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by

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Productive Relations in the Northeast and the Rest of Brazil Regions in 1995: Decomposition & Synergy in Input-Output Systems

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Abstract: Using a set of interregional input-output tables built by Guilhoto (1998) for 1995 for two Brazilian regions (Northeast and rest of the economy), the methodology developed by Sonis *et al*. (1997) is applied in the construction of a series of linkages such that it is possible to examine, through the nature of the internal and external interdependencies, the structure of trading relationships between the two regions. The methodology uses a partitioned input-output system and exploits techniques that produce left and right matrix multipliers of the Leontief Inverse. This procedure facilitates the classification of the types of synergetic interactions within a preset pair-wise hierarchy of economic linkages sub-systems. In general, the results show that the Northeast region has a greater dependence on the rest of the economy region than the rest of the economy has on the Northeast region, and at the same time the rest of the economy region seems to be more developed as it presents a more complex productive structure than the Northeast region.

I. Introduction

In this paper, the methodology developed by Sonis *et al*. (1997) that classifies types of synergetic interactions is used to explore the structure of trading relations among regions. This methodology is applied to a set of interregional input-output tables built by Guilhoto (1998) for two Brazilian regions (Northeast and rest of the economy). The objective is to explore the degree to which the structure of interactions is dominated by intraregional and interregional components and the extent to which the interregional interactions are symmetric in magnitude. The two-region system that has been chosen highlights important, strategic development issues in an economy that is struggling to address both equity and efficiency issues in a spatial context (see Baer, *et al.,* 1998). The Northeast of Brazil has received significant, continuing development initiatives over the past four decades; by 1995, the Northeast's share in GDP had risen to 13.4% from 13.2% in 1960 while per capita GDP grew from 42% to 55% of the national average. When attention is just directed to shares in industrial production, the Northeast declined from 8.3% (1959) to 7.9% (1994). The present paper attempts to explore some

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structural reasons that might shed light on this problem; while the focus will be on the economic structure of the Northeast and the Rest of Brazil (hereafter, NE and RB respectively) at one point in time, 1995, the findings will reflect long-term structural issues that have remained unresolved.

In the next section the theoretical background will be presented. In the third section the theory will be applied to the Brazilian interregional tables, while in the fourth section policy interpretations will be reviewed prior to the presentation of some concluding comments in the final section.

II. Theoretical Background[4]

Consider a two-region, mutually exclusive division of a national economy. Following the adaptation of the Dixit-Stiglitz model by Fujita *et al*., (1999) assume that there are two goods, a tradable and a nontradable, and that there are no external to the national economy interactions. Further assume that labor employed in the tradable commodity is mobile between regions and that labor moves to regions paying higher than average real wages. Given a transportation costs structure in which costs are assumed to be a linear increasing function of distance, then it can be shown that the equilibrium distribution of production will depend in large part on the magnitude of the transportation costs and their interaction with increasing returns at the firm level and labor mobility. Fujita *et al*. (1999) show that with high transportation costs there will be a tendency for production to be divided between the two regions; if labor mobility is limited (by higher transportation or search costs), and the transportation costs are reduced, there is a tendency to develop a core-periphery outcome in which the tradable good becomes concentrated in one of the two regions.

Obviously, with a more complex system in which goods are all tradable to some extent, the search for greater variety by consumers may tend to exacerbate concentration tendencies, tendencies that will be reinforced by the existence of increasing returns. The competition

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[4] This section draws heavily on Sonis, Hewings, and Miyazawa (1997).

between the NE and RB presents a very strikingly familiar scenario. Transportation costs between NE and RB are high but not high enough to create a protective, spatially monopolistic market in the NE; producers in the RB have been able to exploit scale economies and penetrate the NE market to the exclusion of NE producers. In this paper, the resultant interregional structure will be explored and interpreted using a set of input-output tables.

Consider an input-output system represented by the following block matrix, A, of direct inputs:

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{1}$$

where A_{11} and A_{22} are the quadrat matrices of direct inputs within the first and second regions, and A_{12} and A_{21} are the rectangular matrices showing the direct inputs purchased by the second region and vice versa.

The building blocks of the pair-wise hierarchies of sub-systems of intra/interregional linkages of the block-matrix Input-Output system are the four matrices $A_{11,}A_{12}, A_{21}$ and A_{22}, corresponding to four basic block-matrices:

$$A_{11} = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}; \quad A_{12} = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}; \quad A_{21} = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}; \quad A_{22} = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix} \tag{2}$$

This paper will usually consider the decomposition of the block-matrix (1) into the sum of two block-matrices, such that each of them is the sum of the block-matrices (2) $A_{11,}A_{12}, A_{21}$ and A_{22}. From (1), 14 types of pair-wise hierarchies of economic sub-systems can be identified by the decompositions of the matrix of the block-matrix A (see Figure 1 and Table A2 in the appendix).

A set of inner regional multipliers, the set of inverse matrices that are the "building blocks" of the synergetic interactions between the economic sub-systems are presented in table A1 in the appendix. Hereafter, some comments are provided on the entries in this table (the bold numbering refers to the corresponding entries in this table).

1. The matrices $B_1 = (I - A_{11})^{-1}$ and $B_2 = (I - A_{22})^{-1}$ represent the Miyazawa internal matrix multipliers of the first and second regions showing the interindustrial propagation effects within each region, while the matrices, $A_{21}B_1$, $B_1 A_{12}$, $A_{12}B_2$, $B_2 A_{21}$ show the induced effects on output or input activities in the two regions.

