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Jellal, Mohamed and Garoupa, Nuno

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Nuno GAROUPA*
Universidade Nova de Lisboa
CEPR, London

Mohamed JELLAL†
Toulouse Business School
GREI Université Mohammed V, Rabat

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*Faculdade de Economia, Universidade Nova de Lisboa, Campus de Campolide, P-1099-032 Lisboa, Portugal. Phone: 351-21-3801600. Fax: 351-21-3871105. Email: ngaroupa@fe.unl.pt

†Toulouse Business School, 20 Boulevard Lasrosses, BP 7010, 31068 Toulouse, France and GREI, Centre d'études Stratégiques, Université Mohammed V, Rabat, Morocco. Email: jellalmohamed@yahoo.fr

Abstract

This paper modifies a standard model of law enforcement to allow for learning by doing. We incorporate the process of enforcement learning by assuming that the agency's current marginal cost is a decreasing function of its past experience of detecting and convicting. The agency accumulates data and information (on criminals, on opportunities of crime) enhancing the ability of future apprehension at a lower marginal cost.

We focus on the impact of enforcement learning on optimal compliance rules. In particular, we show that the optimal fine could be less than maximal and the optimal probability of detection could be higher than otherwise. It is also suggested that the optimal imprisonment sentence could be higher than otherwise.

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1 Introduction

Economic theory of law enforcement has been primarily built on static models. Since Becker's (1968) seminal paper, the economic theory of compliance and deterrence has been confined to static analysis even though the importance of dynamics has been recognized long time ago. The fundamental problem of a static model is the exclusion of learning and recidivistic behavior[1], as individuals are allowed to break the law only once.

Once we attend to a dynamic model rather than a static model of compliance, we must consider the source of dynamics. In other words, a fundamental issue is to understand why decisions by potential offenders and by the government at a given period are history path dependent. Different sources of history path dependence have been considered in this literature.

One possible source of dynamics is that the pool of potential offenders at time t depends on the pool of offenders at time $t-1$ because (a) some offenders are detected and punished at time $t-1$ and can no longer commit an offense at time t (e.g., they are imprisoned), (b) the offense will continue until detected, (c) offenders solve an optimal stopping problem by choosing a path of offense rate over a temporal horizon, or (d) gains from illegal activities are path dependent (e.g., criminals learn by doing). Compliance rules at time $t-1$ affect the pool of potential offenders at time t. The government should choose compliance rules at time t that optimally deter offenses at the current period and at future periods.[2]

A second source of dynamics comes from the fact that potential offender's perceptions are determined endogenously by incorporating information available to them. This information is generated within the economy as time goes by.[3]Compliance rules should be based on the learning dynamics. For example, raising the probability of detection increases the number of occasions in which offenders get caught giving them more information about law enforcement.

Essentially the literature has considered 'supply side' dynamics, that is, the path dependence is directly related to potential offenders. In this paper, we address 'demand side' dynamics, which means that the path dependence is related to the enforcement agency instead of the offenders. We incorporate the process of enforcement learning by assuming that the agency's current marginal cost is a decreasing function of its past experience of detecting and convicting. The agency accumulates data and information (on criminals, on opportunities of crime) enhancing the ability to apprehend in the future at a lower marginal cost.

The standard model of law enforcement is modified to allow for learning by doing; the learning curve that has been so much discussed in the literature in industrial organization and business strategy.[4]We focus on the impact of enforcement learning on optimal compliance rules. The basic result is, as in familiar learning curve models, that the "true" current marginal cost is lower. In particular, we show that the optimal fine could be less than maximal and the optimal probability of detection could be higher than otherwise. We also show that the same rationale applies to non-monetary versus monetary sanctions: the optimal fine could be less than maximal and the optimal imprisonment sentence could be higher than otherwise.

The objective of the paper, though, is not limited to the determination of the theoretical conditions that can make learning important for optimal compliance rules. Our analysis is also relevant to explain how learning has affected enforcement in practice.

