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CEMA at Central University of Economics and Finance

2011

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March 19, 2012

Abstract

By introducing Keynesian time preference, we reexamine the neoclassical growth model with endogenous time preference. It is shown that the existence, uniqueness, and stability of the steady state guarantee. When introduced by MIU, money is superneutral and the Friedman rule is optimal. Furthermore, the equilibrium real rate of return is equal to the equilibrium rate of time preference and is the unique fixed point of the time preference function.

Keywords: Keynesian Time Preferences, Monetary Superneutrality, the Friedman Rule

JEL Classification Numbers: E1 E52 O42

1 Introduction

In neoclassical economics, the rate of time preference is usually taken as an exogenously given parameter, which equals the equilibrium real interest rate in the long run. As a measure of the degree of patience, the rate captures

*The auther wishes to thank Heng-fu Zou, Liutang Gong, and Yulei Luo for their helpful comments. All errors are my own. Email address: wanggaowang@gmail.com.

agents' tradeoffs between consumption today and consumption in the future. However, in reality it is easy to notice that people save more (consume less) when the rate of return of the financial assets is high, and consume more (save less) with a lower rate of return. The observation implies that time preferences of people are changing with the fluctuations of the real interest rate, even though the time attitude of people is relatively stable as a kind of phychological characteristics or social habits. In his great work, Keynes (1936) had talked about "changes in the rate of time-discounting". He said that "it was convenient to suppose that expenditure on consumption is cet. par. negatively sensitive to changes in the rate of interest, so that any rise in the rate of interest would appreciably diminish consumption. $\cdots\cdots$. Over a long period substantial changes in the rate of interest probably tend to modifiy social habits considerably, thus affecting the subjective propersity to spend–though in which direction it would be hard to say, except in the light of actual experience."

The literature have talked a lot about endogenous time prefences and their macroeconomic effects. By taking the rate of time preference as an increasing and convex function of the level of current utility, Uzawa (1968) sets up an infinitely lived, representative agent model to replicate the Mundell-Tobin effect which tells that monetary growth raises savings and the capital stock. Employing Uzawa's endogenous time preference in small open economies, Obstfeld (1981) further examines the long-run monetary non-superneutrality and the effects of macroeconomic policies, and Obstfeld (1982) shows that the Laursen-Metzler effect does not exist. Epstein and Hynes (1983) have also investigated monetary superneutrality in Sidrauski (1967) model and concluded that a higher rate of monetary expansion increases the steady-state levels of consumption and capital stock, and reduces the steady-state level of real balances. Gootzeit, Schneider and Smith (2002) show that a permanent increase in government expenditure causes "super-crowding-out" of consumption and lowers the steady-state capital stock. By modelling time preference as an increasing function of real wealth, Kam (2005) has also reexamines the existence of the Tobin effect. In an infinitely lived, representative agent model with the Becker and Mulligan (1997) endogenous time preference which taking the rate of time preference as an increasing function of the resources spent on imagining the future, Gong (2006) shows that an increase of inflation reduces the resources spent on imagining the future which increases the rate of time preference and decreases the steady-state value of capital stock. In order to investigate Stockman's (1981) conjecture, Wang and Zou (2011) have reexamined monetary non-superneutrality and the non-optimality of the Friedman rule by taking the rate of time preference as a strictly increasing and strictly convex function of the inflation rate named "inflation aversion". Comparing these models with the standard neoclassical growth model with a constant rate of time preference, we know that in order to ensure the saddle-point stability of the steady state, we have to impose additional conditions. Furthermore, money is not superneutral in these models with endogenous time preference generally speaking.

By modelling the rate of time preference as a strictly decreasing and strictly convex function of the real rate of return of the financial assets in a neoclassical growth model, we draw different conclusions from the literature. It is shown that the existence, uniqueness, and stability of the steady state guarantee in the models withour and with money, and the steady-state rate of return of financial assets is the unique fixed point of the time preference function. And, when money is introduced, money is also superneutral and the optimal monetary policy is the Friedman rule in the sense of Sidrauski (1967) and Friedman (1969). Therefore, the note offers a useful extention of the standard neoclassical growth model.

The rest of this paper is organized as follows. In section 2, we extend the standard neoclassical growth model to a framework with Keynesian time preference and examine the dynamics of the economy. In section 3, by introducing money into the model, we reexamine monetary superneutrality and the optimality of the Friedman rule. The concluding remarks are in section 4.

