Competition and innovation with horizontal RD spillovers

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Competition and Innovation with Horizontal R&D Spillovers

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The paper extends a theoretical framework for analyzing competition and innovation in presence of horizontal spillovers. Introducing two scenarios, it is shown that when firms behave non-cooperatively in both the R&D and production stages the degree of spillover has a negative relationship with the effective and respective R&D expenditures of each firm as well as the level of social welfare. When firms behave cooperatively in the R&D stage, and non-cooperatively in the production stage the relationship between the R&D expenditure of the joint research lab and the number of firms in the market is negative.

1 Introduction

In the economic literature, innovation – also called R&D – is often classified as product innovation or process innovation. A firm carries out a program of product innovation to find a new product that it hopes will generate new demand and lead to large profits. Process innovation, on the other hand, aims at finding a new process to reduce the production cost of a product. A lower production cost, which is the desired outcome of the R&D program, gives the firm a cost advantage over its rivals. Whether a program of innovation will be carried out or not depends on the cost of R&D and the market structure in which the firm finds itself. Knowledge and benefits obtained by a firm from its R&D activities typically leak out to other firms, to consumers and, eventually, to other countries. These leakages – called R&D spillovers – mean that a firm cannot appropriate all the fruit of its R&D activities, especially when spillovers flow to its competitors in the same industry. However, from society's point of view, spillovers represent positive externalities in the sense that they reduce the production costs of other firms, with the ensuing result of lower prices for consumers. In a review of the literature on R&D aimed at providing guidelines for recent efforts to include R&D in the national income accounts, Sveikauskas (2007) indicated that perhaps the private rate of return to R&D is 25%, while it is 65% for social returns.

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In light of the positive externalities generated by R&D activities, the authorities charged with competition policy in Europe and Japan have adopted a rather permissive anti-trust attitude toward R&D cooperation for quite some time. The research – both theoretical and empirical – received the needed impetus in 1984 when the US passed the National Cooperation Act in 1984, allowing firms to cooperate in R&D, but not in product markets. Over the last two decades, the economics of R&D spillovers has been one of the most active fields of research in industrial economics.

The theoretical literature on competition and cooperation in R&D with technological spillovers can be said to begin with the pioneering work of d'Aspremont and Jacquemin (1988) (AJ hereafter), who formulated a two-stage duopoly game of R&D spillovers in which the two firms behave in a non-cooperative manner in the second (production) stage, but can either cooperate or behave in a non-cooperative manner in the first (R&D) stage. The AJ's model has been extended by Kamien *et al.* (1992), Kamien and Zang (1993), Atallah (2000), Amir *et al.* (2003), Escrihuela-Villar (2004), and Cellini (2005), among many others. A survey of the main results of this literature is provided by De Bondt (1997). The predictions of the AJ model, especially the important question of whether spillovers increase firms' incentives to cooperate in R&D, has been addressed by a number of empirical studies with mixed results; see, for example, Cassiman and Veugelers (2002) and Suetens (2004).

In the literature on competition and cooperation in R&D with spillovers just described, the number of firms – often taken to be two symmetric firms – is usually fixed, and thus the relationship between competition and innovation is not addressed. Our paper is intended to fill part of this lacuna. The paper presents an analysis – in the context of an AJ model – of the relationships between competition and innovation when there are horizontal R&D spillovers among firms in a market.

The modified version of the AJ model we formulate is a two-stage static game played by n symmetric firms producing a homogenous good. In the first stage of the game, firms can behave cooperatively or non-cooperatively in their R&D activities to find a new production process with a lower production cost. In the second stage, the firms compete according to the Cournot model of competition. If the firms behave non-cooperatively in the R&D stage, then each firm will run its own research lab, and take into account the natural spillovers that flow among firms in a strategic manner. If the firms cooperate in R&D, then they jointly run a single research lab, with each firm contributing an equal part to the total cost of operating the research lab. The fruit of the joint R&D program, namely the process with a lower production cost that is the results of the efforts by research workers, is available to each firm. In modeling the horizontal spillovers among firms, we follow the pioneering work of Ruff (1969), who analyzed a stylized growth model in which firms compete according to the Cournot model of competition, and in which firms undertake R&D activities by employing research workers. In Ruff's model, a firm recognizes a potential transmission of knowledge from other firms, and the transmission of knowledge is modelled by assuming that the effective input in R&D of a firm consists of its own input plus part of the inputs of all the other firms. Modeling cooperative R&D as research efforts carried out in a single research lab and equal sharing of cost, Ruff found that an economy in which firms cooperate in R&D, but behave non-cooperatively in production, is more progressive than an economy in which firms act non-cooperatively in both R&D and production. Ruff's analytical treatment of R&D spillovers has been adopted by later researchers, such as Spence (1984), Kamien *et al.*, and Amir *et al*.

