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Abstract: This paper accounts for work sharing and unemployment in an efficiency wage model. The Solow condition holds when working hours are exogenous. Under the assumption of endogeneity and using general forms for the effect and cost functions, we prove that work sharing may have a reducing impact on unemployment.

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1. Introduction

According to the efficiency wage literature, there exists a direct and increasing relationship between the wage paid by firms and the level of effort provided by workers (see Akerlof and Yellen, 1986). In equilibrium, firms may find it profitable to pay wage in excess of market clearing. Because of the impact of the wage setting on the workers' effort, profit-maximizing firms are expected to set an optimal wage such that the elasticity of effort with respect to wage is equal to one. This result is known as the Solow condition: each firm hires labor up to the point where the marginal product is equal to the efficiency wage (Solow, 1979).

Unfortunately, several studies have suggested that the Solow condition does not hold in general. Although Akerlof and Yellen (1986, pp. 14-16) suggest that the effort-wage elasticity should be less than one, one can easily show that the elasticity with respect to wage may be greater or lower than one in many situations.

On the one hand, one can modify some basic assumptions of the standard framework. For instance, Lin and Lai (1994) consider an intertemporal maximizing framework with turnover costs, Faria (2000) combine the shirking and the turnover models of efficiency wage with the possibility of managerial supervision, Jellal and Zenou (2000) assume that workers accumulate a stock of knowledge that allows them to increase their effort, and Jellal and Wolff (2003) consider a dual labor markets model in which only the primary sector requires the presence of an efficiency wage, the secondary sector being competitive. On the one hand, one can use more general forms for the production function, as do Rasmaswamy and Rowthorn (1991).

In their contribution on unemployment, Layard et alii (1991, chapter 3) that the efficiency wage theory is useful to explain both the stationary equilibrium level of unemployment and the dynamic path of non-inflationary unemployment after a shock[1]. Jellal and Zenou (1999) introduce the quality of job matching in an efficiency wage model. When the quality of the match is perfectly observable, the equilibrium unemployment level is due to both high wages and mismatch. Conversely, when job matching is unobservable, firms can either set wages such that the effort-wage elasticity is lower or greater than one. There exist inter-industry wage differences because of differences in job complexity and thus in the quality of the job matching.

[1] Concerning the occupational structure of unemployment, efficiency wages explain why there exist job queues, with less unemployment for skilled workers. Concerning its persistence, a supply shock reduces productivity, so that the profit-maximizing wage is expected to rise relative to productivity.

From a public policy perspective, Layard et alii (1991) examine how unemployment could be reduced in developed countries. Drawing on the efficiency wage model, they prove that a work-sharing policy is expected to be counter-productive. At first sight, redistributing the available work to more people would allow to allocate a given amount of work more efficiently. Unfortunately, the available work is not a given, and this is called by the authors the 'lump-of-output' fallacy (Layard et alii, 1991, p. 502-505). In particular, they show that the equilibrium unemployment rate is expected to be independent of hours of work[2]. From an empirical perspective, time-series regressions for 19 0ECD countries indicate that hours do not affect the relation between wage pressure and the unemployment rate. Countries which have reduced hours most are also those where unemployment has grown most, so that shorter working hours is not an efficient way to reduce unemployment.

In this paper, we further examine the relationship between unemployment and working hours in the context of an efficiency wage model. We show that the setting of Layard et alii (1991) is restrictive. With endogenous working hours and general forms for the production and cost functions, their conclusion is no longer relevant and we show that work sharing may have a reducing impact on unemployment. Our approach encompasses their model as a special case, and it may help to understand why there exist country-differences in the relationship between the decrease in working hours and the rise in unemployment (Layard et alii, 1991, figure 3, p. 505).

The remainder of the paper is organized as follows. In section 2, we describe the model of Layard et alii (1991) which introduces working hours in an efficiency wage model. In section 3, we extend their model and examine under which conditions work sharing may have a reducing impact on unemployment. Section 4 concludes.

2. The efficiency wage model with exogenous working hours

Drawing on Layard et alii (1991, chapter 3), we consider an augmented formulation for the aggregated effort function, which depends on the firm's relative wage and on unemployment. Each worker produces $e(w_i / w_0, u)$ units of effort, where w_i is the level of wage in the firm i, w_0 is the reservation wage that a worker could expect to receive elsewhere, and u is the level of

[2] A reduction in working hours along with a higher level of employment will exert a positive pressure on wages, and this will have in turn a negative impact on employment.

unemployment. We have $e_1 > 0$, $e_2 > 0$, $e_{11} < 0$ and $e_{12} < 0$, meaning that a high level of unemployment is expect to raise effort and also to reduce the impact of wages upon effort.

