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Abstract

The paper argues that when a consumer searches for a lower price, a satisficing decision procedure equalizes marginal costs of search with its marginal benefit. The consumer can maximize the utility of his consumption-leisure choice with regard to the equality of marginal values of search. Therefore, the satisficing decision procedure results in the optimizing consumer behavior.

JEL Classification: D11, D83.

The discussion between the search-satisficing concept and the neoclassical paradigm has a long story. In 1957 H.Simon revived the Scottish word *satisficing* to denote decision making "***that sets an aspiration level, searches until an alternative is found that is satisfactory by the aspiration level criterion, and selects that alternative***". The confrontation between two approaches had reached its peak in 1977 when H.Simon presented his Richard T. Ely Lecture. Then, the discussion went into decline, but from time to time researchers in different fields animated it (see for example Slote (1989), Schwartz et al. (2002), Fellner et al. (2006)). As a result, the theory of consumer behavior has accepted the strict distinction between "maximizers" and " satisficers" (Lewer et al. (2009)) Unfortunately, opponents forget the fact that H.Simon himself paid attention to the possibility of matching the satisficing and optimizing procedures. In 1972 he wrote:

"A satisficing decision procedure can be often turned into a procedure for optimizing by introducing a rule for optimal amount of search, or, what amounts to the same thing, a rule for fixing the aspiration level optimally." (Simon (1972), p.170)

This note tries to restore the methodological equilibrium. The rule for optimal amount of search is derived from the reserve maximization model, which emphasizes the role of the need to save for daily expenses and purchases (Malakhov (2011b). This paper shows how a satisficing decision procedure results in an optimal search-stopping rule and in an optimal consumption-leisure choice.

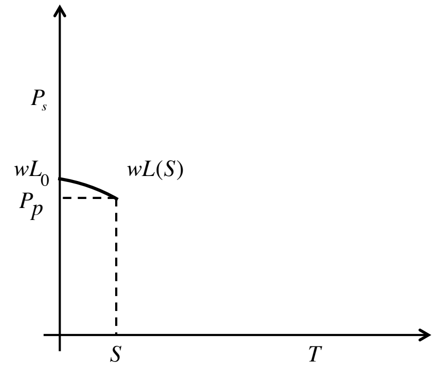
Let us start with the famous distinction between an optimizing model and a satisficing model. In 1978 H.Simon wrote:

"In an optimizing model, the correct point of termination is found by equating the marginal cost of search with the (expected) marginal improvement in the set of alternatives. In a satisficing model, search terminates when the best offer exceeds an aspiration level that itself adjusts gradually to the value of the offers received so far" **(Simon (1978, p.10)).**

Suppose a consumer who ignores the starting price of the search P_S, because he has already reserved the labor income wL_0 for the purchase of an item $Q=1$. He begins to search for a cheaper price and he concludes the search at the satisficing purchase price P_P.

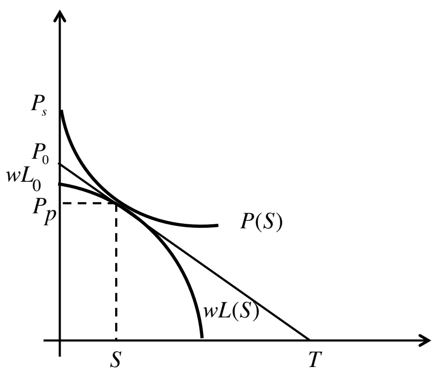
We can expose this procedure, where T is the time horizon of the consumption-leisure choice, $wL=wL(S)$ is the labor income and $\partial L/\partial S<0$, because the best offer P_P exceeds the aspiration level wL_0 (Fig.1):

Fig.1



The points (T,P_P) gives us the value P_0. Then, we can reconstruct the $QP(S)$ ($\partial P/\partial S<0$) and the $wL(S)$ ($\partial L/\partial S<0$), where the (P_0,T) line gives us the value of the price reduction $\partial P/\partial S$ at the point P_P with respect to the time horizon of the consumption-leisure choice (Fig.2):

Fig.2



If the marginal utility of the search is diminishing, we have $\partial^2P/\partial S^2>0$. If the marginal utility of labor is also diminishing, because it leaves fewer productive hours for the search, we have $\partial^2L/\partial S^2<0$. The last consideration is supported by the fact that the search substitutes not only the labor but also the leisure. The substitution of leisure time slows down the decrease in labor time and results in the value $\partial^2 L/\partial S^2<0$.