Our findings are along the lines of the Schumpeterian hypothesis. We show that there is a negative relationship between competition and innovation. More precisely, when there are more firms in the market, there will be less innovation in the sense that the reduction in production cost of each firm that is brought about by R&D is lower when there is more competition. In particular, when the number of firms is large, there will be very little innovation. There is thus a trade-off between dynamic efficiency and allocative efficiency. We argue that when the R&D technology is very productive, monopoly generates a dramatic cost reduction, and yields the highest level of social welfare: the dynamic efficiency generated by monopoly more than offsets the allocative inefficiency it creates. When the R&D technology is slightly productive, we argue that the number of firms that yields the highest level of social welfare is more than one: there is an inverted-U relationship between competition and social welfare. To support our arguments, we provide a numerical example. It is also shown that if firms are allowed to cooperate in the

R&D stage, but not in the production stage, the negative relationship between competition and innovation remains. More precisely, we show that when the firms cooperate by running jointly a single research lab, with each firm contributing equally to the cost of operating the lab, then the industry's total R&D expenditure declines when the number of firms increases. Our model also sheds light on the effects of R&D spillovers on innovation and welfare. We find that R&D spillovers reinforce the negative impact of competition, and make innovation even less attractive, with the ensuing consequence that the higher is the degree of R&D spillovers, the lower will be the level of social welfare.

The paper is organized as follows. In Section 2, the general features of the model are presented. The scenario under which the firms behave non-cooperatively in both the R&D stage and the production stage is analyzed in Section 3. In Section 4, the scenario under which the firms are allowed to cooperate in the R&D stage, but not in the production stage, is analyzed. Some concluding remarks are given in Section 5.

2 The General Features of the Model

Consider the market for a commodity, with

(1) $p = a - bQ$

as the inverse market demand curve. Here Q is the industry output, and p is the market price. Also, a and b are two positive parameters. There are n symmetric firms present in the market, and these firms compete according to the Cournot model of competition. The common initial marginal cost of each firm is $c^0, 0 < c^0 < a.$ We envision two scenarios. Under the first scenario, each firm acts non-cooperatively in R&D, and carries out its own R&D program. Under the second scenario, the firms cooperate with each other in the sense that a single joint program of process innovation is carried out, with each firm contributing an equal part of the cost of the program. The fruit of the joint R&D venture is then available to each contributing member.

We shall suppose that research workers constitute the only input in a program of process innovation. Furthermore, assuming that the wage received by a worker is the numéraire, we can identify the number of research workers with the R&D expenditure. We shall let $y = f[X]$ denote the reduction in marginal cost yielded by a program of process innovation with X as its effective input, with effective input being the sum of the own input provided by the firm that runs the program plus the spillovers from the other innovation programs. The production function of R&D is assumed to have the following properties: (i) $f[0] = 0$. (ii) $f[X] > 0$ for all $X > 0$, (iii) $f'[X] > 0$ for all $X > 0$, and (iv) $f''[X] < 0$ for all $X > 0$. Furthermore, $\ell im_{X \to 0} f'[X] = \infty$, and $\ell im_{X \to \infty} f[X] < c^0$.

If the firms cooperate in R&D, and if $x_i, i = 1,...,n,$ is firm $i's$ contribution to the cost of running the research laboratory, then cost reduction that is the fruit of the joint R&D venture is $f[x_1 + ... + x_i + ... + x_n]$. On the other hand, if there is no cooperation in R&D, then the cost reduction obtained by firm i is assumed to be given by $f[x_i + \beta \sum_{j \neq i} x_j]$, where $0 \leq \beta \leq 1$, is a parameter that represents the degree of spillovers from the R&D activities of all the other firms, and $x_j, j = 1,...,n,$ is firm $j's$ own expenditure on R&D. The expression $\beta \sum_{j \neq i} x_j$ represents the spillovers to firm i from the R&D activities of all the other firms. The sum $X_i = x_i + \beta \sum_{j \neq i} x_j$ thus represents the effective input of firm i in its own program of process innovation. When $\beta = 0$, there are no spillovers, and when $\beta = 1$, there are full spillovers. The intermediate case $0 < \beta < 1$ corresponds to the situation of partial spillovers.

