Optimal Labor-Income Tax Volatility with Credit Frictions.

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Abstract

This paper studies the optimality of labor tax smoothing in a simple model with credit frictions. Firms' borrowing to pay their wage payments in advance is constrained by the value of their collateral at the beginning of the period. The labor tax and the shadow value on the credit constraint lead to a (static) wedge between the marginal product of labor and the marginal rate of substitution between labor and consumption. This paper suggests that while the notion of "wedge smoothing" is carried over to this environment, it is achieved only through a volatile labor-income tax rate. As the shadow value on the financing constraint varies over the business cycle, tax volatility is needed in order to counteract this variation and thus allow for "wedge smoothing". In particular, the optimal labor-income tax rate is lower when the credit market is more tightened and higher when the credit market is less tightened. Therefore, when firms are more credit-constrained and the demand for labor is reduced, optimal fiscal policy calls for boosting labor supply by lowering the labor-income tax rate.

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1 Introduction

A classic result in optimal fiscal policy is that the labor-income tax rate should be virtually constant over the business cycle ("labor tax smoothing"). This paper revisits the optimality of the labor tax rate volatility in a simple growth model in which firms borrow to pay factors of production in advance, and borrowing is constrained by their beginning-of-period collateral. The paper suggests that the labor tax rate should vary over the business cycle. When firms are more constrained in hiring labor, the labor-income tax rate should be lowered to boost labor supply, thus increasing the equilibrium level of labor. Credit frictions, thus, lead to departures from complete smoothing of the labor tax rate.

Besides the government, the baseline setup assumes two types of agents in the economy: households and a representative firm. The firm hires labor from households in a neoclassical labor market at a given real wage. The firm borrows in order to pay at least part of the wage bill at the beginning of the period ("working capital"). Borrowing, in turn, is constrained by the firm's value of real estate. This corresponds to the usual limited enforcement problem as in Kiyotaki and Moore (1997).

The basic intuition behind the result of this paper is a follows. Because of the binding credit constraint, labor demand is inefficiently low and it depends on the tightness of the credit constraint. When the credit constraint tightens more, labor supply should be encouraged by reducing the labor tax rate. When the credit constraint is less tightened, the labor tax rate is relatively higher. In either case, the labor tax rate is lower than in otherwise model with no credit frictions. The labor tax rate thus moves in opposite to the tightness of the credit constraint in order to prevent excessive volatility in the equilibrium amount of labor, hence output and consumption.

An alternative way of viewing the result is by considering the implications of the collateral constraint. Due to the binding credit constraint, the firm hires labor so that the marginal product of labor exceeds the real wage rate, thus generating a "markup". Optimal policy thus aims for offsetting this markup (at least partially) by "subsidizing" labor supply. An increase in labor supply lowers the before-tax wage rate and leads to a higher quantity of labor. If the reduction in the labor tax rate is sufficiently large, the equilibrium level of labor may approach the efficient level.

Also, the labor tax rate and the shadow value on the credit constraint generate a "labor wedge", thus breaking down the full mapping between the wedge and the labor

tax rate. I show that the time-varying labor-income tax rate allows for complete smoothing of the labor wedge when credit frictions are present. Therefore, even though the labor-income tax rate is not completely smoothed, the notion of "static wedge smoothing" remains optimal in this environment.

The idea that the labor-income tax rate should be virtually constant over the business cycle is well-known in the literature since the partial-equilibrium complete-markets analysis of Barro (1979). Lucas and Stokey (1983), Chari, Christiano and Kehoe (1991, 1994) show that this result holds in a general-equilibrium setup that assumes neoclassical labor markets. In an economy with incomplete markets and no capital, Aiyagari et al. (2002) partially affirm the results of Barro (1979). Schmitt-Grohe and Uribe (2004a) show that the volatility of the labor tax rate is very small in a model with flexible prices (with and without imperfect competition in the product market), but it is significantly higher if prices are sticky. Recently, Arseneau and Chugh (2010) have shown that the result of labor tax smoothing does not hold in a model with labor market frictions. Labor tax volatility in their study is optimal in order to induce efficient fluctuations in the labor market by keeping distortions (or wedges) constant over the business cycle.

The economic events of recent years call for studying the effects of various aspects of financial frictions on optimal policies, including optimal fiscal policy. This paper contributes to the existing literature by studying the implications of financial frictions for the optimality of labor-income taxation. In particular, the difficulties of firms in obtaining sufficient credit during the last recession raise questions about the optimal policies that governments should follow during this type of economic episode. This is essentially addressed in this paper in the context of the optimal behavior of the labor-income tax rate.

The remainder of the paper proceeds as follows. Section 2 outlines the model economy with the borrowing constraint and defines the private-sector equilibrium. Section 3 presents the problem of the social planner and section 4 discusses the problem of the Ramsey planner. Section 5 presents some analytical results about the optimal labor-income taxation policy. Section 6 describes the calibration and the solution methodology of the model. Section 7 presents the main quantitative results of this paper. Section 8 presents some robustness analysis and section 9 concludes.

2 The Model

The economy is populated by households, a representative firm and the government. Households consume and supply labor to the firm on spot markets. The firm needs to pay (at least part of) its input costs before production takes place, thus giving rise to borrowing from households. Borrowing is constrained by the value of real estate that the firm owns. This is the source of the credit friction in the baseline model.

2.1 Households

In each period t, the representative household purchases consumption c_t, supplies labor l_t, purchase real estate h_t (in the form of housing) and lends b_t^f to the firm at the beginning of the period at an intra-period gross real interest rate of R_t^f. The household also has access to a standard one-period real government bond b_t that pays a gross real interest rate of R_t.

