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A GAME THEORETIC APPROACH OF WAR WITH FINANCIAL INFLUENCES

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Abstract: During history, an aggressive country seeks to force non-aggressive countries to made many concessions based on military force. In our paper we discuss the situation that one aggressive country is dissatisfied with its current position and try to obtain more concessions from a rival country. To analyze this situation we use a game theory dynamic model in complete and incomplete information. We analyze the countries behavior depending especially on aggressive or non-aggressive strategies and also on battle power. In this context we found conditions to obtain separating and pooling equilibriums for dynamic games in incomplete information. Main result shows that countries behavior depends especially on war costs and on country military power. There are many applications of these types of models, like in Israel - Palestinian war, recent Russian- Georgian conflict or US defense policy.

Keywords: War, Negotiation Game, Repeated Game, Bargaining, Folk theorem, Bounded Rationality
JEL Classification: H56, C73, C78

1. INTRODUCTION

Our paper considers a "negotiation game" which combines the features of two-players alternating offers bargaining and repeated games. The negotiation game in general admits a large number of equilibriums but some of which involve delay and inefficiency. Thus, complexity and bargaining in tandem may offer an explanation for cooperation and efficiency in repeated games.

The Folk Theorem of repeated games is a very used result that shows if players are enough patience then it is possible to obtain a cooperative equilibrium of the infinite repeated game. A few contributions on folk theorem shows that the result survives more or less intact when incomplete (Fudenberg and Maskin, 1986) or imperfect public (Fudenberg, Levine, and Maskin, 1994) information is allowed, or when the players have bounded memory (Sabourian, 1998).

These findings are made precise in numerous *folk theorems*[1]. Each folk theorem considers a class of games and identifies a set of payoff vectors each of which can be supported by some equilibrium strategy profile. There are many folk theorems because there are many classes of games and different choices of equilibrium concept. For example, games may be repeated infinitely or only finitely many times. There are many different specifications of the repeated game payoffs. For example, there is the Cesaro limit of the

[1] The strongest folk theorems are of the following loosely stated form: "Any *strictly individually rational* and *feasible* payoff vector of the stage game can be supported as a subgame-perfect equilibrium average payoff of the repeated game." These statements often come with qualifications such as "for discount factors sufficiently close to 1" or, for finitely repeated games, "if repeated sufficiently many times."

means, the Abel limit (Aumann, 1985), the overtaking criterion (Rubinstein, 1979) as well as the average discounted payoff, which we have adopted. They may be games of complete information or they might be characterized by one of many different specifications of incomplete information. Some folk theorems identify sets of payoff vectors which can be supported by Nash equilibrium; of course, of more interest are those folk theorems which identify payoffs supported by subgame-perfect equilibrium.

In the first part of this paper I present the main folk theorems for finite repeated games.

Finally we present a study-case for negotiation on war condition between one aggressive country and one non-aggressive country and I determine the main equilibrium conditions for static game and for repeated game.

2. LITERATURE

The Folk theorem gives economic theorists little hope of making any predictions in repeated interactions. However, as the aforementioned examples suggest, it seems that negotiation is often a salient feature of real world repeated interactions, presumably to enforce co-operation and efficient outcomes. Can bargaining be used to isolate equilibrium in repeated games?

Busch and Wen (1994) analyze the following game: in each period, two players bargain - in Rubinstein's alternating - offers protocol over the distribution of a fixed and commonly known periodic surplus. If an offer is accepted, the game ends and each player get his share of the surplus according to the agreement at every period thereafter. After any rejection, but before the game moves to the next period, the players engage in a normal form game to determine their payoffs for the period. The Pareto frontier of the disagreement game is contained in the bargaining frontier. The negotiation game generally admits a large number of subgame-perfect equilibrium, as summarized by Busch and Wen in a result that seems to be as the Folk theorem in repeated games.

Considerable effort has gone into introducing considerations that reduce the equilibrium set of a repeated game. For instance, depending on the stage game, the set of equilibrium payoffs is known to shrink by varying degrees when complexity costs are (lexicographically) taken into account (Rubinstein, 1986, Abreu and Rubinstein, 1988, Piccione, 1992, Piccione and Rubinstein, 1993), when strategies and beliefs are restricted to be Turing-computable (Anderlini and Sabourian, 1995, 2001), or when asynchronous choice is allowed (Lagunff and Matsui, 1997).

Obara (2009) proves a folk theorem with private monitoring and communication extending the idea of delayed communication in Compte (1998) to the case where private signals are not correlated.