3 Non-cooperation in both stages

When each firm carries out its own process innovation program, the reduction in the marginal cost of a firm, say firm i, is $f[x_i + \beta \sum_{j \neq i} x_j]$, and its post-innovation marginal cost is given by

(2) $$c_i^1 = c^0 - f[x_i + \beta \sum_{j \neq i} x_j].$$

3.1 The post-innovation equilibrium

Given $c_i^1, i = 1,\dots,n,$ the post-innovation marginal costs of the n firms, let q_i be the output of firm i. Under the combination of strategies $(q_1,\dots,q_i,\dots,q_n),$ the profit – gross of R&D cost – of firm i is

$$q_i\left(a - b(q_1 + \dots + q_i + \dots + q_n) - c_i^1\right).$$

The first-order condition that characterizes the best response of firm i to q_{-i} is

$$a - b(q_1 + \dots + q_i + \dots + q_n) - c_i^1 \text{ - } bq_i = 0. \qquad (i = 1,\dots, n), \tag{3}$$

Letting $Q = q_1 + \dots + q_i + \dots + q_n$ denote the industry output, and then summing (3) over $i = 1,\dots,n,$ we obtain

$$na - nbQ - \sum_{i=1}^{n} c_i^1 - bQ = 0,$$

from which it follows that

$$Q = \frac{na - \sum_{i=1}^{n} c_i^1}{(n+1)b}. \tag{4}$$

Using (4), we obtain the following expression for the equilibrium market price

$$\begin{aligned} p &= a - bQ \\ &= \frac{a + \sum_{i=1}^{n} c_i^1}{n+1}. \end{aligned} \tag{5}$$

Using (4) in (3), we obtain the following expression for the output of firm i:

$$\begin{aligned} q_i &= \frac{a - bQ - c_i^1.}{b} \\ &= \frac{a - nc_i^1 + \sum_{j \neq i} c_j^1}{(n+1)b}. \end{aligned} \tag{6}$$

The profit – gross of innovation cost – made by firm i in the post-innovation stage is then given by

$$\pi_i[c_1^1,...,c_i^1,...,c_n^1] = q_i(p - c_i^1)$$

(7)
$$= \frac{a - nc_i^1 + \sum_{j\neq i} c_j^1}{(n+1)b}\left(\frac{a + \sum_{j=1}^{n} c_i^1}{n+1} - c_i^1\right)$$

$$= \frac{1}{b(n+1)^2}\left(a - nc_i^1 + \sum_{j\neq i} c_j^1\right)^2.$$

3.2 The equilibrium in the innovation stage

Given the list $(x_1,...,x_n)$ of R&D expenditures of the n firms, the profit – net of R&D costs – made by firm i over the two stages of the game is given by

$$-x_i + \pi_i(c_1^1,...,c_i^1,...,c_n^1)$$

(8)
$$= -x_i + \frac{1}{b(n+1)^2}\left(a - n(c^0 - f[X_i]) + \sum_{j\neq i}(c^0 - f[X_j])\right)^2.$$

The first-order condition that characterizes the best response of firm i to $(x_j)_{j\neq i}$ is

(9) $$-1 + \frac{2}{b(n+1)^2}\begin{pmatrix} a - n(c^0 - f[X_i]) \\ + \sum_{j\neq i}(c^0 - f[X_j]) \end{pmatrix}\begin{pmatrix} nf'[X_i] \\ -\beta\sum_{j\neq i} f'[X_j] \end{pmatrix} = 0.$$

The second-order condition is

(10) $$\begin{pmatrix} a - n(c^0 - f[X_i]) \\ + \sum_{j\neq i}(c^0 - f[X_j]) \end{pmatrix}\begin{pmatrix} nf''[X_i] \\ -\beta^2\sum_{j\neq i} f''[X_j] \end{pmatrix} + \begin{pmatrix} nf'[X_i] \\ -\beta\sum_{j\neq i} f'[X_j] \end{pmatrix}^2 < 0.$$

For a symmetric equilibrium, we have $x_1 = ... = x_n = x,$ and the effective R&D expenditure of each firm is given by

(11) $$X_1 = ... = X_n = X = x(1 + (n-1)\beta),$$

and the first-order condition (9) is then reduced to

$$(12) \qquad -1+\frac{2(n-(n-1)\beta)}{b(n+1)^2}\left(a-c^0+f[X]\right)f'[X]=0.$$

We shall let $X[n,\beta]$ denote the equilibrium effective R&D input of each firm, given that the firms act non-cooperatively in the R&D stage. As for the second-order condition, it is reduced to

$$(13) \qquad \left(a-c^0+f[X]\right)\left(\frac{n-(n-1)\beta^2}{\left(n-(n-1)\beta\right)^2}\right)f''[X]+(f'[X])^2<0.$$

Because $f''[X]<0$ and $\left(\left(n-(n-1)\beta^2\right)/\left(n-(n-1)\beta\right)^2\right)<1,$ it follows from (13) that

$$(14) \qquad \left(a-c^0+f[X]\right)f''[X]+(f'[X])^2<0.$$

To study the impact of competition and spillovers on innovation, let us rewrite (12) as follows:

$$(15) \qquad \left(a-c^0+f[X[n,\beta]]\right)f'[X[n,\beta]]=\frac{1}{2}\frac{b(n+1)^2}{n-(n-1)\beta}.$$

