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Jellal, Mohamed and wolff, François charles

Al Makrîzî Institut d'Economie

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Mohamed Jellal[†] François-Charles Wolff[‡]

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Abstract

When focusing on firm's risk-aversion in industry equilibrium, the number of firms may be either larger or smaller when comparing market equilibrium with and without price uncertainty. In this paper, we introduce risk-averse firms under cost uncertainty in a model of spatial differentiation and show that the impact of uncertainty will always increase the number of firms in an industry. This finding is explained by the higher prices that firms charge to consumers under uncertainty. With increased uncertainty, firms have greater incentive to enter the market since they may benefit from higher levels of profit.

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[†]Université Mohammed V, Rabat, Morroco; Toulouse Business School, France; and Conseils-Eco, 10 Impasse de Mansencal, 31500 Toulouse, France. E-mail: jellalmohamed@yahoo.fr

[‡]Corresponding author. LEN-CEBS, Université de Nantes, chemin de la Censive du Tertre BP 52231, 44322 Nantes Cedex 3, France ; CNAV and INED, Paris, France. Tel: 33240141742. Fax: 33240141743. E-mail: wolff@sc-eco.univ-nantes.fr

1 Introduction

When explaining variations in the number of firms across industries, standard arguments drawing on scale economies and entry conditions usually neglect the issue of uncertainty. Unfortunately, the prevalent assumption of risk-neutral firms is not really appropriate. Several theoretical contributions have recently considered a setting where firms behave in a risk-averse manner (see Asplund, 2002, and the references therein). Among the most frequent explanations, one can invoke the presence of liquidity constraints, the management by non-diversified owners or delegation of control to risk-averse supervisors, as well as financial distress (Drèze, 1987). In particular, the extent of corporate hedging activities may be interpreted as a reluctance to bear risk (Nance et alii, 1993). Clearly, the introduction of uncertainty has strong implications for the product market competition.

The pioneering work dealing with the impact of uncertainty on firms' decisions is due to Sandmo (1971). Within a partial equilibrium framework, greater price uncertainty is expected to lower the optimal quantity produced in a perfectly competitive market[1]. Then, the degree and distribution of price uncertainty are significant factors to explain industry structure. At the equilibrium, Sandmo (1971) proves that an increased uncertainty about price lowers the number of firms in the industry. A more general question is to focus on the impact of risk aversion in a model in which the number of firms is determined endogenously. Appelbaum and Katz (1986) were the first to address that issue (see also Haruna, 1996). Once a competitive equilibrium is introduced, they show that the effects of price uncertainty on the number of firms in an industry can no longer be signed, even with additional assumptions about relative or absolute risk aversion.

Despite the ambiguous prediction of price uncertainty on the industry equilibrium, it seems tempting to believe that a negative relationship between uncertainty and the number of firms is more likely[2]. Intuitively, and following the discussion in Sandmo (1971), firms that are characterized by a high value for risk aversion certainly prefer not to operate in a market where price uncertainty prevails. Indeed, uncertainty may be seen as a natural barrier to entry, thereby leading to a decrease in the number of firms

[1] See also Leland (1972) for the effect of uncertainty in a monopoly setting.

[2] And such a negative relationship seems rather supported by the data. Using a cross-section of US manufacturing industries, Ghosal (1996) shows that greater uncertainty exerts a negative impact on the number of firms in an industry when correcting for endogeneity of the price uncertainty measure.

in the industry. However, it is well known since the influential paper of Oi (1961) that variability may also offer opportunities for increasing average profit for risk-averse firms. Average profits of a price taker are increasing in the variability of the output price and Oi's conclusion does generalize to a considerable extent (Friberg and Martensen, 2000). Such positive effects on profits could have a beneficial influence on the entry of firms.

In this paper, following Sandmo (1971) and Appelbaum and Katz (1986), we further examine the effects of uncertainty within an industry equilibrium framework. We examine the problem of free entry and exit of firms in a setting of spatial differentiation with cost uncertainty. Specifically, we draw on the location model originally proposed by Salop (1979), who introduces differentiation using a circular city with consumers uniformly distributed on its circumference. Our main result is to prove that the indeterminate effect of uncertainty on the number of firms in an industry does no longer hold. In a location model with horizontally differentiated products and risk-averse firms, greater cost uncertainty always increases the number of firms operating in the industry.

The intuition of that result is as follows. In a location model (either linear or spatial), it is well known that the competitive price under product differentiation is defined as the sum of the marginal cost and the transportation cost, which leads to a monopoly power for the different firms in the industry (see Tirole, 1988). When one introduces cost uncertainty, the optimal price now includes an additional term corresponding to the risk premium faced by the firms. So, when comparing market equilibrium with and without uncertainty, it turns out that firms charge higher prices to consumers under uncertainty. This leads to higher profits for risk-averse firms, and greater uncertainty increases the number of firms in the industry. Thus, in a certain sense, our theoretical contribution is close to the famous Oi's variability result.