2. The expressions

$$S_1 = I - A_{11} - A_{12}B_2A_{21}, \quad S_2 = I - A_{22} - A_{21}B_1 A_{12} \tag{3}$$

are usually referred to as the Schur complements. The inverses, D_1 and D_2 of the Schur complements (3) are referred to as the *Schur inverses* for the first and second regions. They represent the enlarged Leontief inverse for one region revealing the induced economic influence of the other region; i.e., the Schur inverses represent total propagation effects in the first and second regions.

3. Miyazawa (1966) introduced the concept of left and right external matrix multipliers of the first and second regions, $D_{11}^L, D_{11}^R, D_{22}^L, D_{22}^R$. These multipliers are incorporated in the multiplicative decompositions of the Schur inverses and they represent the total propagation effects in the first and second regions as the products of internal and external regional matrix multipliers.

4, 5. By introducing the abbreviated Schur inverses, D_{11}, D_{22}, and the left and right induced internal multipliers for the first and second regions, $B_1^L, B_1^R, B_2^L, B_2^R$, one can obtain the multiplicative decompositions of the Schur inverses:

$$D_1 = B_1^L D_{11} = D_{11} B_1^R; \quad D_2 = B_2^L D_{22} = D_{22} B_2^R \tag{4}$$

and their corresponding additive representations.

6-10. The formulae for this group of multipliers can be obtained by considering the block-matrices:

$$M=\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix},\ N=\begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ S=\begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix} \tag{5}$$

Those represent the backward and forward linkages of the first region, the second region and the interregional relations of both regions.

The following Schur inverse

$$D_1^{*}=(I-A_{11}-A_{12}A_{21})^{-1} \tag{6}$$

may be referred to as the enlarged Leontief inverse, and the inverses

$$D_{11}^{*L}=(I-B_1\, A_{12}A_{21})^{-1};\quad D_{11}^{*R}=(I-A_{12}A_{21}B_1\,)^{-1} \tag{7}$$

are called the left and right subjoined inverse matrix multipliers.

Consider the hierarchy of Input-Output sub-systems represented by the decomposition $A=A_1+A_2$. Introducing the Leontief block-inverse $L(A)=L=(I-A)^{-1}$ and the Leontief block-inverse $L(A_1)=L_1=(I-A_1)^{-1}$ corresponding to the first sub-system, the outer left and right block-matrix multipliers M_L and M_R are defined by equalities:

$$L=L_1M_R=M_LL_1 \tag{8}$$

The definition (8) implies that:

$$M_L=L(I-A_1)=(I-L_1A_2)^{-1} \tag{9}$$

$$M_R=(I-A_1)\,L=(I-A_2L_1)^{-1} \tag{10}$$

In this paper, the following form of the Leontief block-inverse will be used:

$$L=\begin{bmatrix} D_1 & D_1\,A_{12}B_2 \\ D_2A_{21}B_1 & D_2 \end{bmatrix} \tag{11}$$

This expression can be verified by direct matrix multiplication, using definitions of the Schur inverses and their properties (see table A1, entries **1** and **2**). Further, the application of (9), (10) and (11) will be directed towards the derivation of a taxonomy of synergetic interactions between the two regions. The results are presented in the first and second levels of table A2, while figure 1 shows the schematic representation of the possible forms of the A_1 matrices.

<insert figure 1 here>

Consider the hierarchy of input-output sub-systems represented by the decomposition $A = A_1 + A_2$ and their Leontief block-inverse $L(A) = L = (I - A)^{-1}$ and the Leontief block-inverse $L(A_1) = L_1 = (I - A_1)^{-1}$ corresponding to the first sub-system. The multiplicative decomposition of the Leontief inverse $L = L_1 M_R = M_L L_1$ can be converted to the sum:

$$L = L_1 + (M_L - I)\, L_1 = L_1 + L_1 (M_R - I) \tag{12}$$

If f is the vector of final demand and x is the vector of gross output, then the decomposition (12) generates the decomposition of gross output into two parts: $x_1 = L_1 f$ and the increment $Dx = x - x_1$. Such decomposition is important for the empirical analysis of the structure of actual gross output. In the second level of table A2, the classification is revealed of possible additive decompositions of the Leontief block-inverse for all decompositions of input-output system into pair-vise hierarchies.

While 14 types of pair-wise hierarchies of economic linkages have been developed (figure 1 and table A2), it is possible to suggest a typology of categories into which these types may be placed. The following characterization is suggested:

1. backward linkage type (VI, IX): power of dispersion

2. forward linkage type (V, X): sensitivity of dispersion

3. intra- and inter- linkages type (VII, VIII): internal and external dispersion

4. isolated region vs. the rest of the economy interactions style (I, XIV, IV, XI)

5. triangular sub-system vs. the interregional interactions style (II, XIII, III, XII).

By viewing the system of hierarchies of linkages in this fashion, it will be possible to provide new insights into the properties of the structures that are revealed. For example, the types allocated to category 5 reflect structures that are based on order and circulation. Furthermore, these partitioned input-output systems can distinguish among the various types of dispersion (such as 1, 2 and 3) and among the various patterns of interregional interactions (such as 4 and 5). Essentially, the 5 categories and 14 types of pair-wise hierarchies of economic linkages provide the opportunity to select according to the special qualities of each region's activities and for the type of problem at hand; in essence, the option exists for the basis of a typology of economy types based on hierarchical structure.

III. An Application to Brazil

Using a set of interregional input-output tables built by Guilhoto (1998) at the level of 40 sectors for the year of 1995 for 2 Brazilian regions (NE - Region 1 - and the rest of the economy - Region 2), the methodology presented in section II is applied, and the results are presented in tables 3 to 5 and figures 2 to 4.