Differentiating (15) with respect to $\beta,$ we obtain

$$(16) \qquad \begin{aligned}&\left(\left(a-c^0+f[X[n,\beta]]\right)f''[X[n,\beta]]+\left(f'[X[n,\beta]]\right)^2\right)\frac{\partial X[n,\beta]}{\partial\beta}\\ &\qquad=\frac{1}{2}\frac{b(n+1)^2(n-1)}{\left(n-(n-1)\beta\right)^2}>0.\end{aligned}$$

In (16), the coefficient of $\partial X[n,\beta]/\partial\beta$ is negative according to (14), and this implies that $\partial X[n,\beta]/\partial\beta<0.$ Thus the higher is the degree of spillovers $\beta,$ the lower will be the effective R&D expenditure – and a fortiori the own R&D expenditure. Now a lower effective expenditure on R&D of each firm means that the marginal cost reduction obtained by each firm is smaller, with the ensuing consequence that the equilibrium post-innovation market price will be higher. A higher post-innovation market price obviously implies a lower level of consumer surplus. It also implies a lower level of producer surplus. To see why the latter result is true, first note that under the symmetric equilibrium the producer surplus net of R&D cost is

$$(17) \qquad -X[n,\beta]+\frac{n}{b(n+1)^2}\left(a-c^0+f[X[n,\beta]]\right)^2.$$

Differentiating (17) with respect to β, we obtain

$$(18) \qquad \left(-1+\frac{2n}{b(n+1)^2}\left(a-c^0+f[X[n,\beta]]\right)f'[X[n,\beta]]\right).\frac{\partial X[n,\beta]}{\partial \beta}<0.$$

Note that the inequality (18) has been obtained by using the following facts. First, we have shown $\partial X[n,\beta]/\partial\beta < 0$. Second, note that in light of (12), the expression between the grand pair of parentheses is positive. We summarize the results just obtained in the following proposition:

PROPOSITION 1: *The higher is the degree of spillovers* β,

(i) the lower will be the effective R&D expenditure of each firm,[2]

(ii) the higher will be the equilibrium post-innovation market price;

(iii) the lower will be the consumer surplus;

and

(iv) the lower will be the producer surplus.

In short, the higher is the degree of horizontal spillovers, the lower will be the level of social welfare.

The economic intuition behind this result is not difficult to grasp. When a firm carries out a program of process innovation, part of the fruit of its R&D activities flows to its rivals and lowers their production costs. The lower production costs of the rival firms enable them to compete more effectively with the firm that carries out the program of process innovation, making the innovation less attractive. Furthermore, when spillovers exist, a firm has an incentive to free ride on the R&D activities of the other firms, weakening even more the incentive of a firm to undertake R&D activities. The negative impact of these two effects is more pronounced the higher is the degree of horizontal spillovers. Although from the social perspective more positive externalities will be generated when the degree of spillovers is high, the strategic behaviour of firms that consider only their self interests lead them to curtail R&D. A solution to this problem is to subsidize R&D.

To analyze the relationship between competition and innovation, let us differentiate (15) with respect to n. The result is

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[2] This result is discovered by Amir et al. (2003).

$$\left(\left(a - c^0 + f[X[n,\beta]]\right) f''[X[n,\beta]] + \left(f'[X[n,\beta]]\right)^2\right) \frac{\partial X[n,\beta]}{\partial n} = \frac{b}{2} \frac{(n+1)(n(1-\beta)+3\beta-1)}{\left(n-(n-1)\beta\right)^2} > 0. \tag{19}$$

To see how the strict inequality in (19) is obtained, note that the sign of the right side of (19) is the sign of $n(1-\beta)+3\beta-1$. The derivative of this expression with respect to n is $1-\beta$. If $\beta < 1$, then the expression is strictly increasing in n. Furthermore, for $n = 1$, the expression is reduced to $2\beta \geq 0$. Hence for $\beta < 1$, the expression is strictly increasing in n for $n \geq 1$. For $\beta = 1$, the expression is equal to 2, regardless of the value of n. Thus $\partial X[n,\beta]/\partial n < 0$ for all $n \geq 1$, and we have just proved the following proposition:

PROPOSITION 2: *The effective expenditure on R&D of each firm declines when the number of firms increases. That is, there is a negative relationship between competition and innovation.*

Proposition 2 can be interpreted as follows. In an industry made up of symmetric firms, the market share of a single firm is equal to the inverse of the number of firms. Thus when there is more competition in the sense that there are more firms in the market, the market share of a firm becomes smaller, and thus a given cost reduction will have less and less impact on the firm's profit when more firms are in the market. Thus we can expect a firm's own expenditure on R&D to decline when there is more competition. The first statement of Proposition 2 does confirm this intuition. It asserts an even more dramatic result: the effective R&D expenditure of each firm declines when there are more firms in the industry. This rather surprising result means that the positive externalities that the firms generate among themselves are dominated by the negative impact of competition, and thus the marginal cost reduction that each firm obtains through the program of process innovation gets smaller as the number of firms increase. In particular, (15) indicates that the effective R&D expenditure of each firm will tend to 0 when the number of firms tends to infinity. That is, when the number of firms is high, there is static allocative efficiency – perfect competition with an equilibrium market price that is equal to the initial marginal cost c^0 – but not dynamic efficiency.