2 The Neoclassical Growth Model without Money

2.1 Keynesian Time Preference

In the continous-time framework, based on the idea of Keynes about the relationship between the rate of time preference and the rate of return of the financial asst, we assume that the time preference rate ρ_t of the representative agent is a strictly decreasing and stictly convex function of the real interest rate r_t, namely,

$$\rho_t = \rho(r_t), \tag{1}$$

which satisfies

$$\rho'(r_t) < 0, \rho''(r_t) > 0, \rho(0) = \rho_0 > 0. \tag{2}$$

Asumptions (1) and (2) show that the rate of time preference is endogenously determined by the real rate of return of the financial asset. The higher the real rate of return, the more patience the individual has, but the increase of the level of patience is at a decreasing rate. It is also assumed that the time discount factor of the individual at time t depends not only on the current real rate of return, but also on the entire path of the return rate $\{r_v\}_{v=0}^{t}$. That is,

$$\Theta_t = \int_{v=0}^{t} \rho(r_v) dv. \tag{3}$$

Then, the modelling strategy has generated a new state variable, namely, the time discount factor Θ_t. Differentiating Θ_t with respect to t in equation (3) gives rise to the dynamic equation of the time discount factor, namely,

$$\dot{\Theta}_t = \rho(r_t). \tag{4}$$

2.2 The Neoclassical Model without Money

In this section, we introduce Keynesian time preference into the standard Ramsey-Cass-Koopman (Ramsey, 1928; Cass, 1965; Koopman, 1965) model. The representative consumer's problem is summerized as maximizing

$$\int_{t=0}^{\infty} u(c_t) e^{-\Theta_t} dt, \tag{5}$$

subject to the flow constraint

$$\dot{a}_t = r_t a_t + w_t - c_t, \ a_0 \text{ given}, \tag{6}$$

the dynamic equation of the time discount factor (4), and the no-Ponzi-game condition

$$\lim_{t \to \infty} a_t \exp(-\int_{v=0}^{t} r_v dv) = 0. \tag{7}$$

The Hamiltonian associated with this problem is

$$H = e^{-\Theta_t} u(c_t) + \widetilde{\lambda}_t [f(k_t) - c_t] + \widetilde{\mu}_t \rho(r_t),$$

where $\widetilde{\lambda}_t$ and $\widetilde{\mu}_t$ are two present-valued multipliers associated with constraints (6) and (4), representing the shadow values of capital and the time discount rate, respectively. The optimality conditions include the FOCs[1]

$$
\begin{aligned}
e^{-\Theta}u'(c) &= \widetilde{\lambda}, && (8)\\
\widetilde{\lambda}r &= -\dot{\widetilde{\lambda}}, && (9)\\
e^{-\Theta}u(c) &= -\dot{\widetilde{\mu}}, && (10)
\end{aligned}
$$

and the transversality conditions.

Competition in the markets for capital and labor services gives $r = f'(k)$ and $w = f(k) - kf'(k)$. Define $\lambda = e^{\Theta}\widetilde{\lambda}$ and $\mu = e^{\Theta}\widetilde{\mu}$ as the current-valued multipliers. Substituting these equations and the asset market clearing condition $a = k$ into the optimality conditions leads to the dynamic system that we shall examine as follows:

$$
\begin{aligned}
\dot{c} &= -\frac{u'(c)}{u''(c)}[f'(k) - \rho(f'(k))], && (11)\\
\dot{k} &= f(k) - c, && (12)
\end{aligned}
$$

together with the initial condition k_0 and the TVC. Define the steady state (c^*, k^*) by setting $\dot{c}_t = \dot{k}_t = 0$. We obtain two algebraic equations as follows

$$
\begin{aligned}
f'(k^*) &= \rho(f'(k^*)), && (13)\\
f(k^*) &= c^*. && (14)
\end{aligned}
$$

It is easy to know the existence and uniqueness of the steady state from the assumptions on time preference function (1) and (2), and the neoclassical assumptions of the production function. To check the saddle-point stability of the steady state, we linearize equations (11) and (12) around the steady state calculated from equations (13) and (14)

$$
\begin{bmatrix} \dot{c} \\ \dot{k} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{u'(c^*)}{u''(c^*)}f''(k^*)[1 - \rho'(f'(k^*)] \\ -1 & f'(k^*) \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \end{bmatrix}. \quad (15)
$$

[1] For notational simplicity, we will omit the time subscripts in the following mathematical presentations.