The importance of learning in designing law enforcement policies has been acknowledged by the US government in the last years. The recently created National Institute for Justice works for the Department of Justice as a research and development center for law enforcement policies.[5]At the same time, many universities have developed departments or research units of Police Sciences which has emerged as an important research field. Even though there has been no systematic empirical research on enforcement and learning, anecdotal evidence shows that areas such as tax or environmental compliance have benefited from enforcement learning.[6]In general, we can say that there are three major contributions to law enforcement from research about convicted offenders:

(a) Use of empirical evidence about detection and apprehension to help enforcement. Examples in the US are the National Archive of Criminal Justice Data or the Justice Information Center; in the UK, the British Crime Survey.[7]

(b) Use of empirical evidence about detection and apprehension to test deterrence theories. The empirical analysis of the economics of crime and the development of empirical criminology and social psychology has contributed to the understanding of criminal behavior.

(c) Use of empirical evidence about detection and apprehension to develop new enforcement tools. The creation and development of DNA data pools is probably the example that springs to mind. However, the US government has developed other programs such as the Crime Mapping Research Center that has facilitated detection and apprehension.

The existence of these research programs motivates our focus on learning by doing in enforcement. Nevertheless, we do recognize that the importance of learning in designing enforcement is still an open empirical question.

The paper goes as follows: in section 2, we present the basic model. In section 3, we discuss some possible extensions to our basic results. Section 4 concludes with final remarks. Proofs of propositions are in Appendix.

2 A Model of Law Enforcement

Consider an economy of risk-neutral individuals who choose whether or not to commit an act that benefits the actor by b and harms the rest of society by h. The policy maker does not know the individual's gain b, but knows the distribution of parties by type described by a distribution $G(b)$ with support $[0, B]$, with a positive density $g(b)$. We allow for the possibility that social harm is less than the maximal gain ($h < B$), that is, not every offense is necessarily socially undesirable.

At time t the government chooses a sanction $f(t)$ and a probability of detection and conviction $p(t)$. The announcement by the government concerning law enforcement policy is made at the beginning of each period becoming common knowledge and is credible.[8]At time t, a risk-neutral individual commits an offense if and only if the benefit outweighs the expected cost, $b \geq p(t)f(t)$. We assume that both the individual's gain b and the magnitude of harm h are time invariant, hence we do not model the temporal behavior of offenders as in O'Flaherty (1998) for instance.

The number of offenders at time t in this economy is given by $1 - G(p(t)f(t))$ when the population is normalized to one.[9] The number of detected and convicted offenders at time t in this economy is given by $n(t) = p(t)(1-G(p(t)f(t)))$. The number of convicted offenders rises initially as the probability rises, then peaks, and eventually declines, as the probability gets high. It always declines with the sanction.

The expenditure on detection and conviction to achieve a probability $p(t)$ is given by $C(E(t))p(t)$, where $C(.)$ is the marginal cost and $E(t)$ is a measure of past enforcement experience. In particular, we assume that:

$$E(t) = (1-\rho)E(t-1) + \rho n(t-1) \tag{1}$$

The measure of past experience is developed over past convictions $n(\tau)$, for $\tau < t$. Our measure $E(t)$ is a weighted sum of past convictions $n(\tau)$ with declining weights given to more distant values, where $\rho > 0$ is the rate of memory. More distant history deterrence events carry less weight in the learning mechanism.

From (1), we can observe that if the number of convicted offenders $n(t-1)$ is greater than the weighted sum of past convictions $E(t-1)$, the measure of enforcement experience increases at period t.[10] Conversely, reducing the number of convicted offenders negatively affects deterrence experience. We could think that with less detection and apprehension, the agency forgets about past experience.

The measure of past experience at moment t declines with the sanction from the previous period $(f(t-1))$ because of the deterrence effect. With respect to the probability of the previous period $(p(t-1))$, the measure of past experience rises for small values of the probability, peaks at certain level, and declines as the probability gets higher and higher. Hence we can conclude that severity and probability of punishment have different effects on enforcement experience.[11]

Regarding the cost function we take the following assumptions: (i) the initial value is strictly positive ($C(0) = C > 0$); (ii) it declines as past experience is accumulated ($C'(E(t)) < 0$); and (iii) it satisfies the usual convexity property ($C''(E(t)) \geq 0$). These assumptions pose that the agency must detect and convict offenders in order to learn, and that the cost declines with the habit of detecting but at a decreasing rate. While the agency's technology (cost) at any point in time displays constant returns to scale, it is characterized by dynamic economies to scale.[12]

