{\tau} = \mu \gtreqless \widehat{\tau_k}$.
Proof. *This is obtained from equation (37).* ■

From equation (37), it follows that if one of the factors receives a subsidy, the other factor must be sufficiently taxed such that the optimality rule given by (37) is satisfied.[19]

2.5 Welfare

We are interested in evaluating whether the optimality results for taxes vary when the rule is to maximize welfare and not growth for the model in Section (2.2) and (2.4.1). Since consumption is a fixed fraction of after tax income, it grows at the same gross growth rate of output at the steady state as shown in Lemma 1 and Lemma 3. The total labor supplied is also constant at each time period t. Using this information, we can re-write the life-time utility given in (1) in the planner's version as

$$\Lambda = \Gamma + \Lambda \log(1-\tau) + \Psi \log[(\tau)^{\mu}(1-\tau)^{1-\mu}] \tag{38}$$

where Γ, Λ and Ψ denote all other terms independent of the flat tax rate τ. We show this in Appendix G. It is clear from the above expression for welfare that the maximizing tax rate for the planner's model is less than the tax rate which maximizes growth. This happens because consumption is scaled down by $(1-\tau)$ at every time period even though the balanced growth rate is affected by τ from the last term in (38). This gives us the following result.

[19] We have shown in Lemma 1 and in Lemma 4 that $0 < \Phi < 1$ given by (19), $0 < x < 1$ given by (20) and the supply of labor given by (17) are constants.

Proposition 4 *The welfare maximizing tax rate, τ^w, is less than the growth maximizing tax rate and is given by,*

$$\tau^w = \left[\frac{\beta^2\alpha_1(1-\gamma)}{\beta^2\alpha_1(1-\gamma)+(1-\alpha_1)(2-\gamma)}\right]\mu = \varphi\mu < \mu = \widehat{\tau},$$

where $\varphi = \left[\frac{\beta^2\alpha_1(1-\gamma)}{\beta^2\alpha_1(1-\gamma)+(1-\alpha_1)(2-\gamma)}\right] < 1.$
Proof. *This can be obtained by differentiating (38) with respect to τ.* ■

In the agent's version, the life-time welfare function can be expressed as

$$\Delta = \Xi + +\Lambda\log(\Theta) + \Psi\log[(1-\Theta)^{\mu}(\Theta)^{1-\mu}] \tag{39}$$

where Ξ, Λ and Ψ denote all other terms independent of the tax rates τ_n and τ_k. We get the same result as in the case of the planner's version, as shown in the following proposition

Proposition 5 *There exists a convex combination of factor income taxes, given by*

$$(1-\alpha_1)(\tau_n^w) + \alpha_1(\tau_k^w) = \varphi\mu < \mu = (1-\alpha_1)(\widehat{\tau_n}) + \alpha_1(\widehat{\tau_k}),$$

where this combination maximizes agent's welfare and also yields the planner's welfare associated with τ^w in Proposition (4).
Proof. *We get the above equation from the welfare optimization conditions with respect to τ_n and τ_k.* ■

In sum, we have characterized both the planner's and agent's problem. We have shown that there exists a unique growth maximizing tax rate, which is greater than the tax rate that maximizes long run welfare. This inequality ranking also strictly holds true when agents maximize welfare.

3 Robustness

In order to establish the robustness of our results we first show that if agents are allowed to borrow and lend by participating in the credit market as in Huffman (2008), the planner's allocations can still be decentralized using multiple factor income tax combinations. We then show that the results obtained in Section (2), generalizes to a model where only n_1 enters into the production function of the final good, and n_2 affects investment specific technological change.

3.1 Decentralizing the model under a borrowing-lending setup

As shown in Appendix J, if agents are allowed to borrow and lend in the credit market, the planner's allocations can still be decentralized using multiple factor income tax combinations as above.[20] The crucial difference is that Φ, n and x depend on τ_n and τ_k[21]. If we restrict factor income taxes to be positive, then we can numerically show that the factor income tax mix that decentralizes the planner's allocations are inversely related to each other similar to equation (37). However the relationship is non-linear. If we allow for both τ_n and τ_k to be either a tax or a subsidy, then we can numerically show that such a mix still decentralizes the planner's allocations, although the particular combination of $\tau_n \gtrless 0$ and $\tau_k \gtrless 0$ depends on the parameter values of μ and γ. For the case of positive factor income taxes, Figure (4) plots the set of taxes that decentralizes the planner's growth maximizing allocations.

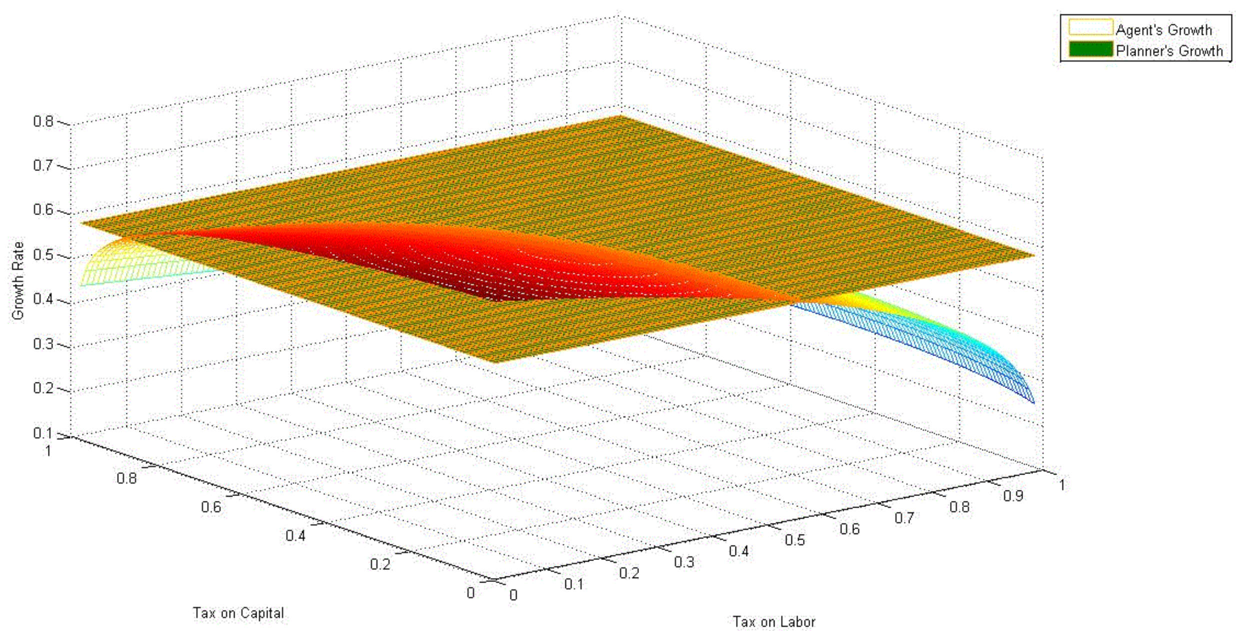

Figure 4: Decentralizing the planner's growth rate - under borrowing and lending

[20] As in Huffman (2008, p. 3456) the agent's budget constraint is given by

$$a_{t+1} = (1+r)a_t + w_t(n_{1t} + n_{2t})(1-\tau_n) - c_t.$$

In equilibrium,

$$a_t = K_t.$$

[21] This is shown in Appendix J

3.2 Decentralizing using only n_1 in production

As shown in Appendix K, the results obtained in Section (2), generalizes to a model where only n_1 enters into the production of the final good, and n_2 affects investment specific technological change, as in Huffman (2008). Our main result - which we show numerically - is:

Proposition 6 *For any given value of μ and one of the factor income taxes, there is at least one feasible value for the other factor income tax that decentralizes the planner's allocations.*

Proof. See Appendix K. ■

In sum, the robustness exercises in Sections (3.1) and (3.2) suggest that decentralizing the planner's allocations leads to indeterminate, or multiple, factor income tax rates.[22] The following Figure [5] illustrates the case with only n_1 in production.

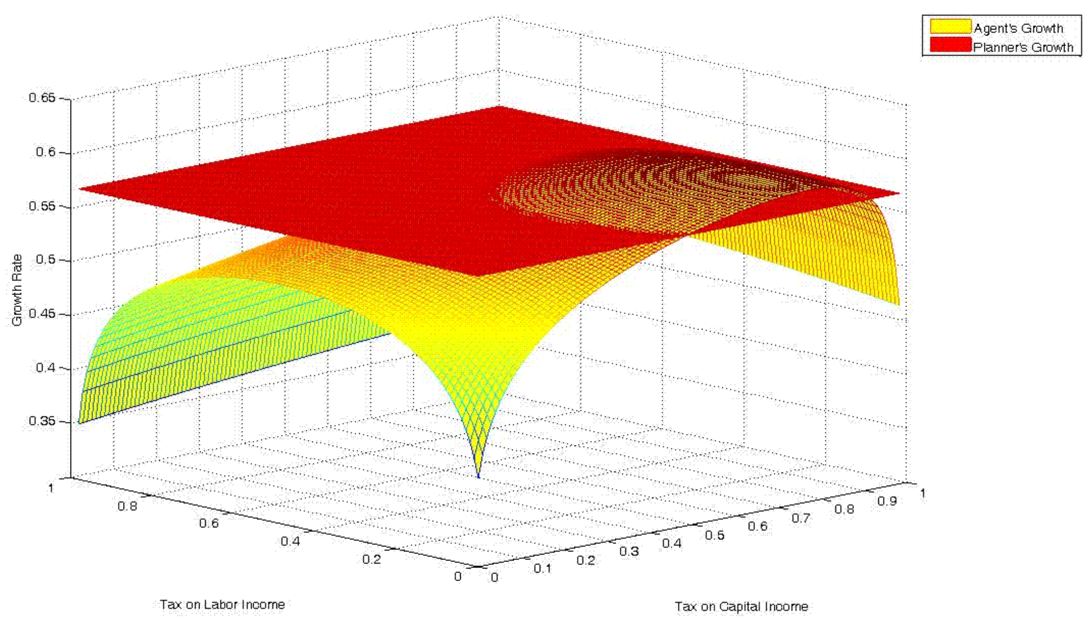

Figure 5: Decentralizing the planner's growth rate without n_1 in production

The above exercises suggest that the optimal factor income tax mix is indeterminate in two natural variants of the model. This indeterminacy result therefore gives the planner the flexibility to choose policy rules from an infinitely large set. As we show in the following

[22] It can be shown that welfare results do not coincide with growth. The results are available from the authors on request.

section, accounting for administrative costs, however, restricts this set of optimal feasible tax mix for the planner and eventually a unique factor income tax mix emerges as the only feasible solution.

4 The Model with Administrative Costs

The presence of administrative costs during tax collection is one of the major reasons (the other being the dead-weight loss of imposing the tax), for the government's inability to change (particularly increase) tax rates with ease for the purpose of raising revenue (see Bovenberg and Goulder (1986), Yitzhaki (1979), Yang (1989)). We provide specific conditions under which the tax rate differs at the optimum – compared to the benchmark model – when administrative costs for implementing public investment occur. In particular, as long as administrative costs are not linear, we show that there will always exists a unique combination of distortionary tax rates on labor income and capital income.

4.1 The Planner's Model

As before, the government collects taxes by imposing a proportional tax rate on income to fund G to contribute to investment specific technological change. It however incurs administrative costs with respect to tax collection. The government budget constraint takes the following form

$$I_t^g = (\tau - \omega(\tau))Y_t$$

where $\omega(\tau)$ represents continuously varying administrative costs with respect to the tax rate τ. Here, $\omega'(\tau) > 0$, which implies that the administrative cost is assumed to be increasing in the tax rate. In what follows, we will assume that these costs could be linear, convex, or concave with respect to the tax rate, and show that these assumptions have different implications for the steady state balanced growth path.

4.1.1 Convex Administrative Costs

Suppose that administrative costs are strictly convex with respect to the tax rate (e.g., Perotti (1993), Buiter and Sibert (2011)), i.e., $\omega''(\tau) > 0$. By strict convexity we mean that a proportional increase in the tax rate causes a more than proportional increase in the collection costs and that governments experience greater difficulty in imposing a higher tax rate on income as compared to a lower tax rate. In such a scenario, if achievable growth rates are much lower than as compared to the case where such costs are absent, the government, at an optimum, would consider imposing a lower tax rate on income.

We parametrize the function, $\omega(\tau) = c\tau^{\chi}$, where $0 < c \leq 1$ is like a "fixed level cost" parameter and $\chi > 1$ is the degree of convexity. The government budget constraint takes the form

$$I_t^g = (\tau - c\tau^{\chi})Y_t. \tag{40}$$

We assume that agents who are subject to these tax rates are unaffected by the administrative costs the government incurs for imposing taxes. It is like as if the government incurs an additional expenditure towards enhancing investment specific technological change. For this reason the first order conditions are the same as in the baseline model and are therefore given by (10), (11), (12) and (13). The decision rules are also the same and are given by (16), (17), (18), (19) and (20) as shown in Lemma 1.

Given the government budget constraint in (40), the law of motion for investment specific technological change according to (7) and the decision rules for consumption, investment and the labor supplies as (16), (17), (18), (19), (20), we can rewrite (7) in terms of the steady state growth rate,

$$\widehat{g_z} = [B((1-\delta_m)(1-x)n)^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}\{(\tau - c\tau^{\chi})^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}. \tag{41}$$

As before, the growth rate, with administrative costs, is increasing in B, decreasing in δ_m, and increasing in n_2. Further, an increase in c reduces the growth rate. We now get the following proposition.

Proposition 7 *The growth maximizing tax rate, τ_{AC} (tax with administrative costs) in a model with convex administrative costs is always less than the growth maximizing tax without such costs, that is, $\widehat{\tau} = \mu$. The optimal tax is obtained from the following expression*

$$(1-\tau_{AC})\mu[1 - c\chi(\tau_{AC})^{\chi-1}] = (1-\mu)(\tau_{AC} - c(\tau_{AC})^{\chi}), \tag{42}$$

where,

$$\tau_{AC} = \widehat{\tau} = \mu \text{ when } c = 0 \text{ or when } \chi = 1. \tag{43}$$

Proof. Shown in Appendix H ■

Given that administrative costs with respect to the tax rates are convex, the steady state optimal growth rate given by (41) will be lower than the steady state optimal growth rate in the baseline model. This is shown in Figure [6] where t is the optimal tax rate as derived in the baseline model. The tax rate t_1 is the growth optimizing tax rate when the government faces convex administrative costs.