In this simple manner we reproduce the search-satisficing procedure in the framework of the reserve maximization model, where $R(S) = wL(S) - QP(S)$ (Malakhov 2011b). However, the analytics of the satisficing procedure changes some properties of the monetary model of the reserve for future purchases, where the value $wL(S)$ represents the total labor income and the value $QP(S)$ represents expenditures on the chosen item. The satisficing procedure reduces the labor income to the level that is required to purchase the chosen item. Here, the reserve for future purchases becomes equal to zero. However, this analytical adjustment doesn't change the key equation of the reserve maximization model, where the maximum of the reserve for future purchases ($\partial R/\partial S=0$) equalizes marginal costs of search, derived from the labor income $wL(S)$ lost during the search, with its marginal benefit, derived from the decrease in expenditures $QP(S)$, or:

$$Q\frac{\partial P}{\partial S}=w\frac{\partial L}{\partial S} \quad (1)$$

The key equation of the reserve maximization model depends on a given price reduction $\partial P/\partial S$ and an individual propensity to search $\partial L/\partial S$. As it has been noted, the propensity to search $\partial L/\partial S$ depends not only on the decrease in labor time, but also on the decrease in leisure time. This consideration re-arranges the value of the individual propensity to search with regard to the total allocation of time:

$$L(S)=T-H(S)-S;$$
$$\partial L/\partial S=-\partial H/\partial S-1;$$
$$dH(S)=dS\frac{\partial H}{\partial S}=-dS\frac{H}{T} \;\Rightarrow\; \frac{\partial L}{\partial S}=-\frac{\partial H}{\partial S}-1=\frac{H-T}{T}. \quad (2)$$

The key equation of the reserve maximization model takes the following form:

$$Q\frac{\partial P}{\partial S}=w\frac{\partial L}{\partial S}=w\frac{H-T}{T}=-w\frac{L+S}{T} \quad (3)$$

The last equation describes the equality of marginal values of search at the level of the purchase price P_P. If we complement this equation by the Fig.2, we can describe the hypothetical P_0 for $Q = 1$ in the following form:

$$P_0 = - T\times\partial P/\partial S = w\times(L+S) \quad (4)$$

Let us suppose that the Fig.2 creates the graphic illusion, the purchase price doesn't produce the equality of marginal values of search, and the marginal costs of search are still less than its marginal benefit at the P_P level, or

$$\left| w\frac{\partial L}{\partial S} \right| < \left| \frac{\partial P}{\partial S} \right| \quad (5)$$

This case results in another hypothetical value $P_0' = w(L'+S') < P_0 = w(L+S)$. However, due to the rule of $\partial^2 L/\partial S^2 < 0$, when $\partial L/\partial S = (H-T)/T$, the inequality $(L'+S') < (L+S)$ results in the following inequalities: $L' > L$ *and* $S' < S$. It means that our assumption is false, because the hypothetical amount of search S' is less than the actual amount of search S.

The same indirect proof can be used when it is supposed that at the purchase price level the marginal costs of search are decreasing already faster than its marginal benefit. The only difference is that this case can be eliminated from the analysis by definition, because it requires recognition that the chosen price is not satisficing.

Now we can say that ***when the consumer chooses the satisficing price, his decision equalizes marginal costs of search with its marginal benefit with respect to the time horizon of his consumption-leisure choice***. In addition, according to the reserve maximization model, ***he maximizes at the purchase price level the utility of his consumption-leisure choice.***

The consumer maximizes the utility $U(Q,H)$ of his consumption-leisure choice when (Appendix):

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H;$$

$$\partial^2 L/\partial S\partial H = \partial\frac{H-T}{T}\Big/\partial H = 1/T$$

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{Q}{T\times\partial L/\partial S} = \frac{Q}{L+S} = -\frac{w}{T\times\partial P/\partial S} = \frac{w}{P_0} \quad (6)$$