By focusing on uncertainty in a location model, our paper is related to the recent literature on risk-averse firms in an oligopoly. In a context of cost uncertainty, Wambach (1999) proves that the Bertrand paradox such that two firms are sufficient for perfect competition does no longer hold with risk-averse firms. In an industry with price competition, the equilibrium price is expected to exceed the competitive price and then increasing the number of firms may lead to an increase in price[3]. Janssen and Rasmusen (2002) also

[3]Specifically, the new price is expected to be higher when there is an increase in the size of the market

consider a Bertrand model with uncertainty on the number of firms operating in the industry. With an uncertain number of competitors, there exists a unique symmetric equilibrium in mixed strategy and again each firms charges a price larger than marginal cost[4]. The question of strategic choices of risk-averse firms is further analyzed in Asplund (2002), who examines how the degree of risk aversion and different types of uncertainty affect competition in an oligopolistic framework. The key feature of this insightful contribution is to propose a general competition model of risk-averse firms that encompasses price competition with differentiated products under various forms of cost and demand uncertainty. In particular, competition is softer in case of marginal cost uncertainty.

Thus, our work may be seen as complementary to the analysis of Asplund (2002). Our contribution is twofold. First, we focus on the consequences of uncertainty in a model with product differentiation and free entry of firms. Second, we present a welfare analysis which accounts for the costs involved by firms in bearing risk. The remainder of the paper is organized as follows. In section 2, we extend the circular location model of Salop (1979) and assume that marginal cost is uncertain. In section 3, we determine the Nash equilibrium in prices for any number of firms and show that firms charge higher prices to consumers because of uncertainty. The Nash equilibrium in the entry game is analyzed in section 4, with a positive impact of uncertainty on the number of firms. Section 5 examines the price equilibrium from a normative viewpoint. Concluding comments are in section 6.

2 The spatial model

We consider a model with firms producing differentiated products, in which consumers are heterogeneous and where firms have uncertain marginal costs. Thus, we relax the prevalent assumption behind the Bertrand paradox that firms produce an homogeneous good, a situation analyzed by Janssen and Rasmusen (2002) and Wambach (1999) in an uncertain setting. With a location model, it follows that firms can raise their price above the marginal cost without losing their entire market share.

and the number of firms in the same proportion (see Wambach, 1999).

[4]The perfectly competitive equilibrium is the limit case when the number of firms becomes large. As the probability of competition increases, each firm reduces its prices.

We restrict our attention to horizontally differentiated products, meaning that brands are not uniformly ranked by all consumers. As usual in the literature, each consumer has a different preference for the brands sold in the market due to different location. In our setting, location corresponds to the physical location of a particular consumer. Each agent observes the prices charged by all the firms, and then decides to purchase the good from the firm at which the price plus the transportation cost is minimized. Another convenient interpretation is that location can also represent a distance between the brand characteristics viewed as ideal by the consumer and the characteristics of the brand actually purchased[5]. Thus, firms choose their products anticipating that their location decision in product space is expected to affect the intensity of price competition.

Our theoretical analysis of the impact of uncertainty on the number of firms draws on the spatial differentiation model originally described by Salop (1979), corresponding to the case of a circular city. In so doing, we are able to examine the problem of firms' entry on the market given marginal cost uncertainty. Specifically, we study entry and location decisions when there exist no barriers to entry other than fixed costs.

We suppose that consumers are located uniformly on a circle C, which has a perimeter equal to L. Clearly, the circumference L is a measure for the heterogeneity of consumers and it may be seen as an indicator for demand intensity. Individuals are continuously and uniformly distributed along this circumference. We assume without loss of generality that the density is constant, and it is denoted by Δ[6]. Thus, the parameter Δ expresses the thickness of the market. Given the location of firms, consumers incur a transportation cost equal to t per unit of length, such that this cost includes the value of time spent in travel. Each consumer buys exactly one unit of the brand that minimizes the sum of the price and the transportation cost. Nevertheless, this generalized cost has to remain lower than the gross surplus that the consumer can obtain from the good. This outside option is denoted by $\overline{s}$. It is assumed to be large enough, so that the market is always covered in equilibrium (goods are bought by all consumers).

Firms are located around the circle. Although the circular model of Salop (1979) is a location model, it does not explicitly explain how firms choose their location (see the

[5]In that case, distance is a measure of the disutility from consuming a less-than-ideal product.

[6]Relaxing this assumption does not modify our theoretical conclusions. See Calvo-Armengol and Zenou (2002) for the case of a general density in a model of differentiated products, but under certainty.

related discussion in Tirole, 1988, p. 285). Indeed, the spatial model has the following two-stage structure. First, the number of firms is endogenously determined. It is assumed that firms are automatically located at an equal distance from one another. Thus, if the number of entering firms is denoted by n and given the circumference L, the distance between any two firms is equal to L/n. Second, firms compete in prices given the previous locations. So, a key feature of this horizontal differentiation model is the focus on firms' entry, and we examine the impact of uncertainty on entry.