Olson (1965) was among the first to formally pose the puzzle of group formation and cooperation, and this has provoked a large literature seeking to understand group behavior. Thorsten and Lim (2009) introduce two incentive mechanisms to sustain intra-group cooperation with prisoner's dilemma payoffs. They examine three-agent groups where relations may either be triadic one person interacting with two others/or tripartite, where all agents interact. Due to shirking incentives, sustained group cooperation requires a system of endogenous enforcement, based on punishments and reward structure and they found that both can ensure cooperation.

Fudenberg and Levine (2007) proves a Nash-threat folk theorem when players' private signals are highly correlated. Ashkenazi-Golan (2004) assumes that deviations are perfectly observable by at least one player with positive probability and proves a Nash-threat folk theorem. These results, as well as the result of this note, apply to repeated games with two or more players. Finally, McLean, Obara and Postlewaite (2005) prove a folk theorem when private signals are correlated and can be treated like a public signal once aggregated. But this result requires at least three players.

Also, there is an existing literature that seeks to model institutions and social networks in terms of endogenous enforcement. The use of incentive slackness in triadic relations to tie strategies across two party games or domains, has been studied by Aoki (2001); Bernheim & Whinston (1990) while exogenous superior information or enforcement capability among group members compared to non- group members is used in (Fearon & Laitin 1996; Ghatak & Guinnane 1999). Moreover, such an institutional arrangement may itself be endogenous (Okada 1993).

Fong and Surti (2008) study also the infinitely repeated Prisoners' Dilemma with side payments and they found that Pareto dominant equilibrium payoffs are implemented by partial cooperation supported by repeated payments. That seems to confirm folk theorems for infinitely repeated games.

Benoit and Krishna (1985, 1993) analyze particular folk theorems for finite repeated games. They show that under such hypothesis it is possible to reinforce collusive equilibrium that not require any binding agreements to ensure that players conform. An important example given by Benoit and Krishna show that for constant cost Cournot duopoly with linear demand it is possible to obtain enterprises cooperation if finite repeated game contains enough stages and discount factor is close to 1.

3. THE MODEL

A (one-shot) game, G, in normal or strategic form, consists of a set of n players, the strategy sets of the players, and their payoff functions.
Thus, we define $G = (S_1, S_2,..., S_n; U_1, U_2 . . . , U_n)$, where S_i is player i's strategy space and $U_i : S \to R$ is i's payoff function, where $S = S_1 \times S_2 \times ... \times S_n$. The strategy space is represented by player's offers in negotiation process.

We may also write $U_i : S \to R^n$ as the function whose i-th component is U_i. We will assume that the strategy spaces are compact sets and that the payoff functions are continuous. $G(T)$ denotes the game that results when G is successively played T times (T is a positive integer). Let $\delta_i \in (0, 1)$ be the i'th player discount factor ant T enough large (eventually ∞).

For $t = 1, 2,.. ., T$ if $s_i \in S$ denotes the outcome of the game $G(T)$ at time t, player i's average payoff in $G(T)$ is given by $u_i(s) = \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(s^{*t})$.

A strategy for player i in the game $G(T)$ is a function s_i which selects, for any history of play, an element of S_i. A Nash equilibrium of G(T) is an n-tuple of strategies s^*, such that for all i, and any strategy or for player i:

$$u_i(s^*) = \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(s^{*t}) \geq \frac{1-\delta_i}{1-\delta_i^{T+1}} \sum_{t=0}^{T} \delta_i^t u_i(a^t) \quad (\forall) a \in S .$$

Let $N(T)$ denote the set of Nash equilibrium outcome paths of $G(T)$. We will assume that $N(1)$ is not empty.

Let $\underline{u}_i$ denote player i's minmax payoff and let $m_i \in S_i$ denote a corresponding strategy combination. A payoff vector u is said to be individually rational if for all i: $u_i > v_i$. Again, for the game G, consider the set of all payoff vectors which may result from players' choices (the range of the function U). The convex hull of this set, denoted by F, will be called the feasible region of payoffs. Note that in both G and $G(T)$, we are restricting attention to pure strategies only. The effect of this restriction is that minmax payoffs, which will play a significant role in what follows, may be higher than those attainable using mixed strategies.