We can see that hypothetical P_0 value gets more and more new features. The consumer maximizes his utility not with respect to the purchase price, but with respect to the price that is equal to his all potential labor income $w(L+S)$. Indeed, the (P_0,T) line represents the set of $(w, \partial L/\partial S)$ pairs, where different wage rates results in different propensities to search. This consideration corresponds to the P_0 value itself. There is a wage rate that makes the search inefficient. This wage rate reduces the amount of search to zero and the P_0 value equalizes itself marginal values of search. However, when the amount of search is equal to zero, it doesn't mean that the propensity to search $\partial L/\partial S$ is also equal to zero. It is still equal to the $(H-T)/T$ value, where $(T-H) = L$. This value, multiplied by the given wage rate, becomes equal to the value of price reduction $\partial P/\partial S$ just at the "zero level" of the search.

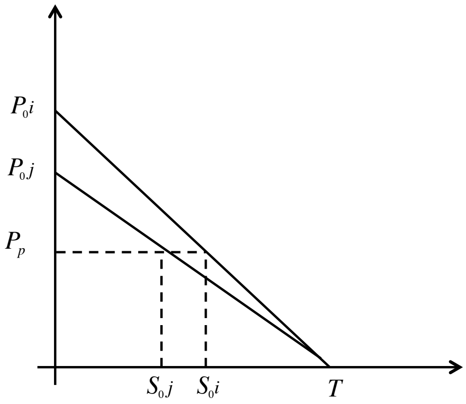
We can presuppose, that when the price P_0 is equal to the potential labor income $w\times(T-H)$, it represents the consumer's ***willingness to pay (WTP***). The consumer doesn't buy at the starting price P_S but he is ready to buy at the wL_0 level. The wL_0 level cannot represent the highest price a buyer is willing to pay for a chosen item. The reserved labor income wL_0 represents the aspiration level, i.e., the price a buyer is really willing to find relatively to his maximum willingness to pay. Therefore, there is a price between the P_S level and the wL_0 level, where the consumer is indifferent whether to buy or not. The value P_0 represents a solution for this problem, because the utility $U(w\times(T-H) - P_0 = 0, Q, H) = U(w\times(T-H), 0, H)$.

If the P_0 value becomes a monopoly price, the consumer should spend all his disposable active ($L+S$) time on work in order to buy the chosen item. However, the increase in the total labor supply will reduce wage rates and will make the P_0 value unattainable. So, the monopoly doesn't enter the market. Indeed, when different consumers with different wage rates have the same willingness to pay it just means that the demand is inelastic and it is not interesting to the monopoly.[1] There, consumers with low wage rate insistently search for chosen items. They are limited only by their physical and/or psychological minimum of leisure time, which creates a choice – either to quit the market or to shift from the 'common model" of behavior to the "leisure model" of behavior, i.e., to cut definitely labor time in order to extend search and to find the chosen item in any way and to increase leisure time (Malakhov 2011b).

On the other hand, when the purchase price P_P represents the equilibrium price, the optimal amount of search S is uniquely defined by the willingness to pay. We can simply repeat the proof for P_{0i} and P_{0j} values in order to demonstrate that P_P value equalizes marginal costs with marginal benefits for the corresponding amounts of search S_{0i} and S_{0j} for the given wage rate w (Fig.3):

[1] Two individuals with different wage rate could have the same willingness to pay when the individual with the higher wage rate has a real chance to buy a substitute, which is unattainable for the individual with the low wage rate (S.M.)

Fig.3



What happens, if the consumer finds the reservation price soon? According to the Equation 4, the short search time results in the lower absolute value of price reduction $|\partial P/\partial S|$ and in the lower willingness to pay P_{0j}. The lower absolute value of price reduction $|\partial P/\partial S|$ increases the monetary $MRS\ (H\ for\ Q) = -\ w/(T\times\partial P/\partial S)$. However, at the same time, due to $\partial^2 L/\partial S^2<0,$ the short search time results in the lower $(L + S)$ value. But this lower $(L + S)$ value increases the physical $MRS\ (H\ for\ Q) = Q/(L + S)$ (Appendix). Even if the consumer finds the interesting price before and if he accepts it, he equalizes marginal costs of search with its marginal benefit and he maximizes the utility of his choice. This situation simply means that he has overestimated his willingness to pay, produced by price uncertainty. And the market corrects his expectations.