For instance, Perotti (1993) and Buiter and Sibert (2011) assume that convex administrative costs are quadratic in nature ($\chi = 2$). Assuming $c = 1$, the optimal tax is now given

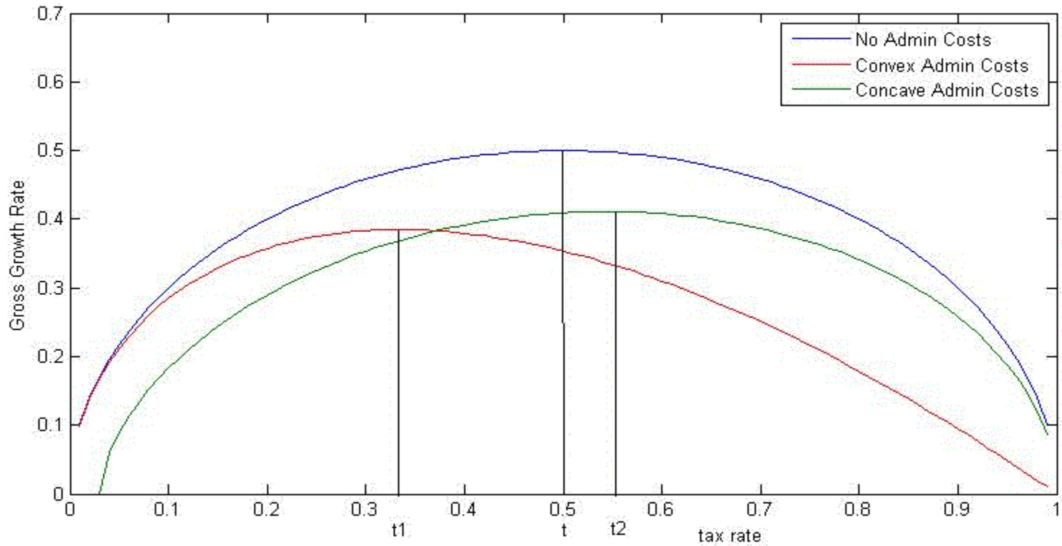

Figure 6: Optimal tax rates across different models - with and without administrative costs

by the following equation

$$(1 - \tau_{AC})\mu(1 - 2\tau_{AC}) = (1 - \mu)(\tau_{AC} - (\tau_{AC})^2) \tag{44}$$

which gives us

$$\tau_{AC} = \frac{\mu}{1 + \mu} < \widehat{\tau} = \mu. \tag{45}$$

Therefore the general result in a model with convex administrative costs is that the planner will choose to charge a lower tax rate compared to the case when there are no administrative costs. These costs hamper the availability of resources for funding public expenditure thereby leading to lower growth rates. But the planner would choose to charge a lower tax rate at the optimum since the costs of imposing higher tax rates are increasing in the tax rate.

4.1.2 Other Cases: Concave or Linear Administrative Costs

The government budget constraint for the planner's version is again given by equation (40) but now instead of having strict convexity in administrative costs with respect to the tax rate, we assume strict concavity ($\omega''(\tau) < 0$) in administrative costs, i.e., $\chi < 1$. Anecdotally, strict concavity of administrative costs is suggestive of a more efficient administrative machinery compared to the previous case with convex costs.[23] Such administrative costs increase with

[23] This may be due to employing better technology that may assist in revenue collection. See Slemrod (1990).

a higher tax rate but at a decreasing rate. Therefore at the optimum, the government has an incentive to impose a higher tax on income as compared to the case of the baseline model even though the steady state growth rates is lower because of the loss due to administrative costs. This again is shown in Figure [6] where now t_2 is the growth optimizing tax rate when the government faces concave administrative costs.

Linear administrative costs simply cause a level downward shift in the optimal growth rate. The optimal tax however remains the same. This is shown in Figure [7].

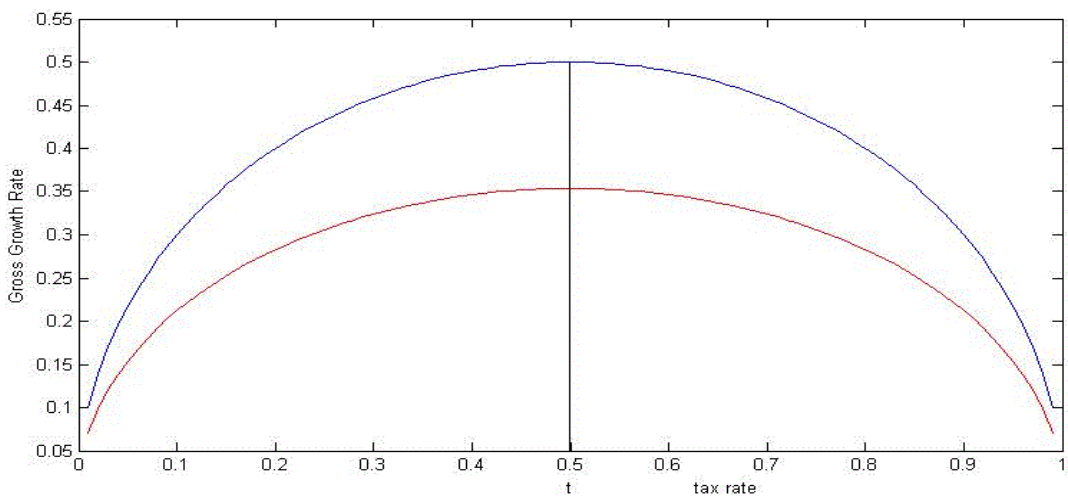

Figure 7: The model with and without linear costs

4.2 The Decentralized Equilibrium

There are two separate factor income taxes imposed on labor and capital income, i.e., τ_n and capital τ_k respectively. The administrative costs incurred by the government for imposing tax rate on labor and capital are assumed to be different, in terms of the fixed level costs although not in terms of the degree of convexity or concavity. Hence the cost of imposing τ_k is $c_1\tau_k^\chi$ while that for τ_n is $c_2\tau_n^\chi$. The following is the government budget constraint[24]

$$I_t^g = [\alpha_1\tau_k + (1-\alpha_1)\tau_k - c_1\tau_k^\chi - c_2\tau_n^\chi]Y_t. \tag{46}$$

The rest of the specification remains the same, just as in the baseline agent's model. The first order conditions are given by equations (30), (31), (32) and (33) and the decision rules

[24] This equation is derived in the same way as in the baseline model, where the firm's profit maximization solutions are substituted into wages and rate of return on capital. In addition, there are also administrative costs of imposing each tax rate.

by (16), (17), (18), (19) and (20) as shown in Lemma 3. We now substitute the decision rules given by equations (17), (18), (19) and (20) as in Lemma 3 and the government budget constraint given by equation (46) into the investment specific technological change equation given by (7) to obtain the following steady state growth rate for Z

$$\widehat{g_z} = [B((1-\delta_m)(1-x)n)^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}\{(1-\Theta-c_1\tau_k^{\chi}-c_2\tau_n^{\chi})^{\mu}(\Theta)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}, \quad (47)$$

where, $\Theta = 1-\alpha_1\tau_k-(1-\alpha_1)\tau_n$. Unlike in the baseline framework where administrative costs were absent, we show that there exists a *unique* combination of the tax on labor income and on capital income given by the following proposition.

Proposition 8 *There exists a unique combination of positive tax rates τ_n and τ_k which maximizes the steady state growth rate given by*

$$\left(\frac{\tau_n^*}{\tau_k^*}\right)^{\chi-1} = \left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{c_1}{c_2}\right). \quad (48)$$

When $c_1 = c_2$,

$$\chi \gtrless 1 \Rightarrow \tau_n^* = \left\{\left(\frac{1-\alpha_1}{\alpha_1}\right)^{\frac{1}{\chi-1}}\right\}\tau_k^* \gtrless \tau_k^*.$$

Proof. *Shown in Appendix I* ■

This inequality result holds for $0 < \alpha_1 < 0.5$. The proposition suggests that if the scale constant, c, and variable costs are identical, the government could maximize efficiency by charging a higher tax on labor income. The tax on capital income could therefore exceed the tax on labor income only under the special case when the fixed level costs of imposing τ_n^* relative to τ_k^* sufficiently exceeds the relative share of total labor to capital in production. In other words, capital income could be subject to a higher tax rate rate compared to that on labor provided it is less costly to impose a higher tax on capital.

This specification gives us uniqueness, which was absent in the baseline model. This happens because μ affects the first order conditions – relating the growth rate to the optimal factor income tax rates – symmetrically (see Appendix I). The ratio of factor income taxes, as seen in equation (48), is therefore independent of μ. However, individual factor income taxes still depend on μ.

4.3 Decentralizing the planner's allocation

While the above results suggest that there can exist a unique combination of factor income taxes that maximizes growth in the agent's problem, it does not guarantee whether such a unique combination also decentralizes the planner's growth maximizing tax rule. While we can't show this analytically, we can show numerically that for feasible yet small values of c and increasing $\chi > 1$ reduces the set of feasible factor income tax mix the planner could choose from. In fact the size of this set shrinks as the convexity of the administrative costs increases, and eventually a unique factor income tax mix is the only feasible solution. Figure [8] illustrates this uniqueness result.

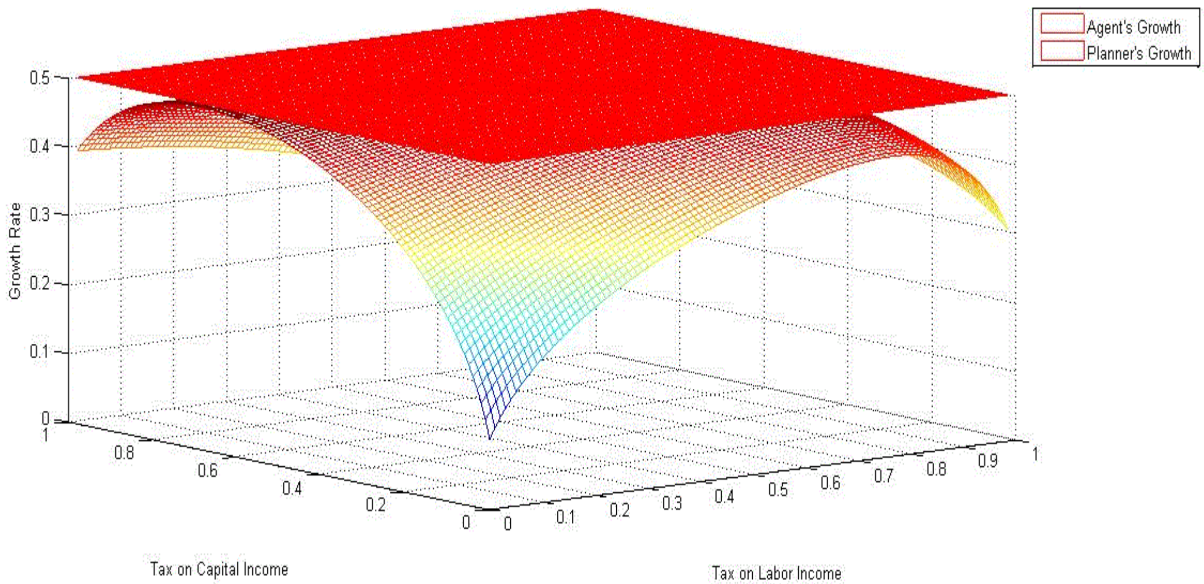

Figure 8: Decentralizing the planner's growth rate in the presence of convex administrative costs

5 A Numerical Example

In this section, we use the baseline model in Section (2.2) and (2.4) to quantify the growth effects of factor income tax changes.[25] Our focus is to assess the growth effects of changing factor income taxes. We use parameter values for the US from Huffman (2008), and other parameters from the literature. We use arbitrary values of μ, γ, and δ_m because of the lack of clear empirical estimates for these parameters. Table [1] contains the parameter values used in the numerical exercise. We are interested in two different policy experiments: the effects of 1) equi-proportionate changes in fact or income taxes and 2) revenue neutral changes on growth and welfare. We then augment these estimates assuming different cost technologies.

[25] These exercises were done in Matlab. The codes are available from the authors on request.

We first analyze the impact of changing the tax on capital on the growth rate for a given value of the labor income tax in the decentralized model using the optimality condition (37),

$$(1-\alpha_1)(\widehat{\tau_n}) + \alpha_1(\widehat{\tau_k}) = \mu = \widehat{\tau}.$$

We earlier quantified the effect of an increase in μ, in the investment specific technological change equation – on the equilibrium growth rate. We showed that a higher μ, leads to a higher growth maximizing tax rate. This effect on growth can be decomposed into two effects: the effect of a higher higher μ on labor effort devoted to research, and the direct effect of a higher weightage on public capital relative to output on increasing the level of investment specific technological change. The quantitative results show that an increase in the government spending share *reduces* the labor input devoted to research effort which reduces growth, but increases the level of technological change which increases growth. Hence, the *net* change in growth depends on the magnitude of these two effects and on the parameter γ.

The above dynamics are replicated when there are tax changes in the decentralized equilibrium, except that at the optimum, there does not exist any unique combination of tax rates on capital and labor income which maximizes growth. For illustrative purposes, we arbitrarily pin down a particular value for the tax on labor, $\widehat{\tau_n} = 0.4$. According to (37) – where $\mu = 0.5$ and $\alpha_1 = 0.35$ – which yields $\widehat{\tau_k} \approx 0.6857$.[26] This can also be replicated if we plot the growth schedule of g_z – fixing $\widehat{\tau_n}$ at 0.4 – and varying $\widehat{\tau_k}$.

This is shown in Figure [9] where the optimum combination of $\widehat{\tau_n}$ and $\widehat{\tau_k}$ is given by $(0.4, 0.6857)$, corresponding to t_1. Suppose we decrease the tax rate on labor income and increase the tax rate on capital by 0.1 in comparison to this combination at the optimum; that is, τ_n and τ_k are now given by $(0.3, 0.7857)$.[27] These tax rates no longer satisfy (37). Growth is no longer at its highest value, although in comparison to the growth maximizing factor income tax mix, $(0.3, 0.87143)$ – given by t_3– which satisfies (37), the gross growth rate is marginally (though not significantly) less: by a magnitude of $\approx 3.303 \times 10^{-4}$. Hence, deviating from the optimal rule by changing factor income taxes in equal proportions has a negligible effect on long run growth and welfare. The reason why this happens is because of the assumed share of output accruing to capital in the final goods production, α_1 where we assume that $0 < \alpha_1 < 0.5$. Under this restriction, the absolute rate of change in growth rate due to a change in the tax rate on capital income will strictly be less than the absolute rate

[26] The revenue neutral rule for changes in factor income taxes is given by, $\frac{\Delta\widehat{\tau_k}}{\Delta\widehat{\tau n}} = -\frac{(1-\alpha_1)}{\alpha_1}$.

[27] The value of $\tau_k = 0.7857$ is given by the point t_2.