In the optimal law enforcement literature, social welfare at time t generally equals the sum of individuals' expected utilities minus the harm caused by offenses minus expenditure on law enforcement[13]:

$$W(t) = \int_{p(t)f(t)}^{B} (b-h)dG(b) - C(E(t))p(t) \tag{2}$$

The monetary sanction is assumed to be costless to impose as conventional in the law enforcement literature, and F is the maximal feasible sanction (e.g., offender's entire wealth).[14]

Suppose each individual lives for two periods only, $t = 1, 2$.[15]The policy maker maximizes the following objective function:

$$\begin{aligned} \mathcal{W} &= W(1) + rW(2) \\ &= \int_{p(1)f(1)}^{B} (b-h)dG(b) - C(E(1))p(1) \end{aligned}$$

$$+ \quad r[\int_{p(2)f(2)}^{B} (b-h)dG(b) - C(E(2))p(2)] \tag{3}$$

where $r > 0$ is the discount rate, $E(1) \geq 0$ is the initial condition, and $E(2) = (1-\rho)E(1) + \rho n(1)$. The maximization problem is subject to the maximal feasible sanction constraint $f(t) \leq F$, for $t = 1, 2$.[16]

If we denote by $\lambda(t)$ the Lagrangean multiplier associated with the maximal feasible sanction at time t, the Lagrangean function $\mathcal{L}$ can be expressed as:

$$\mathcal{L} = \sum_{t=1}^{2} \{ r^{t-1} [\int_{p(t)f(t)}^{B} (b-h)dG(b) - C(E(t))p(t)] + \lambda(t)(F - f(t)) \} \tag{4}$$

Let us define the following elasticity:

$$\epsilon = -E(2)\frac{C'(E(2))}{C(E(2))} \tag{5}$$

which is the elasticity of the marginal cost of enforcement with respect to enforcement experience. We will denote it by elasticity of learning and it is a measure of how sensitive enforcement costs are to learning.

Before presenting our first proposition, we should clarify the meaning of optimal deterrence. Following Polinsky and Shavell (2000), we say that there is complete deterrence if each and every potential offender is deterred, otherwise there is incomplete deterrence. There is perfect deterrence if the expected sanction equals social harm, under-deterrence if the expected sanction is below social harm, over-deterrence if the expected sanction is above social harm. Finally, we have efficient deterrence if the expected sanction maximizes social welfare.

We can now write:

Proposition 1 *At time $t = 2$,*

(a) the optimal fine is maximal, $f(2) = F$;

(b) the optimal probability is given by:

$$p(2) = \frac{h}{F} - \frac{C(E(2))}{F^2 g(.)};$$

(c) some under-deterrence is optimal, $p(2)f(2) < F$.

At $t = 2$ there is no further learning so the classical Becker's maximal fine result always applies, that is, the fine in the second period equals an offender's entire wealth, $f(2) = F$. The optimal probability is adjusted in order to achieve efficient deterrence. As shown previously by Polinsky and Shavell (2000), some under-deterrence is optimal because enforcement costs make perfect deterrence (i.e., expected sanction equals social harm, $p(2)f(2) = h$) too expensive for society.

Proposition 2 *At time $t = 1$,*

(a) the optimal fine can be less than maximal, $f(1) < F$ as long as $\rho > 0$ (strictly positive rate of memory), $r > 0$ (strictly positive discount rate), and $\epsilon > 0$ (strictly positive elasticity of learning);

(b) a less-than-maximal fine is more likely to be optimal if (i) the elasticity of learning ϵ is high, (ii) the discount rate r is high, and (iii) the rate of memory ρ is high;

(c) the optimal probability is given by:

$$p(1) = \frac{h}{f(1)} - \frac{C(E(1))}{f(1)^2 g(.)} - \frac{rC'(E(2))p(2)}{f(1)^2 g(.)} \frac{\partial E(2)}{\partial p(1)};$$

(d) some under-deterrence is optimal, $p(1)f(1) < F$.