The determinant of the Jacobian matrix is negative, i.e.,

$$\det(J) = -\frac{u'(c^*)}{u''(c^*)} f''(k^*)[1 - \rho'(f'(k^*))] < 0, \tag{16}$$

which shows that one of the two eigenvalues is negative and the other one is positive. Hence, the steady state is a saddle. Similar to the standard model with a constant rate of time preference, as is hyperbolic, the linearized system is conjugate to the original nonlinear system in a neighborhood of the steady state.

Proposition 1 *The existence, uniqueness, and stability of the steady state in the neoclassical growth model with Keynesian time preference guarantee, almost the same to the standard model with a constant rate of time preference.*

From equation (13) and the equilibirum condition $r = f'(k)$, we have

$$r^* = \rho(r^*). \tag{17}$$

There exists a unique equilibrium real rate of return satisfying equation (17) because of the monotonicity and convexity of the time preference function.

Proposition 2 *The rate of time preference is equal to the real rate of return in the steady state; furthermore, the equilibrium rate of return is the unique fixed point of the time preference function.*

Proposition 2 does embody not only the original idea that the rate of time preference determines the real rate of return of financial assets, but also Keynes' idea of how the real interest rate affecting the subjective time preference in the long run.

3 The Neoclassical Monetary Growth Model

In this section, we introduce money into the model by MIU putting forward by Sidrauski (1967). The optimization problem of the representative consumer is maximizing

$$W = \int_{t=0}^{\infty} e^{-\Theta_t} u(c_t, m_t) dt, \tag{18}$$

subject to the budegt constraint

$$\dot{a}_t = r_t k_t + w_t + \chi_t - c_t - \pi_t m_t, \tag{19}$$

the dynamic equation of the discount factor (4), the wealth stock constraint

$$a_t = k_t + m_t, \tag{20}$$

and the no-Ponzi-game condition

$$\lim_{t\to\infty} a_t \exp\left(-\int_{v=0}^{t} r_v dv\right) = 0, \tag{21}$$

where c_t , m_t, k_t, and a_t are consumption, real money balances, physical capital stock, and total wealth, respectively; r_t and w_t are the real interest rate and real wages; and χ_t denotes lump-sum real money transfer payments. The wealth stock constraint requires that the total wealth a_t be allocated between capital k_t and real balances m_t. The instantaneous utility function $U_t = u(c_t, m_t)$ satisfies $u_c > 0$, $u_m > 0$, $u_{cc} < 0$, $u_{mm} < 0$, $u_{cc}u_{mm} - u_{cm}^2 > 0$ and the Inada conditions. And we assume that both commodities are not inferior[2].

The corresponding Hamiltonian is

$$H = e^{-\Theta_t}u(c, m) + \widetilde{\lambda}[rk + w + x - c - \pi m] + \widetilde{\mu}\rho(r) + \widetilde{q}(k + m - a),$$

where $\widetilde{\lambda}$ and $\widetilde{\mu}$ are two present-valued multipliers associated with constraints (19) and (4), representing shadow values of the wealth stock and time discount factor, respectively; $\widetilde{q}$ is the Lagrangian multiplier attached to the stock constraint (20), representing the marginal value of the financial wealth.

The optimality conditions for a maximum are given by the FOCs

$$\begin{aligned}
e^{-\Theta}u_c(c, m) &= \widetilde{\lambda}, && (22)\\
e^{-\Theta}u_m(c, m) &= (r + \pi)\widetilde{\lambda}, && (23)\\
\dot{\widetilde{\lambda}} + r\widetilde{\lambda} &= 0, && (24)\\
e^{-\Theta}u(c, m) &= \dot{\widetilde{\mu}}. && (25)
\end{aligned}$$

[2]It is not hard to prove that the normality of the two goods is equivalent to the following two conditions, respectively, $u_{mm} - \frac{u_{cm}u_m}{u_c} < 0$, $\frac{u_{cc}u_m}{u_c} - u_{cm} < 0$.

and the TVCs

$$\lim_{t\to\infty} e^{-\Theta}\widetilde{\lambda} a = 0, \lim_{t\to\infty} e^{-\Theta}\widetilde{\mu}\Theta = 0. \tag{26}$$