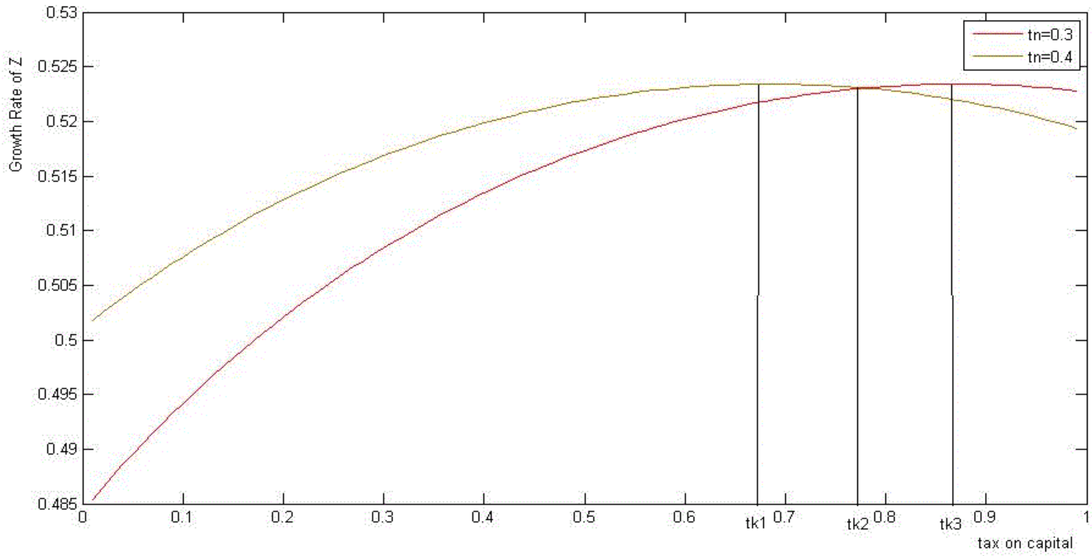

Figure 9: Impact of a change in the tax on labor income on the capital tax Laffer curve

of change in the growth rate due to a change in the tax on labor. That is

$$\left|\frac{\partial g_z}{\partial \tau_k}\right| < \left|\frac{\partial g_z}{\partial \tau_n}\right|.$$

This means, deviating away from the tax rate on capital according to an optimal tax rule will only have a moderate effect on the growth rates. This result is consistent with the results of the policy experiments in Huffman (2008) relating the effect of capital income tax changes on growth. He finds that changes in factor income taxes have a minimal effect on the growth rate. However, as evident from Figure [9], depending upon whether changes to the capital income tax rate are equi-proportionate or revenue neutral, the effect on growth rate would be negative (but marginal), or zero, respectively. This contrasts with Huffman (2008) where a change in the tax on labor income has a negative effect on the growth rate.[28]

If now we reverse the exercise by first fixing τ_k at the arbitrary value of 0.6 and then lower it to 0.3, and then calibrate the equi-proportionate increase and revenue neutral increase in τ_n, we find that changes in growth are of the same order when we compare an equi-proportionate change to a revenue neutral change when we fixed tax τ_n and varied τ_k. However, there is an important difference. An equi-proportionate change in τ_n means that

[28] For a large arbitrary change in taxes, our calibrated results show that the "growth-gap" between equi-proportionate and revenue neutral changes is still not large, but larger than the case for small changes in taxes. For instance, a reduction in τ_n from 0.6 to 0.3 implies that τ_k rises from 0.315 to 0.87143. In contrast, an equiproportionate change in τ_k is 0.61. The growth difference is still roughly 3.303×10^{-4}.

now we increase the tax on labor from 0.44615 (when τ_k is 0.6) to 0.74615 (when τ_k is 0.3) which is significantly higher than the revenue neutral value (at 0.607). Since the equi-proportionate change in τ_n exceeds the revenue neutral value of τ_n, reducing τ_n would increase growth.

We now conduct a similar policy experiment with administrative costs as in Section (5). For simplicity we have assumed that the fixed level cost parameter c and the variable cost parameter χ are the same for both tax rates on labor income and on capital income. We first consider the case with convex administrative costs, and assume (arbitrarily) that $c = 0.4$ and $\chi = 2$. Plugging the value of c and χ into equation (48), and then substituting out the resulting value of τ_n in terms of τ_k into the first order condition derived in Appendix I, the optimal tax rate on capital income τ_k^* is found to be approximately 0.23. Using this value for τ_k^*, and using the equation (48) we get $\tau_n^* = 0.42$. We now get Figure [10].

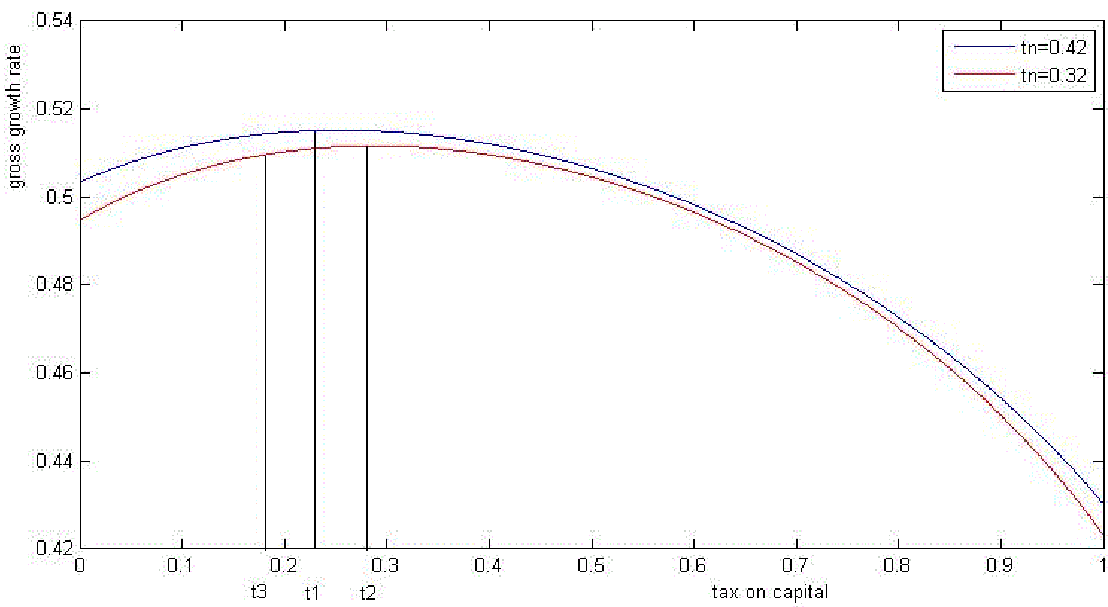

Figure 10: Policy analysis in the model with convex costs

Here t_1 represents the point $(\tau_n^*, \tau_k^*) = (0.42, 0.24)$.[29] This gives us the growth maximizing tax mix with convex administrative costs. Therefore, any change in the tax on labor such that $\tau_n \neq \tau_n^*$, such as at point t_3, where we arbitrarily reduce τ_n by 0.1 to 0.32, and also re-calibrate τ_k according to (48) reduces growth to a sub-optimal value. This is because the new tax mix at t_3 does not satisfy optimality.[30] However, if the choice of τ_k corresponding

[29] Note that there are two solutions but the second solution $\tau_k^* = 0.69$ and $\tau_n^* = 1.28$ is not feasible.

[30] Here, $\frac{\partial g_z}{\partial \tau_k} > 0$.

to $\tau_n = 0.32$ were to satisfy optimality (hence τ_k satisfies $\frac{\partial g_z}{\partial \tau_k}|_{\tau_n=.32} = 0$), then the optimal tax on capital would have to be significantly higher than τ_k^*. This is denoted by point t_2, where growth has marginally fallen (by roughly 3.6×10^{-3}), but by an amount greater than the growth fall in the model without administrative costs. In sum, because of convex administrative costs, deviating from the tax rule, (48), will lead to a greater fall in long run growth and welfare.[31]

We now look at the case with concave administrative costs. With $c = 0.01$ and $\chi = 0.1$ we can show that the optimal tax rate on capital income $\tau_k^* \approx 0.73$.[32] Using this value for τ_k^* and using the equation (48) we can show that $\tau_n^* \approx 0.4013$. This is shown in Figure [11].

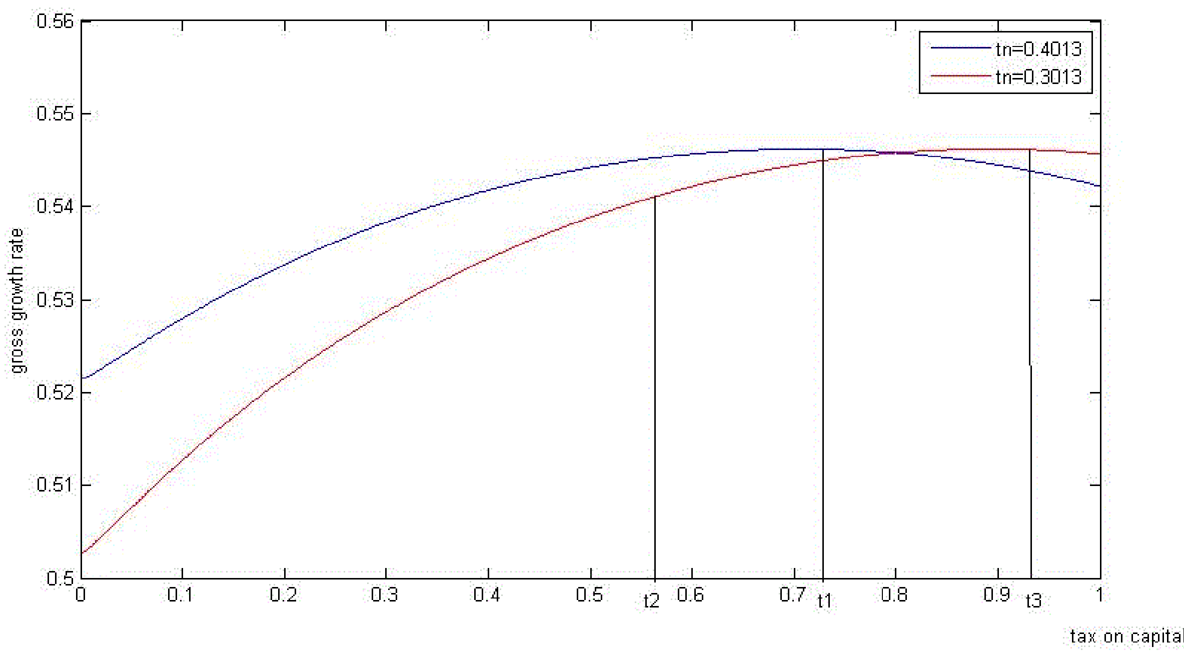

Figure 11: Policy analysis in the model with concave costs

Here t_1 is given by $(\tau_n^*, \tau_k^*) = (0.4013, 0.714)$.[33] We conduct a similar exercise of lowering the tax on labor income by 0.1 and back out the value for the corresponding tax (shown as t_2) on capital income from the equation (48). This will give us a tax rate on capital income equal to 0.594 which does not give us the optimal growth rate. However a tax rate on capital income, much higher than t_1, and given by t_3 in Figure [11] will ensure optimal

[31] If we allow τ_n to fall by 0.2 to 0.22 - a large arbitary amount, we find that a decrease in τ_k – from (48) – is 0.1209. For $\tau_n = 0.22$, $\tau_k = 0.345$. There is larger reduction in the growth rate.

[32] This choice of c and χ ensures feasibility.

[33] The second solutions to τ_k^* and τ_n^* are approximately close to 0 which means that the funding available for the public input will be close to zero. We therefore ignore this solution.

growth rates. The difference in the growth rates by moving from a tax on capital income of 0.594 as in t_3 to 0.908 (moving to t_2) is roughly around 3.4×10^{-3} which is also not large. Hence, having concave administrative costs suggests that there is negligible change in the optimal growth rate when the tax on capital income is increased.[34] Given concave costs, on normative grounds, a planner could charge a higher tax rate on capital without changing the optimal growth rate when there is a fall in the tax on labor income compared to τ_n^*.

6 Conclusion

This paper constructs a dynamic general equilibrium endogenous growth model in which public capital influences investment specific technological change. We characterize the growth and welfare maximizing tax rates in the planner's problem and the decentralized equilibrium. Unlike the existing literature where public input affects current output directly, in our model it affects future output through its effect on investment specific technological change. Our main result is that there exist infinitely many capital and labor tax-subsidy combinations that decentralize the planner's growth rate. Hence the optimal factor income tax mix is indeterminate which gives the planner the flexibility to choose policy rules from an infinitely large set. The indeterminacy in the factor income tax mix, and its robustness, is the main result of our paper.

Accounting for administrative costs, however, reduces the set of optimal feasible tax mix of the planner. In fact the size of this set shrinks as the convexity of the administrative costs increases, and eventually a unique factor income tax mix emerges as the only feasible solution. From a growth-tax policy standpoint, our numerical results are consistent with some other papers in this literature which shows that capital income taxation may increase growth.

While we do not directly solve for the Ramsey optimal fiscal policy allocations, our results are related to a celebrated literature started by Judd (1985) and Chamley (1986), who find that capital taxation decreases welfare and a zero capital tax is thus efficient in the long-run steady state. From a growth standpoint, models analyzing the equilibrium relationship between capital income taxes and growth also typically find that an increase of the capital income tax reduces the return to private investment, which in turn implies a decrease of capital accumulation and thus growth (see Lucas (1990) and Rebelo (1991)). In contrast, our results are consistent with some other papers in this literature which show that the

[34]This result is broadly consistent with the results in Huffman (2008) and Stokey and Rebelo (1995) who show that growth rates are unaffected by a deviation of tax rates on capital income away from their equilibrium values.

optimal capital income tax is positive, i.e., taxation may *increase* growth (see Uhlig and Yanagawa (1996) and Rivas (2003)). On normative grounds, our results suggests that policy makers may want to measure precisely the relative cost associated with factor income tax collection before setting factor income taxes.

In terms of future work, one could formalize the optimal capital income taxation under the Ramsey policy within our environment.

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Technical Appendix

The appendix contains all the calculations and proofs of propositions present in the paper.