A less-than-maximal fine at $t = 1$ is a consequence of enforcement learning being very valuable for the government. Due to the fact that a higher fine diminishes learning (because there are fewer offenders and thus fewer convicted offenders), the government may want to set a lower fine in the first period. The conditions under which setting a lower fine is valuable for the government: (a) when learning affects enforcement costs in an important way (measured by the elasticity), (b) when the government cares for the future (thus the discount rate is high), and (c) when there is learning (thus the memory rate is high). In particular, when $\rho = 0$ (no memory of past experience), $r = 0$ (no discount for the future) or when $\epsilon = 0$ (intertemporal independent enforcement), we have the standard result, that is, the fine equals an offender's entire wealth ($f(1) = F$).

Due to the fact that a higher fine diminishes learning, the marginal social cost of imposing a monetary sanction is no longer zero (as in Becker's model). As it becomes more important, the more likely is an interior solution for the maximization problem.

The effect of learning on the probability is less obvious. The relationship between learning and the probability of conviction depends on if this probability is low or high. If it is small, learning increases with the probability (more offenders are detected and punished thus providing more knowledge to the enforcers). Thus, the government may want to spend more resources on enforcement. However, if it is high, learning decreases with the probability (due to the same deterrence effect that we have described for the sanction). Hence, the government may want to spend less on enforcement.

When the probability in the first period $p(1)$ is low in the absence of learning, the optimal policy will usually be a less-than-maximal fine coupled with a higher-than-otherwise probability. This policy would be the opposite of Becker's result. The reason will be that learning is important and it is easier for enforcers to learn when the fine is lower (there are more offenders to be detected and punished) and the probability is higher because more offenders will be convicted (thus providing more information). This rationale will be more important when learning affects enforcement costs in an important way, when the government cares for the future, and when the memory rate is high.

However, if the probability in the first period $p(1)$ were to be high in the absence of learning, the optimal policy could be a less-than-maximal fine and a lower-than-otherwise probability. The reason is that it is easier to learn when both fine and probability are low because there will be more offenders to be convicted (thus providing more knowledge to the enforcers).

It is shown that some under-deterrence is still optimal. The rationale proposed by Polinsky and Shavell (2000) is reinforced since now both severity and probability are costly. Thus, perfect deterrence is even more expensive for society since it seriously reduces learning (because there will be fewer offenders and thus fewer convicted offenders).

The evolution of the fine as we move from the first to the second period is self-evident from the results obtained previously. As learning is less important, the fine increases until it achieves its maximal value.

The evolution of the probability is not so obvious. The overall result depends on the interaction of two different effects we show in the following proposition and discuss afterwards:

Proposition 3 *(a) The fine is nondecreasing with time;*

(b) The probability is decreasing (increasing) with time if

$$f_1[h - \frac{C(E(2))}{Fg(p(2)F)}] < F[h - \frac{C(E(1))}{f(1)g(p(1)f(1))} - \frac{rC'(E(2))p(2)}{f(1)g(p(1)f(1))}\frac{\partial E(2)}{\partial p(1)}]$$

Marginal Cost Effect

Presumably enforcement is cheaper in the second period (lower marginal cost), thus the probability should increase.[17]The sign and magnitude of this effect is determined by comparing the marginal cost in the second period ($C(E(2))$) with the marginal cost in the first period ($C(E(1))$) plus the marginal reduction in costs due to learning ($rC'(E(2))p(2)\frac{\partial E(2)}{\partial p(1)}$).

The probability at $t = 1$ has a learning effect which is absent at $t = 2$. As explained before, the way this effect plays on the probability depends on the probability being high or low. If the probability in absence of learning is high, we would expect the learning effect to imply a reduction of the probability in the first period (thus, the probability would increase from the first to the second period). If the probability in absence of learning is low, we would expect the learning effect to imply an increase of the probability in the first period (thus, the probability would decrease from the first to the second period).

Marginal Benefit Effect

When determining the optimal probability, it must be taken into account that $f(1)$ can be less than $f(2) = F$. The sign of this effect is determined by comparing the marginal benefit from deterrence in both periods, that is, $F(h - p(2)F)g(p(2)F)$ and $f(1)(h - p(1)f(1))g(p(1)f(1))$ respectively.