The notion of a subgame perfect equilibrium is made precise as follows:

Definition: The strategy profile a is a (subgame) perfect equilibrium of *G(T)* if (i) it is a Nash equilibrium of *G(T),* and (ii) for all *T' < T* and all *T'* period histories h (*T'*), the restriction of *s* to *h(T')* is also a Nash equilibrium of *G(T –T').*

We suppose there exist in our negotiation game three different types of solutions: minmax equilibrium, corresponding to a punishment situation, a cooperation solution and a deviation situation. The relationships between the payoffs of these three strategies are: deviation payoff is greater than cooperation payoff that is greater than minmax payoff.

First case: the three phases of the game are:

- Cooperation phase (*T'* periods) from $t = 0$ to $t = T' - 1$, with cooperation payoffs;
- Deviation phase – one period – for $t = T'$: with deviation payoff for the player that deviate;
- Punishment phase starts from $T' + 1$ phase and continue all the game for the player that deviate from cooperative strategy

The variables:

- v_i – cooperative payoff;
- v_i^D – deviation payoff;
- $\underline{u}_i$ – minmax payoff/punishment payoff ;
- Relationships: $v_i^D \geq v_i > \underline{u}_i$;
- $\underline{\delta}$ - minimum discount factor to cooperate;
- δ_i - player *i* discount factor.
- a parameter $A = \dfrac{u_i^D - v_i}{v_i - \underline{u}_i}$ that shows the relative gap between deviation from cooperation payoff and punishment payoff.
- *T* is the number of game stages and *T'* is the stage where player *i* deviates from cooperative phase.

Infinitely repeated games

First we consider the situation of infinitely repeated game ($T = \infty$). Game solution of infinitely repeated game result from next theorem:

Theorem 1. Folk Theorem

Let *G* be a static, finite game of complete information and *G(∞)* the infinitely repeated game. Let $\underline{u}_i$ the minmax payoff of G for any player *i*, so for any payoff vector v so that $v_i > \underline{u}_i, (\forall) i$, there exists a minimum level of discount factor $\underline{\delta} < 1$, such that $(\forall)\delta \in (\underline{\delta}, 1)$ there exists a subgame perfect Nash Equilibrium that achieves *v* as average payoff. (see proof in Appendix)

This theorem show as also some interesting findings related to player's behavior:

The minimum level of discount factor such that the cooperation strategy depend on relative gain from deviation related on punishment possible to be implemented. Starting on these hypotheses we proof the following results:

- If deviation payoff is close to cooperation payoff then players cooperates in every period of the game;
- If cooperation payoff is close to punishment (minmax) payoff, then cooperative situation is not possible;
- If deviation payoff is very large, then player's cooperation is not possible for any period of the game.

Corollary 1. If there exist a minimum level for discount factor $\underline{\delta}$, then $\underline{\delta} = \dfrac{u_i^D - v_i}{u_i^D - \underline{u}_i}$. (1)

This corollary shows the discount factor depends on deviation payoff, cooperation payoff and punishment payoff.

Corollary 2. If deviation payoff is close to cooperation payoff ($\boldsymbol{u_i^D} \to \boldsymbol{u_i^C}$) then $\underline{\delta} \to 0$ and players cooperates in every period of the game.

Corollary 3. If cooperation payoff is close to punishment payoff ($\boldsymbol{u_i^C} \to \boldsymbol{u_i^P}$), then $\underline{\delta} \to 1$ and cooperative situation is not possible.

Corollary 4. If deviation payoff is very large, ($\boldsymbol{u_i^D} \to \infty$), then $\underline{\delta} \to 1$ and players cooperation is not possible for any period of the game.

4. STUDY-CASE: A BARGAINING APPROACH OF WAR

Economics game models involving goods or money allow assumptions about continuity, risk aversion, or conservation of the total after a division. More theorists have often used 2 x 2 matrices and looked at only the ordinal proprieties of the payoffs to avoid having to justify specific values. Rapaport and Guyer's taxonomy (1966) of 78 ordinal games without tied payoffs has been used repeatedly. The most known study is by Synder and Diesing (1977) who set up 2 x 2 matrices for sixteen historical situations. Other examples are Maoz (1990) on Hitler's expansion and Syria/Israel interactions. One extensive form example on the Falkland War is made by Sexton and Young (1985) who add the interesting analysis of the consequences of misperception. Another example of moving beyond 2 x 2 matrices is Zagare's 3 x 3 x 3 game on Vietnam War negotiation.