Appendix A: The planner's FOCs for the baseline model

The following is the FOC with respect to C_t

$$\beta^t \frac{1}{C_t} - \beta^t \lambda_t = 0.$$

Hence, we get

$$\{C_t\} : \frac{1}{C_t} = \lambda_t.$$

This gives us equation (10). The FOC with respect to K_{t+1} is as follows:

$$\{K_{t+1}\} : \frac{-\lambda_t}{Z_t} + \beta\lambda_{t+1}\frac{\alpha_1 Y_{t+1}(1-\tau)}{K_{t+1}} - \beta\lambda_{t+1}\frac{\partial}{\partial K_{t+1}}(\frac{K_{t+2}}{Z_{t+1}}) - \beta^2\lambda_{t+2}\frac{\partial}{\partial K_{t+1}}(\frac{K_{t+3}}{Z_{t+2}}) - ... = 0.$$

Note that

$$\frac{\partial Z_t}{\partial K_{t+1}} = \frac{\partial Z_{t+1}}{\partial K_{t+1}} = 0,\ \frac{\partial Z_{t+2}}{\partial K_{t+1}} = (1-\gamma)(1-\mu)\frac{Z_{t+2}}{K_{t+1}},\ \frac{\partial Z_{t+3}}{\partial K_{t+1}} = \frac{\gamma Z_{t+3}}{Z_{t+2}}\frac{\partial Z_{t+2}}{\partial K_{t+1}} - \alpha_1(1-\gamma)\frac{Z_{t+3}}{K_{t+1}}.$$

We therefore have

$$\frac{\partial Z_{t+3}}{\partial K_{t+1}} = (1-\gamma)\frac{Z_{t+3}}{K_{t+1}}(\gamma(1-\mu) - \alpha_1).$$

And hence for any other future time period

$$\frac{\partial Z_{t+3+j}}{\partial K_{t+1}} = \gamma^j(1-\gamma)\frac{Z_{t+3+j}}{K_{t+1}}[\gamma(1-\mu) - \alpha_1], \quad for j \geqq 0.$$

Therefore, substituting the above values in $\{K_{t+1}\}$, considering $\frac{1}{C_{t+j}} = \lambda_{t+j}$ and $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$, $j \geqq 0$ and assuming full depreciation (that is $\delta = 1$), we obtain the following FOC for $\{K_{t+1}\}$ which is shown in equation (11)

$$\{K_{t+1}\} : \frac{1}{C_t Z_t} = \frac{\alpha_1 \beta Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \frac{\beta^2 I_{t+2}(1-\gamma)(1-\mu)}{C_{t+2}K_{t+1}} + \frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}}\sum_{j=0}^{\infty}\frac{\beta^j\gamma^j I_{t+j+3}}{C_{t+j+3}}.$$

Next, the FOC with respect to n_{1t} is given by $\frac{-1}{1-n_t} + \frac{\lambda_t\alpha_2 Y_t(1-\tau)}{n_{1t}} - \lambda_t\frac{\partial}{\partial n_{1t}}(\frac{K_{t+1}}{Z_t}) - \beta\lambda_{t+1}\frac{\partial}{\partial n_{1t}}(\frac{K_{t+2}}{Z_{t+1}}) - \beta^2\lambda_{t+2}\frac{\partial}{\partial n_{1t}}(\frac{K_{t+3}}{Z_{t+2}}) - ... = 0$. Given that

$$Z_{t+1} = \frac{B(1-\delta_m)^\theta n_{2t}^\theta Z_t^\gamma (G_t^\mu)^{(1-\gamma)}(K_t^{1-\mu})^{(1-\gamma)}(K_{t-1}^{-\alpha_1})^{(1-\gamma)}(n_{1t-1}^{-\alpha_2})^{(1-\gamma)}(\delta_m^{-\alpha_3}n_{2t-1}^{-\alpha_3})^{(1-\gamma)}}{A^{(1-\gamma)}},$$

this implies

$$\frac{\partial}{\partial n_{1t}}(\frac{K_{t+1}}{Z_t}) = \frac{\partial}{\partial n_{1t}}(\frac{K_{t+2}}{Z_{t+1}}) = 0.$$

We further have

$$\frac{\partial}{\partial n_{1t}}\left(\frac{K_{t+3}}{Z_{t+2}}\right) = \frac{\alpha_2(1-\gamma)K_{t+3}}{n_{1t}Z_{t+2}},$$

and similarly,

$$\begin{aligned}\frac{\partial}{\partial n_{1t}}\left(\frac{K_{t+4}}{Z_{t+3}}\right) &= \frac{\alpha_2(1-\gamma)\gamma K_{t+4}}{n_{1t}Z_{t+3}} \\ \frac{\partial}{\partial n_{1t}}\left(\frac{K_{t+5}}{Z_{t+4}}\right) &= \frac{\alpha_2(1-\gamma)\gamma^2 K_{t+5}}{n_{1t}Z_{t+4}}\end{aligned}$$

for all future time periods. Therefore, substituting the above expressions into the expression for $\{n_{1t}\}$ we get

$$\frac{1}{1-n_t} + \beta^2\lambda_{t+2}\left(\frac{\alpha_2(1-\gamma)K_{t+3}}{n_{1t}Z_{t+2}}\right) + \beta^3\lambda_{t+3}\left(\frac{\alpha_2(1-\gamma)\gamma K_{t+4}}{n_{1t}Z_{t+3}}\right) + ... = \frac{\lambda_t\alpha_2 Y_t(1-\tau)}{n_{1t}}.$$

Recall that for every t, $\frac{1}{C_t} = \lambda_t$. Since $\delta = 1$, $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$, for all $j \geqq 0$. This gives us the final expression of the FOC for n_{1t}, as shown in equation (12)

$$\{n_{1t}\} : \frac{1}{1-n_t} + \frac{\beta^2\alpha_2(1-\gamma)}{n_{1t}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t(1-\tau)}{C_t n_{1t}}.$$

The FOC with respect to n_{2t} is given by

$\{n_{2t}\} : \frac{-1}{1-n_t} + \frac{\lambda_t\alpha_3 Y_t(1-\tau)}{n_{2t}} - \lambda_t\frac{\partial}{\partial n_{2t}}(\frac{K_{t+1}}{Z_t}) - \beta\lambda_{t+1}\frac{\partial}{\partial n_{2t}}(\frac{K_{t+2}}{Z_{t+1}}) - \beta^2\lambda_{t+2}\frac{\partial}{\partial n_{2t}}(\frac{K_{t+3}}{Z_{t+2}})... = 0$. Given

$$\begin{aligned}\frac{\partial Z_t}{\partial n_{2t}} &= 0, \ \frac{\partial Z_{t+1}}{\partial n_{2t}} = \frac{\theta Z_{t+1}}{n_{2t}}; \\ \frac{\partial Z_{t+2}}{\partial n_{2t}} &= \frac{\gamma Z_{t+2}}{Z_{t+1}}\frac{\theta Z_{t+1}}{n_{2t}} - \alpha_3(1-\gamma)\frac{Z_{t+2}}{n_{2t}}\end{aligned}$$

this means a change in n_2 on Z has two effects - a direct effect and an indirect effect. The expression, $\frac{\gamma Z_{t+2}}{Z_{t+1}}\frac{\theta Z_{t+1}}{n_{2t}}$, is the direct effect, while the expression, $-\alpha_3(1-\gamma)\frac{Z_{t+2}}{n_{2t}}$, is the indirect effect. Therefore,

$$\frac{\partial Z_{t+2}}{\partial n_{2t}} = (\gamma\theta - \alpha_3(1-\gamma))\frac{Z_{t+2}}{n_{2t}}.$$

Similarly, the derivative of Z with respect to n_2 at time period $t+3$ is

$$\frac{\partial Z_{t+3}}{\partial n_{2t}} = \gamma(\gamma\theta - \alpha_3(1-\gamma))\frac{Z_{t+3}}{n_{2t}}.$$

Hence, for any future time period $t+j+2$, the derivative is as follows

$$\frac{\partial Z_{t+j+2}}{\partial n_{2t}} = \gamma^j(\gamma\theta - \alpha_3(1-\gamma))\frac{Z_{t+j+2}}{n_{2t}}.$$

Substituting the above expressions into the above expression for $\{n_{2t}\}$, we get

$$\frac{-1}{1-n_t} + \frac{\lambda_t\alpha_3 Y_t(1-\tau)}{n_{2t}} + \beta\lambda_{t+1}\frac{K_{t+2}}{Z_{t+1}^2}\frac{\theta Z_{t+1}}{n_{2t}} + \beta^2\lambda_{t+2}\frac{K_{t+3}}{Z_{t+2}^2}(\gamma\theta - \alpha_3(1-\gamma))\frac{Z_{t+2}}{n_{2t}} + ... = 0.$$

Again, for every t, $\frac{1}{C_t} = \lambda_t$. Since $\delta = 1$, $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$, for all $j \geqq 0$. This implies that

$$\frac{-1}{1-n_t} + \frac{\lambda_t\alpha_3 Y_t(1-\tau)}{n_{2t}} + \beta\theta\frac{\lambda_{t+1}K_{t+2}}{Z_{t+1}n_{2t}} + \frac{\beta^2(\gamma\theta - \alpha_3(1-\gamma))}{n_{2t}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{C_{t+j+2}} = 0.$$

Hence the final expression for $\{n_{2t}\}$ is as follows and as shown in equation (13)

$$\{n_{2t}\} : \frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau)}{C_t n_{2t}} + \beta\theta\frac{I_{t+1}}{C_{t+1}n_{2t}} + \frac{\beta^2(\gamma\theta - \alpha_3(1-\gamma))}{n_{2t}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{C_{t+j+2}}.$$

Appendix B: Decision rules for the planner's problem for the baseline model

We first show that Φ, x and n are constants, and have feasible values. Feasibility requires that $0 < \Phi, x, n < 1$. We also derive a condition that ensures that the balanced growth rate is stable, i.e.,

$$g_{zt+1} = g_{zt} = g_z.$$

We have shown earlier

$$\{K_{t+1}\}: \frac{1}{C_t Z_t} = \beta\frac{\alpha_1 Y_{t+1}(1-\tau)}{C_{t+1}K_{t+1}} + \beta^2\frac{I_{t+2}}{C_{t+2}K_{t+1}}(1-\gamma)(1-\mu) + \frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{C_{t+j+3}}$$

$$\Rightarrow \frac{1}{\Phi Y_t(1-\tau)Z_t} = \beta\frac{\alpha_1 Y_{t+1}(1-\tau)}{\Phi Y_{t+1}(1-\tau)(1-\Phi)Y_t(1-\tau)Z_t} + \beta^2\frac{(1-\Phi)}{\Phi(1-\Phi)Y_t(1-\tau)Z_t}(1-\gamma)(1-\mu) + \frac{\beta^3(1-\gamma)(\gamma(1-\mu)-\alpha_1)}{(1-\Phi)Y_t(1-\tau)Z_t}\left(\frac{1}{1-\beta\gamma}\right)\frac{(1-\Phi)}{\Phi}$$

$$\Rightarrow \quad (1-\Phi) = \frac{\alpha_1\beta(1-\beta\gamma)}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}$$
$$\Rightarrow \quad \Phi = \frac{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)-\alpha_1\beta]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]},$$

as shown in (19).

For $0 < \Phi < 1$, we require that

$$(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)-\alpha_1\beta]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1] > 0$$

and

$$1-\beta\gamma > 0 \Rightarrow \gamma < \frac{1}{\beta}.$$

Note that, $\gamma < \frac{1}{\beta}$, is the condition for stability. From the FOC for n_{1t},

$$\{n_{1t}\} : \frac{1}{1-n_t} + \frac{\beta^2\alpha_2(1-\gamma)}{n_{1t}} \sum_{j=0}^{\infty} \beta^j\gamma^j \frac{I_{t+j+2}}{C_{t+j+2}} = \frac{\alpha_2 Y_t(1-\tau)}{C_t n_{1t}}$$

$$\Rightarrow \quad \frac{n_t}{1-n_t} = \frac{\alpha_2}{\Phi x} - \frac{\beta^2\alpha_2(1-\gamma)(1-\Phi)}{x(1-\beta\gamma)\Phi}$$
$$\Rightarrow \quad n = \frac{\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)]}{(\alpha_2+\Phi x)(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)}$$

which is the expression in equation (17), and Φ is derived above. Note, when

$$\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)] > 0$$

and when

$$0 < \Phi, x < 1,$$

then

$$0 < n < 1.$$

Next, we derive the expression for x. We know the FOC with respect to $\{n_{2t}\}$ is given by

$$\frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau)}{C_t n_{2t}} + \beta\theta \frac{I_{t+1}}{C_{t+1}n_{2t}} + \beta^2(\gamma\theta - \alpha_3(1-\gamma)) \sum_{j=0}^{\infty} \beta^j\gamma^j \frac{I_{t+j+2}}{C_{t+j+2}}$$

$$\Rightarrow (1-x)\left(\frac{n_1}{1-n_t}\right) = \frac{\alpha_3}{\Phi} + \beta\theta\left(\frac{1-\Phi}{\Phi}\right) + \beta^2(\gamma\theta-\alpha_3(1-\gamma))\left(\frac{1}{1-\beta\gamma}\right)\left(\frac{1-\Phi}{\Phi}\right)$$

$$\Rightarrow \quad \left(\frac{1-x}{x}\right) = \frac{\alpha_3(1-\beta\gamma)+\beta\theta(1-\Phi)(1-\beta\gamma)+\beta^2(\gamma\theta-\alpha_3(1-\gamma))(1-\Phi)}{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)}$$
$$\Rightarrow \quad x = \frac{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)}{[\alpha_2+\alpha_3+\beta\theta(1-\Phi)](1-\beta\gamma)+\beta^2(1-\Phi)[\gamma\theta-\alpha_3(1-\gamma)-\alpha_2(1-\gamma)]}$$

which is the expression for x in equation (20). When $0 < \Phi < 1$, $0 < x < 1$ is automatically satisfied. This means feasible values of x require no other additional assumption other than $0 < \gamma < 1$. We first verify that $0 < \Phi < 1$. This means we need to check whether the following two inequalities are satisfied:

$$\begin{aligned}(i)(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)-\alpha_1\beta]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1] &> 0\\ (ii)1-\beta\gamma &> 0 \Rightarrow \gamma < \frac{1}{\beta}.\end{aligned}$$

Since $0 < \gamma < 1$, we have $0 < \gamma < 1 < \frac{1}{\beta}$. This means (ii) is trivially satisfied when $0 < \gamma < 1$. To check (i), we must check whether $(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)-\alpha_1\beta]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1] > 0$, or $(1-\beta\gamma)(1-\alpha_1\beta)-\beta^2(1-\gamma)(1-\mu-\alpha_1\beta) > 0$,

$$\Rightarrow (1-\beta\gamma)(1-\alpha_1\beta) > \beta^2(1-\gamma)(1-\mu-\alpha_1\beta).$$

We know $(1-\alpha_1\beta) > (1-\mu-\alpha_1\beta)$ because $0 < \mu < 1$. Further, $(1-\beta\gamma) > \beta^2(1-\gamma)$ if and only if $(1-\beta\gamma)-\beta^2(1-\gamma) > 0$. Clearly, $(1-\beta)(1+\beta-\beta\gamma) > 0$. Hence, $(1-\beta\gamma)-\beta^2(1-\gamma) > 0$. This implies

$$(1-\beta\gamma)(1-\alpha_1\beta)-\beta^2(1-\gamma)(1-\mu-\alpha_1\beta) > 0.$$

Hence, (i) is satisfied and $0 < \Phi < 1$. We now verify that $0 < x < 1$.We know that

$$x = \frac{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)}{[\alpha_2+\alpha_3+\beta\theta(1-\Phi)](1-\beta\gamma)+\beta^2(1-\Phi)[\gamma\theta-\alpha_3(1-\gamma)-\alpha_2(1-\gamma)]}.$$

To show the above expression for $0 < x < 1$, it is sufficient to show that $\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)] > 0$, since we have already shown, $0 < \Phi < 1$. We also have to show the denominator in the above expression is greater than the numerator so as to ensure that x is a fraction. As shown earlier,

$$(1-\beta\gamma)-\beta^2(1-\gamma) > 0$$

which implies $(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi) > 0$ since we have already shown $0 < \Phi < 1$ (and so $0 < (1-\Phi) < 1$). This implies that, $\alpha_2[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi)] > 0$ is true. The denominator is given by the expression

$$[\alpha_2 + \alpha_3 + \beta\theta(1-\Phi)](1-\beta\gamma) + \beta^2(1-\Phi)[\gamma\theta - \alpha_3(1-\gamma) - \alpha_2(1-\gamma)],$$

which on re-arranging yields $\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)]+\alpha_3[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)]+\beta\theta(1-\Phi)(1-\beta\gamma)+\beta^2(1-\Phi)\gamma\theta$. We earlier showed that, $\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)] > 0$. Similarly, we can obtain

$$\alpha_3[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi)] > 0.$$

Also,

$$\beta\theta(1-\Phi)(1-\beta\gamma) + \beta^2(1-\Phi)\gamma\theta > 0$$

follows from above and the restriction that $(1-\beta\gamma) > 0$. Hence the denominator of x is the numerator plus a sum of two positive terms. This shows that $0 < x < 1$.