<<insert table 1 here>>

Table 1 illustrates the results taking into consideration the vector f of final demand and the vector x of gross output; then the gross output is decomposed into two parts: $x_1 = L_1 f$ and the increment $Dx = x - x_1$. The values for x and x_1 are added for all sectors in regions 1 and 2 such that it is possible to estimate the contribution of each interaction to the total production in each region. As the shares of x_1 in x take also into consideration the value of the final demand, it is interesting to isolate the shares of the final demand in each region to reveal how the pair-wise interaction takes place in the regions.

Focusing on the results presented into table 1, one can see that the value of the final demand in region 1 (NE) is responsible for 63.46 % of the production in this region (the remaining 36.54% is generated in the process of production) while for region 2 (RB), this value is 60.25 % (39.75 % in the process of production). In a certain sense this is an indication that the rest of the economy is more developed than the NE region as the internal transactions in region 2 are responsible for a greater share of the total production than is the case in region 1.

<<insert figure 2 here>>

In figure 2, it is possible to see how intermediation in each region contributes to total production. For the NE region, of the 36.54% share of total production accounted for by intermediate demand, 66.03% of it is the result of intraregional demand, while 11.01% is the result of the NE region selling to the RB. Starting from the isolated regions (block matrices) and then adding the interactions among them it is possible to measure how each interaction adds to the total production. These results are presented in table 2 and figure 3 for the NE and in table 3 and figure 4 for RB.

<<insert tables 2, 3 and figures 3, 4 here>>

Excluding final demand, the following summaries may be provided:

Case I (A_{11}): when the NE region isolated, this value shows how much of the internal production is due to relations only inside the region; in this case, the value is 24.13%, which represents 66.03% of the **p**roduction in the **p**roductive **p**rocess (ppp);

Case II (A_{12}): the purchases made by the industries in the RB region from the NE region generate 4.02% of the production in the NE region, 11.01% of the ppp, and by itself without having any interaction with the other block matrices generates no further production in the RB;

Case III (A_{21}): the purchases made by the industries in the NE region from the RB region generate 0.39% of the production in the RB region, 0.99% of the ppp, and by itself without having any interaction with the other block matrices generates no further production in the NE region;

Case IV (A_{22}): when the RB region is isolated, this value shows how much of the internal production is due to the relations only inside the region and in this case it is 38.65%, 97.24 % of the ppp;

Case V (A_{11} and A_{12}): from the sales of production that the industries in the NE region sell to the production process of both regions, generated a gross value of 30.30% of the production is generated in the NE region, adding 5.88% to the ppp, and 0.00 % in the RB region, as there is no feedback among the regions;

Case VI (A_{11} and A_{21}): from the interactions of the inputs that the industries in the NE region buy from both regions, a gross value of 24.13% of the production in this region is generated, which means a no addition to the ppp, and 0.51% of the production in the RB region, adding 0.30% to the ppp of this region;

Case VII (A_{11} and A_{22}): when both regions are isolated, with no transactions between them, this values shows how much of the internal production is due to the relations only inside each region and in this case they contribute a gross value of 24.13% for the NE region and a gross value of 38.65% for the RB region, with no addition to the ppp of both regions;

Case VIII (A_{12} and A_{21}): considering only the interregional flows among regions one has a gross value of 4.07% of the production in the NE region is due to these flows, adding 0.13% to the ppp, while for the RB region this gross value is 0.42%, adding 0.08% to the ppp, showing again a greater dependence of the production in the NE region in the interrelations among the regions;

Case IX (A_{12} and A_{22}): from the interactions of the inputs that the industries in the RB region buy from both regions, a gross value of 38.65% of the production in this region can be revealed, with no addition to the ppp, and a gross value of 7.89% of the production in the NE region, adding 10.59% to the ppp. When these results are compared with the ones presented in Case VI, this shows a greater dependence of the NE region on the production process of the RB region;

Case X (A_{21} and A_{22}): from the sales of production that the industries in the RB region sell to the production process of both regions, a gross value of 39.37% of the production in the RB region is obtained, adding 0.82% to the ppp, and 0.00% in the NE region as there is virtually no feedback among the regions, in this case showing a greater value of internal multipliers in the RB region than in the NE region;

Case XI (A_{11}, A_{12} and A_{21}): taking the relations inside the NE region and the sales and purchases that it makes from the RB region, a gross value of 30.40% of the production in this region is accounted for, adding 0.15% to the ppp, and a gross value of 0.56% in the RB region, adding 0.03% to the ppp, values greater than the ones presented in case VI since more transactions are now being taking into consideration;

Case XII (A_{11}, A_{12} and A_{22}): taking the relations inside both regions and the purchases that the NE region makes from the RB region, a gross value of 36.33% of the production in the NE region is generated, adding 5.92% to the ppp, and a gross value of 38.65% in the RB region, with no addition to the ppp;

Case XIII (A_{11}, A_{21} and A_{22}): taking the relations inside both regions and the purchases that the NE region makes from the RB region, a gross values of 24.13% of the production in the NE region can be ascertained, with no addition to the ppp, and a gross value of 39.59% in the RB region, adding 0.26% to the ppp;

Case XIV (A_{12}, A_{21} and A_{22}): taking the relations inside the RB region and the sales and purchases that it makes from the NE region, a gross value of 39.49% of the production in this region is generated, adding 0.21% to the ppp, and a gross value of 7.99% in the NE region, adding 0.13% to the ppp, values greater than the ones presented in case IX since more transactions are now being taking into consideration.