To study the impact of competition on welfare – defined as the sum of consumer surplus and producer surplus, let us first analyze the variation in producer surplus when the number of firms increases. Differentiating (17), the industry's profit net of R&D cost, with respect to n, we obtain

$$\text{(20)} \qquad \left(-1+2\frac{n}{b(n+1)^2}\left(a-c^0+f[X[n,\beta]]\right)f'[X[n,\beta]]\right)\frac{\partial X[n,\beta]}{\partial n} - \frac{n^2-1}{b(n+1)^2}\left(a-c^0+f[X[n,\beta]]\right)^2 < 0;$$

that is the producer surplus declines as the number of firms increases.

To analyze the behaviour of the consumer surplus when the number of firms increases, we can look at the equilibrium market price in the post-innovation stage. Using (5), we obtain the following expression for the equilibrium market price – as a function of n – in the post-innovation stage:

$$\text{(21)} \qquad p[n] = \frac{a+n(c^0-f[X[n,\beta]])}{n+1}.$$

If there is only one firm in the market, then (21) is reduced to

$$\text{(22)} \qquad p[1] = \frac{a+(c^0-f[X[1,\beta]])}{2} < \frac{a+c^0}{2},$$

with $(a+c^0)/2$ being the price that the monopolist will charge under the assumption that it does not innovate. If the R&D technology is not very productive in the sense that there is little cost reduction for a given level of effective expenditure, and if the initial marginal cost c^0 is not too high, then the benefits of R&D will be low, and we should expect that $f[X[1,\beta]]$ will be small. Under such a scenario, we have

$$\text{(23)} \qquad c^0 < p[1] = \frac{a+(c^0-f[X[1,\beta]])}{2} < \frac{a+c^0}{2},$$

i.e., with R&D, the monopolist will set a price below the monopoly price without R&D, but above the initial marginal cost. Under this scenario, we expect that the equilibrium market price in the post innovation stage will decline and approach the level c^0 when the number of firms becomes large. Furthermore, as competition increases, consumer surplus will rise to the level of social welfare attained under the competitive equilibrium associated with the high initial marginal cost c^0. Because producer surplus declines as the number of firms rises, social welfare might first rise then decline to the level associated with the static allocative efficiency attained under the competitive equilibrium with the initial marginal cost c^0. There is thus an inverted-U relationship between competition and social welfare under this scenario.

If the R&D technology is very productive, then the cost reduction generated by monopoly will be substantial. Furthermore, if the initial marginal cost is high, say c^0 is very close to a, then we can expect that

(24) $$p[1] = \frac{a + (c^0 - f[X[1,\beta]])}{2} < c^0,$$

i.e., the monopoly price is below the price that will prevail when there are many firms in the market. Under this scenario, we expect the equilibrium market price in the post-innovation stage to rise with the number of firms and reach the level c^0 when $n \to \infty$. Furthermore, along the way, consumer surplus will be declining steadily. Because producer surplus declines when competition increases, social welfare must decline as the number of firms rises under this scenario; that is, monopoly yields the highest level of social welfare under this scenario.

The theoretical predictions of our model are thus along the line of the Schumpeterian hypothesis: the dynamic efficiency generated by market power more than offsets the static allocative inefficiency it creates.

To support the arguments just presented, we now provide a numerical example. In the numerical example, we assume that the cost reduction associated with an effective R&D expenditure X is given by

(25) $$f[X] = 2\sqrt{X/\gamma},$$

where $\gamma > 0$ is a parameter. Also, note that the higher the value of γ, the less productive the R&D technology will be. For the assumed functional form of the cost reduction, the first-order condition (12) can be solved to obtain the following explicit form of the equilibrium effective R&D expenditure of each firm:

$$X[n,\beta] = \frac{4(n-(n-1)\beta)^2\gamma(a-c^0)^2}{\left(4n(\beta-1)-4\beta+b(n+1)^2\gamma\right)^2}. \tag{26}$$

Using (26), we obtain the following expression for the equilibrium cost reduction:

$$\begin{aligned} f[X[n,\beta]] &= 2\sqrt{\frac{X[n,\beta]}{\gamma}} \\ &= \frac{4(n-(n-1)\beta)(a-c^0)}{4n(\beta-1)-4\beta+b(n+1)^2\gamma}. \end{aligned} \tag{27}$$