On one hand, the probability at $t = 1$ should go up to offset the dilution of deterrence caused by a possible reduction in the fine ($p(1)f(1) \leq p(1)F < h$). On the other hand, the probability at $t = 1$ should go down because the value of detection (the return for the government from investing on deterrence) has decreased: if detected and convicted, an offender pays $f(1) \leq F$. The overall effect depends on the parameters of the model.[18]

The time evolution of the probability depends on how these two effects interplay. Whereas the fine is usually increasing with time, the probability can be decreasing or increasing with time. Intuitively, we would expect the probability to be increasing with time due to the fact that learning presumably makes enforcement cheaper. However there are other aspects to consider, namely how the probability in the first period affects learning and possible changes on the fine, that may just work on the opposite direction.

Summing-up, when learning by enforcers is socially important, we should observe less-than-maximal fines. The overall effect on the probability of conviction however is not clear-cut and depends on how different effects (marginal cost and marginal benefit) interplay.

3 Extensions of the Model

In this section we briefly discuss some possible extensions of our model. The first observation concerns enforcement costs. We have specified enforcement costs as $C(E(t))p(t)$, where $C(.)$ is the marginal cost and $E(t))$ is a measure of past enforcement experience. The agency's technology at time t exhibits constant returns to scale, even though it displays dynamic economies to scale. In making enforcement costs a function of the probability of punishment, we follow Polinsky (1980), Garoupa (1997) and Polinsky and Shavell (2000). This specification has the intuitive property that more effective law enforcement (i.e., catching a greater proportion of criminals) costs more money.[19]

In order to assess the robustness of our results we might consider an alternative specification. We could specify that enforcement costs are a function of the number of offenders punished, rather than the probability of punishment (Friedman, 1993): $C(E(t))n(t)$, where $n(t)$ is the number of detected and convicted offenders when the population is normalized to one. However, such cost function has the counter-intuitive property that more effective enforcement might be cheaper: if more offenders are deterred, fewer offenders are detected and convicted, and yet enforcement costs are lower. We can easily show that our conclusions prevail. The optimal fine can be an interior solution for the same reason as before: a higher fine diminishes learning (because there are fewer offenders and thus fewer convicted offenders).

Our second comment takes into consideration other policy objectives. Following Polinsky and Shavell (2000) and Kaplow and Shavell (2001), maximization of social welfare (including illegal gains) is the most 'reasonable' policy objective, the reason being that this is the only criteria that satisfies the Pareto principle. It could be however that enforcers are opportunistic, and rather than maximizing social welfare maximizing, they maximize their own objective function: Law enforcers can be seen as bureaucrats interested in minimizing their effort. Quite naturally that generates the question of delegating optimal compliance rules (Polinsky, 1980; Friedman, 1984; Boyer, Lewis and Liu, 2000; Garoupa and Klerman, 2002).

Suppose bureaucrats choose enforcement effort (the probability of apprehension), but they do not set the sanction. As Polinsky (1980) and Garoupa and Klerman (2002) have shown, optimal compliance rules can be delegated as long as the government sets an appropriate reward to be paid to enforcers by each detected and convicted offender.[20]Suppose however that enforcers choose both severity and probability of punishment. Friedman (1984) has proposed the following solution: let the government decide the crime rate to be targeted and the enforcers to choose the combination of fine and probability to achieve it. Due to the fact that the fine is costless and the probability is costly in terms of effort, the bureaucrats would implement high fines and low effort. In the absence of learning, if the crime rate to be targeted is efficient, the bureaucrats implement the efficient policy. In a model with learning, Friedman's solution still implements the efficient policy. The existence of learning does not seem to alter the delegation problem.

A third extension to be considered is population dynamics. The behavior of the criminal population is constant in our model. Presumably one of the benefits of apprehension at time $t-1$ is that it reduces the population of criminals at time t. Suppose that as the number of convicted criminals at time $t-1$ goes up, the number of potential criminals at time t goes down.[21]Notice however that this is not a model of optimal incapacitation as for example Shavell (1987): Fines cannot incapacitate. Criminals are removed from the population of potential criminals because they are apprehended, not because they are fined. The situation we envisaged would be the government keeping a record of criminals who have been fined. These records would be used to assure tighter monitoring of these criminals. As a consequence, they would have relatively fewer opportunities to commit a crime.