The model

In our model, one country, which we refer to as the aggressive country (player A), is dissatisfied with the status quo and seeks concessions from its rival, which we refer to as the non-aggressive country (player NA). In every period, the aggressive country can either forcibly extract these concessions via war, or it can let the non-aggressive country peacefully make the concessions on its own. While peaceful concession-making is clearly less destructive than war, there are two limitations on the extent to which peaceful bargaining is possible. First, there is limited commitment. Specifically, the non-aggressive country cannot commit to making a concession once it sees that the threat of war has subsided. Moreover, the aggressive country cannot commit to peace in the future in order to reward concession-making by the non-aggressive country today.

Also, we include a steady state situation **(SSS)** such that the aggressive county is forced by international regulations to not attack the non-aggressive country. In this care both payoffs will be zero.

First, we define the strategies for each country as follows: for country aggressive it can attack (**W** strategy) the non-aggressive country or let peacefully the player B (**NW** strategy). For non-aggressive country, in can fight back (**FB** strategy) or it may surrender (**S** Strategy).

Starting on these strategies we define the payoffs for each country as follows:

- GR represents the gain of aggressive country from keeping his reputation if attack the non aggressive country;
- GW represents the gain of aggressive country from non aggressive country after the war (that is a cost for non-aggressive country);
- CWA represents the financial cost of war for aggressive country;
- LWA represents the cost for aggressive country due on life loss during the war if the non-aggressive country fight back;
- GC represents the gain of aggressive country if the non-aggressive country surrender with no war and make initial concessions (that is a cost for non-aggressive country);
- CWNA represents the costs of fighting back for non-aggressive country;
- LWNA represents the cost for non- aggressive country due on life loss during the war if the non-aggressive country fight back;
- All these variables are positives;
- We suppose also that in the case of the war, the aggressive country win;
- The gain for aggressive country in the case of war (GW) is greater like its gain in the case of non aggressive concessions, GC.

The payoff matrix of this game is depicted in figure 1.

		Country NA	
		FB	**S**
Country A	**W**	**GR+GW-CWA-LWA, -CWNA-LWNA-GW**	**GR+GC-CWA, -GC**
	NW	**0, -CWNA**	**GC, - GC**

Figure 1.

Case 1. The static game

The possible equilibrium for this game are described as follows:

Corollary 1. If the reputation gain for aggressive country is smaller like aggressive country cost of war **(GR < CWA)** and the concession loss for non aggressive country is smaller like the cost of war for non-aggressive country **(GC < CWNA),** then the equilibrium is **(NW,S),** respectively the aggressive country do not attack and the non-aggressive country surrender.

Corollary 2. If the reputation gain for aggressive country is greater like aggressive country cost of war **(GR > CWA)** and the concession loss for non aggressive country is smaller like the cost of war for non-aggressive country **(GC < CWNA),** then the equilibrium is **(W,S)**, respectively the aggressive country attack and the non-aggressive country surrender.

Corollary 3. If the reputation gain for aggressive country is smaller like aggressive country cost of war **(GR < CWA),** the concession loss for non aggressive country is greater like the cost of war for non-aggressive country **(GC > CWNA),** and **GR+GW-CWA-LWA < 0**

then the equilibrium is **(NW,FB),** respectively the aggressive country do not attack and the non-aggressive country prepare to fight back.

Corollary 4. If the reputation gain for aggressive country is greater like aggressive country cost of war **(GR > CWA),** the concession loss for non aggressive country is greater like the cost of war for non-aggressive country **(GC > CWNA),** and **GR+GW-CWA-LWA > 0** then the equilibrium is **(W,S),** respectively the aggressive country attack and the non-aggressive country surrender.

Corollary 5. If the reputation gain for aggressive country is smaller like aggressive country cost of war **(GR < CWA),** the concession loss for non aggressive country is greater like the cost of war for non-aggressive country **(GC > CWNA),** and **GR+GW-CWA-LWA > 0** then the equilibrium only in mixed strategies *((p₁,p₂), (q₁,q₂),* where:

$$(p_1,p_2)=\left(\frac{GC-CWNA}{LWNA+GW},\frac{LWNA+GW-GC+CWNA}{LWNA+GW}\right) \text{ and}$$

$$(q_1,q_2)=\left(\frac{GR-CWA}{LWA+GW},\frac{LWA+GW-GR+CWA}{LWA+GW}\right)$$

Comment 1. If the gain of war increase, then we the attack probability for aggressive country is closer to 1 and also the surrender probability for non-aggressive country is closer to 0.

Comment 2. In any mixed strategy solution the concession gain (GC) does not appear.