Case XV (A_{11}, A_{12}, A_{21} and A_{22}): this case is not displayed in table 2 because it considers all the interactions in the economy, it is listed here only to call attention for the contribution that this last case has to the ppp, i.e., adding 0.14% to the ppp in the NE region and 0.08% to the ppp in the RB region.

Tables 2 and 3 and figures 3 and 4 show for both regions the contribution that each block matrix in each pair wise decomposition has to the ppp; they also present the total contribution of each block matrix. From these data, it is possible to see a greater dependence of the NE region on the RB region, for while 71.03% of the ppp in the NE region is due to interactions inside the region, the corresponding value for the RB region is 97.82%. Hence, it is possible to observe and to measure how the relations between the 2 Brazilian regions take place. The NE region has a greater dependence on the rest of the economy region than the rest of the economy has on the NE region, and at the same time the rest of the economy region seems to be more developed as it presents a more complex productive structure than the NE region.

IV Policy Implications

One of the major changes that has occurred within the economic structure of many economies is the apparent increase in specialization and diversification at the same time. Overall, regional economies are becoming more diversified, in terms of their macro structure. However, establishments (plants) within sectors are becoming more specialized, responding in large part to consumer demands for greater product variety. As a result, trade between regions tends to be concentrated in *intraindustry* rather than *interdindustry* trade (see Krugman, 1990). However, these developments are associated with trade between regions with similar levels of per capita income and with excellent transportation connections. Neither is the case for the NE-RB interaction; transportation costs are low enough to allow penetration from the other region but not sufficiently low enough to allow for the full realization of the benefits of increasing returns.

Having discerned significant imbalances in the trading relationships and the complexity of internal to the region intermediation, the next issue centers on the policy implications. Comparative analysis recently conducted for the NE economy with that of the Midwest of the US (Magalhães *et al.*, 2000) revealed dramatically significant differences in the level and volume of interactions for two regions. While both regions account for about the same percentage of their nation's GDP, the Midwest US economy's GDP per capita is above the national average in contrast to the NE Brazil economy (about 55% of the Brazil GDP per capita). While the Midwest region is highly connected to the rest of the US economy (with an overall positive balance of trade), a huge volume of interactions flow between the member states; in the NE, the level of internal intermediation is lower and there is a negative balance of trade (imports > exports) with the RB. Clearly, appeals to development of clusters of activities to enhance the level of intermediation may not reflect the realities of an economy whose capacity to sustain further levels of activity may be circumscribed by poor *internal* transportation connectivities that reduce the effective demand for goods and services.

In addition, as noted by Baer *et al*. (1998), the promotion of more open markets within the context of WTO guidelines may make traditional forms of market intervention less feasible; in any case, the record from prior interventions suggest that the prior policies had little success in significantly changing the structure of the NE region's economy to ensure that it would be in a position to compete successfully in the national and international marketplace in the next several decades.

V. Conclusions

The main contribution of this paper was to show, using different synergetic interactions, that it is possible to analyze and to measure how the trading relationship between two regions takes place. This was accomplished using a two-region interregional input-output table constructed for the Brazilian economy for the year of 1995. From the results, it was possible to see that NE region has a greater dependence on the rest of the economy region than the rest of the economy has on the NE region, and at the same time the rest of the economy region seems to be more developed as it presents a more complex productive structure than the NE region.

This study was conducted using one point in time and two regions; it would be interesting to compare how the relations between the two regions change trough time and also how these relations would evolve if the RB region were to be divided into several subregions.

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Table A1: Inner Regional Multipliers and Their Properties

1. Internal regional multipliers:

$B_1 = (I\text{-}A_{11})^{-1}$; $\quad B_2 = (I\text{-}A_{22})^{-1}$

2. Schur complements and Schur inverses:

$S_1 = I\text{-}A_{11}\text{-}A_{12}B_2A_{21}$; $\quad S_2 = I\text{-}A_{22}\text{-}A_{21}B_1A_{12}$

$D_1 = (I\text{-}A_{11}\text{-}A_{12}B_2A_{21})^{-1}$; $\quad D_2 = (I\text{-}A_{22}\text{-}A_{21}B_1A_{12})^{-1}$

3. Left and right Miyazawa external matrix multipliers:

$D_{11}^{L} = (I\text{-}B_1A_{12}B_2A_{21})^{-1}$; $\quad D_{22}^{L} = (I\text{-}B_2A_{21}B_1A_{12})^{-1}$

$D_{11}^{R} = (I\text{-}A_{12}B_2A_{21}B_1)^{-1}$; $\quad D_{22}^{R} = (I\text{-}A_{21}B_1A_{12}B_2)^{-1}$

Main Properties:

$D_1 = B_1D_{11}^{R} = D_{11}^{L}B_1$; $\quad D_2 = B_2D_{22}^{R} = D_{22}^{L}B_2$

4. Abbreviated Schur inverses:

$D_{11} = (I\text{-}A_{12}B_2A_{21})^{-1}$; $\quad D_{22} = (I\text{-}A_{21}B_1A_{12})^{-1}$

5. Left and right induced internal multipliers:

$B_1^{L} = (I\text{-}D_{11}A_{11})^{-1}$; $\quad B_2^{L} = (I\text{-}D_{22}A_{22})^{-1}$

$B_1^{R} = (I\text{-}A_{11}D_{11})^{-1}$; $\quad B_2^{R} = (I\text{-}A_{22}D_{22})^{-1}$

Main Properties:

$D_1 = B_1^{L}D_{11} = D_{11}B_1^{R}$; $\quad D_2 = B_2^{L}D_{22} = D_{22}B_2^{R}$