Households maximize their expected discounted lifetime utility given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, l_t),\tag{1}$$

where E_0 is the expectation operator, $\beta < 1$ is the subjective discount factor and $u(c_t, h_t, l_t)$ is the period utility function from consumption, labor and real estate. This function satisfies: $\dfrac{\partial u(\cdot)}{\partial c} > 0$, $\dfrac{\partial^2 u(\cdot)}{\partial c^2} < 0$, $\dfrac{\partial u(\cdot)}{\partial h} > 0$, $\dfrac{\partial^2 u(\cdot)}{\partial h^2} < 0$, $\dfrac{\partial u(\cdot)}{\partial l} < 0$ and $\dfrac{\partial^2 u(\cdot)}{\partial l^2} < 0$.

Maximization is subject to the sequence of budget constraints of the form:

$$(1 - \tau_t^l) w_t l_t + (1 - \tau_t^\pi) \Pi_t + q_t h_t + R_{t-1} b_t + R_t^f b_t^f = c_t + q_t h_{t+1} + b_{t+1} + b_t^f,\tag{2}$$

where c_t denotes consumption, w_t is the real wage rate, q_t is the market price of housing τ_t^l and is the labor-income tax rate, Π_t denotes lump-sum profits from the ownership of the firm and τ_t^π is the tax rate on those profits.

The optimal choices of consumption, bonds, lending to firms, labor supply and real estate yield:

$$R_t^f = 1,\tag{3}$$

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l)w_t, \tag{4}$$

$$u_{c,t} = \beta R_t E_t(u_{c,t+1}), \tag{5}$$

$$q_t u_{c,t} = \beta E_t(u_{h,t+1} + q_{t+1}u_{c,t+1}), \tag{6}$$

where $u_{c,t}$ is the marginal utility of consumption in period t, $u_{h,t}$ is the marginal utility of housing in period t and $u_{l,t}$ is the marginal disutility of supplying labor in period t. Equation (3) governs the lending of households to firms. As is in Carlstrom and Fuerst (1998), households are basically passive suppliers of credit to the firm. Equation (4) is the standard labor-supply condition, equation (5) is the standard consumption Euler equation and condition (6) is an asset pricing-type condition. This condition states the marginal utility from consumption is equalized to the marginal gain from real estate. The latter has two components- a direct utility from real estate and the possibility to expand future consumption by the realized resale value of real estate.

2.2 The Firm

The representative firm hires labor and uses real estate to produce a homogenous good using the following production function:

$$y_t = z_t f(\tilde{h}_t, l_t), \tag{7}$$

where y_t is output, z_t is total factor productivity and $\tilde{h}_t$ denotes the stock of real estate of the firm at the beginning of the period.

Due to a mismatch between the timing of the realization of revenues and wage payment, at least part of labor costs are paid before the realization of revenues, which requires the firm to borrow at the beginning of period t. This assumption has some similarity with the assumption of Carlstrom and Fuerst (1998), but with differences in the specifics of the model. Borrowing, however, is constrained by the value of the firm's assets, which are entirely held in the form of real estate. Therefore, the firm's collateral is equal to the beginning-of-period market value of its real estate.

Assuming that firms use real estate as collateral is common in the literature: for example, Kiyotaki and Moore (1997) assume that borrowing is tied to the value of land and Iacoviello (2005) assumes that entrepreneurs use housing as collateral. Chaney,

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Sraer and Thesmar (2011) show that, for U.S. firms over 1993-2007, appreciation in the real estate values of firms led to increases in investment, which is mainly financed through additional debt issuance. This effect is particularly strong for credit-constrained firms. I, therefore, follow those studies and use the value of real estate as the firm's collateral.

As shown in Appendix A, the firm's problem with credit frictions can be reduced to the following maximization problem:

$$Max[z_t f(\tilde{h}_t, l_t) - w_t l_t],\tag{8}$$

subject to

$$\phi w_t l_t \leq \kappa q_t \tilde{h}_t,\tag{9}$$

where $\tilde{h}_t$ is the firm's beginning-of-period stock of real estate, κ is the share of assets that can be used as collateral (or the loan-to-value ratio), ϕ is the fraction of factor payments that has to be paid in advance. Clearly, if $\phi = 0$, then the model collapses to the standard model with neoclassical labor markets.

Denoting the Lagrange multiplier on (9) by μ_t, profit maximization gives the following labor demand condition:

$$z_t f_{l,t} = (1 + \phi \mu_t) w_t,\tag{10}$$

The firm thus hires labor so that its marginal product is a "markup" over the real wage. The net markup is given by $\phi \mu_t$ and it arises only due to the external financing needs of the firm. This result is similar to the result in the "output model" of Carlstrom and Fuerst (1998). In their model, agency costs, which arise due to the monitoring activity of lenders, induce differences between the marginal products of labor and capital and their respective factor prices. The use of the term "markup" in this paper is borrowed from their own study.

In the second part of the period, the firm chooses the next-period real estate taking into account the role of real estate as collateral and subject to the budget constraint $z_t f(\tilde{h}_t, l_t) - w_t l_t + q_t \tilde{h}_t = q_t \tilde{h}_{t+1} + \Pi_t$. The left-hand side is the total resources of the firm after production takes place, and they are equal to the sum of operating profits $z_t f(\tilde{h}_t, l_t) - w_t l_t$ and the market value of assets $q_t \tilde{h}_t$. Those resources are first used to

finance the purchasing of next-period real estate $\widetilde{h}_{t+1}$. Then, any remaining profits (or resources), denoted by Π_t, are remitted to households in a lump-sum fashion.

I also assume that in the process of accumulating assets, the firm is more impatient than households (one may think about this firm as being managed by an entrepreneur who is more impatient that households). For this reason, the firm's stochastic discount factor is $\gamma \Xi_{t,t+1}$, where $\Xi_{t,t+1} = \beta \dfrac{u_{c,t+1}}{u_{c,t}}$ is the households' stochastic discount factor and $\gamma < 1$. The parameter γ is introduced to avoid self financing by the firm.