The behavior of the firm is also summarized by the two conditions on marginal productivity

$$r = f'(k), w = f(k) - kf'(k). \tag{27}$$

In order to examine macroeconomic equilibrium, we introduce the government's behavior. It is assumed that the government maintains a constant rate of monetary growth

$$\frac{\dot{M}}{M} = \theta, \tag{28}$$

and keeps its budget balanced

$$\chi = \frac{\dot{M}}{P}. \tag{29}$$

Substituting equation (28) and $m = \frac{M}{P}$ into equation (29) results in $\chi = \theta m$. Taking the derivative of $m = \frac{M}{P}$ with respect to t, rearranging, and substituting equtions (28) into it, we have

$$\dot{m} = (\theta - \pi)m.^3 \tag{30}$$

Equations (23) and (27) imply that:

$$\frac{u_m(c,m)}{u_c(c,m)} = (f'(k) + \pi). \tag{31}$$

From equation (31), we solve π as a function of c, m, and k, i.e., $\pi_t = \pi(c,k,m)$. And it is easy to know that

$$\pi_c = \frac{u_{mc}u_c - u_{cc}u_m}{u_c^2} > 0, \pi_m = \frac{u_{mm}u_c - u_{cm}u_m}{u_c^2} < 0, \pi_k = -f''(k) > 0. \tag{32}$$

Putting $\pi_t = \pi(c,k,m)$ into equation (30) gives the dynamics of real money balances

[3] We assume that the representative agent has perfect foresight, i.e., $\frac{\dot{P}}{P} = \pi$.

$$\dot{m} = (\theta - \pi(c, k, m))m. \tag{33}$$

Equations (22), (24), (27), and (33) result in the dynamics of consumption

$$\dot{c} = -\frac{u_c(c, m)}{u_{cc}(c, m)}[f'(k) - \rho(f'(k))] - \frac{u_{cm}(c, m)}{u_{cc}(c, m)}[\theta - \pi(c, k, m)]m. \tag{34}$$

From equation (19), (20), (27) and (33), we have

$$\dot{k} = f(k) - c. \tag{35}$$

Therefore, equations (33), (34) and (35) describe the whole dynamics of the model together with the initial and transversality conditions.

The steady state (c^*, k^*, m^*) of the dynamic system is defined by $\dot{c} = \dot{k} = \dot{m} = 0$. The resulting form of algebraic equations is

$$\begin{aligned} f'(k^*) &= \rho(f'(k^*)), && (36)\\ f(k^*) &= c^*, && (37)\\ \theta &= \pi(c^*, k^*, m^*). && (38) \end{aligned}$$

Equation (36) gives the familiar modified golden-rule level of capital accumulation, which shows that, in the steady state, the marginal product of physical capital equals the subjective time preference rate; equation (37) tells that the steady state level of production equals the steady state level of consumption; and equation (38) shows that the steady state level of inflation is equal to the exogenously given rate of monetary growth.

Similar to the previous section, equation (36) determines uniquely the steady state level of capital k^* and real interest rate r^* by the assumptions on the time preference and production functions. Then equation (37) gives the steady state level of consumption and equation (38) determines implicitly the steady state level of real money balances. To examine the stibility of the steady state, we linearize equations (33)-(35) around the steady state (c^*, k^*, m^*)

$$\begin{bmatrix} \dot{c} \\ \dot{k} \\ \dot{m} \end{bmatrix} = \begin{bmatrix} \frac{u^*_{cm}}{u^*_{cc}}\pi^*_c m^* & -\frac{u^*_c}{u^*_{cc}} f''(1-\rho') + \frac{u^*_{cm}}{u^*_{cc}}\pi^*_k m^* & \frac{u^*_{cm}}{u^*_{cc}}\pi^*_m m^* \\ -1 & f'(k^*) & 0 \\ -\pi^*_c m^* & -\pi^*_k m^* & -\pi^*_m m^* \end{bmatrix} \begin{bmatrix} c - c^* \\ k - k^* \\ m - m^* \end{bmatrix}. \tag{39}$$