Finally, we need to check that $0 < n < 1$. Recall that,

$$n = \frac{\alpha_2[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\Phi)]}{(\alpha_2 + \Phi x)(1-\beta\gamma) - \alpha_2\beta^2(1-\gamma)(1-\Phi)}.$$

This means if we just show the numerator is greater than zero, $0 < n < 1$ is true. This is because the denominator is simply the numerator + $\Phi x(1-\beta\gamma)$. From above, this term ($\Phi x(1-\beta\gamma)$) is positive. We have also seen that $\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)] > 0$. Hence, $0 < n < 1$ is true.

Therefore, $0 < \Phi, x, n < 1$.

Appendix C : Balanced Growth Rates

In the planner version,

$$Z_{t+1} = B((1-\delta_m)n_{2t})^\theta Z_t^\gamma \left\{ \left(\frac{G_t}{Y_{t-1}}\right)^\mu \left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu} \right\}^{(1-\gamma)}.$$

Since under full depreciation $K_t = I_{t-1}Z_{t-1}$ and $G_t = I^g_{t-1}Z_{t-1}$, we can substitute for G and K in the law of motion and further substitute the decision rules for I and I^g, leading to

$$I_t = (1-\Phi)Y_t(1-\tau)$$

and

$$I_t^g = \tau Y_t.$$

Further,

$$n_2 = (1-x)n.$$

Hence,

$$\begin{aligned} Z_{t+1} &= B((1-\delta_m)(1-x)n)^{\theta} Z_t^{\gamma} \left\{ \left(\frac{\tau Y_{t-1} Z_{t-1}}{Y_{t-1}} \right)^{\mu} \left(\frac{(1-\Phi) Y_{t-1}(1-\tau) Z_{t-1}}{Y_{t-1}} \right)^{1-\mu} \right\}^{(1-\gamma)} \\ &\Rightarrow \ Z_{t+1} = \widehat{M} Z_t^{\gamma} Z_{t-1}^{(1-\gamma)} \{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)} \end{aligned}$$

where

$$\widehat{M} = B((1-\delta_m)(1-x)n)^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}$$

is a constant. This implies

$$\begin{aligned} \frac{Z_{t+1}}{Z_t} &= g_{z_{t+1}} = \widehat{M} \frac{Z_t^{\gamma-1}}{Z_{t-1}^{\gamma-1}} Z_{t-1}^{(1-\gamma)+\gamma-1} \{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)} \\ &\Rightarrow \ g_{z_{t+1}} = g_{z_t}^{-(1-\gamma)} \widehat{M} \{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}. \end{aligned}$$

The assumption $0 < \gamma < 1$, makes the system defined by

$$g_{z_{t+1}} = \frac{\Omega}{g_{z_t}^{(1-\gamma)}}$$

(where $\Omega = \widehat{M}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}$ is a constant) dynamically stable. Any deviation from the point of intersection of the 45-degree line with the plot for $g_{z_{t+1}} = \frac{\Omega}{g_{z_t}^{(1-\gamma)}}$ will eventually result in the system converging to the 45-degree line. This is shown in Figure [1]. Note that $(1-\gamma) = 1$ will give an oscillating system which will never ever converge. Likewise $(1-\gamma) > 1$ will lead to an explosive system. In our model, it is therefore sufficient to have $0 < \gamma < 1$ to ensure a steady state BGP.

At the steady state therefore,

$$g_{z_{t+1}} = g_{z_t} = \widehat{g_z}$$

and hence,

$$\widehat{g_z} = \Omega^{\frac{1}{1+(1-\gamma)}} = [\widehat{M}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}$$

is the steady state growth rate, where $\widehat{M} = B((1-\delta_m)(1-x)n)^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}$ is a constant.

To derive the growth rates of the other variables on the BGP, note that

$$\begin{aligned}
K_{t+1} &= I_t Z_t \\
&\Rightarrow \frac{K_{t+1}}{K_t} = \frac{I_t Z_t}{I_{t-1} Z_{t-1}} \\
&\Rightarrow \widehat{g_k} = \frac{(1-\Phi)Y_t(1-\tau)Z_t}{(1-\Phi)Y_{t-1}(1-\tau)Z_{t-1}} = \frac{Y_t Z_t}{Y_{t-1} Z_{t-1}} = \widehat{g_y}\widehat{g_z}.
\end{aligned}$$

Now,

$$\begin{aligned}
Y_t &= AK_t^{\alpha_1} n_{1t}^{\alpha_2}(\delta_m n_{2t})^{\alpha_3} \\
&\Rightarrow \frac{Y_t}{Y_{t-1}} = \widehat{g_y} = \frac{AK_t^{\alpha_1} n_{1t}^{\alpha_2}(\delta_m n_{2t})^{\alpha_3}}{AK_{t-1}^{\alpha_1} n_{1t-1}^{\alpha_2}(\delta_m n_{2t-1})^{\alpha_3}} = \frac{K_t^{\alpha_1}}{K_{t-1}^{\alpha_1}} = \widehat{g_k}^{\alpha_1} \\
&\Rightarrow \widehat{g_y} = \widehat{g_k}^{\alpha_1} \\
&\Rightarrow \widehat{g_y} = \widehat{g_k}^{\alpha_1} = \widehat{g_z}^{\frac{\alpha_1}{1-\alpha_1}}.
\end{aligned}$$

Appendix D - Optimal tax rate in the baseline model - the planner's problem

$$\begin{aligned}
\frac{\partial \widehat{g_z}}{\partial \tau} &= \frac{\partial[\widehat{M}\{(\tau)^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}}{\partial \tau} = 0 \\
&\Rightarrow \widehat{M}^{\frac{1}{(2-\gamma)}}\frac{(1-\gamma)}{(2-\gamma)}[(\tau)^{\mu}(1-\tau)^{1-\mu}]^{\frac{-1}{(2-\gamma)}}\frac{\partial[(\tau)^{\mu}(1-\tau)^{1-\mu}]}{\partial \tau} = 0 \\
&\Rightarrow (1-\tau)^{1-\mu}\mu(\tau)^{\mu-1} = (\tau)^{\mu}(1-\mu)(1-\tau)^{-\mu} \\
&\Rightarrow (1-\tau)\mu = (\tau)(1-\mu) \\
&\Rightarrow \widehat{\tau} = \mu.
\end{aligned}$$

Hence the steady state growth optimizing tax rate in the planner version is $\widehat{\tau} = \mu$.

Appendix E - Optimal tax rate in the baseline model - the agent's version

$$\begin{aligned}
\{\tau_n\} \quad : \quad & \frac{\partial \widehat{gz}}{\partial \tau_n} = \frac{\partial[\widehat{M}\{(1-\Theta)^{\mu}(\Theta)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{(2-\gamma)}}}{\partial \tau_n} = 0 \\
&\Rightarrow \widehat{M}^{\frac{1}{(2-\gamma)}}\frac{(1-\gamma)}{(2-\gamma)}[(1-\Theta)^{\mu}(\Theta)^{1-\mu}]^{\frac{-1}{(2-\gamma)}}\frac{\partial[[(1-\Theta)^{\mu}(\Theta)^{1-\mu}]]}{\partial \tau_n} = 0 \\
&\Rightarrow (\Theta)^{1-\mu}\mu(1-\Theta)^{\mu-1}\frac{\partial[(1-\Theta)]}{\partial \tau_n} + (1-\Theta)^{\mu}(1-\mu)(\Theta)^{-\mu}\frac{\partial[(\Theta)]}{\partial \tau_n} = 0.
\end{aligned}$$

Since

$$\begin{aligned}
\frac{\partial[(1-\Theta)]}{\partial\tau_n} &= \frac{\partial[(\alpha_2+\alpha_3)(\tau_n)+\alpha_1(\tau_k)]}{\partial\tau_n} = \alpha_2+\alpha_3, \\
\frac{\partial[(\Theta)]}{\partial\tau_n} &= \frac{\partial[(\alpha_2+\alpha_3)(1-\tau_n)+\alpha_1(1-\tau_k)]}{\partial\tau_n} = -(\alpha_2+\alpha_3).
\end{aligned}$$

Substituting the above expressions yields,

$$\begin{aligned}
&\Rightarrow\ (\Theta)^{1-\mu}\mu(1-\Theta)^{\mu-1}-(1-\Theta)^{\mu}(1-\mu)(\Theta)^{-\mu}=0 \\
&\Rightarrow\ \frac{\mu}{1-\mu}=\frac{1-\Theta}{\Theta} \\
&\Rightarrow\ \widehat{1-\Theta}=\mu.
\end{aligned}$$

Likewise

$$\{\tau_k\}:\frac{\partial\widehat{gz}}{\partial\tau_k}=\frac{\partial[\widehat{M}\{(1-\Theta)^{\mu}(\Theta)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{(2-\gamma)}}}{\partial\tau_k}=0.$$

Since

$$\begin{aligned}
\frac{\partial[(1-\Theta)]}{\partial\tau_k} &= \frac{\partial[(\alpha_2+\alpha_3)(\tau_n)+\alpha_1(\tau_k)]}{\partial\tau_k} = \alpha_1, \\
\frac{\partial[(\Theta)]}{\partial\tau_k} &= \frac{\partial[(\alpha_2+\alpha_3)(1-\tau_n)+\alpha_1(1-\tau_k)]}{\partial\tau_k} = -(\alpha_1).
\end{aligned}$$

Substituting the above yields the same FOCs just as in the case of $\{\tau_n\}$. This implies, both $\{\tau_n\}$ and $\{\tau_k\}$ give the same FOC

$$\begin{aligned}
\widehat{1-\Theta} &= \mu \\
&\Rightarrow\ (\alpha_2+\alpha_3)(\widehat{\tau_n})+\alpha_1(\widehat{\tau_k})=\mu=\widehat{\tau}.
\end{aligned}$$

Appendix F - Comparative statics - the planner's problem

We know from equations (17) and (19)

$$n=\frac{\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)]}{(\alpha_2+\Phi x)(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)}.$$

And

$$\Phi=\frac{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)-\alpha_1\beta]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}.$$

Further

$$\widehat{\tau} = \mu.$$

We shall first see how Φ changes with μ. This implies, the relationship between $1 - \Phi$ and μ gets reversed.

$$\begin{aligned}
\frac{\partial \Phi}{\partial \mu} &= \frac{\partial}{\partial \mu}\left[\frac{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)-\alpha_1\beta]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}\right] \\
&= \frac{\partial}{\partial \mu}\left[1-\frac{\alpha_1\beta(1-\beta\gamma)}{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}\right] \\
&= \frac{\alpha_1\beta(1-\beta\gamma)}{\{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]\}^2} \times \\
&\quad \frac{\partial\{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]\}}{\partial \mu}
\end{aligned}$$

$$= \frac{\alpha_1\beta(1-\beta\gamma)[(1-\beta\gamma)\beta^2(1-\gamma)+\beta^3(1-\gamma)\gamma]}{\{(1-\beta\gamma)[1-\beta^2(1-\gamma)(1-\mu)]-\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]\}^2} > 0.$$

Hence

$$\frac{\partial \Phi}{\partial \mu} > 0,$$

and therefore

$$\frac{\partial (1-\Phi)}{\partial \mu} < 0.$$

Since

$$\begin{aligned}
C_t &= \Phi Y_t (1-\tau) \\
&\Rightarrow \frac{\partial C_t}{\partial \tau} < 0.
\end{aligned}$$

Now we take the partial derivative of

$$x = \frac{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)}{[\alpha_2+\alpha_3+\beta\theta(1-\Phi)](1-\beta\gamma)+\beta^2(1-\Phi)[\gamma\theta-\alpha_3(1-\gamma)-\alpha_2(1-\gamma)]}$$

with respect to μ. We will use $\frac{\partial \Phi}{\partial \mu} > 0$ in our analysis. Suppose we consider the value of $\frac{1}{x}$,

$$\begin{aligned}
\frac{1}{x} &= 1 + \frac{\alpha_3(1-\beta\gamma)-\beta^2\alpha_3(1-\gamma)(1-\Phi)}{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)} + \frac{\beta\theta(1-\Phi)}{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)} \\
\Rightarrow \frac{\partial\left(\frac{1}{x}\right)}{\partial \mu} &= \frac{\partial}{\partial \mu}\left(\frac{\alpha_3(1-\beta\gamma)-\beta^2\alpha_3(1-\gamma)(1-\Phi)}{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)}\right) + \frac{\partial}{\partial \mu}\left(\frac{\beta\theta(1-\Phi)}{\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi)}\right).
\end{aligned}$$

On opening up the brackets and simplifying the above expression, we get

$$\Rightarrow \frac{\partial\left(\frac{1}{x}\right)}{\partial\mu} = \frac{\alpha_2(1-\beta\gamma)\beta\theta\frac{\partial}{\partial\mu}(1-\Phi)}{(\alpha_2(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\Phi))^2} < 0$$

because

$$\frac{\partial\Phi}{\partial\mu} > 0 \text{ and so } \frac{\partial(1-\Phi)}{\partial\mu} < 0.$$

Hence,

$$\begin{aligned}
\frac{\partial\left(\frac{1}{x}\right)}{\partial\mu} &< 0 \\
\frac{\partial\left(x\right)}{\partial\mu} &> 0.
\end{aligned}$$

We now look at the partial derivative of n with respect to μ. We have shown earlier that

$$n = \frac{\alpha_2[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\Phi)]}{(\alpha_2+\Phi x)(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)}.$$

Hence, on applying the quotient rule and on re-arranging the terms, we get

$$\begin{aligned}
\frac{\partial}{\partial\mu}(n) &= \frac{[\alpha_2\beta^2(1-\gamma)\Phi x(1-\beta\gamma)]\frac{\partial\Phi}{\partial\mu}}{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)+\Phi x(1-\beta\gamma)]^2} \\
&\quad -\frac{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)](1-\beta\gamma)\Phi\frac{\partial x}{\partial\mu}}{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)+\Phi x(1-\beta\gamma)]^2} \\
&\quad -\frac{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)](1-\beta\gamma)x\frac{\partial\Phi}{\partial\mu}}{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)+\Phi x(1-\beta\gamma)]^2}.
\end{aligned}$$

Given that $\frac{\partial x}{\partial\mu} > 0$ we can easily see that the second term in the above expression

$$-\frac{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)](1-\beta\gamma)\Phi\frac{\partial x}{\partial\mu}}{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)+\Phi x(1-\beta\gamma)]^2} < 0.$$

This is because from an earlier exercise we have shown that $\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi) >$