The assumption that the borrower (firm/entrepreneur in this case) is less patient than the lender is standard in this class of models (see, for example, Carlstrom and Fuerst 1997, 1998). In addition, assuming that profits are transferred to households simplifies the optimal policy problem as it reduces the objective function of the Ramsey planner to only the utility function of households. This formulation also allows for better comparisons with the standard neoclassical model.

With this characterization of the firm's problem, the choice of $\widetilde{h}_t$ gives the following dynamic equation in the price of real estate:

$$q_t u_{c,t} = \gamma \beta E_t \left[u_{c,t+1}[z_{t+1} f_{\widetilde{h},t+1} + (1 + \kappa \mu_{t+1}) q_{t+1}] \right], \tag{11}$$

which makes explicit the roles of the credit friction and the additional discount factor.

Since profits Π_t are transferred to households, one could alternatively assume that the objective function of the firm is to choose labor and real estate in order to maximize $\Pi_t = z_t f(\widetilde{h}_t, l_t) - w_t l_t + q_t \widetilde{h}_t - q_t \widetilde{h}_{t+1}$. In this case, the firm's problem is to maximize $E_0 \sum_{t=0}^{\infty} \gamma^t \Xi_{0,t} \Pi_t$ subject to the financing constraint (9). The first-order conditions with respect to labor and real estate of this problem are exactly the same as (10)-(11). In this respect, both approaches are identical.

2.3 The Government
The government collects labor-income and profit taxes and issues real debt to finance an exogenous stream of real government expenditures g_t.

The government budget constraint in period t is thus given by:

$$\tau_t^l w_t l_t + \tau_t^\pi \Pi_t + b_{t+1} = g_t + R_{t-1} b_t \qquad (12)$$

2.4 Market Clearing

In equilibrium, the resource constraint of the economy reads:

$$z_t f(\tilde{h}_t, l_t) = c_t + g_t, \qquad (13)$$

and the market for real estate clears:

$$h_t + \tilde{h}_t = 1. \qquad (14)$$

which, as in Kiyotaki and Moore (1997) and Iacoviello (2005), implies a fixed supply of real estate.

2.5 The Private Sector Equilibrium

Definition 1: *Given the exogenous processes* $\{z_t, g_t, \tau_t^l, \tau_t^\pi\}$, *the private-sector equilibrium is a state-contingent sequence of allocations* $\{c_t, l_t, h_t, \tilde{h}_t, w_t, q_t, R_t^f, R_t, b_t, \mu_t\}$ *that satisfy the equilibrium conditions (3)-(6) and (9)-(14).*

3 Efficient Allocations

It is useful to consider the optimal tax results that emerge as a solution to the social planner's problem in order to better understand the results of the Ramsey planner later. I refer to the allocations of the social planner as the "efficient allocations" or the "first-best allocations", interchangeably. Those are the allocations the planner will choose when lump-sum taxes are available.

Definition 2: *Given the exogenous processes* $\{z_t, g_t\}$, *the problem of the social planner is to choose consumption, labor and real estate to maximize (1) subject to (13)-(14).*

As Appendix B shows, the choice of labor and consumption yield:

$$-\frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t}, \qquad (15)$$

which state that the social planner chooses consumption and labor so that the marginal rate of substitution between labor and consumption is equalized to the marginal product of labor. This the usual efficiency condition in this class of models.

4 Optimal Labor Taxation- The Ramsey Problem

In this section, I present the solution to the second-best labor taxation problem using the standard Ramsey approach (maximizing the utility of households subject to the private-sector equilibrium conditions and the resource constraint). Following Lucas and Stokey (1983) and Chari and Kehoe (1999), I use the primal approach, in which the government only chooses allocations after prices and taxes have been substituted out using the private-sector equilibrium conditions. To do so, I derive the present-value implementability constraint (PVIC) by substituting the equilibrium conditions into the households' budget constraint. Differently from standard Ramsey models, however, the PVIC in this paper does not capture all of the equilibrium conditions of the private sector (in addition to the resource constraint, of course). Therefore, the Ramsey problem will be enlarged beyond just maximizing utility subject to the PVIC and the resource constraint.

As shown in Appendix C, the PVIC in this problem reads:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{c,t} c_t + u_{l,t} l_t + \beta u_{h,t+1} h_{t+1} - u_{c,t} (1 - \tau_t^{\pi}) \Pi_t \right] = u_{c,0} R_{-1} b_0 + u_{c,0} q_0 h_0, \tag{16}$$

Definition 3: *Given the exogenous processes* $\{z_t, g_t\}$, *the Ramsey planner chooses sequences of allocations* $\{c_t, l_t, h_t, \tilde{h}_t, q_t, \mu_t\}$ *to maximize (1) subject to (11), (13)-(14) and (16).*

I assume that $\tau_t^{\pi} = 1$ (which is the standard assumption in this class of models). Confiscating all profits has the advantage of generating tax revenues that allow for reducing distortionary taxes without influencing households' decisions at the margin. Setting $\tau_t^{\pi} = 1$, the solution to Ramsey problem yields:

$$-\frac{u_{l,t} + \zeta(u_{l,t} + u_{ll,t} l_t + u_{cl,t} c_t + u_{hl,t} h_t)}{u_{c,t} + \zeta(u_{c,t} + u_{cc,t} c_t + u_{lc,t} l_t + u_{hc,t} h_t)} = z_t f_{l,t}, \tag{17}$$

with ζ being the Lagrange multiplier on the PVIC and $u_{xy,t}$ being the second derivative of u with respect to any two arguments. Comparing (17) with (15), the solutions to the Ramsey problem and the social planner problem coincide if $\zeta = 0$ as the problem of the Ramsey planner is essentially reduced to the problem of the social planner.

Finally, the combination of labor supply (4) and labor demand (10) gives:

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$$-\frac{u_{l,t}}{u_{c,t}} = \left(\frac{1-\tau_t^l}{1+\phi\mu_t}\right)z_tf_{l,t}, \tag{18}$$

which suggests that the labor tax rate and the credit friction drive a wedge between the marginal rate of substitution between labor and consumption and the marginal product of labor. We may refer to this wedge as the "labor wedge".