Comment 3. If the cost for aggressive country due on life loss during the war (LWA) is large, then the probability fight back for non-aggressive country is close to 0.

Comment 4. Only in the case described in Corollary 3 it is possible to impose the steady state solution. For any different situation the aggressive country will threat the non aggressive country.

Case 2. The repeated game

We consider now the previous game, depicted in figure 1, where our players repeat this game finitely of infinitely.

For the cases described in Corollary 1,2,4 and 5 it is not possible to implement a cooperative equilibrium (the steady state solution), respectively the players will play each stage of the game the equilibriums depicted previously.

So, we consider only the case of Corollary 3, which the conditions: the reputation gain for aggressive country is smaller like aggressive country cost of war **(GR < CWA),** the concession loss for non aggressive country is greater like the cost of war for non-aggressive country **(GC > CWNA),** and **GR+GW-CWA-LWA < 0.** The equilibrium of stage game is **(NW,FB),** with the payoffs **(0, -CWNA)**

For this case we determine the discount factor for aggressive country to not attack the non aggressive country.

The cooperative solution for non-aggressive country is **(NW,FB),** with the payoffs (0, - CWNA). The punishment solution of the game is (NW, S), with the payoffs **(GC, -GC).** The deviation solution is for non-aggressive country to implement the steady state solution, with the payoffs **(0,0).**

In terms of the first part of the paper, we have:

- the deviation payoff: $u_i^D = 0$;
- the cooperative payoff: v_i = -CWNA;
- the punishment payoff: $\underline{u}_i = -GC$.

The discount factor such that the non-aggressive country accepts the cooperative solution is:

$$\underline{\delta} = \frac{u_i^D - v_i}{u_i^D - \underline{u}_i} = \frac{0 + CWNA}{0 + GC} = \frac{CWNA}{GC}$$

Comment: If the cost of defense for non-aggressive country is small (close to 0) then it is possible to implement the cooperative solution. If the cost of preparing war is closer to the cost of concession, then it is very difficult to achieve the cooperative solution and the non-aggressive country will choose the deviation solution, respectively to implement the steady state equilibrium.

5. CONCLUSIONS

In this paper I present a theoretical study that analyzes the behavior of aggressive and non aggressive countries in conflict situations. The model solutions indicate that the aggressive country prefers in 2 cases from 5 to attack and in other two cases to let peaceful the non aggressive country. In one case I obtained a equilibrium in mixed strategies

For the nonaggressive country the preferred solution is to surrender (three cases from five), in one case to fight back and in another case a equilibrium in mixed strategies.

Also, based on folk theorem I found the conditions for cooperation between countries if the game is infinitely repeated. In this case if the cost of defense for non-aggressive country is relatively small, then it is possible to obtain a cooperative behavior.

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APPENDIX.

Proof of Theorem 1 (Folk Theorem).

Suppose that there exists a pure strategy such that $u(a) = v$ (with $v > \underline{u}$) and every player will play next strategy*: „ I will play a_i at stage 0 and I will continue to play a_i such time previous period all players played a. Anywhere I'll play minmax strategies for the rest of the game."* How it is this possible for player i to improve his payoff playing this strategy?

We suppose also there exists a deviation payoff, $v_i^D = \max_a u_i(a) >$ v_i. So $v_i^D \geq v_i > \underline{u_i}$.

Player i will play a_i for t periods with v_i payoff, then deviate, and his payoff will be $v_i^D = \max_a u_i(a)$, and for the rest of the game all other players will punish player i and he will receive minmax payoff $\underline{u_i}$.

So average deviation payoff at t stage is:

$$u_D = (1-\delta_i^t)u_i + \delta^t(1-\delta)\cdot v_i^D + \delta^{t+1}\underline{u}_i$$

This payoff is greater like v_i as long as discount factor δ_i is smaller like a minimum level of discount factor $\underline{\delta}_i$, given by relationship:

$$(1-\underline{\delta}_i)\cdot v_i^D + \underline{\delta}_i \cdot \underline{u}_i = v_i$$

So $\underline{\delta}_i = \dfrac{v_i^D - v_i}{v_i^D - \underline{u}_i}$.

Let $\underline{\delta} = \max_i \underline{\delta}_i$. So there exists a minimum level of discount factor $\underline{\delta} < 1$, such that $(\forall)\delta \in (\underline{\delta}, 1)$ there exists a subgame perfect Nash Equilibrium that achieves v as average payoff. q.e.d.