However, the reverse case, when he cannot find the interesting price in time, cannot exist. The extended search produces P_0-values that are higher than his willingness to pay and, therefore, the extended search results in the corner solution. But another consumer, who lives far away, can undertake the extended search, because he has higher willingness to pay.

Finally, let us pay attention to the situation when the same amount of search results in a price $P' < P_P$, i.e., when the best offer significantly exceeds the aspiration level – the case that challenges the optimizing approach. Here we realize that the absolute value of the actual price reduction $|\Delta P/\Delta S|$ is greater than its planned value. It seems that if the consumer accepts this price, he doesn't equalize marginal costs of the search $|w\times\partial L/\partial S|$ with its marginal benefit $|Q\times\partial P/\partial S|$, because the allocation of time and, therefore, the propensity to search $\partial L/\partial S=(H-T)/T$ have not been changed.

However, his decision nonetheless changes both the allocation of time and the propensity to search. The equation (4) shows that for the given willingness to pay P_0 the greater absolute value of price reduction $|\partial P/\partial S|$ decreases the value T of the time horizon. However, the time horizon of the consumption-leisure choice depends on the products' lifecycles. The lower price can exhibit the coming expiration date for pork sausages, for example.

If we go back to the Friedman's metaphor, we should say that billiards is played by two people. The seller doesn't bother about consumer's marginal values of search, but he either cut the price for yesterday's "fresh" sausages, or he offers packed pork sausages with extended shelf life. In addition, if the consumer buys yesterday's "fresh" sausages, he should quickly eat them.

The integration of the satisficing decision procedure with the reserve maximization model produces the general relationship between prices, savings on purchases, the search, and the time horizon of consumption-leisure choice:

$$\frac{\Delta S}{T} = -\frac{\Delta P}{P_0} \quad (7)$$

We see that the absolute value of price reduction $|\Delta P/\Delta S|$ is equal to the P_0/T ratio. If the consumer finds the lower price and if he accepts it, he inevitably decreases the time horizon of his consumption-leisure choice. But now the short time horizon increases the absolute value of price reduction $|\partial P/\partial S|$. However, while the value $P_0 = w(L+S)$ remains constant, the decrease in the time horizon is equal to the decrease in the leisure time, or $\Delta T = \Delta H$. Therefore, the decrease in the time horizon also rises the absolute value of propensity to search $|\partial L/\partial S|$, where $\partial L/\partial S =((H\text{-}\Delta H) - (T\text{-}\Delta T))/(T\text{-}\Delta T) = (H - T)/(T\text{-}\Delta T)$. It is easy to show that the change in the time horizon increases proportionally both absolute values of the propensity to search $|\partial L/\partial S|$ and of the price reduction $|\partial P/\partial S|$, because now $\partial P/\partial S = -P_0/(T\text{-}\Delta T)$. Therefore, the marginal values of search become equal again.

The last example is very important from both methodological and practical points of view. The *MRS(H for Q),* produced by the reserve maximization model, takes the value of the time horizon *T* as the independent variable. This methodological advantage results in simple re-calculations of the *MRS (H for Q)* with regard to the time horizon of the shelf life.

However, if we take the planned $\Delta P/\Delta S$ value as the starting point of the sufficient decision procedure, the value of the time horizon becomes the dependent value, now on the planned efficiency $\Delta P/\Delta S$ of the search itself.