6. Enlarged Leontief inverses:

$D_1^{*} = (I\text{-}A_{11}\text{-}A_{12}A_{21})^{-1}$; $\quad D_2^{*} = (I\text{-}A_{22}\text{-}A_{21}A_{12})^{-1}$

7. Induced external multipliers:

$D_{11}^{*} = (I\text{-}A_{12}A_{21})^{-1}$; $\quad D_{22}^{*} = (I\text{-}A_{21}A_{12})^{-1}$

Table 1 (Continued)

9. Left and right subjoined inverses:

$D_{11}^{*L} = (I - B_1 A_{12} A_{21})^{-1}$; $\quad D_{22}^{*L} = (I - B_2 A_{21} A_{12})^{-1}$

$D_{11}^{*R} = (I - A_{12} A_{21} B_1)^{-1}$ $\quad D_{22}^{*R} = (I - A_{21} A_{12} B_2)^{-1}$

Main Properties:

$D_1^{*} = B_1 D_{11}^{*R} = D_{11}^{*L} B_1$; $\quad D_2^{*} = B_2 D_{22}^{*R} = D_{22}^{*L} B_2$

10. Left and right induced subjoined inverses:

$D_{11}^{**L} = [I - D_{11}(A_{11} - A_{12} B_2 A_{22} A_{21})]^{-1}$; $\quad D_{22}^{**L} = [I - D_{22}(A_{22} - A_{21} B_1 A_{11} A_{12})]^{-1}$;

$D_{11}^{**R} = [I - (A_{11} - A_{12} B_2 A_{22} A_{21}) D_{11}]^{-1}$; $\quad D_{22}^{**R} = [I - (A_{22} - A_{21} B_1 A_{11} A_{12}) D_{22}]^{-1}$

Main Properties:

$D_1^{*} = D_{11}^{**L} D_{11} = D_{11} D_{11}^{**R}$; $\quad D_2^{*} = D_{22}^{**L} D_{22} = D_{22} D_{22}^{**R}$

Table A2
Taxonomy of Synergetic Interactions between Economic Sub-Systems

[Each entry consists of two levels: in the first level, a description of the structure and the corresponding form of the A_1 matrix is shown. In the second level the additive decompositions of the Leontief block-matrix are shown]

Level 1 *Description* *Form of the* A_1 *matrix*

Level 2 $L = L_1 + (M_L - I)L_1 = L_1 + L_1(M_R - I)$

I. *Hierarchy of isolated region versus the rest of economy* $A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & 0 \end{bmatrix}$:

$$L = \begin{bmatrix} B_1 & 0 \\ 0 & I \end{bmatrix} + \begin{bmatrix} B_1 A_{12} & 0 \\ 0 & I \end{bmatrix} D_2 \begin{bmatrix} I & I \\ I & I - S_2 \end{bmatrix} \begin{bmatrix} A_{21}B_1 & 0 \\ 0 & I. \end{bmatrix}$$

II. *The order replaced hierarchy of interregional linkages of second region versus lower triangular sub system* $A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & 0 \end{bmatrix}$:

$$L = \begin{bmatrix} I & A_{12} \\ 0 & I \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} I - S_1 & A_{12}B_2 - S_1 A_{12} \\ A_{21}B_1 & I - S_2 \end{bmatrix}$$

III. *The order replaced hierarchy of interregional linkages of first region versus upper triangular sub system* $A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & 0 \end{bmatrix}$:

$$L = \begin{bmatrix} I & 0 \\ A_{21} & I \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ 0 & D_2 \end{bmatrix} \begin{bmatrix} I - S_1 & A_{12}B_2 \\ A_{21}B_1 - S_2 A_{21} & I - S_2 \end{bmatrix}$$

IV. *The order replaced hierarchy of backward and forward linkages of the first region versus rest of economy* $A_1 = \begin{bmatrix} 0 & 0 \\ 0 & A_{22} \end{bmatrix}$:

$$L = \begin{bmatrix} I & 0 \\ 0 & B_2 \end{bmatrix} + \begin{bmatrix} I & 0 \\ 0 & B_2 A_{21} \end{bmatrix} D_1 \begin{bmatrix} I - S_1 & I \\ I & I \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & A_{12}B_2 \end{bmatrix}$$

V. *Hierarchy of forward linkages of first and second regions* $A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & 0 \end{bmatrix}$:

$$L = \begin{bmatrix} B_1 & B_1 A_{12} \\ 0 & I \end{bmatrix} + \begin{bmatrix} B_1 A_{12} \\ I \end{bmatrix} D_2 \begin{bmatrix} A_{21}B_1 & I - S_2 \end{bmatrix}$$

Table A2 (Continued)

VI. *Hierarchy of backward linkages of first and second regions*

$$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & 0 \end{bmatrix}:$$

$$L = \begin{bmatrix} B_1 & 0 \\ A_{21}B_1 & I \end{bmatrix} + \begin{bmatrix} B_1 A_{12} \\ I - S_2 \end{bmatrix} D_2 \begin{bmatrix} A_{21}B_1 & I \end{bmatrix}$$

VII. *The hierarchy of intra- versus inter- regional relationships*

$$A_1 = \begin{bmatrix} A_{11} & 0 \\ 0 & A_{22} \end{bmatrix}::$$

$$\mathrm{L} = \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix} + \begin{bmatrix} D_1 A_{12} B_2 & 0 \\ 0 & D_2 A_{21} B_1 \end{bmatrix} \begin{bmatrix} A_{21} & I - A_{22} \\ I - A_{11} & A_{12} \end{bmatrix} \begin{bmatrix} B_1 & 0 \\ 0 & B_2 \end{bmatrix}$$

