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# Education and Social Returns An Optimal Policy

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## 1. Introduction

This paper is a direct extension of the paper of Jones (2007). This author presents a simple and tractable Mincerian approach to endogenizing schooling time in market economy. His specification is closest to that in Mincer (1958) which does not take into account of social benefits of education. Our short note extends his paper on the social returns to accumulation of human capital, with particular emphasis on the social returns to education which are given by the sum of the private and external marginal benefits of a unit of human capital. In other words, we study the problem of human capital externalities which comes from social interactions. The theoretical literature claims that there are indeed positive educational externalities arising from accumulation of human capital. For instance, according to the market externalities (Lucas, 1988), a high level of average human capital helps to increase the speed of diffusion of knowledge among workers. The main idea underlying these externalities is the contrasting private return to education or the effect of individual school time on individual income with the social return given by the effect of average schooling time on everyone's income.

## 2. The Model

Following Jones (2007), let the aggregate human capital  $H$  be the labor in efficiency units:  $H = hL$ , where  $h$  is human capital per worker and  $L$  is the number of workers. Assume the constant population in a country is distributed exponentially by age and faces a constant death rate  $\delta > 0$ : the density is  $f(a) = \delta e^{-\delta a}$ . We suppose that an individual attending school for  $s$  years

obtains human capital  $h(s, S)$ , where  $S$  is the average human capital externality. A recent literature has recognized the existence of such educational positive externality. Indeed, a number of authors (Lucas (1988), Rauch (1993), Acemoglu and Angrist (2000), Moretti (2003, 2004) have argued that education has large and substantial external benefits.

The representative individual ignores the human capital externality and his problem is to choose school time  $s$  to maximize the expected present discounted value of net income:

$$\text{Max}_s \int_s^\infty (w(t)h(s, S)(1 - \tau) + T(t))e^{-(\delta+\bar{r})t} dt \quad (1)$$

$$T(t) = \tau w(t)h(S, S)$$

$$\text{Ex post } s = S$$

where the base wage  $w(t)$  is assumed to grow exponentially at rate  $\bar{g}$ ,  $\tau$  is rate of income tax and  $T(t)$  is lump sum transfer.

Solving this maximization problem leads to the extended Mincerian return equation:

$$(1 - \tau) \frac{\frac{\partial h(s^*, S)}{\partial s}}{h(s^*, S)} = \bar{r} - \bar{g} + \delta \quad (2)$$

The left side of this equation is the extended standard Mincerian return: the percentage increase in the wage if schooling increases by a year. The first order condition says that the optimal choice of schooling equates the extended Mincerian return to the effective discount rate. In this case, the effective discount rate is the interest rate, adjusted for wage growth and the probability of death. The original Mincer (1958) specification pinned down the Mincerian

return by the interest rate. The generalization here shows the additional role played by economic growth and limited horizons. Rather than being an exogenous parameter, as in the simple version of Bils and Klenow (2000) used by Hall and Jones (1999) and others, the Mincerian return in this specification is related to fundamental economic variables.

### Definition 1

Let the following constant elasticity functions:

$$\epsilon = \frac{\frac{\partial h(s^*, s^*)}{\partial s}}{h(s^*, s^*)} \quad \text{and} \quad \eta = \frac{\frac{\partial h(s^*, s^*)}{\partial S}}{h(s^*, s^*)}$$

This definition helps to rewrite at equilibrium the extended Mincerian return as follows:

$$(1 - \tau) \frac{\frac{\partial h(s^*, S)}{\partial s}}{h(s^*, S)} = (1 - \tau) \frac{\epsilon}{s^*} = \bar{r} - \bar{g} + \delta \quad (3)$$

Therefore we have the following result.

### Proposition 1 (Extended Jones 2007)

Under decentralized economy, the optimal time for schooling is given by the following extended Mincerian Return:

$$s^* = \frac{\epsilon(1 - \tau)}{\bar{r} - \bar{g} + \delta}$$

### Corollary 1

The human capital of labor force in efficiency units is:

$$h^* = \left( \frac{\epsilon(1 - \tau)}{\bar{r} - \bar{g} + \delta} \right)^{\epsilon + \eta}$$

Proof:

If we assume that  $h(s, S) = s^\epsilon \cdot S^\eta$ , where  $S^\eta$  is the schooling time externality size, we obtain the announced result by simple substitution.