5 Analytical Results

The main analytical results about the optimal labor-income tax rate are presented in this section. I start by describing the solution to the social planner's problem and then turn to the solution of the Ramsey planner's problem.

5.1 First-Best Labor-Income Taxation Policy

In this subsection, I show that the market solution with a constant labor tax rate is not efficient. Comparing condition (18) to condition (15), the market allocation is efficient only if

$$\tau_{FB,t}^l = -\phi\mu_t, \tag{19}$$

with $\tau_{FB,t}^l$ being the first-best labor tax rate. Condition (19) suggests that for the market allocation to be efficient, labor income should be subsidized by the size of the credit friction. More importantly, the size of this subsidy is not constant as μ_t varies over the business cycle. Clearly, this subsidy is not needed when credit frictions are absent; in this case, the first-best labor tax rate is zero in all dates and states.

5.2 Second-Best Labor-Income Taxation Policy

In order to provide an analytical solution to the Ramsey taxation problem, I assume the following separable period utility function:

$$u(c_t, h_t, l_t) = logc_t + \psi logh_t - \chi l_t, \tag{20}$$

with ψ and χ being parameters that measure the relative weights on real estate and the disutility from labor, respectively. Given this functional form, condition (17) reads:

$$-\frac{u_{l,t}}{u_{c,t}} = \frac{z_tf_{l,t}}{1+\zeta}, \tag{21}$$

which differs from the solution of the Ramsey planner only due to the shadow value on the PVIC.

The combination of (18) and (21) gives the following optimal labor tax rate:

$$\tau_{SB,t}^{l} = \frac{\zeta - \phi\mu_t}{1+\zeta},$$ (22)

where $\tau_{SB,t}^{l}$ is the optimal (second-best) labor-income tax rate that is chosen by the Ramsey planner. Equation (22) is the key expression characterizing the optimal labor tax rate in this section.

The main insights that come out of this condition can be summarized as follows:

Proposition 1: *In an economy with no credit frictions (ϕ =0), the optimal labor-income tax is constant over the business cycle.*

Proof: Setting ϕ =0 in condition (22), we have $\tau_{SB,t}^{l} = \frac{\zeta}{1+\zeta}$, which is completely constant. This is re-affirmation to the classical result in optimal labor taxation with neoclassical labor markets. *QED.*

Proposition 2: *In an economy with credit frictions (ϕ >0), the optimal labor-income tax rate is non constant. Moreover, the optimal labor tax rate is decreasing in the degree of tightness of the credit constraint.*

Proof: When ϕ >0, condition (22) suggests that, to the extent that μ_t is time varying, the labor tax rate is time varying as well. Clearly, the labor tax rate is lower whenever the shadow value on the financing constraint is higher, and vice versa. *QED.*

By reducing the labor tax rate more in periods of tighter credit markets, optimal policy in this setup "leans against the wind". The optimal labor tax rate is lower than in an otherwise model with no credit imperfections; the Ramsey planner sets a lower labor tax rate to boost labor supply whenever labor demand is reduced due to the binding credit constraint.

Finally, the assumption that the credit constraint is always binding does not alter the main insights of this subsection. If the constraint was assumed to only occasionally bind, the value of μ_t will be either zero or positive, hence not constant. In turn, the labor tax rate will not be constant. In order to simplify matters and to make the computational solution more tractable, I do not consider this case here.

6 Computational Strategy and Calibration

6.1 Parameterization and Functional Forms

The time unit is a quarter and hence the discount factor β is set to 0.99, implying an annual interest rate of roughly 4 percent. I also assume the following period utility function for households:

$$u(c_t, h_t, l_t) = \log c_t + \psi \log h_t - \chi \frac{l_t^{1+\theta}}{1+\theta}. \tag{23}$$

The parameter θ is set to zero, implying a linear disutility function of labor. The implied labor supply elasticity helps in capturing the volatility of total hours in a model with no extensive margin, as is the case in this paper. The parameter χ is calibrated so that the steady state value of l is 0.33 and ψ is set so that the steady state of h is 0.8.

Firms produce using the Cobb-Douglas production function:

$$f(\tilde{h}_t, l_t) = \tilde{h}_t^{\alpha} l_t^{1-\alpha}, \tag{24}$$

with α, the share of real estate in the production function, being of 0.03, in line with Iacoviello (2005).

Total factor productivity is governed by the following AR(1) process:

$$log(z_t) = (1 - \rho_z) log(z) + \rho_z log(z_{t-1}) + u_t, \tag{25}$$

with the innovation term u_t being normally distributed with zero mean and a standard deviation of σ_u. The coefficient ρ_z is set to 0.95 and the standard deviation σ_v is set to 0.0075, in line with the literature. The deterministic steady state value of z_t is normalized to 1.

Similarly, government expenditures evolves according to the following AR(1) process:

$$log(g_t) = (1 - \rho_g) log(g) + \rho_g log(g_{t-1}) + v_t, \tag{26}$$

where v_t is normally distributed with zero mean and a standard deviation of σ_v and g is set so that the deterministic steady state value of government spending is 20 percent of deterministic steady state output (which is the average government-GDP ratio over 1960-2007). In line with the literature, ρ_g and σ_v are set to 0.90 and 0.018, respectively.

The steady state value of b is obtained so that b/y is 0.36. This is the average of the gross federal debt held by the public as percentage of GDP over the period 1960-2007

(see Table B79 of the *2011 Economic Report of the President*). I choose 2007 as the final year of the sample because this ratio has increased dramatically in the last three years; including those years in the sample may only bias my results without adding any further insights.

The additional discount factor γ is set to 0.99, implying an annual discount rate of about 0.98 for the firm, in line with Iacoviello (2005). I set ϕ set to 1 in the benchmark calibration of the model, but I also consider other values of this parameter in the robustness analysis section. I set the loan-to-value ratio κ to 0.89, which equals the entrepreneurial loan-to-value ratio as reported in Iacoviello (2005).