There are many potential firms in the location model, which have all the same technology. To address the issue of entry, we suppose that each firm is characterized by a fixed cost of entry denoted by $\overline{f}$. Once the firm is located at a point on the product space, it faces a marginal cost c that is supposed to be constant. We depart from the model of Salop (1979) by assuming that this marginal cost is uncertain, so that firms face supply-induced cost fluctuations in our setting. To formalize this type of uncertainty, we assume that the marginal cost is described by a random variable $\tilde{c}$ whose mean is $E(\tilde{c}) = c$ and the corresponding variance is $Var(\tilde{c}) = \sigma^2$. As usual, greater cost uncertainty is measured by an increase in the variance σ^2 (a mean preserving spread in costs).

It seems important to note that our way to include uncertainty in the location model is absolutely not restrictive. Indeed, there are numerous examples in the industry of sources of uncertainty arising by the marginal cost of production. For instance, Wambach (1999, p. 946) mentions the case of insurance corporations where the probability of accident is imperfectly known to the insurers, firms which provide guarantees for new products (given random breakdown), or simply firms which import brands and then face exchange-rate uncertainty. Other explanations concern poor climatic conditions for firms that produce or use agricultural goods or uncertain wages linked to efficiency wage considerations and shirking behaviors as well as uncertainty over the number of active workers (due to illness).

Each firm is labelled by subscript i ($i = 1, \ldots, n$), and the firm's location is denoted by x_i. A firm is fully described by the list of prices charged on consumers $(p_1, \ldots, p_i, \ldots, p_n)$. A consumer is located at the distance $x \in C$. Then, the generalized price to buy the brand is equal to $p_i + t|x - x_i|$ under linear transportation costs[7]. Firms anticipate that

[7]While we restrict our attention to the case of linear transportation costs for the sake of simplicity, our theoretical results remains unchanged with quadratic transportation costs.

consumers choose to buy the brands to the firms which give them the lowest full price. In the circular model, a representative firm has only two competitors. Given two level of prices p_{i-1} and p_{i+1}, the demand pool for the firm i is composed of two sub-segments. The outside boundaries of the pool are given by two marginal consumers, respectively denoted by $\underline{x}$ and $\overline{x}$, for whom the generalized price is identical between two adjacent firms : respectively between $i-1$ and i for $\underline{x}$, and between i and $i+1$ for $\overline{x}$. Thus, the marginal value $\underline{x}$ is the solution of the following equation :

$$p_i + t(x_i - \underline{x}) = p_{i-1} + t(\underline{x} - x_{i-1}) \tag{1}$$

Hence, the consumer which is indifferent between purchasing the brand from firm i and purchasing it from its closest neighbor $i-1$ is characterized by :

$$\underline{x} = \frac{(p_i - p_{i-1}) + t(x_i + x_{i-1})}{2t} \tag{2}$$

So, the firm i faces a demand from all the consumers whose location belong to the interval $[\underline{x}; x]$, since the generalized price these consumers obtain from firm i is lower than the one they would obtain from firm $i-1$. In a similar way, the marginal consumer $\overline{x}$ is such that $p_i + t(\overline{x} - x_i) = p_{i+1} + t(x_{i+1} - \overline{x})$, which implies :

$$\overline{x} = \frac{(p_{i+1} - p_i) + t(x_i + x_{i+1})}{2t} \tag{3}$$

Finally, the demand pool for the firm i consists of all consumers whose location is comprised in the closed interval $[\underline{x}; \overline{x}]$.

Now, let Π_i be the profit level of the firm i. Knowing the firm's demand, the presence of a fixed cost and given the uncertainty on marginal cost, the profit for the firm is also a random variable which is given by :

$$\tilde{\Pi}_i = \int_{\underline{x}}^{\overline{x}} \Delta(p_i - \tilde{c})dx - \overline{f} \tag{4}$$

so that the random profit $\tilde{\Pi}_i$ can be expressed as :

$$\tilde{\Pi}_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f} \tag{5}$$

Given the uncertain environment, we assume that firms are risk averse following some recent extensions in oligopoly theory (see Asplund, 2002, Haruna, 1996, Mai et alii, 1993, Tessiotore, 1994, Wambach, 1999). Relaxing the standard assumption that firms are risk-neutral has strong implications for the product market competition.

There are several reasons that may explain why firms behave in a risk-averse manner. The existence of fixed costs means that firms are making costly investment before producing, so that risk aversion is driven by liquidity constraints (see Drèze, 1987). Many firms have an imperfect access to the capital markets, and thus they have to bear part of the risk associated with their production. Another reason deals with non-diversified owners. Although owners may be tempted to maximize expected profits, the delegation of control to managers in hierarchical structure favors the reluctance to bear risk since the managers' income is clearly related to the firm's performance. Others arguments in the prevalent literature are linked to costly financial distress and to non-linear tax systems. Some studies have suggested that the extent of corporate hedging activities may be interpreted as the result of risk-averse behavior (Nance et alii, 1993, Gézci et alii, 1997).