The marginal cost of each firm in the post-innovation stage is then given by

$$\begin{aligned} c_1^1[n] = \ldots = c_n^1[n] = c^1[n] &= c^0 - f[X[n,\beta]] \\ &= c^0 - \frac{4(n-(n-1)\beta)(a-c^0)}{4n(\beta-1)-4\beta+b(n+1)^2\gamma}. \end{aligned} \tag{28}$$

Using (4) and (28), we obtain the following expression for the equilibrium industry output in the post innovation stage:

$$Q[n] = \frac{na - \sum_{i=1}^{n}\left(c^0 - \frac{4(n-(n-1)\beta)(a-c^0)}{4n(\beta-1)-4\beta+b(n+1)^2\gamma}\right)}{(n+1)b}. \tag{29}$$

Using (29), we obtain the following expression for the equilibrium market price in the post innovation stage

$$\begin{aligned} p[n] &= a - bQ[n] \\ &= \frac{a + \sum_{i=1}^{n}\left(c^0 - \frac{4(n-(n-1)\beta)(a-c^0)}{4n(\beta-1)-4\beta+b(n+1)^2\gamma}\right)}{n+1}. \end{aligned} \tag{30}$$

The equilibrium consumer surplus in the post-innovation stage is

(31) $$CS[n] = \frac{1}{2}(a - p[n])Q[n].$$

Using (26) and (27) in (17), we obtain the following expression for the equilibrium producer surplus net of R&D cost

(32) $$\begin{aligned} PS[n] &= -X[n,\beta] + \frac{n}{b(n+1)^2}\left(a - c^0 + f[X[n,\beta]]\right)^2 \\ &= -\frac{4(n-(n-1)\beta)^2\gamma(a-c^0)^2}{\left(4n(\beta-1) - 4\beta + b(n+1)^2\gamma\right)^2} \\ &\quad + \frac{n}{b(n+1)^2}\left(a - c^0 + \frac{4(n-(n-1)\beta)(a-c^0)}{4n(\beta-1) - 4\beta + b(n+1)^2\gamma}\right)^2. \end{aligned}$$

Social welfare is then

(33) $$SW[n] = CS[n] + PS[n].$$

The following values of the parameters are used in the calculations:

$$\{a \to 2, b \to 1, c^0 \to 1.75, \beta \to 0.10, \gamma \to 1.25\}.$$

For these values of the parameters, the choke price is $a = 2.$ The initial marginal cost is $c^0 = 1.75,$ which has been chose rather close to the choke price to make the cost reduction more dramatic.

The variations of the consumer surplus, the producer surplus, and social welfare as competition increases are depicted in the following figure:

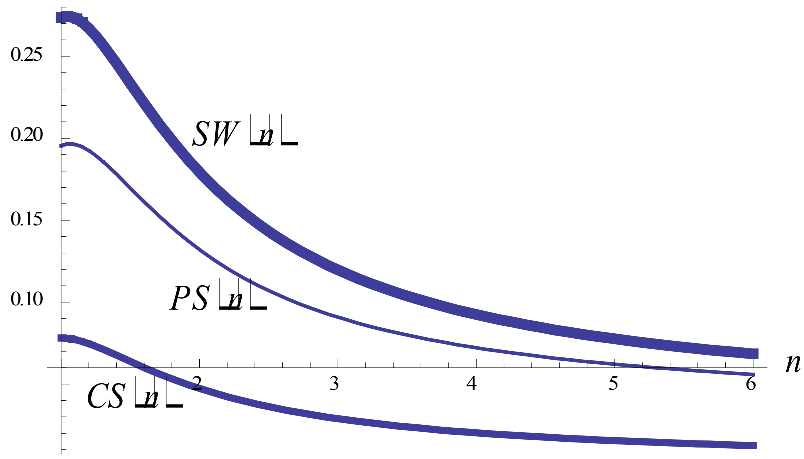

Figure 1.—The impact of competition on consumer surplus, producer surplus, and social welfare: $\gamma = 1.25$

As can be seen from Figure 4, both consumer and producer surplus decline when competition increases. Given the values of the parameters, monopoly generates the most innovation, and yields the highest level of social welfare. The following table presents the equilibrium effective R&D expenditures and the cost reductions as competition increases.

Table 1.

Effective R&D Expenditure and Cost Reduction: $\gamma = 1.25$

Number of firms	X **(effective R&D expenditure)**	$f[X]$ **(reduction in marginal cost)**
1	**0.3125**	**1**
2	**0.0846782**	**0.520548**
3	**0.0316374**	**0.318182**
4	**0.0158096**	**0.224924**
5	**0.0093455**	**0.172932**

As can be seen from Table 4, the cost reduction under monopoly is equal to 1, which is quite dramatic. When there are two firms in the market, the cost reduction of each firm is only about half of the cost reduction when there is only one firm. The gain in allocative efficiency is not sufficient to offset the loss in dynamic efficiency. This is the reason why monopoly yields the highest level of social welfare.