6.2 Solution Methodology

The decision rules that solve this problem are obtained through a second-order approximation to the optimality conditions of the Ramsey planner around the non-stochastic steady state of the model. I apply the second-order approximation procedure that was developed by Schmitt-Grohe and Uribe (2004b).

7   Quantitative Results

This section presents the main numerical results regarding the optimal labor-income tax rate.

7.1 Second-Best Labor Taxation Policy

Table 1 presents the mean and the second moments of the labor tax rate following shocks (of one standard deviation size) to total factor productivity only, government expenditures only and to simultaneous shocks to TFP and government expenditures.

With credit frictions, optimal policy calls for a time-varying path of the labor tax rate. The standard deviation of the labor tax rate is significantly high (and higher than models with neoclassical labor markets usually predict). In all cases considered, the standard deviation of the labor tax rate is more than twice as large as the standard deviation of output. Interestingly, this high volatility of the labor tax rate is observed even though the volatility of output is empirically plausible. The case of simultaneous shocks suggests a relatively high volatility of output, in which case the *relative* volatility of the labor tax rate to the volatility of output is perhaps a better indicator for

the non-constant path of the labor-income tax rate. The labor tax rate also displays little persistence over the business cycle as a result of the borrowing constraint. As the shadow value of the binding borrowing constraint changes, the labor-income tax rate fluctuates as well, leaving little room for persistence in the labor tax rate.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
Government Expenditures and TFP shocks				
τ^l	0.1740	0.0551	0.1062	-0.9590
y	0.3285	0.0230	0.1990	1.0000
l	0.3346	0.0223	0.1240	0.9479
TFP Shock				
τ^l	0.1955	0.0360	0.1397	-0.9260
y	0.3160	0.0168	0.2729	1.0000
l	0.3210	0.0143	0.0916	0.8933
Government Expenditures Shock				
τ^l	0.1882	0.0483	0.0827	-0.9949
y	0.3247	0.0186	0.2435	1.0000
l	0.3297	0.0195	0.2445	0.9806

Table 1: Optimal fiscal policy with credit frictions. The standard deviation of the U.S. GDP over 1964:1-2007:4 is 0.0152.

The labor-income tax rate falls in recessions and rises in booms. Fiscal policy thus "leans against the wind". The fall in the labor tax rate following a negative shock to government expenditures is not surprising- the planner cuts the labor-income tax rate accordingly. In this paper, the fall in the labor tax rate is also due to the binding credit constraint, as discussed above.

The volatility of the labor tax rate in this paper allows for the more general result of "wedge smoothing", which is a very central result in optimal taxation, to hold. In the lack of credit frictions, labor taxation is the only source of the labor wedge (see condition 18). Therefore, smoothing the labor wedge is equivalent to smoothing the labor tax rate. In this model, however, the credit friction is another source of the labor wedge and complete smoothing of the labor tax rate is not translated into complete smoothing of the wedge. Keeping the labor tax rate constant in the face of exogenous

shocks induces variations in the wedge over the business cycle. Under the optimal tax policy characterized in section 4 and numerically evaluated in Table 1, the labor wedge is completely smoothed following exogenous shocks. We thus conclude that even though labor tax smoothing is not optimal in this setup, smoothing distortions over the business cycle remains optimal.

7.2 Impulse Responses

Figure 1 displays the response of the economy to a one standard deviation shock to TFP (for illustration purposes, I only consider a negative shock). A negative TFP shock reduces the demand of labor and the real wage, but at the same time the real price of real estate and the demand of the entrepreneur for real estate fall as well. The overall effect is an increase in the shadow value on the credit constraint μ, which in turn leads to a fall in the labor tax rate (of about 2 percent). The fall in the equilibrium amount of labor and the fall in real estate held by entrepreneurs lead to a fall in output and consumption.
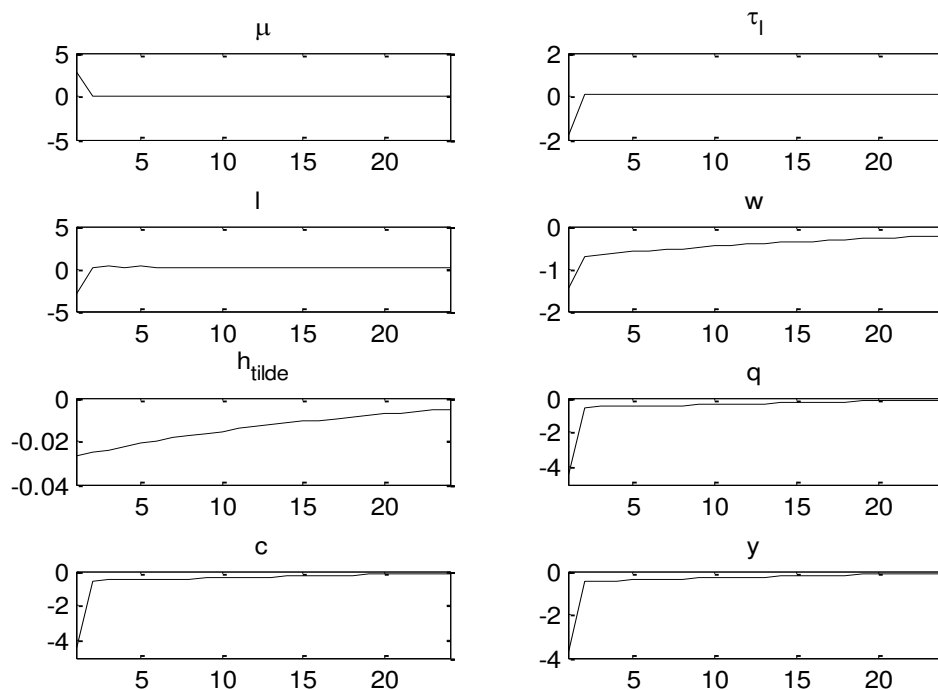