Given the uncertainty on the marginal cost, the firm i is characterized by a Von Neumann-Morgenstern utility function denoted by U_i, so that the objective function for the firm may be expressed as :

$$\max V_i = E[U_i(\tilde{\Pi}_i)] \tag{6}$$

where U_i is a continuous, twice-differentiable and concave utility function ($U_i' > 0$, $U_i'' < 0$). From the definition of $\tilde{\Pi}_i$, the representative firm i seeks to maximize the expected utility function :

$$V_i = E\left[U_i\left(\Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}\right)\right] \tag{7}$$

Let us finally remind the definition of the monopolistic-competition equilibrium in the circular city. At the optimum, each firm behaves as a monopoly on its brand, meaning that the firm chooses the price that maximizes its utility function given the demand for brand i and given that all other firms charge the same price, and then free entry of firms results in zero profit. So, we solve the model by first determining the Nash equilibrium in prices for any number of firms, then by calculating the Nash equilibrium in the entry game (see Salop, 1979, Tirole, 1988).

3 The monopolistic-competition equilibrium

Let us assume that n firms have entered the market. Since these different firms are located symmetrically around the circle, we examine an equilibrium in which each firm charges the same price. We restrict our attention to the case of a covert market, which means that there are enough firms in the market. This corresponds to a situation where the value of the fixed cost $\overline{f}$ is not too high.

Thus, the maximization program for the firm i is $\max_{p_i} V_i$, so that the corresponding first-order condition given by $\partial V_i / \partial p_i = 0$ under marginal cost uncertainty is :

$$E\left[U_i'(.)\left(\Delta(p_i - \tilde{c})\left(\frac{\partial \overline{x}}{\partial p_i} - \frac{\partial \underline{x}}{\partial p_i}\right) + \Delta(\overline{x} - \underline{x})\right)\right] = 0 \tag{8}$$

with $U_i'(.) = U_i'\left(\Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}\right)$ for the notation. We also check that the second-order condition $\partial^2 V_i / \partial p_i^2 < 0$ for a maximum is satisfied since :

$$E\left[U_i''(.)\left(\Delta(p_i - \tilde{c})\left(\frac{\partial \overline{x}}{\partial p_i} - \frac{\partial \underline{x}}{\partial p_i}\right) + \Delta(\overline{x} - \underline{x})\right)^2 + 2\Delta\left(\frac{\partial \overline{x}}{\partial p_i} - \frac{\partial \underline{x}}{\partial p_i}\right)U_i'(.)\right] < 0$$

using $U_i''(.) < 0$ and $\partial \overline{x} / \partial p_i - \partial \underline{x} / \partial p_i < 0$. Since Π_i is continuous in (p_{i-1}, p_i, p_{i+1}) and since Π_i is strictly concave in p_i, we deduce that there always exists a Nash equilibrium in prices and that this Nash equilibrium is unique.

Proposition 1 *The symmetric Nash equilibrium price denoted by p_i^* is given by :*

$$p_i^* = c + \frac{tL}{n} + \frac{cov[\tilde{c}, U_i'(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}{E[U_i'(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]} \tag{9}$$

Proof : The optimal price is given by condition (8). First, we know that firms are symmetrically located and thus the distance between two firms is L/n, so that the market area for each firm is $\overline{x} - \underline{x} = L/n$. Second, given the definition of the marginal consumers $\overline{x}$ and $\underline{x}$, using (2) and (3) leads to $\partial \overline{x} / \partial p_i - \partial \underline{x} / \partial p_i = -1/t$. Thus, we get :

$$E\left[U_i'(.)\Delta\left(\frac{L}{n} - \frac{p_i - \tilde{c}}{t}\right)\right] = 0$$

Given the properties of the expectancy operator, it follows that :

$$p_i^* = \frac{tL}{n} + \frac{E[\tilde{c}U_i'(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}{E[U_i'(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}$$

Since $\tilde{c}$ is an argument of $U_i'(.)$, we can further simplify the optimal price using the fact that $E(XY) = E(X)E(Y) + cov(X, Y)$ for two variables X and Y. Since the mean of the random marginal cost is $E(\tilde{c}) = c$, we finally deduce (9). QED

Clearly, the sign of the covariance $cov[\tilde{c}, U_i'(.)]$ is positive since Baron (1971) has shown that the inequality $cov[\tilde{p}, U_i'(.)] < 0$ holds under price uncertainty and provided that the marginal utility $U_i'(.)$ is decreasing. Proposition 1 gives us a first result concerning the role of cost uncertainty on the spatial monopolistic-competition equilibrium. A greater cost uncertainty when producing brands leads to higher generalized prices charged to consumers. At the equilibrium, the price p_i^* is the sum of three elements : the marginal cost of production c, the transportation cost tL/n, which measures the monopsonistic behavior of firms, and the risk premium given by $cov[\tilde{c}, U_i'(.)]/E[U_i'(.)]$.