When γ rises from 1.25 to 5.25, the R&D technology becomes much less productive. The following figure depicts the variations of the consumer surplus, the producer surplus, and social welfare as competition increases in this case. As can be seen from Figure 5, consumer surplus rises with competition, but producer surplus declines when competition increases. This results in an inverted-U relationship between competition and social welfare, with a rather flat curvature.

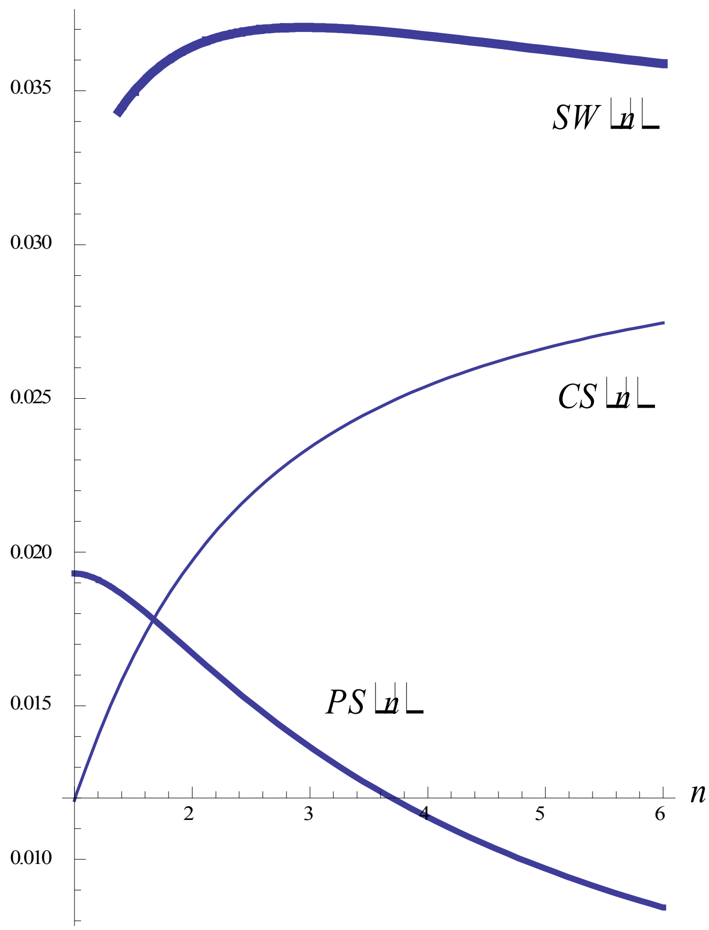

Figure 2.—The impact of competition on consumer surplus, producer surplus, and social welfare: $\gamma = 5.25$

The reason why more than one firm is needed to maximize social welfare when $\gamma = 5.25$ can be seen in Table 5. Under monopoly, the cost reduction obtained is quite small. When there are two firms the cost reduction is about 80% of the cost reduction under monopoly, and the gain in allocative efficiency more than offsets the loss in dynamic efficiency. The trade-off between dynamic efficiency and allocative efficiency yields $n = 3$ as the number of firms that maximizes social welfare.

Table 2.

Effective R&D Expenditure and Cost Reduction: $\gamma = 5.25$

Number of firms	X **(effective R&D expenditure)**	$f[X]$ **(reduction in marginal cost)**
1	**0.00454152**	**0.0588235**
2	**0.00301384**	**0.0479193**
3	**0.00194157**	**0.0384615**
4	**0.00132502,**	**0.0317733**
5	**0.000954238**	**0.0269637**

4 Cooperation in R&D, but non-cooperation in production

Suppose that the firms cooperate in R&D by running a single R&D lab, but behave non-cooperatively in the production stage. Let X be the total expenditure of the joint R&D program, with each firm contributing equally to the cost of the research lab. Under the assumption of symmetric firms, the industry's total profit net of R&D cost is given by

(34) $$-X + n\pi_i[c^0 - f[X],...,c^0 - f[X]] = -X + \frac{n}{b}\left(\frac{a - c^0 + f[X]}{n+1}\right)^2.$$

The following first-order condition characterizes the optimal R&D expenditure of the joint research lab:

(35) $$-1 + \frac{2n}{b(n+1)^2}\left(a - c^0 + f[X]\right)f'[X] = 0.$$

The second-order condition is

(36) $$\left(a - c^0 + f[X]\right)f''[X] + (f'[X])^2 < 0.$$

If we let $X^*[n]$ denote the optimal level of R&D expenditure when the firms cooperate in a joint research venture, then the first-order condition (35) can be rewritten as

$$(37)\qquad \left(a - c^0 + f[X^*[n]]\right)f'[X^*[n]] = \frac{b(n+1)^2}{2n}.$$

It follows directly from (37) that the R&D expenditure $X^*[n]$ tends to 0 when the number of firms tends to infinity. Differentiating (36) with respect to n, we obtain

$$(38)\qquad \left(\left(a - c^0 + f[X^*[n]]\right)f''[X^*[n]] + \left(f'[X^*[n]]\right)^2\right)\frac{dX^*[n]}{dn} = \frac{b}{2}\left(1 - \frac{1}{n^2}\right) \geq 0.$$