0. Clubbing the other two terms, we get,

$$\begin{aligned}\frac{\partial}{\partial\mu}(n) &= \frac{-\alpha_2 x[(1-\beta\gamma)^2-\beta^2(1-\gamma)\Phi(1-\beta\gamma)-\beta^2(1-\beta\gamma)(1-\gamma)(1-\Phi)]\frac{\partial\Phi}{\partial\mu}}{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)+\Phi x(1-\beta\gamma)]^2} \\ &\quad -\frac{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)](1-\beta\gamma)\Phi\frac{\partial x}{\partial\mu}}{[\alpha_2(1-\beta\gamma)-\alpha_2\beta^2(1-\gamma)(1-\Phi)+\Phi x(1-\beta\gamma)]^2}.\end{aligned}$$

If $[(1-\beta\gamma)^2-\beta^2(1-\gamma)\Phi(1-\beta\gamma)-\beta^2(1-\beta\gamma)(1-\gamma)(1-\Phi)]>0$ then $\frac{\partial n}{\partial\mu}<0$. To see this, we re-arrange

$$[(1-\beta\gamma)^2-\beta^2(1-\gamma)\Phi(1-\beta\gamma)-\beta^2(1-\beta\gamma)(1-\gamma)(1-\Phi)]$$

to get

$$(1-\beta\gamma)[(1-\beta\gamma)-\beta^2(1-\gamma)\Phi-\beta^2(1-\gamma)(1-\Phi)]$$

which is equal to

$$(1-\beta\gamma)(1-\beta)[1-\beta\gamma+\beta]>0$$

because we have shown earlier that $[1-\beta\gamma-\beta^2(1-\gamma)]>0$. Hence

$$\frac{\partial n}{\partial\mu}<0.$$

We now need to verify whether, $\frac{\partial n_1}{\partial\mu}<0$. Note that

$$\frac{\partial}{\partial\mu}(n_1)=\frac{\partial}{\partial\mu}(xn)=x\frac{\partial}{\partial\mu}(n)+n\frac{\partial}{\partial\mu}(x),$$

the sign of which is ambiguous because x is increasing in μ while n is decreasing in μ. Therefore whichever term dominates will determine the way n_1 behaves with μ. However we can show

$$\frac{\partial}{\partial\mu}(n_2)<0,$$

since

$$\frac{\partial}{\partial\mu}(n_2)=\frac{\partial}{\partial\mu}((1-x)n)=(1-x)\frac{\partial}{\partial\mu}(n)+n\frac{\partial}{\partial\mu}(1-x)<0.$$

Appendix G: Welfare analysis

In the planner's problem, we know

$$C_t=\Phi Y_t(1-\tau)$$

$$\Rightarrow \quad \frac{C_t}{C_{t-1}} = \frac{\Phi Y_t(1-\tau)}{\Phi Y_{t-1}(1-\tau)} = \widehat{g}_y$$
$$\Rightarrow \quad \widehat{g}_c = \widehat{g}_y.$$

Since $\widehat{g}_c$ is a constant, $C_t = C_0 \widehat{g}c^t$. On the BGP, the supply of labor is the same across time. We denote welfare by Λ, where,

$$\Lambda = \sum_{j=0}^{\infty} \beta^t [\log C_t + \log(1-n_t)]$$
$$\Lambda = \sum_{j=0}^{\infty} \beta^t \log C_t + \frac{\log(1-\widehat{n})}{1-\beta}$$

$$\Rightarrow \Lambda = \log C_o + \beta \log C_1 + \beta^2 \log C_2 + \beta^3 \log C_3 + \beta^4 \log C_4 + ... + \tfrac{\log(1-\widehat{n})}{1-\beta}$$

$$\Rightarrow \Lambda = \frac{\log C_o}{1-\beta} + \frac{\beta^2}{1-\beta} \log \widehat{g}_c + \frac{\log(1-\widehat{n})}{1-\beta}$$

$$\Rightarrow \Lambda = \tfrac{\log C_o}{1-\beta} + \tfrac{\beta^2 \alpha_1}{(1-\beta)(1-\alpha_1)} \log[\widehat{M}\{(\tau)^\mu (1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} + \tfrac{\log(1-\widehat{n})}{1-\beta}$$
$$\Rightarrow \Lambda = \tfrac{\log(\Phi Y_o(1-\tau))}{1-\beta} + \tfrac{\beta^2 \alpha_1}{(1-\beta)(1-\alpha_1)} \log[\widehat{M}\{(\tau)^\mu (1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} + \tfrac{\log(1-\widehat{n})}{1-\beta}$$
$$\Rightarrow \Lambda = \tfrac{\log(\Phi Y_o)}{1-\beta} + \tfrac{\log(1-\tau)}{1-\beta} + \tfrac{\beta^2 \alpha_1}{(1-\beta)(1-\alpha_1)} \log[\widehat{M}\{(\tau)^\mu (1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} + \tfrac{\log(1-\widehat{n})}{1-\beta}$$

$$\Rightarrow \Lambda = \Gamma + \Lambda \log(1-\tau) + \Psi \log[(\tau)^\mu (1-\tau)^{1-\mu}],$$

where Γ is independent of the tax rate and

$$\Psi = \frac{\beta^2 \alpha_1 (1-\gamma)}{(1-\beta)(1-\alpha_1)(2-\gamma)}, \text{ and } \Lambda = \frac{1}{1-\beta}.$$

On differentiating the above welfare function, we get

$$\tau^w = \frac{\beta^2 \alpha (1-\gamma)\mu}{\beta^2 \alpha(1-\gamma) + (1-\alpha)(2-\gamma)}.$$

In the agent's version, we denote welfare by Δ.

$$\begin{aligned}\Delta &= \sum_{j=0}^{\infty}\beta^t[\log C_t+\log(1-\widehat{n})] \\ &\Rightarrow \quad \Delta=\sum_{j=0}^{\infty}\beta^t\log C_t+\frac{\log(1-\widehat{n})}{1-\beta} \\ &= \quad \frac{\log C_o}{1-\beta}+\frac{\beta^2}{1-\beta}\log\widehat{gc}+\frac{\log(1-\widehat{n})}{1-\beta}\end{aligned}$$

$$\Rightarrow \Delta=\frac{\log(\Phi Y_o)}{1-\beta}+\frac{\log\Theta}{1-\beta}+\frac{\beta^2\alpha_1}{(1-\beta)(1-\alpha_1)}\log[\widehat{M}\{(1-\Theta)^{\mu}(\Theta)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}+\frac{\log(1-\widehat{n})}{1-\beta}$$

$$\Rightarrow \Delta=\Xi+\Lambda\log\Theta+\Psi\log[(1-\Theta)^{\mu}(\Theta)^{1-\mu}],$$

where Ξ is independent of the tax rates and

$$\Psi=\frac{\beta^2\alpha_1(1-\gamma)}{(1-\beta)(1-\alpha_1)(2-\gamma)},\ \text{and}\ \Lambda=\frac{1}{1-\beta}.$$

Appendix H - The model with administrative costs: growth optimization in planner's version

$$\begin{aligned}\widehat{g_z} &= [B((1-\delta_m)(1-x)(n))^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}\{(\tau-c\tau^{\chi})^{\mu}(1-\tau)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}} \\ &\Rightarrow \quad \frac{\partial\widehat{gz}}{\partial\tau}=\frac{\partial[\{(\tau-c\tau^{\chi})^{\mu}(1-\tau)^{1-\mu}\}]^{\frac{(1-\gamma)}{(2-\gamma)}}}{\partial\tau}=0 \\ &\Rightarrow \quad (\tau-c\tau^{\chi})^{\mu}\frac{\partial(1-\tau)^{1-\mu}}{\partial\tau}+(1-\tau)^{1-\mu}\frac{\partial(\tau-c\tau^{\chi})^{\mu}}{\partial\tau}=0\end{aligned}$$

$$\Rightarrow -(1-\mu)(\tau-c\tau^{\chi})^{\mu}(1-\tau)^{-\mu}+\mu(1-\tau)^{1-\mu}(\tau-c\tau^{\chi})^{\mu-1}(1-c\chi\tau^{\chi-1})=0$$

$\Rightarrow \mu(1-\tau^{AC})[1-c\chi(\tau^{AC})^{\chi-1}]=(1-\mu)(\tau^{AC}-c(\tau^{AC})^{\chi})$ as shown in equation (42). Substituting $c=0$, we get

$$\begin{aligned}\mu(1-\tau^{AC}) &= (1-\mu)\tau^{AC} \\ &\Rightarrow \quad \tau^{AC}=\mu.\end{aligned}$$

Substituting $\chi = 1$

$$\begin{aligned}\mu(1-\tau^{AC})[1-c] &= (1-\mu)(\tau^{AC} - c\tau^{AC})\\ &\Rightarrow \mu(1-\tau^{AC}) = (1-\mu)\tau^{AC}\\ &\Rightarrow \tau^{AC} = \mu.\end{aligned}$$

Appendix I - The model with administrative costs: growth optimization in the agent's version

We have,

$$\widehat{g_z} = [B((1-\delta_m)(1-x)(n))^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}\{(1-\Theta - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}(\Theta)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}$$

where $1-\Theta = \alpha_1\tau_k + (1-\alpha_1)\tau_n$. Substituting for $(1-\Phi)$ in $\widehat{g_z}$ we get,

$\widehat{g_z} = [B((1-\delta_m)(1-x)(n))^{\theta}(1-\Phi)^{(1-\mu)(1-\gamma)}\{(\alpha_1\tau_k + (1-\alpha_1)\tau_n - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}(1-\alpha_1\tau_k - (1-\alpha_1)\tau_n)^{1-\mu}\}^{(1-\gamma)}]^{\frac{1}{2-\gamma}}$.

The FOC of $\widehat{g_z}$ with respect to τ_n is given by

$$\{\tau_n\} : \frac{\partial\widehat{gz}}{\partial\tau_n} = 0$$

$$\Rightarrow \frac{\partial\{(\alpha_1\tau_k + (1-\alpha_1)\tau_n - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}(1-\alpha_1\tau_k - (1-\alpha_1)\tau_n)^{1-\mu}\}}{\partial\tau_n} = 0$$

$$\Rightarrow (1-\Theta - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}\frac{\partial(1-\alpha_1\tau_k-(1-\alpha_1)\tau_n)^{1-\mu}}{\partial\tau_n} + (\Theta)^{1-\mu}\frac{\partial(\alpha_1\tau_k+(1-\alpha_1)\tau_n - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}}{\partial\tau_n} = 0$$

$$\Rightarrow \mu(1-\alpha_1 - c_2\chi\tau_n^{\chi-1})(\Theta) = (1-\alpha_1)(1-\mu)(1-\Theta - c_1\tau_k^{\chi} - c_2\tau_n^{\chi}). \quad (1)$$

Similarly, the FOC with respect to τ_k is given by

$$\{\tau_k\} : \frac{\partial\widehat{gz}}{\partial\tau_k} = 0$$

$$\Rightarrow \frac{\partial\{(\alpha_1\tau_k + (1-\alpha_1)\tau_n - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}(1-\alpha_1\tau_k - (1-\alpha_1)\tau_n)^{1-\mu}\}}{\partial\tau_k} = 0$$

$$\Rightarrow (1-\Theta - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}\frac{\partial(\Theta)^{1-\mu}}{\partial\tau_k} + (\Theta)^{1-\mu}\frac{\partial(1-\Theta - c_1\tau_k^{\chi} - c_2\tau_n^{\chi})^{\mu}}{\partial\tau_k} = 0$$

$$\Rightarrow \mu(\alpha_1 - c_1\chi\tau_k^{\chi-1})(\Theta) = \alpha_1(1-\mu)(1-\Theta-c_1\tau_k^{\chi}-c_2\tau_n^{\chi}). \tag{2}$$

Using FOC (a) and FOC (b), we have

$$\begin{aligned}
\frac{\mu(1-\alpha_1-c_2\chi\tau_n^{\chi-1})(\Theta)}{\mu(\alpha_1-c_1\chi\tau_k^{\chi-1})(\Theta)} &= \frac{(1-\alpha_1)(1-\mu)(1-\Theta-c_1\tau_k^{\chi}-c_2\tau_n^{\chi})}{\alpha_1(1-\mu)(1-\Theta-c_1\tau_k^{\chi}-c_2\tau_n^{\chi})} \\
&\Rightarrow \frac{(1-\alpha_1-c_2\chi\tau_n^{\chi-1})}{(\alpha_1-c_1\chi\tau_k^{\chi-1})} = \frac{(1-\alpha_1)}{\alpha_1} \\
&\Rightarrow \alpha_1(1-\alpha_1-c_2\chi\tau_n^{\chi-1}) = (1-\alpha_1)(\alpha_1-c_1\chi\tau_k^{\chi-1}) \\
&\Rightarrow \alpha_1 c_2\chi\tau_n^{\chi-1} = (1-\alpha_1)c_1\chi\tau_k^{\chi-1} \\
&\Rightarrow \frac{\tau_n^{\chi-1}}{\tau_k^{\chi-1}} = \frac{(1-\alpha_1)c_1}{\alpha_1 c_2} \\
&\Rightarrow \left(\frac{\tau_n}{\tau_k}\right)^{\chi-1} = \left(\frac{1-\alpha_1}{\alpha_1}\right)\left(\frac{c_1}{c_2}\right)
\end{aligned}$$

as shown in equation (48). We now summarize the results for linear administrative costs. The following is the FOC with respect to τ_n

$$\{\tau_n\} : \mu(1-\alpha_1-c_2\chi\tau_n^{\chi-1})(\Theta) = (1-\alpha_1)(1-\mu)(1-\Theta-c_1\tau_k^{\chi}-c_2\tau_n^{\chi})$$

and the following is the FOC with respect to τ_k

$$\{\tau_k\} : \mu(\alpha_1-c_1\chi\tau_k^{\chi-1})(\Theta) = \alpha_1(1-\mu)(1-\Theta-c_1\tau_k^{\chi}-c_2\tau_n^{\chi}).$$

Substituting $\chi = 1$, we now get the following as the FOCs with respect to τ_n and τ_k respectively

$$\{\tau_n\} : \mu(1-\alpha_1-c_2)(\Theta) = (1-\alpha_1)(1-\mu)(1-\Theta-c_1\tau_k-c_2\tau_n)$$

$$\{\tau_k\} : \mu(\alpha_1-c_1)(\Theta) = \alpha_1(1-\mu)(1-\Theta-c_1\tau_k-c_2\tau_n).$$

This suggests that there does not exist any unique solution to the combination of tax rates on labor and capital.