As the optimal price stands, it seems at first sight difficult to interpret the last term dealing with risk aversion. To find a more explicit result and get closed form solutions for our problem, we have to make an additional assumption concerning the marginal cost.

Assumption 1 *The marginal cost $\tilde{c}$ follows a Normal distribution, with $E(\tilde{c}) = c$ and $Var(\tilde{c}) = \sigma^2$.*

Under assumption 1, we can use the Stein's lemma (Huang and Litzenberger, 1988). Let us consider two variables X and Y such that they are bivariate normally distributed. If the function $f(Y)$ is continuously differentiable, Rubinstein (1976) prove that $cov[X, f(Y)] = E[f'(Y)]cov(X, Y)$. Now, if we apply the lemma of Stein to our problem, it follows that :

$$cov[\tilde{c}, U_i'(\Delta(p_i - \tilde{c})L/n - \overline{f})] = E[U_i''(\Delta(p_i - \tilde{c})L/n - \overline{f})]cov[\tilde{c}, \Delta(p_i - \tilde{c})L/n - \overline{f}]$$

Since we have $cov[\tilde{c}, \Delta(p_i - \tilde{c})L/n - \overline{f}] = -\Delta\sigma^2 L/n$, this implies :

$$cov[\tilde{c}, U'(\Delta(p_i - \tilde{c})L/n - \overline{f})] = -E[U_i''(\Delta(p_i - \tilde{c})L/n - \overline{f})]\frac{\Delta L}{n}\sigma^2$$

and thus the symmetric Nash equilibrium price may be expressed as[8] :

$$p_i^* = c + \frac{tL}{n} - \frac{E[U_i''(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}{E[U_i'(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}\frac{\Delta L}{n}\sigma^2 \tag{10}$$

[8]The derivation of the first-order condition in the case of normally distributed uncertainty is also derived in Asplund (2002) as a special case.

Let us define the parameter a such that :

$$a = -\frac{E[U_i''(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}{E[U_i'(\Delta(p_i^* - \tilde{c})L/n - \overline{f})]}$$

In the literature, a is known as the Rubinstein's measure of absolute risk aversion[9]. Rubinstein (1973, 1976) has proved that this measure based on the expectations of $U_i''(.)$ and $U_i'(.)$ remains constant.

Proposition 2 *Under assumption 1, the Nash symmetric price p_i^* is given by:*

$$p_i^* = c + \frac{tL}{n} + \frac{\Delta L}{n} a\sigma^2 \tag{11}$$

Assumption 1 leads to a closed-form solution for the positive risk premium, which is now equal to $\Delta La\sigma^2/n$. It is an increasing function of the density Δ of consumers on the circle and of the demand intensity L, but it is negatively related to the number of firms n. In that case, the risk due to uncertain marginal cost is spread over a larger number of firms. A novel result in our analysis is that firms charge higher prices for consumers given cost uncertainty. When firms are characterized by risk aversion (a>0), we obtain $\partial p_i^*/\partial a = \Delta L\sigma^2/n > 0$. Also, the optimal price is positively related to the variance σ^2 of the marginal cost since the derivative $\partial p_i^*/\partial\sigma^2 = \Delta La/n$ is positive. Both results indicate that firms share with consumers the risk generated by cost fluctuations. In industries characterized by greater cost uncertainty, higher prices for brands are expected since the risk premium increases.

Another interesting result is that the optimal price is an increasing function of the demand intensity L and of the consumer density Δ (only in an uncertain context), with increased opportunities of differentiation for firms. Other findings concerning the variables that affect the optimal price are more standard. With risk-averse firms in the industry ($a > 0$), a larger product market exerts a positive effect on the equilibrium price, given the higher possibility of differentiation for firms (the market area for each firm is fixed, given by L/n). Each firm faces the same degree of uncertainty on its marginal cost and

[9]Asplund (2002, appendix 1) also uses the measure $-EU_i''(\tilde{\Pi}_i - \overline{f})/EU_i'(\tilde{\Pi}_i - \overline{f})$. The author defines this ratio as the Arrow-Pratt measure of global absolute risk aversion. However, as pointed out by an anonymous referee, this expression cannot be considered as the Arrow-Pratt measure which is given by $-U_i''(\tilde{\Pi}_i - \overline{f})/U_i'(\tilde{\Pi}_i - \overline{f})$.

the risk premium is an increasing function of the density of consumers, which leads to a higher price. Also, the optimal price increases with t since the market power of firms is increased for consumers who are located close to the firms (Salop, 1979). Finally, given the increased competition, we basically observe that the price decreases with the number of firms in the market since $\partial p_i^*/\partial n = -t/n^2 - \Delta La\sigma^2/n^2 < 0$[10].