The efficiency wage hypothesis is relevant in the productive sector and there are job rationing and voluntary payments by firms of wages in excess of market clearing. The output is a function of labor efficiency units, which are defined as the product of effort and employment. The profit function for the firm is given by :

$$\Pi_i = F_i\left(e\left(\frac{w_i}{w_0}, u\right)N_i H\right) - w_i N_i H \tag{1}$$

where $F_i(.)$ is the production function of the firm, with the standard assumption of concavity ($F_i' > 0$, $F_i'' < 0$), N_i is the number of workers in the firm, and H is the exogenous number of hours per worker. The problem for the firm is then to maximize the profit function (1) with respect to w_i and N_i. From the first-order conditions given by $e'(.)HN_iF_i'(.)/w_0 = N_iH$ and $e(.)HF_i'(.) = w_iH$, we get the following result :

$$\frac{e'\left(\frac{w_i}{w_0}, u\right)}{e\left(\frac{w_i}{w_0}, u\right)}\frac{w_i}{w_0} = 1 \tag{2}$$

which is the Solow condition. As pointed out by Solow (1979), the efficiency wage minimizes the employer's wage cost per effective units of service employed and each firm hires labor up to the point where the marginal product is equal to the efficiency wage.

Let us now turn to a general equilibrium approach, so that unemployment has to be high enough to stop the firm setting excessive wage. This implies that $w_i = w_0$. Following the notation of Layard et alii (1991, p. 151-152), the equilibrium unemployment $u*$ is given by :

$$e_1(1, u*) = e(1, u*) \tag{3}$$

As demonstrated by these authors (p. 503), working hours do not affect the natural rate of unemployment. Understanding why work sharing is inefficient is simply due to the fact that hours of work do not affect the desired wage mark-up[3].

[3] Layard et alii (1991, pp. 503-504) also note that a similar result holds under bargaining. Unemployment is again independent of hours, since hours do not affect the wage mark-up.

3. The model with endogenous working hours

With respect to the previous case, we make the following changes. First, the number of working hours is now endogenous. Second, we rely on general forms for both the technology and the cost functions. As pointed out in Akerlof and Yellen (1986, pp. 14-15), the Solow condition depends on a production function of the sort $F(eN)$, while other plausible production functions are expected to have a lower wage-equilibrium wage elasticity[4]. We also prove that in our framework, the efficiency wage can be greater, equal or lower than the standard one. Also, assuming that the firm bears a general cost function seems not unrealistic (see Oï, 1962).

Hence, the maximization program for the firm i is :

$$\max_{w_i, N_i, H} \Pi_i = F_i\left(e\left(\frac{w_i}{w_0}, u\right), N_i, H\right) - C(w_i, N_i, H) \tag{4}$$

where $C(.)$ is the cost function which is continuous, twice differentiable and concave. The corresponding first-order conditions are given by :

$$e'(.)F_e(.)/w_0 = C_w(.) \tag{5}$$

$$F_N(.) = C_N(.) \tag{6}$$

$$F_H(.) = C_H(.) \tag{7}$$

According to (5), the marginal benefit of adjusting wages is equalized with its marginal cost, which is the optimal condition for wage setting. Equalities (6) and (7) respectively indicate that the firm hires labor up to the point where the marginal cost of labor is equal to its marginal revenue and that the marginal benefit of the endogenous working hours is equal to its marginal cost. Using (5), (6) and (7), we easily obtain:

$$\frac{F_e e'/w_0}{F_N + F_H} = \frac{C_w}{C_N + C_H} \tag{8}$$

so that after some manipulations, the optimal efficiency wage can be expressed as :

$$\frac{eF_e}{F}\frac{e'}{e}\frac{w}{w_0} = \frac{H(wC_w/C)(NF_N/F) + N(wC_w/C)(HF_H/F)}{H(NC_N/C) + N(HC_H/H)} \tag{9}$$

[4] Rasmaswamy and Rowthorn (1991) have clearly shown that the Solow condition does not hold with a general production function.