Appendix J - The Agent's Version - solved as a borrowing-lending problem

The Firm's Problem

The firm's value function is given by

$$\underset{K_{t+1},n_{1t},n_{2t}}{Max} V(k_t,Z_t)=\{AK^{\alpha_1}n_{1t}^{\alpha_2}(\delta_m n_{2t})^{\alpha_3}-w_t(n_{1t}+n_{2t})\}(1-\tau_k)-\frac{K_{t+1}}{Z_t}+\left(\frac{1}{1+r}\right)V(k_{t+1},Z_{t+1}).$$

The FOC with respect to K_{t+1} is as follows:

$$\{K_{t+1}\}:\frac{-1}{Z_t}+\left(\frac{1}{1+r}\right)\left[\frac{\alpha_1 Y_{t+1}(1-\tau_k)}{K_{t+1}}+\left(\frac{1}{1+r}\right)\left[-\frac{\partial}{\partial K_{t+1}}(\frac{K_{t+3}}{Z_{t+2}})-\left(\frac{1}{1+r}\right)\frac{\partial}{\partial K_{t+1}}(\frac{K_{t+4}}{Z_{t+3}})-..\right]\right]=0.$$

Note that

$$\frac{\partial Z_t}{\partial K_{t+1}}=\frac{\partial Z_{t+1}}{\partial K_{t+1}}=0,\ \frac{\partial Z_{t+2}}{\partial K_{t+1}}=(1-\gamma)(1-\mu)\frac{Z_{t+2}}{K_{t+1}},\ \frac{\partial Z_{t+3}}{\partial K_{t+1}}=\frac{\gamma Z_{t+3}}{Z_{t+2}}\frac{\partial Z_{t+2}}{\partial K_{t+1}}-\alpha_1(1-\gamma)\frac{Z_{t+3}}{K_{t+1}}.$$

We therefore have

$$\frac{\partial Z_{t+3}}{\partial K_{t+1}}=(1-\gamma)\frac{Z_{t+3}}{K_{t+1}}(\gamma(1-\mu)-\alpha_1).$$

And hence for any other future time period,

$$\frac{\partial Z_{t+3+j}}{\partial K_{t+1}}=\gamma^j(1-\gamma)\frac{Z_{t+3+j}}{K_{t+1}}[\gamma(1-\mu)-\alpha_1],\quad for j\geqq 0.$$

Therefore, substituting the above values in $\{K_{t+1}\}$, and since $\delta=1$, $I_{t+j}=\frac{K_{t+1+j}}{Z_{t+j}}$, for all $j\geqq 0$. This implies

$$\{K_{t+1}\}:\frac{1}{Z_t}=\left(\frac{1}{1+r}\right)\left[\begin{array}{c}\frac{\alpha_1 Y_{t+1}(1-\tau_k)}{K_{t+1}}+\left(\frac{1}{1+r}\right)(1-\gamma)(1-\mu)\left(\frac{I_{t+2}}{K_{t+1}}\right)\\ +\left(\frac{1}{1+r}\right)^2(1-\gamma)[\gamma(1-\mu)-\alpha_1]\sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j\frac{I_{t+j+3}}{K_{t+1}}\end{array}\right].$$

The FOC with respect to n_{1t} is as follows:

$$\{n_{1t}\}:\frac{\alpha_2 Y_t(1-\tau_k)}{n_{1t}}-w_t(1-\tau_k)+\left(\frac{1}{1+r}\right)\left[-\frac{\partial}{\partial n_{1t}}(\frac{K_{t+3}}{Z_{t+2}})-\left(\frac{1}{1+r}\right)\frac{\partial}{\partial n_{1t}}(\frac{K_{t+4}}{Z_{t+3}})-..\right]=0$$

Note that

$$\frac{\partial Z_t}{\partial n_{1t}}=\frac{\partial Z_{t+1}}{\partial n_{1t}}=0,\ \frac{\partial Z_{t+2}}{\partial n_{1t}}=-(1-\gamma)\alpha_2\frac{Z_{t+2}}{n_{1t}},\ \frac{\partial Z_{t+3}}{\partial n_{1t}}=\frac{\gamma Z_{t+3}}{Z_{t+2}}\frac{\partial Z_{t+2}}{\partial n_{1t}}\text{ and so on.}$$

Therefore, substituting the above expressions (and $\delta = 1$, $I_{t+j} = \frac{K_{t+1+j}}{Z_{t+j}}$, for all $j \geqq 0$) into the expression for $\{n_{1t}\}$ we get

$$\{n_{1t}\} : w_t(1-\tau_k) = \frac{\alpha_2 Y_t(1-\tau_k)}{n_{1t}} - \left(\frac{1}{1+r}\right)^2 \left[\frac{(1-\gamma)\alpha_2}{n_{1t}}\right] \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+r}\right)^j I_{t+j+2}.$$

The FOC with respect to n_{2t} is given by

$$\{n_{2t}\} : -w_t(1-\tau_k) + \frac{\alpha_3 Y_t(1-\tau_k)}{n_{2t}} + \left(\frac{1}{1+r}\right)\left[-\frac{\partial}{\partial n_{2t}}\left(\frac{K_{t+2}}{Z_{t+1}}\right) - \left(\frac{1}{1+r}\right)\frac{\partial}{\partial n_{2t}}\left(\frac{K_{t+3}}{Z_{t+2}}\right) - ..\right] = 0$$

Given

$$\begin{aligned}
\frac{\partial Z_t}{\partial n_{2t}} &= 0,\ \frac{\partial Z_{t+1}}{\partial n_{2t}} = \frac{\theta Z_{t+1}}{n_{2t}}; \\
\frac{\partial Z_{t+2}}{\partial n_{2t}} &= \frac{\gamma Z_{t+2}}{Z_{t+1}}\frac{\theta Z_{t+1}}{n_{2t}} - \alpha_3(1-\gamma)\frac{Z_{t+2}}{n_{2t}} \\
&= [\gamma\theta - \alpha_3(1-\gamma)]\frac{Z_{t+2}}{n_{2t}} \\
\text{and } \frac{\partial Z_{t+2+j}}{\partial n_{2t}} &= \gamma^j[\gamma\theta - \alpha_3(1-\gamma)]\frac{Z_{t+2+j}}{n_{2t}}, \forall j \geq 0 \text{ and so on.}
\end{aligned}$$

Substituting the above expressions into the above expression for $\{n_{2t}\}$, we get

$$\{n_{2t}\} : w_t(1-\tau_k) = \frac{\alpha_3 Y_t(1-\tau_k)}{n_{2t}} + \left(\frac{\theta}{1+r}\right)\frac{I_{t+1}}{n_{2t}} + \frac{[\gamma\theta - \alpha_3(1-\gamma)]}{(1+r)^2 n_{2t}} \sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j I_{t+j+2}.$$

The Agent's Problem

The agent is modelled as solving a borrowing-lending problem, as follows

$$Max \sum_{t=0}^{\infty} \beta^t \left[\log c_t + \log(1-n_t\right]$$

subject to

$$a_{t+1} = (1+r)a_t + w_t(n_{1t} + n_{2t})(1-\tau_n) - c_t.$$

The following are the FOCs

$$
\begin{aligned}
\{c_t\} &: \frac{1}{c_t} = \lambda_t, \text{ where } \lambda_t \text{ is the Lagrangian multiplier} \\
\{a_{t+1}\} &: \frac{\beta(1+r)}{c_{t+1}} = c_t \\
\{n_{1t}\} &: \frac{w_t(1-\tau_n)}{c_t} = \frac{1}{1-n_t} \\
\{n_{2t}\} &: \frac{w_t(1-\tau_n)}{c_t} = \frac{1}{1-n_t}
\end{aligned}
$$

In equilibrium,

$$a_t = K_t. \ \forall t$$

Now, the firm's FOC $\{K_{t+1}\}:$

$$\{K_{t+1}\} : \frac{1}{Z_t} = \left(\frac{1}{1+r}\right)\left[\begin{array}{c} \frac{\alpha_1 Y_{t+1}(1-\tau_k)}{K_{t+1}} + \left(\frac{1}{1+r}\right)(1-\gamma)(1-\mu)\left(\frac{I_{t+2}}{K_{t+1}}\right) \\ +\left(\frac{1}{1+r}\right)^2 (1-\gamma)[\gamma(1-\mu)-\alpha_1]\sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j \frac{I_{t+j+3}}{K_{t+1}} \end{array}\right].$$

Substituting for $(1+r)$ from $\{a_{t+1}\}$

$$
\begin{aligned}
\frac{1}{Z_t} = {} & \frac{\beta c_t}{c_{t+1}}\left[\frac{\alpha_1 Y_{t+1}(1-\tau_k)}{K_{t+1}}\right] + \frac{\beta c_t}{c_{t+1}}\frac{\beta c_{t+1}}{c_{t+2}}(1-\gamma)(1-\mu)\left(\frac{I_{t+2}}{K_{t+1}}\right) \\
& + \frac{\beta c_t}{c_{t+1}}\frac{\beta c_{t+1}}{c_{t+2}}\beta c_{t+2}\frac{(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{c_{t+j+3}}.
\end{aligned}
$$

This implies

$$
\begin{aligned}
\{K_{t+1}\} : \frac{1}{c_t Z_t} = {} & \frac{\alpha_1 \beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} + \frac{\beta^2(1-\gamma)(1-\mu)}{K_{t+1}}\left(\frac{I_{t+2}}{c_{t+2}}\right) \\
& + \frac{\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{c_{t+j+3}}.
\end{aligned}
$$

The firm's FOC $\{n_{1t}\}:$

$$\{n_{1t}\} : w_t(1-\tau_k) = \frac{\alpha_2 Y_t(1-\tau_k)}{n_{1t}} - \left(\frac{1}{1+r}\right)^2\left[\frac{(1-\gamma)\alpha_2}{n_{1t}}\right]\sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j I_{t+j+2}.$$

Substituting for the FOC $\{n_{1t}\}$ from the agent's problem, we get

$$\{n_{1t}\} : \frac{1}{1-n_t} = \frac{\alpha_2 Y_t(1-\tau_n)}{c_t n_{1t}} - \beta^2\left[\frac{\alpha_2(1-\gamma)(1-\tau_n)}{n_{1t}(1-\tau_k)}\right]\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{c_{t+j+2}}.$$

Likewise,

$$\{n_{2t}\}: \frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau_n)}{c_t n_{2t}} + \left(\frac{\beta\theta}{n_{2t}}\right)\frac{I_{t+1}}{c_{t+1}}\frac{(1-\tau_n)}{(1-\tau_k)} + \frac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)]}{n_{2t}}\frac{(1-\tau_n)}{(1-\tau_k)}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{c_{t+j+2}}.$$

Hence we can summarize the required FOCs as follows

$$\begin{aligned}
\{K_{t+1}\} &: \frac{1}{c_t Z_t} = \frac{\alpha_1\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} + \frac{\beta^2(1-\gamma)(1-\mu)}{K_{t+1}}\left(\frac{I_{t+2}}{c_{t+2}}\right) \\
&\quad + \frac{\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{c_{t+j+3}} \\
\{n_{1t}\} &: \frac{1}{1-n_t} = \frac{\alpha_2 Y_t(1-\tau_n)}{c_t n_{1t}} - \beta^2\left[\frac{\alpha_2(1-\gamma)(1-\tau_n)}{n_{1t}(1-\tau_k)}\right]\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{c_{t+j+2}} \\
\{n_{2t}\} &: \frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau_n)}{c_t n_{2t}} + \left(\frac{\beta\theta}{n_{2t}}\right)\frac{I_{t+1}}{c_{t+1}}\frac{(1-\tau_n)}{(1-\tau_k)} + \frac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)]}{n_{2t}}\frac{(1-\tau_n)}{(1-\tau_k)}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{c_{t+j+2}}.
\end{aligned}$$

It is easy to see that $\tau_n = \tau_k = \tau$ gives us the planner's version.

The Decision Rules

We will now derive the decision rules from the above FOCs. The following are the decision rules which we will derive using the method of undetermined coefficients.

$$\begin{aligned}
c_t &= \Phi AY_t, \text{ where } AY_t\text{, is the after tax income accruing to agents} \\
i_t &= (1-\Phi)AY_t \\
n_{1t} &= x n_t \\
n_{2t} &= (1-x)n_t \\
n_t &= \overline{n} \text{ that is, labor supply turns out to be constant}
\end{aligned}$$

We will also consider the following

$$\{Y_t - w_t(n_{1t}+n_{2t})\}(1-\tau_k) + w_t(n_{1t}+n_{2t})(1-\tau_n) = AY_t.$$

Substituting for w_t from the firm's FOCs $\{n_{1t}\}$ and $\{n_{2t}\}$,

$$\left[Y_t - \alpha_2 Y_t - \alpha_3 Y_t + \tfrac{\alpha_2(1-\gamma)\beta^2\Phi AY_t}{(1-\tau_k)(1-\beta\gamma)}\left(\tfrac{1-\Phi}{\Phi}\right) - \tfrac{\beta\theta\Phi AY_t}{(1-\tau_k)}\left(\tfrac{1-\Phi}{\Phi}\right) - \tfrac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)]\Phi AY_t}{(1-\tau_k)(1-\beta\gamma)}\left(\tfrac{1-\Phi}{\Phi}\right)\right](1-\tau_k)$$
$$+\left[\alpha_2 Y_t + \alpha_3 Y_t - \tfrac{\alpha_2(1-\gamma)\beta^2\Phi AY_t}{(1-\tau_k)(1-\beta\gamma)}\left(\tfrac{1-\Phi}{\Phi}\right) + \tfrac{\beta\theta\Phi AY_t}{(1-\tau_k)}\left(\tfrac{1-\Phi}{\Phi}\right) + \tfrac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)]\Phi AY_t}{(1-\tau_k)(1-\beta\gamma)}\left(\tfrac{1-\Phi}{\Phi}\right)\right](1-\tau_n) = AY_t$$

$$\Rightarrow (1-\tau_k)Y_t\left[\alpha_1 + \tfrac{\alpha_2(1-\gamma)\beta^2 A(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)} - \tfrac{\beta\theta A(1-\Phi)}{(1-\tau_k)} - \tfrac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)](1-\Phi)A}{(1-\tau_k)(1-\beta\gamma)}\right]$$
$$+\left[(1-\alpha_1) - \tfrac{\alpha_2(1-\gamma)\beta^2 A(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)} + \tfrac{\beta\theta A(1-\Phi)}{(1-\tau_k)} + \tfrac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)](1-\Phi)A}{(1-\tau_k)(1-\beta\gamma)}\right]Y_t(1-\tau_n) = AY_t.$$

This gives us

$$Y_t\left[\begin{array}{c} \alpha_1(1-\tau_k) + (1-\alpha_1)(1-\tau_n) + \frac{\alpha_2(1-\gamma)\beta^2 A(1-\Phi)(\tau_n-\tau_k)}{(1-\tau_k)(1-\beta\gamma)} - \frac{\beta\theta A(1-\Phi)(\tau_n-\tau_k)}{(1-\tau_k)} \\ -\frac{\beta^2[\gamma\theta-\alpha_3(1-\gamma)](1-\Phi)A(\tau_n-\tau_k)}{(1-\tau_k)(1-\beta\gamma)} \end{array}\right] = AY_t,$$

which means

$$A = \left[\alpha_1(1-\tau_k) + (1-\alpha_1)(1-\tau_n) + \frac{(\tau_n-\tau_k)\beta A(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}\{(1-\alpha_1)(1-\gamma)\beta - \theta\}\right].$$

We will now derive decision rules for consumption and investment. From the FOC of $\{K_{t+1}\}$

$$\begin{aligned} \{K_{t+1}\} \;:\; \frac{1}{c_t Z_t} &= \frac{\alpha_1\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} + \frac{\beta^2(1-\gamma)(1-\mu)}{K_{t+1}}\left(\frac{I_{t+2}}{c_{t+2}}\right) \\ &+\frac{\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{c_{t+j+3}}. \end{aligned}$$