Before finding the equilibrium number of brands (n is endogenous), we briefly examine the situation where firms are risk neutral. When cost fluctuations have no impact on the utility derived by the firms ($a = 0$), the optimal price is :

$$p_i^* = c + \frac{tL}{n}$$

which is the result obtained by Salop (1979) in a spatial model under certainty[11]. In the case of risk neutrality, we note that the consumer density does not influence the equilibrium price. This conclusion does not longer hold when firms share with consumers part of the risk generated by cost volatility, as shown below.

So, at this first-stage of the location model, our main conclusion is that prices are higher with cost uncertainty. The cost of an increase in uncertainty is supported by consumers with differentiated products. As a consequence, greater cost uncertainty increases average profits for firms, and this positive effect of variability on firms' profit should be linked to the influential contribution of Oi (1961), who evidences a positive relationship between the variability of the output price and average profits of a price taker.

4 Free entry of firms

We now turn to the determination of the endogenous number of firms n^*, assuming that there are enough potential entrants to cover the market. Let us briefly detail the condition for the market to be covert[12]. We know that the equilibrium price has to be lower than the gross surplus $\overline{s}$. Since the maximum distance for a consumer is $L/2n$, the corresponding

[10]The competitive outcome can be regarded as a limit case of our model when the number of firms becomes very large.

[11]In the original presentation of Salop(1979), the length of the circle is set to one.

[12]On this issue of covert market in spatial model, see the further discussion of Jellal et alli (1998) in the context of a labor market.

condition of positive surplus is :

$$p^* + \frac{tL}{2n} \leq \overline{s} \tag{12}$$

Using the definition of p^*, it can also be expressed as :

$$a\sigma^2 \frac{\Delta L}{n} \leq (\overline{s} - c)^2 - \frac{3}{2}\frac{tL}{n} \tag{13}$$

so that the condition ensuring that the market is covered at the price equilibrium is :

$$0 < \sigma^2 < \frac{2n(\overline{s} - c)^2 - 3tL}{2a\Delta L} \tag{14}$$

Thus, the variance σ^2 has to take intermediate values for each consumer to buy the brand at the equilibrium. The interpretation of this result is as follows. When the variance σ^2 is small, the equilibrium price is above the price under uncertainty, but the increase in price remains limited since firms charge a low risk premium to the consumers. Hence, the market is covert. Conversely, when the risk premium becomes important, the firms are expected to set prices that are excessively high. Then, some consumers will no longer purchase anything.

By definition, the equilibrium number of firms n^* is given by :

$$E[U_i(\tilde{\Pi}_i)] = 0 \tag{15}$$

Ignoring assumption 1, let us suppose more generally that the uncertain cost $\tilde{c}$ is distributed according to a density function $g(\tilde{c})$ defined over the support $\Omega = [\underline{c}; \overline{c}]$. Thus, the previous condition may be expressed as $\int_\Omega U_i[\Pi(\tilde{c})]dg(\tilde{c}) = 0$, the reservation profit being normalized to 0. Again, the difficulty for our problem is to find an explicit solution for the optimal number of firms n^*, which involves additional restrictions either on the distribution of $\tilde{c}$ or on the functional form for U.

Recall that to derive the optimal price p_i^*, we have used the Stein's lemma by assuming that the marginal cost is normally distributed. It is well known that the mean and the variance provide a complete characterization of a random variable which is normally distributed. Thus, under assumption 1, we can rely on the mean-variance specification for the utility function U_i[13]. Thus, the problem for a firm may be expressed as :

$$V_i = E(\tilde{\Pi}_i) - \frac{a}{2}Var(\tilde{\Pi}_i) - \overline{f} \tag{16}$$

[13]The mean-variance approach can be used if the stochastic distribution of the marginal cost belongs to a particular parametrized family, normal or elliptical random variable.

where a is the degree of absolute risk aversion ($a \geq 0$) and the profit is $\tilde{\Pi}_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \overline{f}$. It follows that :

$$V_i = \Delta(p_i - \tilde{c})(\overline{x} - \underline{x}) - \frac{a}{2}(\Delta(\overline{x} - \underline{x}))^2\sigma^2 - \overline{f} \tag{17}$$

One can easily check that with the mean-variance utility, the optimal symmetric price is $p_i^* = c + tL/n + \Delta La\sigma^2/n$ as claimed in Proposition 2. Using this optimal value for p_i^*, we finally obtain V_i such that :

$$V_i = t\Delta\left(\frac{L}{n}\right)^2 + \frac{a}{2}\sigma^2\Delta^2\left(\frac{L}{n}\right)^2 - \overline{f} \tag{18}$$

Since the number of firms n^* is given by $V_i(n^*) = 0$, we get $\left(\frac{L}{n}\right)^2\left(t\Delta + \frac{a}{2}\Delta^2\sigma^2\right) = \overline{f}$.