Figure 1: Response to a TFP shock (percentage deviations from SS levels).
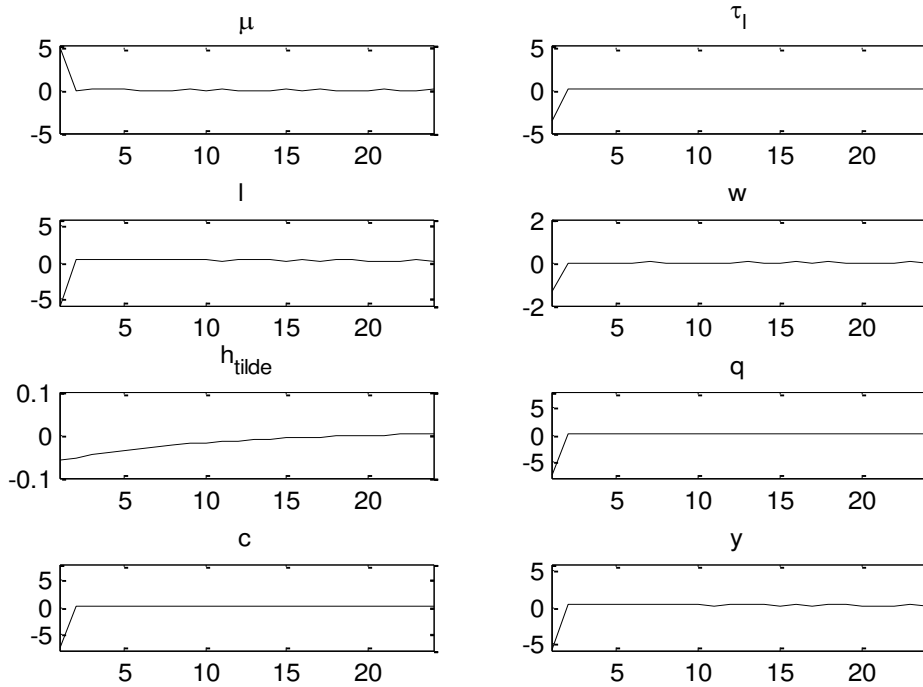
Figure 2: Response to a Government expenditure shock (percentage
deviations from SS levels).

The fall in the labor tax rate is bigger following a fall in government expenditures
(Figure 2). In this case, the fall in government expenditures and the increase in the
tightness of the credit constraint lead to a stronger fall in the labor tax rate. Other
variables display similar patterns as in Figure 1, but with different magnitudes.

8   Robustness Analysis

I first show the volatility of the labor tax rate for different values of the parameter ϕ. I
then show the results under a finite elasticity of labor supply. Finally, I study the case
when profits are not taxed at the optimal rate of 100 percent.

8.1   Changing the Value of the Parameter ϕ

Figure (3) presents the standard deviation of the labor tax rate for various values of ϕ
between 0.2 and 1 following all types of shocks considered in Table 1. For illustration

purposes, the results for $\phi=0$ are not presented, but with the note the labor tax rate is completely constant in this case.
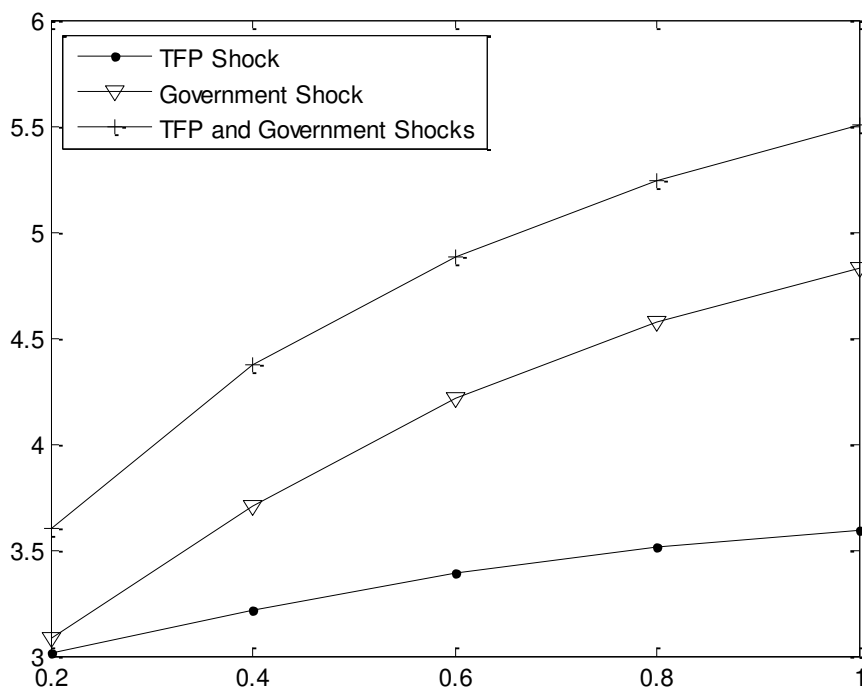

Figure 3: The standard deviation of the labor-income tax rate for various values of ϕ (in percentage terms).

The main observations can be summarized as follows. First, following all types of shocks, the standard deviation of the labor tax rate is significantly meaningful even if ϕ is relatively low. For example, for $\phi=0.2$, the standard deviation of the tax rate is roughly 3 percent following TFP and government shocks and more than 3.5 percent following simultaneous shocks. Second, the volatility of the labor tax rate is increasing in the value of ϕ, but at a lower rate. Third, the differences between the volatilities of the tax rate following different types of shocks are increasing in ϕ. We can better understand this result by first considering the case with no credit frictions- the volatility is zero following all shocks. As ϕ becomes positive but remains low, the volatilities remain highly similar. However, the differences start to increase when this parameter increases more as the type of the shock becomes more important for the behavior of the labor tax rate.