VIII. *The hierarchy of inter versus intra regional relationships*

$$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & 0 \end{bmatrix}:$$

$$L = \begin{bmatrix} D_{11}^* & D_{11}^* A_{12} \\ D_{22}^* A_{21} & D_{22}^* \end{bmatrix} + \begin{bmatrix} I & B_1 A_{12} \\ B_2 A_{21} & I \end{bmatrix} \begin{bmatrix} D_1 A_{11} D_{11}^* & 0 \\ 0 & D_2 A_{22} D_{22}^* \end{bmatrix} \begin{bmatrix} I & A_{12} \\ A_{21} & I \end{bmatrix}$$

IX. *Order replaced hierarchy of backward linkages*

$$A_1 = \begin{bmatrix} 0 & A_{12} \\ 0 & A_{22} \end{bmatrix}:$$

$$L = \begin{bmatrix} I & A_{12} B_2 \\ 0 & B_2 \end{bmatrix} + \begin{bmatrix} 1 - S_1 \\ B_2 A_{21} \end{bmatrix} D_1 \begin{bmatrix} I & A_{12} B_2 \end{bmatrix}$$

X. *Order replaced hierarchy of forward linkages*

$$A_1 = \begin{bmatrix} 0 & 0 \\ A_{21} & A_{22} \end{bmatrix}:$$

$$L = \begin{bmatrix} I & 0 \\ B_2 A_{21} & B_2 \end{bmatrix} + \begin{bmatrix} 1 \\ B_2 A_{21} \end{bmatrix} D_1 \begin{bmatrix} I - S_1 & A_{12} B_2 \end{bmatrix}$$

XI. *The hierarchy of backward and forward linkages of the first region versus rest of economy*

$$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & 0 \end{bmatrix}:$$

$$L = \begin{bmatrix} D_1^* & D_1^* A_{12} \\ A_{21} D_1^* & D_{22} \end{bmatrix} + \begin{bmatrix} B_1 A_{12} \\ I \end{bmatrix} D_2 D_{22} A_{22} \begin{bmatrix} A_{21} B_1 & I \end{bmatrix}$$

Table A2 (Continued)

XII. *The hierarchy of upper triangular sub system versus interregional linkages of first region*	$A_1 = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix}$:
$L = \begin{bmatrix} B_1 & B_1 A_{12} B_2 \\ 0 & B_2 \end{bmatrix} + \begin{bmatrix} B_1 A_{12} \\ I \end{bmatrix} D_2 A_{21} B_1 \begin{bmatrix} I & A_{12} B_2 \end{bmatrix}$	
XIII. *The hierarchy of lower triangular sub system versus interregional linkages of second region*	$A_1 = \begin{bmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{bmatrix}$:
$L = \begin{bmatrix} B_1 & 0 \\ B_2 A_{21} B_1 & B_2 \end{bmatrix} + \begin{bmatrix} 1 \\ B_2 A_{21} \end{bmatrix} D_1 A_{12} B_2 \begin{bmatrix} A_{21} B_1 & I \end{bmatrix}$	
XIV. *Hierarchy of the rest of economy versus second isolated region*	$A_1 = \begin{bmatrix} 0 & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$:
$L = \begin{bmatrix} D_{11} & A_{12} D_2^* \\ D_2^* A_{21} & D_2^* \end{bmatrix} + \begin{bmatrix} 1 \\ B_2 A_{21} \end{bmatrix} D_{11} A_{11} D_1 \begin{bmatrix} I & A_{12} B \ 2 \end{bmatrix}$	

Table 1
Results of the Synergetic Interactions Between the NE and the RB Regions

Pair-Wise Hierarchy	Share (%) of x_1 in x NE	Share (%) of x_1 in x Rest of BR	Share (%) of $(x_1\text{-}f)$ in x NE	Share (%) of $(x_1\text{-}f)$ in x Rest of BR	Share (%) of f in x NE	Share (%) of f in x Rest of BR
I	87.59	60.25	24.13	0.00	63.46	60.25
II	67.49	60.25	4.02	0.00	63.46	60.25
III	63.46	60.64	0.00	0.39	63.46	60.25
IV	63.46	98.90	0.00	38.65	63.46	60.25
V	93.76	60.25	30.30	0.00	63.46	60.25
VI	87.59	60.76	24.13	0.51	63.46	60.25
VII	87.59	98.90	24.13	38.65	63.46	60.25
VIII	67.54	60.67	4.07	0.42	63.46	60.25
IX	71.36	98.90	7.89	38.65	63.46	60.25
X	63.46	99.62	0.00	39.37	63.46	60.25
XI	93.87	60.81	30.40	0.56	63.46	60.25
XII	99.79	98.90	36.33	38.65	63.46	60.25
XIII	87.59	99.84	24.13	39.59	63.46	60.25
XIV	71.45	99.74	7.99	39.49	63.46	60.25

Source: Estimated by the authors

Table 2
Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of $(x_1\text{-}f)$ in x -NE

Pair-Wise Hierarchy	Pair-Wise	Block Matrix A11	Block Matrix A12	Block Matrix A21	Block Matrix A22
I	66.03	66.03	-	-	-
II	11.01	-	11.01	-	-
III	-	-	-	-	-
IV	-	-	-	-	-
V	5.88	2.94	2.94	-	-
VI	-	-	-	-	-
VII	-	-	-	-	-
VIII	0.13	-	0.07	0.07	-
IX	10.59	-	5.30	-	5.30
X	-	-	-	-	-
XI	0.15	0.05	0.05	0.05	-
XII	5.92	1.97	1.97	-	1.97
XIII	-	-	-	-	-
XIV	0.13	-	0.04	0.04	0.04
XV	0.14	0.04	0.04	0.04	0.04
Total	100.00	71.03	21.42	0.20	7.35