For the notation, we define the following elasticities : $\eta_e = eF_e / F$, $\eta_N = NF_N / F$ and $\eta_H = HF_H / F$ represent the elasticity of the production with respect to effort, to employment and to working hours ; $\gamma_w = wC_w / C$, $\gamma_N = NC_N / C$ and $\gamma_H = HC_H / C$ are the elasticity of the cost function with respect to wage, to employment and to working hours ; $\varepsilon_w = e' w / ew_0$ is the elasticity of effort with respect to wage.

Proposition 1. *The effort-wage elasticity with endogenous working hours is given by :*

$$\varepsilon_w = \frac{\gamma_w}{\eta_e} \frac{(H\eta_N + N\eta_H)}{(H\gamma_N + N\gamma_H)} \tag{9}$$

Before examining the general equilibrium setting, we observe that the Solow condition does not hold with general forms for both production and cost functions. Our results indicate that ε_w can be lower, equal or greater than one depending on the values of the different elasticities.

Corollary 1. *The value of effort-wage elasticity is given by the following equivalence:*

$$\varepsilon_w <> 1 \quad \Leftrightarrow \quad \frac{(H\eta_N + N\eta_H)}{(H\gamma_N + N\gamma_H)} <> \frac{\eta_e}{\gamma_w} \tag{10}$$

Corollary 2. *For a production function of the type $F(eNH)$ and with a linear cost function $C = wNH$, the Solow condition holds.*

In fact, the results presented in Layard et alii (1991) are restrictive since they imply $\gamma_N = 1$, $\gamma_H = 1$, $\gamma_w = 1$, $\eta_N = 1$ $\eta_H = 1$ and $\eta_e = 1$. In this partial equilibrium approach, a less stringent assumption is to consider that the cost function is of the form $C(eNH)$. In that case, the wage-elasticity for effort is higher than one when the elasticity of production with respect to effort is sufficiently low. Hence, the level of wage is expected to be set at a low value, since it is useless to provide incentives for workers to work hard.

Let us turn to the general equilibrium setting. When each firm chooses $w_i = w_0$, we define the function $\Phi(u)$ so that $\Phi(u) = e_1(1,u) / e(1,u)$. Hence, we have :

$$\Phi(u) = \frac{\gamma_w}{\eta_e} \frac{(H\eta_N + N\eta_H)}{(H\gamma_N + N\gamma_H)} \tag{11}$$

With respect to the framework of Layard et alii (1991), we now observe that the level of unemployment depends on the number of working hours, which is endogenous in the model.

Proposition 2. *Work sharing reduces unemployment only if :*

$$\frac{\gamma_H}{\gamma_N} < \frac{\eta_H}{\eta_N} \tag{12}$$

Proof. Let us calculate the derivative du / dH. By differentiating (11), we obtain :

$$\Phi'(u)\frac{du}{dH} = \frac{\gamma_w}{\eta_e}\left(\frac{(\eta_N\gamma_H - \eta_H\gamma_N)N}{(H\gamma_N + N\gamma_H)^2}\right) \tag{13}$$

Given the definition of $\Phi(u)$, we have $\Phi'(u) = (e_{12}e - e_1e_2)/e^2$, which is clearly negative given the underlying assumption $e_{12} < 0$. Hence, we deduce that $\operatorname{sgn} du / dH = \operatorname{sgn}(\eta_H\gamma_N - \eta_N\gamma_H)$, so that the derivative du / dH is positive only if $\eta_H / \eta_N > \gamma_H / \gamma_N$. *QED*

Importantly, our result shows that cost considerations matter to explain the effect of work sharing on unemployment. Any employment variations that are obtained from shorter working hours depend on the elasticities of cost and production with respect to employment and working hours. In particular, when the cost-hours elasticity is sufficiently low, one expects that a decrease in working hours may be useful to reduce unemployment. Otherwise, a negative relationship between the two variables is expected. Finally, the general effort-wage elasticity in Proposition 1 may be helpful to explain differences in the magnitude of the relationship between the decrease in working hours and the rise in unemployment observed for developed countries.

4. Conclusion

Drawing on the model of Layard et alii (1991) who claim that work-sharing is not able to reduce unemployment, we analyze in this paper the question of work-sharing in an efficiency wage model. We show that their result is no longer valid when using general forms for the cost and production functions and endogenous working hours. Our framework generalizes the Solow condition and we prove that a work-sharing policy is not necessarily counter-productive, at least when the cost-hours elasticity is sufficiently low.

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