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8.2 A Lower Elasticity of Labor Supply

The analysis so far assumed that $\theta=0$, implying infinite labor supply elasticity, to give numerical predictions to support the analytical results of section 5. In this subsection, I consider a lower labor supply elasticity since the size of this elasticity can matter for the volatility of labor and the volatility of the labor-income tax rate.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
Government Expenditures and TFP shocks				
τ^l	0.1823	0.0514	0.2958	-0.8497
y	0.3277	0.0186	0.4938	1.0000
l	0.3324	0.0191	0.3103	0.8277
TFP Shock				
τ^l	0.1967	0.0345	0.5012	-0.6293
y	0.3184	0.0133	0.7392	1.0000
l	0.3228	0.0127	0.4899	0.5759
Government Expenditures Shock				
τ^l	0.1908	0.0421	0.2641	-0.9926
y	0.3250	0.0158	0.5264	1.0000
l	0.3298	0.0168	0.5316	0.9723

Table 2: Optimal fiscal policy with credit frictions and $\theta=1$.

Table 2 shows the results for unitary labor supply elasticity. I keep using a relatively high labor supply elasticity to better account for the volatility of total hours in this setup. In general, the results of this subsection support my earlier findings about the volatility of the labor tax rate despite a slight decrease in the volatility of the labor tax rate compared to the results reported in Table 1. The labor tax rate remains highly volatile and significantly more volatile than output (which also becomes less volatile given the same magnitudes of shocks). Therefore, the choice of the labor supply elasticity behind the results of Table 1 is not significant for the main result of this paper: credit frictions induce, optimally, high volatility in the labor-income tax rate.

8.3 Zero Taxation of Profits

The analyses above assumed that the government confiscates all profits (i.e. $\tau_t^\pi = 1$). Since after-tax profits do not affect households' decision at the margin, it is optimal to tax them on the rate of 100 percent and thus allow for other taxes to be reduced. In this subsection, I show the results when this assumption is relaxed. Specifically, I consider the other pillar case of $\tau_t^\pi = 0$.

	Mean	Standard Deviation	Auto-correlation	Correlation with output
Government Expenditures and TFP shocks				
τ^l	0.1948	0.0625	0.5581	-0.9482
y	0.3163	0.0237	0.5447	1.0000
l	0.3207	0.0226	0.4494	0.9348
TFP Shock				
τ^l	0.2046	0.0481	0.6147	-0.9290
y	0.3125	0.0187	0.6974	1.0000
l	0.3171	0.0170	0.5166	0.8816
Government Expenditures Shock				
τ^l	0.1973	0.0572	0.5114	-0.9885
y	0.3152	0.0200	0.4390	1.0000
l	0.3193	0.0210	0.4406	0.9753

Table 3: Optimal fiscal policy with credit frictions and $\tau_t^\pi = 1$.

The results, obtained under the benchmark calibration of the model, are presented in Table 3. Labor tax rate volatility remains optimal in this case. In fact, the volatility of the labor tax rate in this case is considerably higher than in the benchmark case presented in Table 1. In addition, the average of the labor tax rate is higher in all cases considered since the lack of profit taxation requires heavier taxation of labor income to generate sufficient government revenues. Zero taxation of profits also requires the planner to vary the labor tax rate even more over the business cycle, thus reducing the degree to which the labor tax rate can be smoothed.

9   Conclusions

This paper revisits an old question in optimal fiscal policy- the optimality of labor tax smoothing- in a model with credit frictions. Firms' borrowing to finance the hiring of labor at the beginning of the period is constrained by their collateral. The credit constraint induces an inefficiently low demand for labor. In this environment, keeping the labor tax rate constant over the business cycle is not optimal. When the credit constraint tightens more, it is optimal to hold a relatively lower labor-income tax rate in order to boost labor supply. When the credit constraint is less tightened, the optimal labor tax rate should be relatively higher. The labor-income tax rate is thus moving in opposite to the tightness of the credit constraint.

Quantitatively, the volatility of the labor tax rate is considerably higher in a model with credit frictions than in an otherwise model with frictionless credit markets and significantly higher than the (empirically-plausible) volatility of output. The volatility of the labor tax rate allows for stabilizing labor (thus output) over the business cycle, which on itself induces a smoothed path of consumption.

Since the borrowing constraint induces inefficiently low demand for labor, the firm hires labor so that the marginal product of labor is a "markup" over the real wage rate. The tax reduction in more tightened credit markets helps in offsetting this markup, thus positioning the economy closer to the efficient allocation.

This paper is part of the very timely line of research that studies the implications of credit frictions for macroeconomic policies in general, and for optimal taxation in particular. Credit frictions are proven as important factors in shaping macroeconomic policies in addition to their traditional role in magnifying the effects of exogenous shocks on the macroeconomy.

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Appendix: Mathematical Derivations

A   The Firm's Problem

The firm chooses labor and loans to maximize:

$$z_t f(\tilde{h}_t, l_t) + b_t^f - w_t l_t - R_t^f b_t^f,$$ (A1)

subject to

$$b_t^f - \phi w_t l_t \geq 0,$$ (A2)

and,

$$\kappa q_t \tilde{h}_t - b_t^f \geq 0,$$ (A3)

Letting υ_t and γ_t denote the Lagrange multipliers on the constraints (A2) and (A3), respectively, the optimality condition with respect to b_t^f reads:

$$\gamma_t = R_t^f + \upsilon_t - 1.$$ (A4)

Similarly, the first order condition with respect to l_t yields:

$$z_t f_{l,t}(\tilde{h}_t, l_t) = (1 + \phi \upsilon_t) w_t,$$ (A5)

Recalling that $R_t^f = 1$, equation (A4) becomes:

$$\upsilon_t = \gamma_t.$$ (A6)

Alternatively, conditions (A2) and (A3) can be combined to get:

$$\kappa q_t \tilde{h}_t - \phi w_t l_t \geq 0,$$ (A7)

which is condition (9) in the text. Furthermore, substituting $R_t^f = 1$ in (A1), the profit function is now given by:

$$z_t f(\tilde{h}_t, l_t) - w_t l_t,$$ (A8)

which is condition (8) in the text. Therefore, the optimization problem of the firm is to maximize (A8) subject to (A7). Letting μ_t be the Lagrange multiplier on (A7), the demand function of labor reads:

$$z_t f_{l,t}(\tilde{h}_t, l_t) = (1 + \phi \mu_t) w_t,$$ (A9)

which is condition (10) in the text. This condition coincides with (A5) with $\upsilon_t = \gamma_t = \mu_t$.