Define the Jacobian matrix of the linearized system as J. It is not hard to find that

$$\prod_{i=1}^{3} \lambda_i = \det(J) = \frac{u_c^*}{u_{cc}^*} f''(k^*) \pi_m^* m^* (1 - \rho'(f'(k^*))) < 0, \tag{40}$$

and

$$\sum_{i=1}^{3} \lambda_i = tr(J) = f'(k^*) + \frac{(u_{cc}^* u_{mm}^* - u_{mc}^{*2})}{-u_c^* u_{cc}^*} > 0. \tag{41}$$

Equation (40) implies that there exists one negative real eigenvalue or three eigenvalues with negative real parts, and equation (41) shows that there exists at least one eigenvalue with a positive real part. Hence there exists an eigenvalue with a negative eigenvalue exactly and the steady state is a saddle.

Proposition 3 *In the neoclassical monetary growth model with Keynesian time preference, the steady state exists uniquelly and is locally saddle-point stable, similar to the original Sidrauski model.*

In order to reinvestigate the macroeconomic effects of the permanent monetary policy, totally differentiating equations (36)-(38) leads to a system of linear equations as follows:

$$\begin{bmatrix} 0 & (1 - \rho'(f'(k^*)))f''(k^*) & 0 \\ 1 & -f'(k^*) & 0 \\ \pi_c^* & \pi_k^* & \pi_m^* \end{bmatrix} \begin{bmatrix} \frac{dc^*}{d\theta} \\ \frac{dk^*}{d\theta} \\ \frac{dm^*}{d\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Applying Cramer's rule results in

$$\frac{dc^*}{d\theta} = 0, \tag{42}$$

$$\frac{dk^*}{d\theta} = 0, \tag{43}$$

$$\frac{dm^*}{d\theta} = \frac{1}{\pi_m^*} < 0. \tag{44}$$

Equations (42) and (43) give the result of monetary superneutrality in the sense of Sidrauski (1967), i.e., a permanent increase of monetary growth has no effect on the steady-state levels of consumption and capital in the long run. Equation (44) shows that a positive monetary policy has negative effects on real money balances because of the increase of the opportunity cost of holding money.

The superneutrality result has direct implications for the optimal rate of monetary growth. To examine the optimality of the Friedman rule, we write down the steady-state level of utility as follows:

$$W^* = \int_{t=0}^{\infty} e^{-\rho(r^*)t} u(c^*, m^*) dt = \frac{u(c^*, m^*)}{\rho(r^*)}. \tag{45}$$

Taking the derivative of W^* with respect to θ in equation (45) yields

$$\frac{dW^*}{d\theta} = \frac{(u_c^* \frac{dc^*}{d\theta} + u_m^* \frac{dm^*}{d\theta}) r^* + u^* f''(k^*) \frac{dk^*}{d\theta}}{r^{*2}} = \frac{u_m^*}{\pi_m^* r^*}. \tag{46}$$

Because monetary growth does not affect consumption in the steady state, the steady-state utility is maximized by making real balances large enough that their marginal utility equals zero. Setting $\frac{dW^*}{d\theta} = \frac{u_m^*}{\pi_m^* r^*} = 0$ and reminding equations (27), (31) and (38) lead to

$$\theta (= \pi^*) = -\rho(r^*) (= -r^*). \tag{47}$$

Equation (47) gives the similar result of Friedman's rule for optimum quantity of money, i.e., the optimal monetary growth rate is equal to the negative of the time preference rate, or equivalently the nominal interest rate equals zero, $i^* (= \pi^* + r^*) = 0$. Moreover, the equilibrium real interest rate equals the equilibrium time preference rate in the long run and also is the unique fixed point of the time preference function. Hence, we have the following proposition.

Proposition 4 *In the neoclassical monetary growth model with endogenous time preference, money is superneutral in the sense of Sidrauski (1967), and optimal monetary policy is Friedman's rule for optimum quantity of money. Furthermore, the equlibrium interest rate is equal to the equilibrium time preference rate and also the unique fixed point of the time preference function.*

4 Concluding Remarks

In this note, by introducing Keynesian time preference, we have reexamined the neoclassical growth model. It is shown that the existence, uniqueness and stability of the steady state guarantee, almost the same to the original neoclassical model. And, when money is introduced by money-in-utility, money is superneutral in the long run and Friedman's rule for optimum quantity of money holds. Furthermore, the equilibrium interest rate is equal to the equilibrium time preference rate and is the unique fixed point of the time preference function.

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