The effect of convicted criminals at time $t-1$ on potential criminals at time t is quite similar to that of the learning effect. More convicted criminals at $t-1$ means less costly enforcement and fewer potential criminals at time t. The population of potential criminals at t increases with the sanction from the previous period ($f(t-1)$) because of the deterrence effect (fewer criminals and hence fewer convicted criminals at time $t-1$). With respect to the probability enforced on the previous period ($p(t-1)$), the population of potential criminals declines for small values of the probability (because more offenders are convicted on the previous period), achieves a minimum, and increases as the probability gets higher and higher (because of the deterrence effect). Thus, severity and probability of punishment have different effects on the population of potential criminals as before.

Due to the fact that the learning effect and effect on population of potential criminals are analytically similar, the consequences in terms of compliance policy are reinforced. Our preliminary results are very different from those of Shavell (1987), the reason being that in his model a higher sanction at time $t-1$ incapacitates criminals at time t. In our model, more incapacitation is achieved by more detection rather than by more severe punishment.

A final word concerning nonmonetary sanctions. In our model, the possibility of a less-than-maximal fine happens because the marginal cost is no longer zero (due to the fact that higher sanctions diminish learning). In a model such as the one by Kaplow (1990), where nonmonetary sanctions are considered, there is a positive social marginal cost from imposing a sanction. As a consequence, our result should be re-interpreted as suggesting a less-than-otherwise severe sanction because of a dynamic enforcement policy feature.

An interesting extension of the model is to consider that an imprisonment term could also benefit the government in providing information about criminal opportunities. Setting longer imprisonment terms at time $t-1$ reduces the cost of enforcement at time t. Consequently, the optimal imprisonment term at $t-1$ could be higher than otherwise. Polinsky and Shavell (1984) have argued that the monetary sanction should be taken to its highest value, and an imprisonment term should be used as a complement when the maximal fine is not very large. Using the rationale we have provided in this paper, we could have a higher-than-otherwise imprisonment term with a less-than-otherwise severe fine if the future marginal gains from learning more than compensate the current marginal cost of imprisonment.

4 Conclusion

We have shown that a less-than-maximal sanction is possible as a response to a dynamic feature of enforcement policy. The marginal cost of enforcement depends on a measure of learning provided by past enforcement experience. Setting a lower sanction at time $t-1$ provides further gain in reducing the marginal cost of enforcement at time t. A similar rationale has been applied to imprisonment sentences.

Although the model is tailored to flesh out the learning process of the enforcement agency, a more comprehensive analysis should recognize a unified set of instances in which learning may occur, and would generalize our result. In actual enforcement regimes, both the determination of liability and the sentencing process involve the display of expertise on part of decision makers such as juries, judges and attorneys. It can be argued, in the spirit of our article, that it could be socially desirable to increase the number of instances in which individuals are prosecuted, in order to provide the adjudicators with more opportunities to learn how to apply the law, to develop a more fine-tuned set of legal principles, and thereby both reduce adjudication cost in the future, as well as improve the quality of legal rules.[22]

While its theoretical validity is established cleanly in the paper, the extent to which learning by enforcing actually takes place is an empirical open question. In fact, research programs have benefited from collecting data about those criminals who have been detected and eventually convicted. This observation suggests that it has been useful for research to enforce the law with a probability superior to the one predicted by the classical deterrence model or to have criminals for longer periods in jail than prescribed by the economic theory. In this light, our paper suggests a new explanation for these well-known stylized facts.

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Appendix

Proof of Propositions 1 and 2

Let us start by defining the following useful elasticities:

$$\sigma_p = \frac{\partial E(t)}{\partial p(t-1)} \frac{p(t-1)}{E(t)} \frac{1}{\rho}$$

$$\sigma_f = \frac{\partial E(t)}{\partial f(t-1)} \frac{f(t-1)}{E(t)} \frac{1}{\rho}$$

Define the relative cost gain to be:

$$\gamma = \frac{C(E(2))p(2)}{C(E(1))p(1)}$$

Assume that second-order conditions are satisfied (for example, this would be trivially true if the distribution of illegal gains were to be uniform, $g(b) = 1$, $G(b) = b$, and $B = 1$).