B Efficient Allocations

The social planner chooses consumption, labor and real estate for the next period to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, l_t, h_t),$$ (B1)

subject to the sequence of resource constraints:

$$z_t f(1 - h_t, l_t) = c_t + g_t.$$ (B2)

Letting η_t be the Lagrange multiplier associated with (B2), the first-order conditions with respect to c_t, l_t and h_{t+1}, respectively, read:

$$u_{c,t} = \eta_t,$$ (B3)

$$u_{l,t} + \eta_t z_t f_{l,t} = 0.$$ (B4)

and

$$\beta E_t u_{h,t+1} - \beta E_t [\eta_{t+1} z_{t+1} f_{h,t+1}] = 0.$$ (B5)

Combining (B3) and (B4) yields

$$-\frac{u_{l,t}}{u_{c,t}} = z_t f_{l,t}(\tilde{h}_t, l_t),$$ (B6)

and hence efficiency requires the marginal rate of substitution (the left hand side of condition (B6)) to be equal to the marginal product of labor (given by the right-hand side of condition (B6)).

Similarly, combining (B3) and (B5) gives

$$\frac{u_{h,t}}{u_{c,t}} = z_t f_{h,t},$$ (B7)

which is another efficiency condition when households derive utility from real estate and firms produce using real estate.

C The Present- Value Implementability Constraint

I show here the derivation of the PVIC for the Ramsey problem. Recalling that $R_t^f = 1$, the households' budget constraint becomes:

$$(1 - \tau_t^l) w_t l_t + (1 - \tau_t^\pi) \Pi_t + q_t h_t + R_{t-1} b_t = c_t + q_t h_{t+1} + b_{t+1}.$$ (C1)

By introducing $E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t}$ to (C1) and rearranging, we have:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} (1-\tau_t^l) w_t l_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} (1-\tau_t^\pi) \Pi_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} q_t h_t$$

$$+ E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} R_{t-1} b_t - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} c_t - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} q_t h_{t+1} - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} b_{t+1} = 0$$

(C2)

Recall that, from the solution to the households' problem, we have:

$$-\frac{u_{l,t}}{u_{c,t}} = (1-\tau_t^l) w_t ,$$

(C3)

$$u_{c,t} = \beta R_t E_t (u_{c,t+1}) ,$$

(C4)

$$u_{c,t} q_t = \beta E_t (u_{h,t+1} + u_{c,t+1} q_{t+1}) ,$$

(C5)

Substituting (C3) in the first term of (C2), (C5) in the sixth term of (C2) and (C4) in the last term of (C2) yield:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} \left(-\frac{u_{l,t}}{u_{c,t}} \right) l_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} (1-\tau_t^\pi) \Pi_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} q_t h_{tt} + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} R_{t-1} b_t$$

$$- E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} c_t - E_0 \sum_{t=0}^{\infty} \beta^t \beta E_t (u_{h,t+1} + u_{c,t+1} q_{t+1}) h_{t+1} - E_0 \sum_{t=0}^{\infty} \beta^t \beta R_t u_{c,t+1} b_{t+1} = 0.$$

(C6)

Combining the third and sixth terms of (C6) yield:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} q_t h_{tt} - E_0 \sum_{t=0}^{\infty} \beta^t \beta E_t u_{h,t+1} h_{t+1} - E_0 \sum_{t=0}^{\infty} \beta^t \beta E_t (u_{c,t+1} q_{t+1} h_{t+1})$$

$$= u_{c,0} q_0 h_0 - E_0 \sum_{t=0}^{\infty} \beta^{t+1} u_{h,t+1} h_{t+1}$$

(C7)

Similarly, combing the fourth and seventh terms of (C6) gives

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} R_{t-1} b_t - E_0 \sum_{t=0}^{\infty} \beta^t \beta R_t u_{c,t+1} b_{t+1} = u_{c,0} R_{-1} b_0 + E_0 \sum_{t=1}^{\infty} \beta^t u_{c,t} R_{t-1} b_t - E_0 \sum_{t=0}^{\infty} \beta^t \beta R_t u_{c,t+1} b_{t+1}$$

$$= u_{c,0} R_{-1} b_0$$

(C8)

Also, the combination of the first and fifth terms of (C6) gives:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} \left(-\frac{u_{l,t}}{u_{c,t}} \right) l_t - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} c_t = -E_0 \sum_{t=0}^{\infty} \beta^t u_{l,t} l_t - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} c_t .$$

(C9)

Finally, substituting (C7)-(C9) into (C6) yield:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{l,t} l_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} c_t + E_0 \sum_{t=0}^{\infty} \beta^{t+1} u_{h,t+1} h_{t+1} - E_0 \sum_{t=0}^{\infty} \beta^t u_{c,t} (1-\tau_t^\pi) \Pi_t - u_{c,0} R_{-1} b_0 - u_{c,0} q_0 h_0 = 0$$

or,

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[u_{c,t} c_t + u_{l,t} l_t + \beta u_{h,t+1} h_{t+1} - u_{c,t} (1-\tau_t^\pi) \Pi_t \right] = u_{c,0} R_{-1} b_0 + u_{c,0} q_0 h_0 ,$$

(C10)

which is condition (16) in the text.