Proposition 3 *Under assumption 1 and with a mean-variance utility function, the optimal number of firms n^* in a situation of imperfect competition with free entry is :*

$$n^* = \sqrt{\frac{(t\Delta + \frac{a}{2}\Delta^2\sigma^2)L^2}{\overline{f}}} \tag{19}$$

Proposition 4 *Under assumption 1 and with a mean-variance utility function, the optimal price value p^* under free entry is given by :*

$$p^* = c + \sqrt{\frac{t\overline{f}}{\Delta}}\sqrt{\frac{(1 + a\sigma^2\frac{\Delta}{t})^2}{(1 + \frac{a}{2}\sigma^2\frac{\Delta}{t})}} \tag{20}$$

Now, let us define $\phi(a, \sigma)$ such that :

$$\phi(a, \sigma) = \frac{1 + a\sigma^2\frac{\Delta}{t}}{\sqrt{1 + \frac{a}{2}\sigma^2\frac{\Delta}{t}}}$$

Clearly, we have $\phi(a, \sigma) > 1$, $\phi(0, \sigma) = 1$ and $\phi(a, 0) = 1$. Thus, the optimal price under certainty p_0^* is simply $p_0^* = c + \sqrt{\frac{t\overline{f}}{\Delta}}$ and we are now able to compare p_0^* and p^*.

Corollary 1 *With free entry of firms, the price is higher under uncertainy.*

In this model of spatial differentiation, the main contribution of our paper is to formally prove that greater uncertainty increases the number of firms in an industry. There are more firms because of uncertainty *and* risk aversion[14]. Clearly, both the degree of risk aversion a and the measure of variance σ^2 exert a positive effect on the optimal number of firms. That uncertainty positively affects free entry may be surprising, since it is usually admitted that greater uncertainty is rather expected to decrease the number of firms in an industry. For instance, in the context of price uncertainty, Sandmo (1971) argues that firms characterized by a large value for risk aversion will choose not to enter in an industry facing a high degree of uncertainty. Only low risk-averse firms are expected to enter in industries with greater uncertainty, thereby reducing the number of firms.

Then, how can we justify that greater uncertainty does not act as a barrier to entry under spatial competition ? In fact, we have previously shown that firms can charge a higher price to consumers under marginal cost uncertainty, since they shift the risk to the consumers. So, with greater uncertainty, the risk premium becomes larger and risk-averse firms have greater incentives to enter the market since entering firms may benefit from a higher price. This positive relationship between entry and uncertainty under monopolistic competition is a novel result with respect to the previous literature for models in which the number of firms in the market is endogenously determined[15].

5 Welfare analysis

We now consider the price equilibrium under uncertainty from a normative viewpoint. In particular, we examine the impact of marginal cost uncertainty in a free-entry and exit equilibrium in order to know whether uncertainty produces a larger or a smaller variety of brands than the optimal variety level[16].

With respect to the previous literature, we have to account for the additional cost involved in bearing risk since the firms are risk-averse. From the definition of V_i such that $V_i = E(\tilde{\Pi}_i) - \frac{a}{2}Var(\tilde{\Pi}_i) - \overline{f}$, we note that the term $\frac{a}{2}Var(\tilde{\Pi}_i)$ indicates the risk supported

[14]When the degree of risk aversion a is set to 0 (or $\sigma^2 = 0$), we find that the optimal number of firms is $n^* = \sqrt{t\Delta L^2/\overline{f}}$, which is the original result of Salop (1979).

[15]Also, we observe that an increase in the fixed cost value causes a decrease in the number of firms in the market and that a rise in the transportation cost leads to an increase in the profit margin since there is a higher probability of differentiation for firms.

[16]Under certainty, it is well known that private and social incentives do not necessarily coincide and the market is expected to generate too many firms (see Tirole, 1988).

by each firm given the randomness of $\tilde{\Pi}_i$. Using the definition of the profit level $\tilde{\Pi}_i$, we deduce that $Var(\tilde{\Pi}_i) = \Delta^2 L^2 \sigma^2 / n^2$. Thus, the cost of risk bearing by a firm denoted by B_i is given by :

$$B_i = \frac{a}{2} \left(\frac{\Delta L}{n} \right)^2 \sigma^2 \tag{21}$$

We note that this cost increases with the absolute degree of risk aversion a, with the demand intensity L and with the variance of the marginal cost σ^2. Conversely, risk bearing costs are a decreasing function of the number of firms n. The aggregate cost of risk bearing is simply nB_i.