Hence $dX^*[n]/dn < 0.$

We summarize the results just obtained in the following proposition:

PROPOSOTION 3: *The more firms there are in the market, the lower will be the R&D expenditure for the joint research lab, and a fortiori the lower will be the reduction in production cost. Furthermore, when the number of firms tends to infinity, there will be no innovation in the market.*

We note in passing that the industry's profits net of R&D costs decline when the number of firms increases. This result can be obtained by applying the envelope theorem to the industry's total profits net of R&D costs (37).

One of the major results of the model is that there is more innovation when the firms are allowed to cooperate in R&D than when they behave non-cooperatively in both stages of the game, i.e., $X^*[n] > X[n,\beta]$ for $n > 1.$ This result has been derived by assuming that $\left(a - c^0 + f[X]\right)f'[X]$ is strictly declining in $X,$ an assumption that is clearly satisfied if the production function of R&D has the form represented by (25). To see why $X^*[n] > X[n,\beta]$ for $n > 1,$ divide (15) by (37) to obtain

$$(39) \qquad \frac{\left(a-c^0+f[X[n,\beta]]\right)f'[X[n,\beta]]}{\left(a-c^0+f[X^*[n]]\right)f'[X^*[n]]}=\frac{n}{n-(n-1)\beta}>1.$$

The statement $X^*[n]>X[n,\beta]$ for $n>1$ follows from (39) and the assumption that $\left(a-c^0+f[X]\right)f'[X]$ is strictly declining in X.[3]

The result $X^*[n]>X[n,\beta]$ for $n>1$ implies that consumer surplus is higher when the firms cooperate in R&D than when they act non-cooperatively in R&D. Because non-cooperation in R&D is also a possible choice for the firms when they cooperate in the R&D stage, producer surplus is obviously higher under the cooperative R&D scenario than under the scenario of non-cooperation in both stages of the game. Hence when there are more than one firm in the market, allowing them to cooperate in the R&D stage, but not in the production stage, yields a level of social welfare that is higher than the social welfare level obtained when the firms behave non-cooperatively in both stages of the game.

Although social welfare can be raised by allowing the firms – which are forbidden to collude in the production stage – to cooperate in the R&D stage, this policy is not necessarily socially optimal. When R&D activities are particularly productive, our analysis suggests that monopoly yields the highest level of social welfare; that is, allowing firms to collude in both stages of the game might be socially optimal. Proposition 3 describes another facet of the negative impact of competition on innovation. The more firms there are in the market, the less will all the firms together spend on the joint research venture, even when they cooperate in the R&D stage.

5 Conclusion

In this paper, we have extended a theoretical framework for analyzing competition and innovation in presence of horizontal spillovers. Introducing two scenarios, we have shown that when firms behave non-cooperatively in both the R&D and production stages, (i) the degree of

[3] In the AJ model the firms' cooperation in R&D activities does not entail running a single research lab, nor is there information sharing among the firms, and hence the relationship depends on the degree of spillover; there is more innovation when firms are allowed to cooperate in R&D compared to when they behave non-cooperatively in both stages of the game when the degree of spillover is high. It is the reverse when the degree of spillover is low.

spillovers has a negative relationship with the effective and respective R&D expenditures of each firm as well as the level of social welfare; (ii) the relationship between the number of firms and the respective and effective R&D expenditure of each firm is negative. It also has a negative relationship with the level of social welfare: an inverted-U relationship when the benefits of R&D are low and a steady negative relationship when R&D technology is very productive. The theoretical predictions of our model are thus along the line of the Schumpeterian hypothesis in a sense that the dynamic efficiency generated by market power more than offsets the static allocative inefficiency it creates.

When firms behave cooperatively in the R&D stage, and non-cooperatively in the production stage, (i) the relationship between the R&D expenditure of the joint research lab and the number of firms in the market is negative, and (ii) in the limit, there would be no innovation in the market when the number of firms tends to infinity. Furthermore, it is also shown that when there are more than one firm in the market, allowing them to cooperate in the R&D stage, but not in the production stage, yields a level of social welfare that is higher than the social welfare level obtained when the firms behave non-cooperatively in both stages of the game.

A discussion of the protection of intellectual property rights with certain significant effects on the relationship deserves more attention in a future work. Uncertainty and risk have been ignored in the literature and therefore merit more attention. A more complete modeling of the innovation process should include an examination of the major drivers influencing the degree of spillovers: distance between the innovators, property rights, and the extent of telecommunication network, *inter alia*.

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