This implies,

$$\begin{aligned} \frac{1}{\Phi AY_tZ_t} &= \frac{\alpha_1\beta Y_{t+1}(1-\tau_k)}{\Phi AY_{t+1}(1-\Phi)AY_tZ_t} + \frac{\beta^2(1-\gamma)(1-\mu)}{(1-\Phi)AY_tZ_t}\left(\frac{1-\Phi}{\Phi}\right) \\ &+\frac{\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha_1]}{(1-\Phi)AY_tZ_t(1-\beta\gamma)}\left(\frac{1-\Phi}{\Phi}\right). \end{aligned}$$

This gives us

$$\Rightarrow (1-\Phi) = \frac{\alpha_1\beta(1-\beta\gamma)(1-\tau_k)}{A[(1-\beta\gamma) - \beta^2(1-\gamma)(1-\mu) + \alpha_1\beta^3(1-\gamma)]}. \tag{3}$$

And hence expressions for consumption and investments follow. Substituting this back into the expression for A,

$$\begin{aligned} A &= \alpha_1(1-\tau_k) + (1-\alpha_1)(1-\tau_n) + \frac{(\tau_n-\tau_k)\beta A(1-\Phi)}{(1-\tau_k)(1-\beta\gamma)}\{(1-\alpha_1)(1-\gamma)\beta-\theta\} \\ &= \alpha_1(1-\tau_k) + (1-\alpha_1)(1-\tau_n) + \frac{\alpha_1\beta^2(1-\beta\gamma)(1-\tau_k)(\tau_n-\tau_k)[(1-\alpha_1)(1-\gamma)\beta-\theta]}{(1-\tau_k)(1-\beta\gamma)[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)+\alpha_1\beta^3(1-\gamma)]} \\ &= \alpha_1(1-\tau_k) + (1-\alpha_1)(1-\tau_n) + \frac{\alpha_1\beta^2(\tau_n-\tau_k)[(1-\alpha_1)(1-\gamma)\beta-\theta]}{[(1-\beta\gamma)-\beta^2(1-\gamma)(1-\mu)+\alpha_1\beta^3(1-\gamma)]}. \end{aligned}$$

Clearly, when $\tau_n = \tau_k = \tau$

$$\begin{aligned} A &= [\alpha_1(1-\tau_k) + (1-\alpha_1)(1-\tau_n)] \\ &= (1-\tau). \end{aligned}$$

We will now derive decision rules for labor supply and the way it is distributed between n_1 and n_2. Using the following FOCs $\{n_{1t}\}$ and $\{n_{2t}\}$

$$\begin{aligned} \{n_{1t}\} &: \frac{1}{1-n_t} = \frac{\alpha_2 Y_t(1-\tau_n)}{c_t n_{1t}} - \beta^2 \left[\frac{\alpha_2(1-\gamma)(1-\tau_n)}{n_{1t}(1-\tau_k)}\right] \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{c_{t+j+2}} \\ \{n_{2t}\} &: \frac{1}{1-n_t} = \frac{\alpha_3 Y_t(1-\tau_n)}{c_t n_{2t}} + \left(\frac{\beta\theta}{n_{2t}}\right) \frac{I_{t+1}}{c_{t+1}} \frac{(1-\tau_n)}{(1-\tau_k)} + \frac{\beta^2[\gamma\theta - \alpha_3(1-\gamma)]}{n_{2t}} \frac{(1-\tau_n)}{(1-\tau_k)} \sum_{j=0}^{\infty} \beta^j \gamma^j \frac{I_{t+j+2}}{c_{t+j+2}} \end{aligned}$$

and the definitions

$$\begin{aligned} n_{1t} &= x n_t \\ n_{2t} &= (1-x) n_t \\ n_t &= \overline{n} \text{ that is, labor supply turns out to be constant} \end{aligned}$$

and the decision rules for consumption and investments, we get

$$\begin{aligned} \{n_{1t}\} \quad : \quad \frac{x\overline{n}}{1-\overline{n}} &= \frac{\alpha_2 Y_t(1-\tau_n)}{\Phi A Y_t} - \beta^2 \left[\frac{\alpha_2(1-\gamma)(1-\tau_n)}{(1-\tau_k)(1-\beta\gamma)}\right] \left(\frac{1-\Phi}{\Phi}\right) \\ \Rightarrow \quad \frac{x\overline{n}}{1-\overline{n}} &= \frac{\alpha_2(1-\tau_n)}{\Phi A} - \beta^2 \left[\frac{\alpha_2(1-\gamma)(1-\tau_n)}{(1-\tau_k)(1-\beta\gamma)}\right] \left(\frac{1-\Phi}{\Phi}\right) \\ \Rightarrow \quad \frac{x\overline{n}}{1-\overline{n}} &= \frac{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\tau_n)A(1-\Phi)}{\Phi A(1-\tau_k)(1-\beta\gamma)} \\ \Rightarrow \quad \frac{\overline{n}}{1-\overline{n}} &= \frac{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\tau_n)A(1-\Phi)}{x\Phi A(1-\tau_k)(1-\beta\gamma)} \end{aligned}$$

This implies

$$\Rightarrow \overline{n} = \frac{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\tau_n)(1-\Phi)A}{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma) - \beta^2\alpha_2(1-\gamma)(1-\tau_n)(1-\Phi)A + x\Phi A(1-\tau_k)(1-\beta\gamma)}. \tag{4}$$

From the FOC $\{n_{2t}\}$

$$\{n_{2t}\} : \frac{(1-x)\overline{n}}{1-\overline{n}} = \frac{\alpha_3 Y_t(1-\tau_n)}{\Phi A Y_t} + \beta\theta \left(\frac{1-\Phi}{\Phi}\right) \frac{(1-\tau_n)}{(1-\tau_k)} + \frac{\beta^2[\gamma\theta - \alpha_3(1-\gamma)](1-\tau_n)}{(1-\tau_k)(1-\beta\gamma)} \left(\frac{1-\Phi}{\Phi}\right)$$

$$\Rightarrow \quad \frac{(1-x)[\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\tau_n)A(1-\Phi)]}{x\Phi A(1-\tau_k)(1-\beta\gamma)}$$
$$= \quad \frac{\alpha_3(1-\tau_n)(1-\tau_k)(1-\beta\gamma)+\beta\theta(1-\Phi)(1-\tau_n)A(1-\beta\gamma)+A\beta^2[\gamma\theta-\alpha_3(1-\gamma)](1-\Phi)(1-\tau_n)}{\Phi A(1-\tau_k)(1-\beta\gamma)}.$$

This implies
$\Rightarrow \frac{1-x}{x} = \frac{\alpha_3(1-\tau_n)(1-\tau_k)(1-\beta\gamma)+\beta\theta(1-\Phi)(1-\tau_n)A(1-\beta\gamma)+A\beta^2[\gamma\theta-\alpha_3(1-\gamma)](1-\Phi)(1-\tau_n)}{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\tau_n)A(1-\Phi)}$
which gives us
$\Rightarrow x = \frac{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\tau_n)A(1-\Phi)}{(\alpha_2+\alpha_3)[(1-\tau_n)(1-\tau_k)(1-\beta\gamma)-\beta^2(1-\gamma)(1-\tau_n)A(1-\Phi)]+\beta\theta(1-\tau_n)(1-\Phi)A(1-\beta\gamma)+A\beta^2\gamma\theta(1-\tau_n)(1-\Phi)}$.
This implies

$$x = \frac{\alpha_2(1-\tau_n)(1-\tau_k)(1-\beta\gamma)-\beta^2\alpha_2(1-\gamma)(1-\tau_n)A(1-\Phi)}{(\alpha_2+\alpha_3)[(1-\tau_n)(1-\tau_k)(1-\beta\gamma)-\beta^2(1-\gamma)(1-\tau_n)A(1-\Phi)]+\beta\theta(1-\tau_n)(1-\Phi)A}. \tag{5}$$

The expressions for n_1 and n_2 follow.

Appendix K: The Model without $\mathbf{n_2}$ in production

The Firm's Problem

The firm's value function

$$\underset{K_{t+1},n_{1t},n_{2t}}{Max} \ V(k_t,Z_t) = \{AK_t^{\alpha}n_{1t}^{1-\alpha} - w_t(n_{1t}+n_{2t})\}(1-\tau_k) - \frac{K_{t+1}}{Z_t} + \left(\frac{1}{1+r}\right)V(k_{t+1},Z_{t+1})$$

where,

$$Z_{t+1} = BZ_t^{\gamma}n_{2t}^{\theta}\left[\left(\frac{G_t}{Y_{t-1}}\right)^{\mu}\left(\frac{K_t}{Y_{t-1}}\right)^{1-\mu}\right]^{1-\gamma}$$

The final expressions for the FOCs are as follows:

$$\{K_{t+1}\}: \frac{1}{Z_t} = \left(\frac{1}{1+r}\right)\left[\begin{array}{c} \frac{\alpha Y_{t+1}(1-\tau_k)}{K_{t+1}} + \left(\frac{1}{1+r}\right)(1-\gamma)(1-\mu)\left(\frac{I_{t+2}}{K_{t+1}}\right) \\ +\left(\frac{1}{1+r}\right)^2(1-\gamma)[\gamma(1-\mu)-\alpha]\sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j\frac{I_{t+j+3}}{K_{t+1}} \end{array}\right].$$

$$\{n_{1t}\}: w_t(1-\tau_k) = \frac{(1-\alpha)Y_t(1-\tau_k)}{n_{1t}} - \left(\frac{1}{1+r}\right)^2\left[\frac{(1-\gamma)(1-\alpha)}{n_{1t}}\right]\sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j I_{t+j+2}.$$

$$\{n_{2t}\} : w_t(1-\tau_k) = \left(\frac{\theta}{1+r}\right)\left(\frac{1}{n_{2t}}\right)\sum_{j=0}^{\infty}\left(\frac{\gamma}{1+r}\right)^j I_{t+j+1}.$$

The Agent's Problem

The agent is modelled as solving a borrowing-lending problem similar to Appendix J:

$$Max\sum_{t=0}^{\infty}\beta^t\left[\log c_t + \log(1-n_t)\right]$$

subject to

$$a_{t+1} = (1+r)a_t + w_t(n_{1t}+n_{2t})(1-\tau_n) - c_t.$$

Solving the FOCs the same way as done in Appendix J, we can summarize them as follows

$$\begin{aligned}
\{K_{t+1}\} \quad &: \quad \frac{1}{c_t Z_t} = \frac{\alpha\beta Y_{t+1}(1-\tau_k)}{c_{t+1}K_{t+1}} + \frac{\beta^2(1-\gamma)(1-\mu)}{K_{t+1}}\left(\frac{I_{t+2}}{c_{t+2}}\right) \\
&\quad + \frac{\beta^3(1-\gamma)[\gamma(1-\mu)-\alpha]}{K_{t+1}}\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+3}}{c_{t+j+3}} \\
\{n_{1t}\} \quad &: \quad \frac{1}{1-n_t} = \frac{(1-\alpha)Y_t(1-\tau_n)}{n_{1t}} - \beta^2\left[\frac{(1-\gamma)(1-\alpha)(1-\tau_n)}{n_{1t}(1-\tau_k)}\right]\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{c_{t+j+2}} \\
\{n_{2t}\} \quad &: \quad \frac{1}{1-n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1-\tau_n}{1-\tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}}.
\end{aligned}$$

It is again easy to see that $\tau_n = \tau_k = \tau$ gives us the planner's version.

The Decision Rules

We will now derive the decision rules from the above FOCs. The following are the decision rules which we will derive using the method of undetermined coefficients.

$$\begin{aligned}
c_t &= \Phi AY_t, \text{ where } AY_t, \text{ is the after tax income accruing to agents} \\
i_t &= (1-\Phi)AY_t \\
n_{1t} &= xn_t \\
n_{2t} &= (1-x)n_t \\
n_t &= \overline{n} \text{ that is, labor supply turns out to be constant.}
\end{aligned}$$

We will also consider the following

$$\{Y_t - w_t(n_{1t} + n_{2t})\}(1 - \tau_k) + w_t(n_{1t} + n_{2t})(1 - \tau_n) = AY_t.$$

Solving for the above decision rules, we get

$$(1 - \Phi) = \frac{\alpha\beta(1 - \beta\gamma)(1 - \tau_k)}{A[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \mu) + \alpha\beta^3(1 - \gamma)]}.$$

$$\begin{aligned}
A &= \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{(\tau_n - \tau_k)}{(1 - \tau_k)(1 - \beta\gamma)}\beta A\,(1 - \Phi)\,[(1 - \gamma)\beta - \theta] \\
&= \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{\alpha\beta^2(1 - \beta\gamma)(1 - \tau_k)(\tau_n - \tau_k)[(1 - \gamma)\beta - \theta]}{(1 - \tau_k)(1 - \beta\gamma)[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \mu) + \alpha\beta^3(1 - \gamma)]} \\
&= \alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n) + \frac{\alpha\beta^2(\tau_n - \tau_k)[(1 - \gamma)\beta - \theta]}{[(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \mu) + \alpha\beta^3(1 - \gamma)]}
\end{aligned}$$

Clearly, when $\tau_n = \tau_k = \tau$

$$\begin{aligned}
A &= [\alpha(1 - \tau_k) + (1 - \alpha)(1 - \tau_n)] \\
&= 1 - \tau.
\end{aligned}$$

The decision rules for total labor supply and the way it is distributed between n_1 and n_2 can be derived from the following FOCs,

$$\begin{aligned}
\{n_{1t}\} &: \frac{1}{1 - n_t} = \frac{(1 - \alpha)Y_t(1 - \tau_n)}{c_t n_{1t}} - \beta^2\left[\frac{(1 - \gamma)(1 - \alpha)(1 - \tau_n)}{n_{1t}(1 - \tau_k)}\right]\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+2}}{c_{t+j+2}} \\
\{n_{2t}\} &: \frac{1}{1 - n_t} = \left(\frac{\beta\theta}{n_{2t}}\right)\left(\frac{1 - \tau_n}{1 - \tau_k}\right)\sum_{j=0}^{\infty}\beta^j\gamma^j\frac{I_{t+j+1}}{c_{t+j+1}},
\end{aligned}$$

with the following definitions

$$\begin{aligned}
n_{1t} &= x n_t \\
n_{2t} &= (1 - x)n_t \\
n_t &= \overline{n} \text{ that is, labor supply turns out to be constant}
\end{aligned}$$

as follows

$$\overline{n} = \frac{(1 - \alpha)(1 - \tau_n)(1 - \tau_k)(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \alpha)(1 - \tau_n)(1 - \Phi)A}{(1 - \alpha)(1 - \tau_n)(1 - \tau_k)(1 - \beta\gamma) - \beta^2(1 - \gamma)(1 - \alpha)(1 - \tau_n)(1 - \Phi)A + x\Phi A(1 - \tau_k)(1 - \beta\gamma)}.$$

The expression for x is given by

$$x = \frac{[(1-\alpha)(1-\tau_k)(1-\beta\gamma) - \beta^2(1-\gamma)(1-\alpha)(1-\Phi)A]}{[(1-\alpha)(1-\tau_k)(1-\beta\gamma) - \beta^2(1-\gamma)(1-\alpha)(1-\Phi)A] + \beta\theta(1-\Phi)A}.$$