The necessary and sufficient conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial p(2)} &= rf(2)(h - p(2)f(2))g(.) - rC(E(2)) = 0 \\
\frac{\partial \mathcal{L}}{\partial f(2)} &= rp(2)(h - p(2)f(2))g(.) - \lambda(2) = 0 \\
\frac{\partial \mathcal{L}}{\partial p(1)} &= f(1)(h - p(1)f(1))g(.) - C(E(1)) - rC'(E(2))\frac{\partial E(2)}{\partial p(1)}p(2) = 0 \\
\frac{\partial \mathcal{L}}{\partial f(1)} &= p(1)(h - p(1)f(1))g(.) - \lambda(1) - rC'(E(2))\frac{\partial E(2)}{\partial f(1)}p(2) = 0 \qquad (6)
\end{aligned}$$

We need to invoke the Kuhn-Tucker conditions to deal with the inequality constraint. We must have $f(t) \leq F$, $\lambda(t) \geq 0$, and $\lambda(t) = (F - f(t))$.

For the second period, the proof is standard. See Garoupa (1997). Notice that $\lambda(2) = rC(E(2))p(2)/F$. Also the efficiency of under-deterrence is clear from $rp(2)(h - p(2)F)g(.) = \lambda(2) > 0$.

Let us now consider the first period. As compared to the second period, there are new terms for $f(1)$ and $p(1)$. Whereas for $f(1)$ it is a marginal cost, it could be a marginal benefit for $p(1)$ given the relationship between severity and probability of enforcement with learning.

In order to show that a less-than-maximal sanction could be optimal, we must prove that the marginal cost of the probability is no longer always superior to the marginal cost of the fine (as in the second period). Suppose $f(1) < F$. It must be the case that $\lambda(1) = 0$ and:

$$p(1)(h - p(1)f(1))g(.) = rC'(E(2))\frac{\partial E(2)}{\partial f(1)}p(2)$$

$$f(1)(h - p(1)f(1))g(.) = C(E(1)) + rC'(E(2))\frac{\partial E(2)}{\partial p(1)}p(2)$$

Using the definitions of elasticities and relative cost gain:

$$(h - p(1)f(1))g(.) = -r\epsilon\gamma\rho\sigma_f C(E(1)/f(1) \tag{7}$$

$$(h - p(1)f(1))g(.) = C(E(1))/f(1) - r\epsilon\gamma\rho\sigma_p C(E(1))/f(1) \tag{8}$$

Putting together (7) and (8) we derive the following expression which is satisfied if the optimal fine is less than maximal:

$$r\epsilon\rho(\sigma_p - \sigma_f) = 1/\gamma \tag{9}$$

In conclusion, if when $f(1) = F$, $r\epsilon\gamma\rho(\sigma_p - \sigma_f) < 1$, then the optimal fine is maximal. Otherwise, the optimal fine is less than maximal and satisfies (9). Thus, a less-than-maximal sanction will be more likely as $r\epsilon\gamma\rho$ is higher, *ceteris paribus.*

From $p(1)(h - p(1)f(1))g(.) = \lambda(1) + rC'(E(2))\partial E(2)/\partial f(1)p(2) > 0$, some under-deterrence is still optimal.□

Proof of Proposition 3

It has been shown that $f(1) \leq F$ and $f(2) = F$. Thus, the fine is time invariant if $f(1) = f(2) = F$ or increasing with time $f(1) < f(2) = F$.

The proof of the second part of the proposition is obtained by comparing $p(1) = \frac{h}{f(1)} - \frac{C(E(1))}{f(1)^2 g(.)} - \frac{rC'(E(2))p(2)}{f(1)^2 g(.)} \frac{\partial E(2)}{\partial p(1)}$ and $p(2) = \frac{h}{F} - \frac{C(E(2))}{F^2 g(.)}$.□

Endnotes

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[1] On recidivistic behavior, see Polinsky and Rubinfeld (1991), Burnovski and Zafra (1994), Polinsky and Shavell (1998), Chu, Hu and Huang (2000), Emons (2003).

[2] See Rubinstein (1980), Davis (1988), Leung (1991, 1995), Nash (1991), O'Flaherty (1998).

[3] See Sah (1991) and Ben-Shahar (1997).