Source: Estimated by the authors

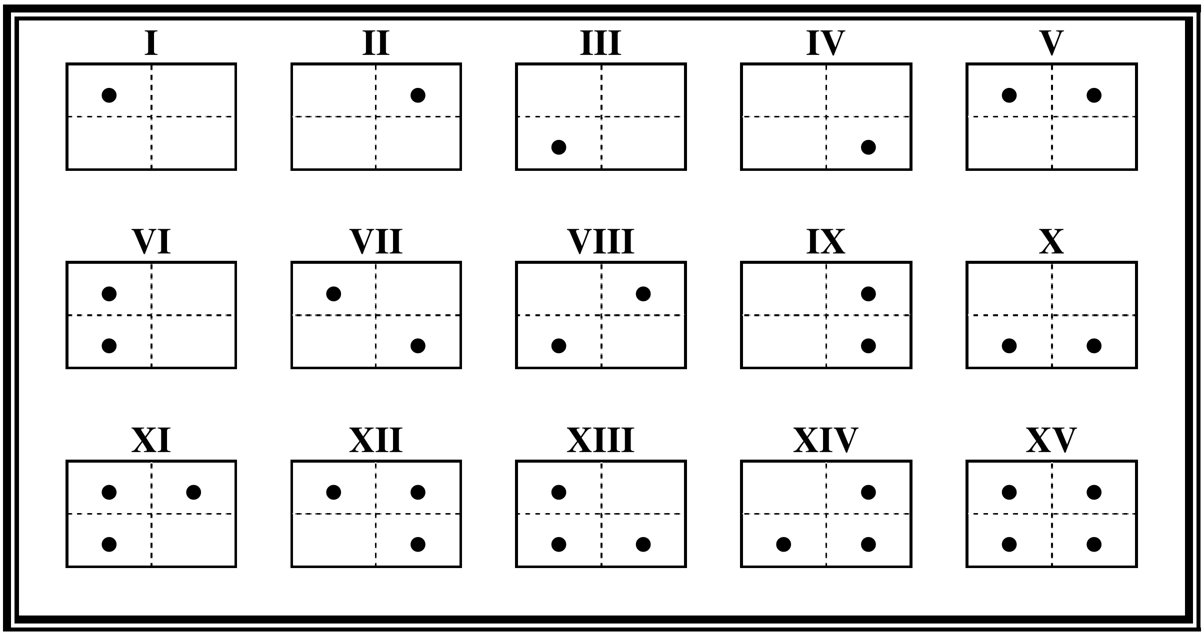
Table 3

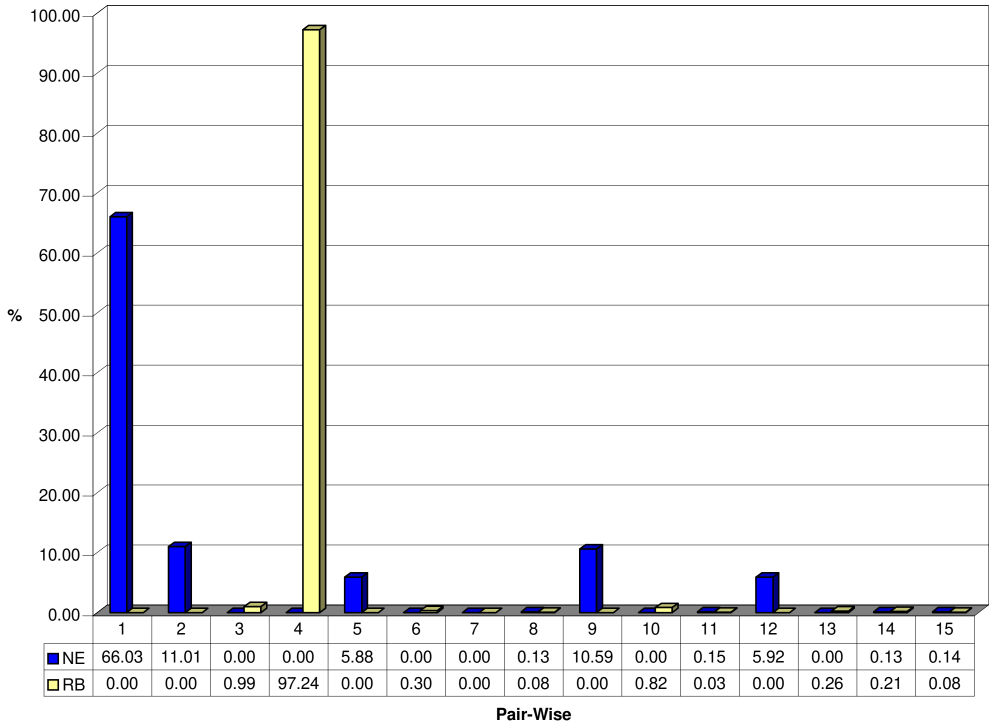
Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of *(x_1-f)* in *x* – RB