Since the agents do not internalize the externality of schooling induced by social interactions, in market economy the human capital of labor force is suboptimal. Indeed, given the assumptions and constraints facing economic agents ex ante, in centralized economy, a benevolent planner recognizes that individuals are identical and that their choice will be the same ex post, he is then leads to internalize the schooling time externality by assuming ex ante that educational choice are identical :  $s = S$ . Hence, in a centralized economy, the optimal human capital of labor force is a solution of the planner's problem. This planner chooses school time  $s$  that maximizes the representative agent's expected present discounted value of income:

$$\text{Max}_s \int_0^\infty (w(t)h(s, S)(1 - \tau) + T(t))e^{-(\delta+\bar{r})t} dt \quad (4)$$

$$T(t) = \tau w(t)h(S, S)$$

$$\text{Ex ante } s = S$$

This program is equivalent to the planner to the following one:

$$\text{Max}_S \int_0^\infty w(0)h(S, S) e^{-(\delta+\bar{r}-\bar{g})t} dt \quad (5)$$

This simple problem leads to the following result.

### Proposition 2

Under centralized economy, the optimal time for schooling is given by the following extended Mincerian Return:

$$S^* = \frac{\epsilon + \eta}{\bar{r} - \bar{g} + \delta}$$

Proof:

The standard optimization leads to the equilibrium condition:

$$\frac{\frac{\partial h(S^*, S^*)}{\partial s}}{h(S^*, S^*)} + \frac{\frac{\partial h(S^*, S^*)}{\partial S}}{h(S^*, S^*)} = \frac{\epsilon + \eta}{S^*} = \bar{r} - \bar{g} + \delta \quad (6)$$

Therefore, from (6) we obtain the announced result:

$$S^* = \frac{\epsilon + \eta}{\bar{r} - \bar{g} + \delta}$$

### Corollary 2

The school time under centralized economy is greater than that chosen under market economy, indeed, we have:

$$S^* = s^* + \frac{\eta + \epsilon\tau}{\bar{r} - \bar{g} + \delta}$$

Proof

Under decentralized economy the schooling time is given by  $s^* = \frac{\epsilon(1-\tau)}{\bar{r} - \bar{g} + \delta}$  while the optimal schooling time is given by  $S^* = \frac{\epsilon + \eta}{\bar{r} - \bar{g} + \delta}$ , then we have the announced result:  $S^* - s^* = \frac{\eta + \epsilon\tau}{\bar{r} - \bar{g} + \delta}$ .

We observe that the distortion from the first best is given by the term  $D = \frac{\eta + \epsilon\tau}{\bar{r} - \bar{g} + \delta}$  which depends on the externality size impact  $\eta$  and the rate of income taxation given by  $\tau$ .

### Corollary 3

The optimal human capital of labor force in efficiency units is:

$$H^* = \left( \frac{\epsilon + \eta}{\bar{r} - \bar{g} + \delta} \right)^{\epsilon + \eta}$$

Proof:

Let  $H^* = h(S^*, S^*) = S^{*\epsilon+\eta}$  then with  $S^* = \frac{\epsilon+\eta}{\bar{r}-\bar{g}+\delta}$ , we have the announced result.

Although there is a distortion in educational allocation, this distortion can be corrected through a taxation subvention policy. Indeed, if available, an optimal policy exists and is given by the following result.

### Proposition 3

The educational market allocation may be decentralized through a subvention taxation scheme given by :

$$\tau^* = -\frac{\eta}{\epsilon}$$

Proof:

We know that schooling time under centralized economy and that which is chosen under market economy are related by the following equation:

$$S^* = s^* + \frac{\eta + \epsilon\tau}{\bar{r} - \bar{g} + \delta}$$

Thus we have  $S^* = s^*$  if and only if  $D = \frac{\eta + \epsilon\tau}{\bar{r} - \bar{g} + \delta} = 0$ , therefore:

for  $(\bar{r} - \bar{g} + \delta) > 0$ , we have  $\eta + \epsilon\tau = 0$  which implies  $\tau = \tau^* = -\frac{\eta}{\epsilon}$ .

From this finding, we note that in order to implement an optimal education policy in the presence of positive educational externalities, we should subsidize the accumulation of human capital, and, the rate of the subsidy depends on the relative size of the externality.

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