Sometimes high aspiration levels result in unrealistic $\Delta P/\Delta S$-expectations. However, unrealistic $\Delta P/\Delta S$-values usually result in corner solutions. It means, that the market, when it "sells" products' lifecycles or shelf lives, tries to adjust $\Delta P/\Delta S$-expectations in order to restore the equation (5). In 1979 Kapteyn et al. presented the brilliant example of this kind of adjustment. The authors demonstrated that purchase decisions concerned durables were satisficing rather than maximizing (Kapteyn et al. 1979, p.559.). Now we can say that consumers' reports for that study had simply documented the adjustment of their aspiration levels to the time horizons of their optimal consumption-leisure choices.[2]

We see that the search-satisficing concept and the optimizing approach complements each other. When the consumer does not have accurate knowledge of prices he reserves the labor income wL_0 and begins to search for a satisficing price. When he concludes the search at the satisficing level, the purchase price one way or another equalizes marginal costs of search with its marginal benefit and maximizes the utility of the consumption-leisure choice.

Appendix.

Setting the Lagrangian expression $\Lambda = U(Q,H) + \lambda(w - Q\frac{\partial P/\partial S}{\partial L/\partial S}.)$, the first-order conditions for a maximum are

$$\frac{\partial \Lambda}{\partial Q} = \frac{\partial U}{\partial Q} - \lambda \frac{\partial P/\partial S}{\partial L/\partial S} = 0; \quad \frac{\partial \Lambda}{\partial H} = \frac{\partial U}{\partial H} - \lambda Q \frac{\partial \frac{\partial P/\partial S}{\partial L/\partial S}}{\partial H} = 0.$$

Trying to determine the marginal rate of substitution of leisure for consumption, we get[3]

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = \frac{\partial Q\frac{\partial P/\partial S}{\partial L/\partial S} \Big/ \partial H}{\frac{\partial P/\partial S}{\partial L/\partial S}} = -Q\frac{\partial P/\partial S \times \partial^2 L/\partial S \partial H}{\frac{\partial P/\partial S}{\partial L/\partial S}(\partial L/\partial S)^2} = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$Q\frac{\partial P}{\partial S} = w\frac{\partial L}{\partial S} \Rightarrow \frac{Q}{\partial L/\partial S} = \frac{w}{\partial P/\partial S} \Rightarrow$$

$$\frac{\partial U/\partial H}{\partial U/\partial Q} = MRS(HforQ) = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H \quad (8).$$

The physical form of the *MRS (H for Q)* results in the following equation:

$$MRS(HforQ) = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = -\frac{Q \times T}{T(H-T)} = \frac{Q}{L+S} \quad (9).$$

Now we can present the *MRS (H for Q*) with regard to the elasticity of substitution between leisure and consumption:

[2] The analysis of the paradox of little pre-purchase search for durables is presented in (Malakhov 2012a).

[3] If we presuppose that an individual can always adjust price reduction to a pre-allocated quantity *(∂P/∂S=∂P/∂S(Q))* and to target leisure time *(∂P/∂S=∂P/∂S(H))*, consumption and leisure become perfect complements. The model implies that consumers can choose a market with certain price dispersion, but they are still price-takers there—now, *price-reduction takers*.

$$MRS(HforQ) = -\frac{\partial Q}{\partial H} = -\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = -\frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H;$$

$$\frac{\partial Q}{\partial H} = \frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H = \frac{Q}{\partial L/\partial S}\partial^2 L/\partial S\partial H = \frac{QT}{(H-T)}\frac{1}{T} = \frac{Q}{H}\frac{H}{(H-T)} = \frac{Q}{H}\frac{(H-T+T)}{(H-T)}; \quad (10)$$

$$\frac{\partial Q}{\partial H} = \frac{Q}{H}(1+\frac{T}{H-T}) = \frac{Q}{H}(1+\frac{1}{\partial L/\partial S}) = \frac{Q}{H}(\frac{\partial L/\partial S+1}{\partial L/\partial S});$$

$$if\ \partial L/\partial S = -\alpha \Rightarrow \frac{\partial Q}{\partial H} = -\frac{Q}{H}(\frac{1-\alpha}{\alpha})$$

$$MRS(HforQ) = (\frac{1-\alpha}{\alpha})\frac{Q}{H} = (\frac{-\partial H/\partial S}{-\partial L/\partial S})\frac{Q}{H}$$

We can get the same result for the following Cobb-Douglas utility function:

$$U(Q,H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S}.$$

If we follow the $\partial L/\partial S+\partial H/\partial S+1=0$ rule, the elasticity of substitution between leisure and consumption is $\sigma =1$.

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