Pair-Wise Hierarchy	Pair-Wise	Block Matrix A11	Block Matrix A12	Block Matrix A21	Block Matrix A22
I	-	-	-	-	-
II	-	-	-	-	-
III	0.99	-	-	0.99	-
IV	97.24	-	-	-	97.24
V	-	-	-	-	-
VI	0.30	0.15	-	0.15	-
VII	-	-	-	-	-
VIII	0.08	-	0.04	0.04	-
IX	-	-	-	-	-
X	0.82	-	-	0.41	0.41
XI	0.03	0.01	0.01	0.01	-
XII	-	-	-	-	-
XIII	0.26	0.09	-	0.09	0.09
XIV	0.21	-	0.07	0.07	0.07
XV	0.08	0.02	0.02	0.02	0.02
Total	100.00	0.27	0.14	1.77	97.82

Source: Estimated by the authors

Figure 1: Schematic Representation of the Possible Forms of the A_1 Matrix

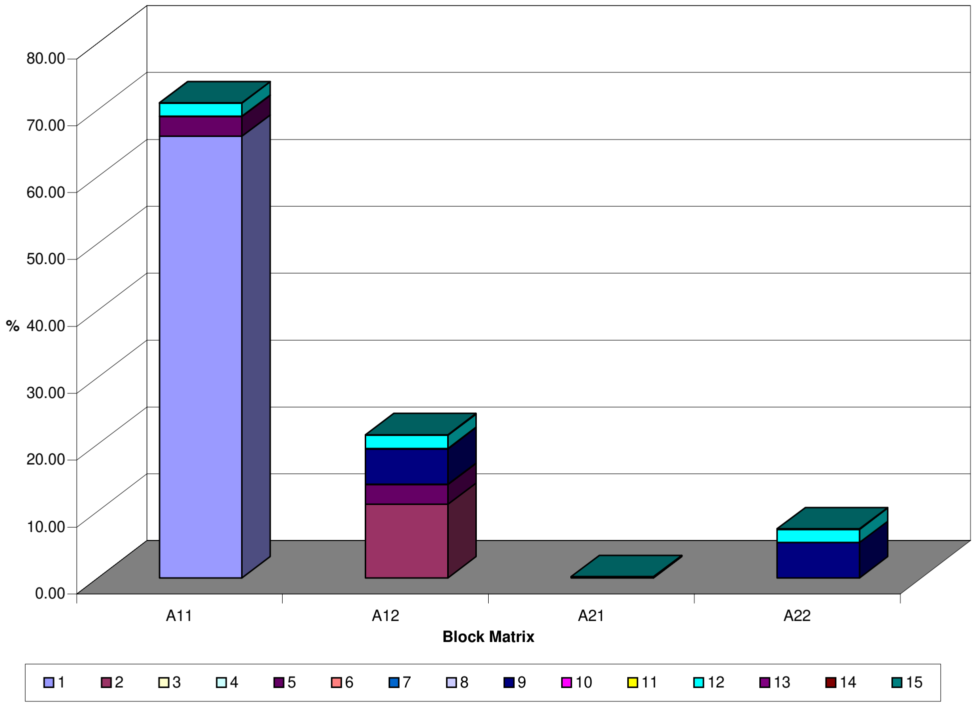




Source: Tables 1, 2, and 3

Figure 2

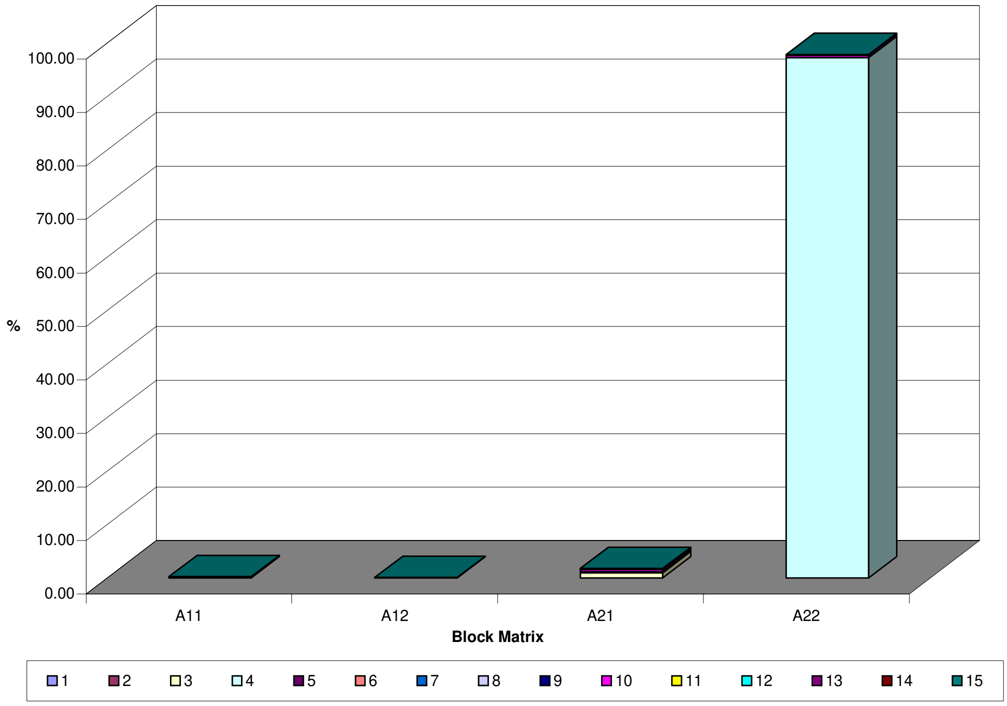
Results of the Synergetic Interactions Between the NE and the RB Regions



Source: Table 2

Figure 3

Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of $(x_1\text{-}f)$ in x – NE



Source: Table 3

Figure 4

Contribution (%) of Each Pair-Wise and Block Matrix to the Total Share of *(x_1-f)* in *x* – RB