In the spatial model of Salop (1979), the aggregate transportation cost T is :

$$T = 2nt \int_0^{L/2n} \Delta x dx \tag{22}$$

since all consumers purchasing the brand from a firm are located between 0 and $L/2n$ units of distance from that firm. So, the average consumer has to travel $L/4n$ units of distance, which leads to the following aggregate transportation cost :

$$T = \frac{t \Delta L^2}{4n} \tag{23}$$

Now, the problem for the social planner is to minimize the sum of fixed costs paid by the producing firms, aggregate transportation costs and aggregate costs of risk bearing. The social aggregate cost S is then equal to $S = n\overline{f} + T + nB_i$. Formally, the problem for the social planner may be expressed as :

$$\min_n n\overline{f} + \frac{t \Delta L^2}{4n} + \frac{a}{2} \frac{(\Delta L)^2}{n} \sigma^2 \tag{24}$$

Proposition 5 *Under cost uncertainty, the optimal number of firms $\hat{n}$ chosen by an omniscient planner is :*

$$\hat{n} = \sqrt{\frac{L^2}{\overline{f}} \left(\frac{t\Delta}{4} + \frac{a}{2} \sigma^2 \Delta^2 \right)} \tag{25}$$

Proof. Since the problem for the social planner is $\min_n S$, we solve the corresponding first-order condition $\partial S / \partial n = 0$ and obtain :

$$\overline{f} - \frac{1}{\hat{n}^2} \left(\frac{t \Delta L^2}{4} + \frac{a}{2} \sigma^2 (\Delta L)^2 \right) = 0$$

which gives the optimal number of firms $\hat{n}$. *QED*

Corollary 2 *The market generates too many firms at the equilibrium, i.e.* $\hat{n} < n^*$.

When comparing the number of firms chosen by the social planner and the decentralized equilibrium, it follows that :

$$\hat{n} < n^* = \sqrt{\frac{L^2}{\overline{f}}\left(t\Delta + \frac{a}{2}\sigma^2\Delta^2\right)} \tag{26}$$

So, in the free-entry location model, we note that the market generates too many firms at the equilibrium. Clearly, too many brands are produced since firms have too much of an incentive to enter. Of course, such a result also holds in the model of Salop (1979) under certainty. But with respect to spatial differentiation under certainty, we observe that the social planner chooses a higher number of firms in order to achieve an optimal risk-sharing among firms. Increasing the number of firms in the markets leads to an implicit hedging. Finally, when the transportation cost is very low, we find that n^* is approximately equal to $\hat{n}$. In that case, the number of firms only depends on costs involved in bearing risk, and this factor which is equal to $\frac{a}{2}\sigma^2\Delta^2$ is identical in n^* and $\hat{n}$[17].

Since entry of firms is socially justified by the savings in transportation costs and costs of risk bearing, we suggest that there are some policy solutions for the social planner in order to reduce the excessive entry of firms in the market. In particular, any policy designed to decrease the level of risk in industries may be an effective way to regulate the market. Resources devoted to the pooling of industrial risks should significantly contribute to the decline of prices charged by the firms, by lessening the production risk premium supported by consumers when buying the goods given spatial differentiation.

6 Concluding comments

In this paper, we have analyzed a location model to examine the effects of uncertainty in an industry equilibrium. We extend the model of spatial differentiation proposed by Salop (1979) by introducing marginal cost uncertainty and examine the free-entry equilibrium. Accounting for horizontal product differentiation strongly affects the effects of uncertainty on the number of firms in an industry, which is indeterminate in a standard

[17] When $t \to 0$, we get $n^* = \hat{n} = \sqrt{\frac{a}{2}\frac{\sigma^2\Delta^2L^2}{\overline{f}}}$.

framework with homogeneous goods and price uncertainty (Appelbaum and Katz, 1986). Our analysis is a contribution to the recent developments on the theory of oligopolistic firms under uncertainty with differentiated products presented in Asplund (2002).

In our setting, the optimal price charged to consumers includes an additional term corresponding to a measure of the risk premium faced by risk-averse firms, so that the cost of uncertainty is supported by consumers with differentiation. As a consequence, when there are no barriers to entry other than fixed costs, firms have greater profits opportunities and then incentives to enter the market are increased. Finally, comparing the number of goods in a market economy and a social economy indicates that too many brands are produced in a free-entry location model, cost uncertainty having an additional positive impact on the distortion.

A final comment deals with empirical testing. Our framework suggests a positive relationship between cost uncertainty and entry of firms in industries with differentiated products. However, evidence on the effects of uncertainty on the industry equilibrium remains scarce. Using a cross-section of American manufacturing industries, Ghosal (1996) finds that greater price uncertainty has a significant and large negative effect on the number of firms in an industry. Focusing on the intertemporal dynamics of industry structure again for manufacturing firms in the United States, Ghosal (2002) shows that greater uncertainty does not affect large establishments, while it has a negative impact on the number of small firms in an industry (see also Ghosal and Loungani, 2000).

Nevertheless, this observed negative relationship between uncertainty and industry equilibrium should not necessarily be interpreted against our model of spatial competition. For instance, Ghosal (1996) only includes a price uncertainty measure and does not account for cost uncertainty. Asplund (2002) clearly shows that different types of uncertainty may have opposite effects on competition for risk-averse firms in oligopolies. Also, the issue of differentiated products is not specifically addressed in the previous empirical literature. Thus, it would be useful to investigate the effects of uncertainty on the number of firms for markets with differentiated products and significant cost uncertainty. Such markets could be identified with uncertainty measures based on the standard deviations of residuals in price equations for most important inputs. This empirical issue, which could provide valuable information for public policy, is left for future research.

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