[4] See Cabral and Riordan (1994) and references therein for literature on the learning curve.

[5] There is information available concerning research on law enforcement at the NIJ site, www.ojp.usdoj.gov

[6] As far as we know, there is no estimation of the efficiencies and cost reductions arising from experience in law enforcement. However some empirical examples of the importance of learning to enforcement can be given. See MacDonald (2002) for how the British Crime Survey might provide useful information to enforcement agencies. On tax compliance, see Braithwaite and Braithwaite (2001) and references therein for a description of the Australian Tax Office (ATO) compliance model adopted in April 1998. This approach incorporates research on the different motivational sources of non-compliance (drawing from survey and interview data) and provides useful signals that

are used by the tax authority when making auditing decisions.

[7] In the US, most research in this area has been published by the NIJ Journal and the FBI Law Enforcement Bulletin.

[8] For a discussion about credible enforcement announcements and temporal consistency, see Boadway, Marceau and Marchand (1996). Notice also that we take a normative approach to the problem, thus we do not consider how to motivate law enforcers in order to detect efficiently. On this aspect, see Boyer, Lewis and Liu (2000).

[9] The population is time invariant. See section three for some comments on population dynamics. Notice however that in (3) the interest rate r could be interpreted as the population growth rate.

[10] Notice that $E(t) - E(t-1) = \rho[n(t-1) - E(t-1)]$.

[11]These results are straightforward from:

$$\begin{aligned}\frac{\partial E(t)}{\partial p(t-1)} &= \rho \frac{\partial n(t-1)}{\partial p(t-1)} \\ &= \rho(1 - G(p(t-1)f(t-1)) - p(t-1)f(t-1)g(p(t-1)f(t-1)))\end{aligned}$$

$$\frac{\partial E(t)}{\partial f(t-1)} = \rho \frac{\partial n(t-1)}{\partial f(t-1)} = -\rho p(t-1)^2 g(p(t-1)f(t-1))$$

Whereas more experience is gained with less severe sanctions, more experience is gained with more enforcement if $G(p(t-1)f(t-1)) + p(t-1)f(t-1)g(p(t-1)f(t-1)) < 1$.

[12] The enforcement cost function is standard, see Polinsky and Shavell (2000). In section three, we discuss other possible specifications.

[13] See Garoupa (1997) and Polinsky and Shavell (2000). It is conventional in this literature to include all gains in social welfare. Some argue that the offender's gains should be excluded for moral reasons. However, following Kaplow and Shavell (2001), this is the only 'reasonable' social welfare function. See section three for further considerations.

[14] See section three for some comments on nonmonetary sanctions.

[15] An infinite continuous time version of the model was presented in a previous draft of the paper, Jellal and Garoupa (1999). In order to fully answer the suggestions by the referee and the editor to provide a more comprehensive characterization of the problem, we have decided to present a two-periods' model in the tradition of Rubinstein (1980), Polinsky and Rubinfeld (1991), Ben-Shahar (1997), Polinsky and Shavell (1998), and Emons (2003). Davis (1988) and Nash (1991) consider more than two periods, but essentially stop the game at the first conviction. O'Flaherty (1998) analyzes a multi-period model but in a stochastic context.

[16] Since there is no intertemporal inconsistency, it does not really matter if the severity and probability of punishment to be enforced at $t = 2$ are chosen at the same time or after the choice of severity and probability of punishment to be enforced at time $t = 1$.

[17] Presumably but not surely since $E(2)$ could be less than $E(1)$. Notice that $E(2) - E(1) = \rho(n(1) - E(1))$.

[18] See Garoupa (2001) for a discussion of these two effects. There is a substitution effect and an income effect that altogether generate the counterintuitive result that fine and probability may go down at the same

time.

[19] Also, because our population is normalized to one, our cost function can be interpreted as varying with the population of potential criminals.

[20] See Garoupa and Klerman (2002) for a short summary of the literature and the eventual problems of using rewards to delegate optimal law enforcement.

[21] An explanation has been suggested to us by Tracy Lewis. Think of criminals as fish, who are removed from the population, whenever they are "caught" by enforcers. But the fish population produces new recruits each year so the population of fish never vanishes entirely.

[22] See Shavell (1993) on related observations.