Categorized production function: a note

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Categorized Production Function: A Note

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ABSTRACT

On the basis of the current production function, this paper categorizes the industries in an economy into the traditional and emerging industries, builds a categorized production function (**CPF**) and defines the innovation efficiency. We can repeat all the researches about original production function by the categorized production function, and will get many different results.

Key Wards: economic output; traditional and emerging industries; categorized production function (CPF); innovation efficiency

***JEL* classification code**: E23; O40; O47

1. Introduction

Although the traditional industres are still the fundamental industries to support economic growth, emerging industries are or will be the active industries to promote economic growth.

We can analyse the output of traditional industries and emerging industries by current production function. But, traditional industries have many different characteristics with those of emerging industries, for example, technical progress growth rate, so their production function should be different. The studies in this aspect seem not to be found much.

2. Categorized Production Function

The industries in an economy can generally be categorized as traditional and emerging. Traditional industries are those that mostly involve labor and basic manufacturing, whereas emerging industries are those that mostly involve new science and technology. Traditional industries require large quantities of labor and equipment; these resources constitute the essential foundation of the industry. In a traditional industry, capital often takes a material form (e.g., in production equipment or buildings), whereas labor is produced by workers with standardized skills and effort. Technical progress is measured by the technologies that are used in production equipment, processing techniques and products. Traditional industries usually employ advanced processing techniques and complete equipment systems and enjoy stable product markets. Traditional industries often require a higher cost of capital and better technology. Furthermore, their technology levels tend to remain stable for long periods of time. In contrast, powerful technology are fundamental to emerging industries. In an emerging industry, capital may take a material or immaterial form; it may include equipment, patents, software, intangible assets and workers with standardized professional skills and effort levels. Technology develops rapidly in emerging industries, and technical level tends to change relatively rapidly.

For the sake of convenience, the capital input is the value of the capital required for production, the labor input is the number of workers required for production and the technology input is the funds required for the research and development of technologies for production. Thus, the production function (Solow, 1956 & 1957; Barro & Sala-i-Martin, 1995) for an economy can be expressed as follows:

$$Y=A\cdot F(K, L)$$

where Y is the output of the economy, K is the capital, L is the labor and A is the technology level. For given quantities of capital and labor, the increase in the technology level will yield an increase in output. Thus, economies with higher technology levels have greater productivity efficiency.

In reality, because capital, labor and innovation change over time, $K=K(t)$, $L=L(t)$, $A=A(t)$. Technical progress $A(t)$, because it changes with time, can also be expressed as $\frac{dA(t)}{dt}=h(t)$ if it is differentiable; thus, $A(t)=\int h(t)dt=H(t)+a$ with a as a

constant. Therefore, the output of an economy can be expressed as

(1) $$Y=Y_1+Y_2,$$

where $Y_1= a\cdot F[K(t), L(t)]$, $Y_2=H(t)\cdot F[K(t), L(t)]$.

In Model (1), the technology level of the output $Y_1=a\cdot F[K(t), L(t)]$, a, is a constant, which signifies that the level of technology is stable, as in traditional industries. Therefore, Y_1 represents the output of traditional industries. The technology level of the output $Y_2=H(t)\cdot F[K(t), L(t)]$, $H(t)$, is a time function, which signifies that the level of technology is variable, as in emerging industries. Therefore, Y_2 represents the output of emerging industries. Model (1) is called the categorized production function (CFP) for traditional and emerging industries, and $A(t)=H(t)+a$ is the categorized total factor productivity (CTFP). ***CFP*** (1) indicates that traditional industries have two input factors, capital and labor, whereas emerging industries have three input factors: capital, labor and innovation. Further, ***CFP*** (1) can also be concisely expressed as follows:

(2) $$Y=\mu+\sigma$$

where $\mu=a\cdot F[K(t), L(t)]$ is the production function of traditional industries, $\sigma=q(t)\cdot\mu=$ $=q(t)\cdot a\cdot F[K(t), L(t)]$ is the production function of emerging industries and $q(t)=H(t)/a$ is the ratio of the technology level of emerging industries to that of traditional industries, indicating the degree of innovation of the former, referred to as innovation efficiency, or simply as innovation. Innovation efficiency is a dimensionless quantity. It embodies the advantage and the production efficiency of emerging industries on traditional industries.

Generally, traditional industries and emerging industries have different capital and labor requirements. ***CPF*** (1), however, shows that the input factors of both traditional and emerging industries stem from the economy's overall capital and labor. This finding can be explained as follows. Each unit of capital can be divided into two parts, with one used for traditional industries and another for emerging industries. Similarly, each unit of labor can have two skill types: one applicable to traditional industries and another applicable to emerging industries. Thus, capital and labor can flow between traditional industries and emerging industries. When the production efficiency of emerging industries increases, capital and labor will flow toward those industries.

CPF (1) or (2) points out, for an economy, a part of its total output comes from the traditional industries, and the rest comes from the emerging industries. If emerging industries is low in innovation efficiency, economic output is mainly contributed by traditional industries. If emerging industries is higher in innovation efficiency, economic output is mainly contributed by emerging industries. At this time, the economy should be a developed one, because its growth is mainly promoted by innovations.

3. Discussion

There are many outstanding researches based on current production function (1),

including economic growth analysis. By ***CPF*** (1) or (2), we may get a lot of different results, because differences between the traditional industry and emerging industry. We do not discuss them fully here, just list a small part of them. For example:

(i) Solow Residual. According to ***CPF*** (1), we have:

- Due to $\Delta a/a = 0$, there is no Solow residual in traditional industries.
- Due to $\Delta H/H \neq 0$, there is Solow residual in emerging industries.
- Due to $\Delta H/(H+a) \neq 0$, there is Solow residual in an economy.

There are Solow residual in some industries, and There are no Solow residual in others, the result should be more reasonable.

(ii) Growth accounting equation. According to Cobb-Douglas production function and Solow growth model, we have $Y=A\cdot F[K(t), L(t)]= A\cdot K^{\alpha}L^{1-\alpha}$, and growth accounting equation is as follows:

$$\Delta Y/Y = \Delta A/A + \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot(\Delta L/L)$$

According to ***CPF*** (2), growth accounting equation can be divided as:

- growth accounting equation of traditional industry is:
  $$\Delta Y_1/Y_1 = \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot(\Delta L/L)$$
- growth accounting equation of emerging industry is:
  $$\Delta Y_2/Y_2 = \Delta H/H + \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot(\Delta L/L)$$
- growth accounting equation of economyis:
  $$\Delta Y/Y = \Delta H/(H+a) + \alpha\cdot(\Delta K/K) + (1-\alpha)\cdot(\Delta L/L)$$

Industries are generally different in their technical progress levels. The productivities of some industries are stable in a longer period, and others changes quickly, this is a fact. Therefore, it is more reasonable to describe the industrial productivities and their growth accounting equation in different way. So the division above may be valuable.

So we see, many conclusions may be different for traditional industries and emerging industry.

Although ***CPF*** (1) and (2) are right in logic, they need to be empirically researched further.

Reference

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